Theoretical analysis of multiparticle production on nuclei at high energy

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The multiplicity distribution of shower particles from emulsion exposures at a 200-GeV/c proton beam at NAL is analyzed using Gottfried's model. The data give information on the value of a fundamental parameter of the model.

Can high-energy collisions on nuclei provide us with some basic information on hadron-hadron dynamics? Spurred by this question and the advent of high-energy accelerator data on nuclear targets, theoretical activity on this subject has steadily increased.[1] Though it is perhaps too early to give an answer to the opening question, attempts at interpreting data on nuclear targets in terms of basic hadronic processes deserve close examination.

Recently Gottfried[2] has argued that because of time dilation, at high energy there is not enough time between two successive collisions within a nucleus for most of the produced hadronic matter to reach asymptotic states.

From the point of view of the second target nucleon, most of the incident matter looks just like a single excited hadron, and the second collision should have features similar to those of a single hadron-hadron interaction. The fact that a number of would-be hadrons behave at early stages of their production like a single hadron provides a substantial decrease in the number of produced particles, thus explaining the surprisingly low ratio between the average multiplicity of shower particles in emulsions $\bar{n}_{sh}$ and the observed charge multiplicity on hydrogen $\bar{n}_H$.

We outline the model[2] which provides a realization of these ideas according to the following scheme.

1. The spectrum of secondaries produced in hadron-hadron collisions is divided into two parts: Particles with rapidity y larger than some prescribed value y_c, which are supposed to carry the quantum numbers of the incident particle, are collectively denoted by "head," and particles with $y < y_c$ by "tail." The dividing value y_c is taken to be proportional to the incoming rapidity Y, i.e.,

$$y_c = \eta Y. \tag{1}$$

Actually, in the Gottfried model[2] η is set equal to $\frac{1}{3}$, as a result of a detailed dynamical description of the process. In our analysis η is regarded to be a parameter whose value has to emerge from the experimental data.

2. If there is a subsequent collision within the same nucleus, all head particles, which are still strongly interacting with one another, are assumed to behave just as a single proton of rapidity Y. (A correction of this value due to the loss of the tail and evaluated using energy-momentum conservation does not appreciably change the results.)

3. All tail particles are supposed to emerge from the nucleus without producing any additional hadrons: Rescattering among particles belonging to different tails is likely to be a low-energy process, not leading to new emission of secondaries, whereas rescattering with other nucleons is neglected.

In the present work we compare the prediction of the model with the multiplicity distribution of shower particles in an emulsion exposure in a 200-GeV/c proton beam at NAL.[3,4] We take the distribution of charged particles in a tail emerging from the collision with a proton to be equal to the slower half of the distribution produced in a pp collision of rapidity $Y' = 2\eta Y$. Thus, we write for the probability of finding n charged tail particles

$$P_n^{(t)}(\eta Y) = P_{2n}(2\eta Y), \tag{2}$$

where $P_m(Y)$ is the probability of having m charged particles in an inelastic pp collision with incident rapidity Y. Equation (2) takes into account the fact that the tails are made of the low-energy part of the distribution and that there can be "end effects." $P_m(2\eta Y)$ will have the same end effects both for the low- and high-energy components, and in Eq. (2) we take half of those effects.

For a detailed comparison with experiment it turns out to be crucial to subtract from the predicted charge distribution the slow particles with velocity less than 0.7, which give rise to heavy tracks and are not counted as shower particles.

For pp collisions we calculate the average number of such particles by integrating the inclusive spectrum of slow protons and charged pions inferred from NAL and CERN ISR measurements.[5] We conclude that there are approximately 0.48 slow protons and 0.14 slow charged pions per tail. For pn collisions we assume that the number of slow charged pions produced is the same as in pp, whereas the number of slow protons is taken to be equal to the number of slow neutrons in pp collisions. The latter is inferred from the inclusive $pp \to p + X$ cross section and baryon-number conservation. When all this is taken into account, we estimate that the shower-particle average multiplicity per tail is $\frac{2}{3}$ of a charge below what would follow from Eq. (2). We can take this into account by writing for the distribution of tail shower particles

$$P^{(t)}_{n,\mathrm{sh}}(\eta Y) = \tfrac{1}{3}P^{(t)}_{n}(\eta Y) + \tfrac{2}{3}P^{(t)}_{n+1}(\eta Y). \tag{3}$$

The last collision within a nucleus is entirely equivalent to a proton-nucleon collision, and there is no point in separating head from tail. Still, we have to perform the correction for slow secondaries and neutron targets, and we may write

$$P^{(\mathrm{last})}_{n,\mathrm{sh}}(Y) = \tfrac{1}{3}P_n(Y) + \tfrac{2}{3}P_{n+1}(Y). \tag{4}$$

This expression ensures the desired reduction of the average multiplicity of shower particles for the last collision in a nucleus, but it favors odd values of n, because P_m is nonzero for m even and larger than or equal to 2. Alternatively we may obtain the same multiplicity by writing

$$P^{(\mathrm{last})}_{n,\mathrm{sh}}(Y) = \tfrac{5}{12}P_n(Y) + \tfrac{1}{2}P_{n+1}(Y) + \tfrac{1}{12}P_{n+2}(Y), \tag{5}$$

which does not favor even or odd values of n. When we recall the origin of the effect that we are trying to take into account, Eq. (5) appears to be more adequate than Eq. (4), and we adopt it in the following calculation. Similarly we modify Eq. (3) so as to get a form analogous to Eq. (5). [When we use Eqs. (2) and (5) we take into account that $\sum_n P_{2n+4} = 1 - P_2$ and we correct accordingly.]

Following Gottfried, we write for the probability $\mathcal{G}_\nu$ of occurrence of ν collisions in a nucleus of radius $R = r_0 A^{1/3}$

$$\mathcal{G}_\nu = (\nu+1)\left[1 - e^{-\xi}\left(1 + \xi + \cdots + \frac{\xi^{\nu+1}}{(\nu+1)!}\right)\right] \times \left[\tfrac{1}{2}\xi^2 + e^{-\xi}(1+\xi) - 1\right]^{-1}, \tag{6}$$

where

$$\xi = 2R/\lambda, \quad \lambda = 4\pi r_0^3/3\sigma_{\mathrm{in}}, \tag{7}$$

and assume that, in the emulsion, collisions with hydrogen, $A = 11$ nuclei, and $A = 95$ nuclei occur with probabilities 0.05, 0.25, and 0.70, respectively. We take $\sigma_{\mathrm{in}} = 32$ mb and $r_0 = 1.2$ F. The calculation of the number of secondaries is now straightforward. For a given kind of nucleus we have the distribution of shower prong numbers $\Pi_n(Y)$ expressed in terms of the tail and last collision distributions,

$$\Pi_n(Y) = \sum_\nu \mathcal{G}_\nu \times \sum_{n_1 + \cdots + n_\nu = n} P^{(t)}_{n_1}(\eta Y)\cdots P^{(t)}_{n_{\nu-1}}(\eta Y) P^{(\mathrm{last})}_{n_\nu}(Y). \tag{8}$$

It is understood that for $P^{(t)}_m$ and $P^{(\mathrm{last})}_m$ we have to take the shower distribution. We have tried two values for η, namely the Gottfried prescription $\eta = \frac{1}{3}$ and $\eta = \frac{1}{2}$, which corresponds to a cut at zero rapidity in the c.m. system for the proton-nucleon collision. In the case $\eta = \frac{1}{2}$ the calculation was performed taking, as input, directly the pp data of Ref. 6, and in the case $\eta = \frac{1}{3}$ we used the interpolating formulas of Refs. 7 and 8. We also checked numerically that for $\eta = \frac{1}{2}$ the use of those interpolating formulas makes very little difference with respect to the direct use of the data.

The results of the calculation are shown in Figs. 1(a) and 1(b) compared with experimental data reported in Refs. 3 and 4, respectively. It is clear that the value $\eta = \frac{1}{2}$ is much more favored than $\eta = \frac{1}{3}$, and indeed for $\eta = \frac{1}{2}$ the theory shows a good

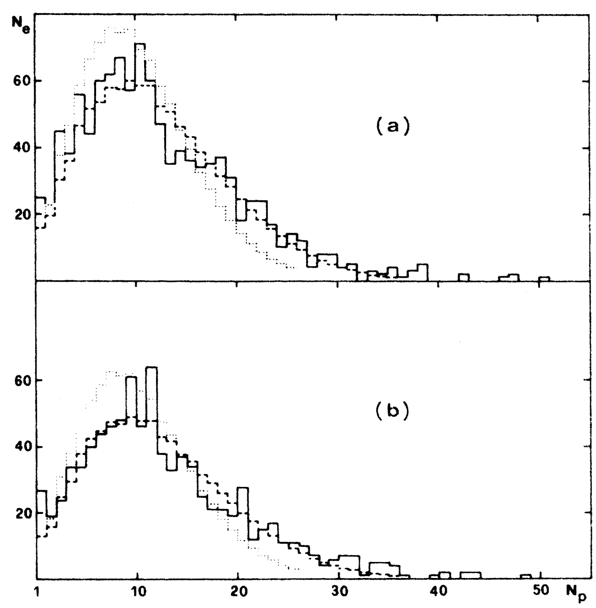

FIG. 1. Number of events versus number of prongs (showers) at $p_{\mathrm{lab}} = 200$ GeV/c. Solid lines: (a) experimental data from Ref. 3 (1071 events), (b) experimental data from Ref. 4 (876 events, coherent production subtracted). Dashed lines: theoretical results for $\eta = \frac{1}{2}$. Dotted lines: theoretical results for $\eta = \frac{1}{3}$.

agreement with the data. We recall that data shown in Fig. 1(b) were obtained after subtracting the events corresponding to coherent production. Before the subtraction, the data of Ref. 4 showed the same peak structure at low odd multiplicities as the data of Ref. 3, where no subtraction has been performed. With regard to this point one should keep in mind that the theory we are investigating does not describe the nuclear coherent production. There is a structure present in the data of both Ref. 3 and Ref. 4 which is not reproduced by the model, namely a sort of step near $n=13$. We think this is probably a physical effect which is not taken into account in the version of the theory we are investigating, which is still in its early stages. On the other hand, the shape of the curve along which the distribution goes to zero for high n is fairly well reproduced for $\eta=\frac{1}{2}$. This part of the distribution is mostly determined by the "tails," and therefore it depends critically on a typical feature of the theory,[9] whereas it is very little sensitive to the detailed way in which the slow charges are lost because of the cut at $\beta=0.7$, e.g., it would be almost the same choosing Eq. (4) instead of Eq. (5).

The multiplicity of the shower tracks for $\eta=\frac{1}{2}$ is given by

$$\bar{n}_{sh}(Y)=\bar{n}_H(Y)-\tfrac{2}{3}+(\bar{\nu}-1)[\tfrac{1}{2}\bar{n}_H(Y)-\tfrac{2}{3}],$$

where $\bar{n}_H$ is the multiplicity on hydrogen and $\bar{\nu}$ is the average number of collisions in the emulsion, which we evaluate to be 3.06 from Eq. (6). This gives, for $p_{lab}=200$ GeV/c, $\bar{n}_{sh}=13.5\pm0.4$ to be compared with the experimental values 12.9 ± 0.2 for Ref. 3 and 13.3 ± 0.3 for Ref. 4. We obtain for $p_{lab}=3000$ GeV/c and $p_{lab}=8000$ GeV/c the multiplicities $\bar{n}_{sh}=23\pm2$ and $\bar{n}_{sh}=27\pm2$, respectively, to be compared with the experimental values from cosmic-ray data[2] 22.5 ± 1.5 and 23.3 ± 1.0.

It would be very interesting to test the model with data at different energies, since one of the crucial features is its energy dependence. Data from an emulsion exposure at NAL with $p_{lab}=300$ GeV/c will be available in the future. This energy is, however, not substantially higher, so one would not expect an important variation of the experimental results.

A prong distribution for $p_{lab}=67$ GeV/c is reported in Ref. 4. Even if in this case the incident energy is relatively low with respect to the expected range of validity of the model, we have compared the theoretical results with the data and we have found a surprisingly good agreement for $\eta=\frac{1}{2}$, whereas $\eta=\frac{1}{3}$ seems again ruled out. This comparison is presented in Fig. 2. The multiplicity calculated at this energy for $\eta=\frac{1}{2}$ is $\bar{n}_{sh}=9.9\pm0.8$, and the experimental value is 9.73 ± 0.23.

In summary, we conclude that the evidence examined supports Gottfried's model for particle production on nuclei at high energy, but that $\eta=\frac{1}{2}$ is strongly favored.

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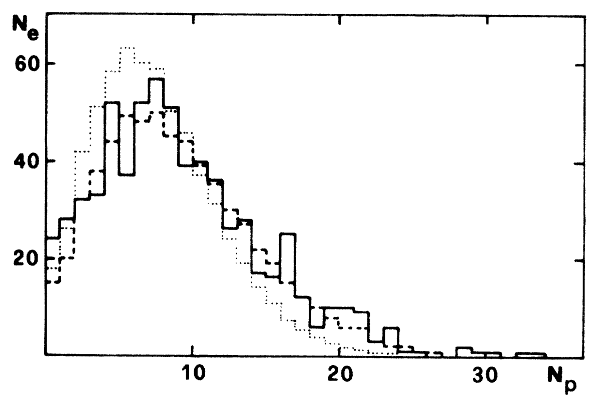

FIG. 2. Number of events versus number of prongs (showers) at p_{lab} = 67 GeV/c. Solid line: experimental data from Ref. 4 (657 events, coherent production subtracted). Dashed line: theoretical result for $\eta=\frac{1}{2}$. Dotted line: theoretical result for $\eta=\frac{1}{3}$.

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[9]The actual computation was done with the sum in Eq. (8) stopped at $\nu = 7$. This makes the result lower than the exact value, but the effect is in general very small, becoming slightly more important for very high multiplicities.

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Optical activity for neutrons

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When a neutron passes through an optically active medium, the transverse component of its polarization should precess around the direction of propagation while the longitudinal component should increase (or decrease) monotonically. For a representative medium, in a typical case, the rotatory power for neutron polarization is expected to be of order 10^{-5} cm^{-1}. The rate of acquisition of longitudinal polarization is also expected to be of similar magnitude, and the γ rays associated with thermal neutron capture should have circular polarization of order 10^{-7}.

It has been known for a long time that plane-polarized light, while traversing certain media which are said to be optically active, suffers a rotation of its plane of polarization by an amount and in a sense (right or left) which is characteristic of the medium. This seemingly arbitrary preference of a medium for one handedness over another is due not to any asymmetry of the laws of physics but, as shown by Pasteur, to the handed structure of the constituents of the medium. Since there is a one-to-one correspondence between the states of polarization of a light beam and those of spin-$\frac{1}{2}$ particles, one expects that similar effects should occur when polarized spin-$\frac{1}{2}$ particles[1] traverse an optically active medium.[2] For definiteness, we shall treat the case of slow neutrons transmitted through an optically active fluid,[3] i.e., one made up of handed molecules, and find that the detection of the phenomenon should be within reach of present experimental techniques.

The analog of optical activity for neutrons is the rotation of the neutron polarization, by an amount proportional to the distance traversed, about the direction of propagation $\hat{k}$. Such an effect requires that the mean forward scattering amplitudes f_R and f_L of longitudinally polarized neutrons be unequal for the two opposite helicities $(\vec{\sigma}\cdot\hat{k})=\pm 1$. The refractive index for neutrons of wavelength $\lambda = 2\pi/k$ in a medium comprising a number density N of scatterers with a mean forward-scattering amplitude f is

$$n = 1 + \frac{2\pi N}{k^2} f \tag{1}$$

for $Nf \ll k^2$, a condition which is well satisfied for thermal neutrons in matter of normal densities. The rotatory power, viz., the amount by which the transverse component of the neutron spin precesses while traversing unit distance, is then

$$\begin{aligned}\Phi &= \lambda N \,\mathrm{Re}(f_L - f_R)\\ &= \lambda N \,\mathrm{Re} f_S\,,\end{aligned} \tag{2}$$

where f_S is the spin-dependent part, proportional to $-\frac{1}{2}(\vec{\sigma}\cdot\hat{k})$, of the mean forward-scattering amplitude for neutrons from the molecules of the medium. In addition, the neutrons will acquire a degree of longitudinal polarization in traversing a distance x, amounting to

$$P_L(x) = \tanh(\lambda N x \,\mathrm{Im} f_S)\,. \tag{3}$$

For small x, this is proportional to x, i.e., the acquired polarization is

$$\Psi = \lambda N \,\mathrm{Im} f_S \tag{4}$$

per unit distance, for $x \ll \Psi^{-1}$. Conversely, there will be differential absorption of longitudinally polarized neutrons with opposite helicities. This is the analog of the Cotton effect.

With parity-conserving interactions,[4] a difference f_S between the forward scattering amplitudes for left-handed and right-handed neutrons can arise only if the medium contains molecules having a