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Massive-muon-pair production at high energy

R. A. Brandt, A. Kaufman,[†] and G. Preparata[‡]
Department of Physics, New York University, New York, New York 10003
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A detailed analysis of the inclusive massive-μ-pair production process in high-energy proton-proton collisions is presented based on the combined assumptions of light-cone (LC) expansions and multi-Regge theory. The scaling limit is LC dominated and the assumed strongly convergent Regge theory leads to dominance by the leading LC singularities. The resulting amplitude is expressed as a sum of two distinct contributions, a "pionization" piece, which dominates at large dimuon mass M, and a "fragmentation" piece, which dominates at smaller M. The result of the combination of these two contributions, each of fast decrease in M, can produce a shoulder in the $d\sigma/dM$ cross section, as seems to be present experimentally. This requires a small, perhaps vanishing, Pomeron-particle-Pomeron coupling at $t = 0$. A phenomenological model, which simply incorporates the derived behaviors of the scattering amplitude, is introduced to fit the data quantitatively. A good fit to the $d\sigma/dM$ data (at fixed energy) fixes the (five) parameters. The model then is compared with the experimental curves for the transverse and longitudinal dimuon cross sections and the total (energy-dependent) cross section. Good agreement is found. A comparison with the parton model is also given.

I. INTRODUCTION

The last few years have seen a substantial development in understanding the electromagnetic and weak interactions of hadrons at very large momentum transfers. It has become apparent that configuration space is a most suitable framework for the description of these processes.[1,2] In fact, what invariably turn out to be of importance here are Fourier transforms of products of local operators that describe the interaction of a hadronic system with an electromagnetic or a weak current. These Fourier transforms are light-cone-dominated when the mass of the currents becomes very large.[2] The crucial point is then that near and on the light cone $x^2=0$ a very important simplification occurs in the description of the product of two operators, in the form of operator-product expansions near the light cone (LCOPE).[3]

We present in this paper an analysis, within such a framework, of the BNL-Columbia μ-pair production process in high-energy proton-proton scattering[4]; the basic ingredients of this analysis have already been presented in Ref. 6. In this process two particles, of momenta p and p', collide to produce an observed lepton pair of momentum q and an unobserved hadronic final state. (See Fig. 1.) The existing experimental data involve the production of μ pairs in proton-proton collisions,[4] and further experiments of this type are in progress at the CERN Intersecting Storage Rings (ISR) or being planned for the National Accelerator Laboratory (NAL).

Several aspects of the light-cone treatment of the massive μ-pair production process have already been reported elsewhere.[5,6,7] Notably, a significant connection between the SLAC-MIT deep-inelastic scattering experiment[8] and the BNL-Columbia massive μ-pair production has been emphasized. This phenomenological connection is simply a statement of light cone "universality," whereby a given LCOPE describes a variety of different physical processes according to what specific matrix elements of it are taken. Hence, the light-cone expression measured in the SLAC-MIT electroproduction experiments can be used to describe the BNL-Columbia experiments.

The larger number of variables involved in the lepton-pair process make it much more complicated and difficult to analyze than the simpler electroproduction process. This greater complexity is, however, obviously a possible source of a much greater insight into hadronic structure. All of the physics of electroproduction, and much more, is involved here. In addition to the lepton-hadron interaction, we have here a rich background of purely hadronic reactions. We can expect a subtle interplay between the lepton-hadron (structure-probing, scale-invariant) phenomena and the hadron-hadron (Regge-behaved, non-scale-invariant) phenomena. Compared with electroproduction, we have the additional possibility of utilizing and exploring the nucleon-nucleon interaction, and, compared with purely hadronic single-particle production, we have the additional possibility of varying the mass of the produced particle. Our analysis will make much use of all these possibilities.

Apart from spin complication, which will be fully treated in the text, the amplitude under con-

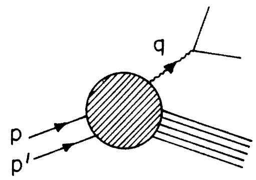

FIG. 1. The μ-pair production kinematics.

sideration is

$$W(q^2, s, \nu, \nu')=\int d^4x\, e^{iq\cdot x}\,{}_{\rm in}\langle pp'|j(x)j(0)|pp'\rangle_{\rm in}\,. \tag{1.1}$$

The variables are

$$s=(p+p')^2, \quad \nu=p\cdot q, \quad \nu'=p'\cdot q\,, \tag{1.2}$$

and we have taken $p^2=p'^2=1$.

We consider the behavior of W in the generalization of the scaling (A) limit in which each of the four variables (1.2) becomes large with the three ratios fixed:

$$A \text{ limit: } q^2, s, \nu, \nu' \to \infty, \quad \text{with} \quad \frac{\nu}{q^2}, \frac{\nu'}{q^2}, \frac{s}{q^2} \text{ fixed.} \tag{1.3}$$

From phase arguments of the type used in electroproduction,[2] the region of configuration space which controls the behavior of W in the A limit is again seen to be the light cone (LC)[5,9,10]:

$$|x^2| \leq 1/|q^2|\,. \tag{1.4}$$

In spite of (1.4), it does *not* follow in the present case, contrary to the situation for electroproduction, that the *leading* LC singularity dominates in the A limit. If the LC behavior is

$$\langle p, p'|j(x)j(0)|p, p'\rangle \underset{x^2\to 0}{\sim} \frac{1}{x^2} f(x\cdot p, x\cdot p', s) + g(x\cdot p, x\cdot p', s)+\cdots\,, \tag{1.5}$$

the nonleading term g can be as important as the leading term $(1/x^2)f$ if it grows fast enough with s.

We will see that in all presently known models and, more generally, if multi-Regge theory describes the high-energy behavior, the leading singularities do in fact dominate. (Actually, since in the parton model[11] the only configurations considered *exclude* those which contribute to the *leading* LC singularity, there it is the *second leading* contribution which dominates.) More generally, any uniform bound on the large-s behavior at fixed q^2, such as that provided by Regge theory, is sufficient to ensure leading LC dominance. Strictly speaking, such a bound would only be relevant in the Regge limit $s\to\infty$ with q^2 fixed, but commutativity relations[12] make these bounds relevant in the A limit as well.[6] Furthermore, in the LC treatment the large q^2 and large-s dependencies are effectively decoupled and only the behavior of the five-point functions $\langle pp'|O_{\alpha_1\cdots\alpha_n}(0)|p, p'\rangle$ in the R limit are relevant, provided the sum over n is sufficiently convergent.[5,6]

It follows that under the quite general circumstances described above, the leading LC singularity dominates the A limit and we have

$$W(q^2, s, \nu, \nu') \underset{A}{\sim} W_A(q^2, s, \nu, \nu')\,, \tag{1.6}$$

where W_A is obtained from (1.1) by keeping only the leading LC singularity $[(1/x^2)f(x\cdot p, x\cdot p', s)$ in the canonical case$]$.

The result of the LC multi-Regge analysis is

$$W_A \underset{A}{\sim} s^{\alpha}(\ln s)\int_0^1 da \int_{-1}^1 d\alpha \int_{-1}^1 d\alpha'\, s^{1-a} s^a \Psi(\alpha s^a, \alpha' s^{1-a}; a)\, \delta(q^2-2\alpha\nu-2\alpha'\nu'+\alpha\alpha' s)\, \theta(\alpha\alpha'-q^2/s)\,, \tag{1.7}$$

where $\Psi(\beta, \beta'; a)$ is a rapidly decreasing function of its first two arguments and is independent of its third argument for $\epsilon < a < 1-\epsilon$. [Here $\epsilon=\epsilon(s)$ is such that $s^{\epsilon(s)} \simeq 2$ GeV^2.] W_A can therefore be conveniently decomposed into the sum of a "pionization" piece W^P coming from $\epsilon < a < 1-\epsilon$ and a "fragmentation" piece W^F coming from $0 \leq a \leq \epsilon$ and $1-\epsilon \leq a \leq 1$ (see Fig. 6):

$$W_A = W^P + W^F\,, \tag{1.8}$$

with

$$\Psi(\beta, \beta'; a) \cong \Psi^P(\beta, \beta')\,, \quad \epsilon < a < 1-\epsilon \tag{1.9}$$

and

$$\Psi(\beta, \beta'; a) \cong \Psi^F(\beta, \beta'), \quad a < \epsilon \text{ or } a > 1-\epsilon\,. \tag{1.10}$$

Note that since $\Psi(\beta, \beta')$ must be symmetric under interchange of β and β' ($\nu \rightleftarrows \nu'$ symmetry of W), we can write

$$\Psi(\beta, \beta') = \Phi(\tfrac{1}{2}(\beta+\beta'), (\beta\beta')^{1/2})\,. \tag{1.11}$$

One integral in (1.7) can be done with the δ function and the others can be estimated if Φ is a sufficiently smooth function of its arguments. The result for W^P and W^F gives the expression

$$\left(\frac{d\sigma}{dq^2}\right)^P \sim \Phi(\sqrt{q^2},\sqrt{q^2}),\ \left(\frac{d\sigma}{dq^2}\right)^F \sim \Phi(q^2,\sqrt{q^2}), \tag{1.12}$$

for the contributions of the pionization region, $(d\sigma/dq^2)^P$, and the fragmentation regions, $(d\sigma/dq^2)^F$, to the cross section:

$$\frac{d\sigma}{dq^2} = \left(\frac{d\sigma}{dq^2}\right)^P + \left(\frac{d\sigma}{dq^2}\right)^F . \tag{1.13}$$

The interesting feature is the difference in the behavior of the sum variable. The sum (1.13) can thus easily appear as the superposition of two different rapidly decreasing contributions, one, $(d\sigma/dq^2)^P$, dominating at large q^2 and the other, $(d\sigma/dq^2)^F$, dominating at small q^2. The result of the combination of these two contributions can produce a shoulder, as seems to be present experimentally.

When spin is correctly included, the result is an expression of the form (1.7) with Ψ replaced by a sum over Ψ_i, $i=1-4$. To compare with experiment, we take the simplest possible phenomeenological model for the Ψ_i's. We take

$$\Psi_i(\beta,\beta')\underset{P}{\sim} P_i' e^{-h(\beta+\beta')} \tag{1.14}$$

in the pionization limit with β and β' both large, and

$$\Psi_i(\beta,\beta')\underset{F}{\sim} F_i' e^{-k\beta}(1-\beta')^n \tag{1.15}$$

in the fragmentation limit with β large and $\beta'\sim 1$. In (1.15) we have included a threshold factor analogous to the one in electroproduction. With these explicit forms, the integrals over α, α', and a can be performed in the A limit. The final result is the sum of a P contribution, with an unknown over-all coefficient P and an F contribution, with another unknown over-all coefficient F. So, in this simplest case, we obtain a representation in terms of the five free parameters P, F, h, k, and n.

To compare with the experimental data, we must integrate over phase space, respecting the experimental cuts. We obtain expressions for $d\sigma/d\sqrt{q^2}$, $d\sigma/d(\cos\theta)$, $d\sigma/dq_{\parallel}$, and $\sigma(E_p)$ to be compared with the experimental results. Our procedure was to fix our five parameters by fitting the $d\sigma/d\sqrt{q^2}$ curve. A typical hand fit, with

$$P=1.67,\quad F=10^4,\quad h=0.10,\quad k=2.0,\quad n=4, \tag{1.16}$$

is seen to be quite good. The shoulder appears, as expected, from the interference of the two exponentials and the final rapid decrease of the curve is due to phase space. Using the same values (1.16), we obtain predictions for the other curves. The agreement is seen to be very good in all cases. It is possible to obtain still better fits using more sophisticated fitting methods, but this hardly seems warranted at present because our assumed forms (1.14) and (1.15) are only guesses and because of the crudity of the existing data.

Accepting at least the gross features of the present data, a few remarks about the significance of the fit (1.16) are in order. The small value $\sim 10^{-4}$ obtained for the ratio P/F suggests that the Pomeron-particle-Pomeron coupling at $t=0$ is very small and perhaps vanishes. This must be the case if the Pomeron is an isolated pole at $\alpha(0)=1$.[13] The value $n=4$ for the threshold-power decrease is similar to the value $n=3$ found in electroproduction. We also obtain acceptable fits with $n=3$. We finally note the smooth experimental falloff of $d\sigma/d(\cos\theta)$. The behavior is $\sim e^{-500(1-\cos\theta)}$ and we fit this nicely. This should be compared to the behavior $e^{-2000(1-\cos\theta)}$ which one finds for hadronic single-particle production in similar experimental conditions and which is the parton-model prediction.[11]

Detailed derivations and discussions of these results are given in the following sections: Preliminary information is collected in Sec. II. Light-cone expansions are reviewed in Sec. II A, the kinematics and notations are given in Sec. II B, and model calculations are reviewed in Sec. II C. The behavior of the scalar amplitude (1.1) in the Regge (Sec. III A) and scaling (Sec. III B) limits is given in Sec. III and the commutativity relation is stated in Sec. III C. In Sec. III B, LC dominance is kinematically established but it is stressed that this does *not* imply leading LC dominance in this case. In Sec. IV it is shown how strongly convergent multi-Regge behavior at the five-point function level does give leading LC dominance. The momentum-space form of this leading LC contribution is given and discussed in Sec. V. The pionization and fragmentation contributions to the amplitude are estimated in Sec. VI, assuming a certain amount of smoothness. In Sec. VII, the effect of the photon spin is taken into account. The specific phenomenological model is treated in Sec. VIII. The motivation is given in Sec. VIII A, the implications are deduced in Sec. VIII B, and the resulting cross sections, respecting the experimental cuts, are calculated in Sec. VIII C. The model is compared with experiment in Sec. IX. A fit to the experimental $d\sigma/d\sqrt{q^2}$ curve is seen to provide good fits to the other measured cross sections: $d\sigma/dq_{\parallel}$, $d\sigma/d\cos\theta$, and $\sigma(E)$. Section X contains a general discussion of our assumptions and results.

II. PRELIMINARIES

A. The light cone and Regge behavior

As is well known, a LC expansion is generally a sum of terms constructed out of c-number functions of x^2, singular on the LC, with bilocal operator coefficients that can be expanded in infinite convergent series of local operators,[3]

$$A(x)B(0) \underset{x^2\to 0}{\sim} E(x^2)\sum_n x^{\alpha_1}x^{\alpha_2}\cdots x^{\alpha_n}O^{(n)}_{\alpha_1\alpha_2\cdots\alpha_n}(0) + \text{LS}, \qquad (2.1)$$

where LS denotes less singular terms. Here, from the assumption that the leading LC singularities are mass-independent, $E(x^2)$ has the singular structure

$$E(x^2) \underset{x^2\to 0}{\sim} (x^2)^{-(d_A+d_B-d_0)/2}, \qquad (2.2)$$

with

$$d_A \equiv \dim A(x), \quad d_B \equiv \dim B(x)$$

and

$$\dim O^{(n)}_{\alpha_1\alpha_2\cdots\alpha_n}(x) \equiv n + d_0 .$$

The value of LC expansions is that they considerably simplify the structure of scattering amplitudes in the kinematical limit where the LC dominates.

Furthermore, it has been argued[9,14] that Regge behavior of current-hadron amplitudes plays an essential role in establishing kinematical dominance of the LC in the appropriate Bjorken limit. We shall see later on that the Regge-behavior assumption also plays a crucial role in establishing dynamical LC dominance of the corresponding amplitudes, that is, in settling the question of whether or not nonleading LC singularities contribute significantly to the Bjorken[1] limit.[15]

This connection between the Regge limit and the LC-dominated Bjorken limit has been studied[16] in some detail on the scattering amplitude $T(q^2,\nu)$ for

$$A(q)+c(p)\to A(q)+c(p),$$

where A's are scalar currents of dimension two, c's are on-shell scalar particles, and $\nu\equiv p\cdot q$. In fact, if one assumes scaling and Regge pole behavior for $T(q^2,\nu)$ in the corresponding limits, one can show that for a large class of models defined by DGS (Deser-Gilbert-Sudarshan) representations with suitable spectral functions, the following symbolic commutativity relation holds[12]:

$$\lim_{q^2/2\nu\to 0}\ \lim_{\substack{q^2,\nu\to\infty \\ q^2/2\nu \text{ fixed}}} T(q^2,\nu) = \lim_{q^2\to\infty}\ \lim_{\substack{\nu\to\infty \\ q^2 \text{ fixed}}} T(q^2,\nu). \qquad (2.3)$$

The content of (2.3) is that the forward intercept of the Regge trajectory $\alpha(t)$ is given by the large-$(p\cdot x)$ behavior of the coefficient to the leading LC singularity of the amplitude. It also follows from (2.3) that the leading LC singularity determines the large q^2 piece of the Regge residue functions $\beta(q^2)$.

These basic results, suitably generalized, will be seen to play an essential role in the LC analysis of the μ-pair production process and will be discussed in some detail later on.

B. Kinematics

Now that most of the relevant concepts have been stated, we begin the LC analysis of the BNL-Columbia experiments. The process of interest is the high-energy reaction

$$\text{proton}(p)+\text{proton}(p')\to\mu^+(q^+)+\mu^-(q^-)+\text{anything},$$

where two protons, of momenta p and p', collide to produce an observed muon pair of momenta q^+ and q^-, respectively, and an unobserved hadronic final state.

Our notation will be

$$q\equiv q^+ + q^- = (q_0, \vec{q}_\perp, q_\parallel),$$

with $q^2>0$ and the direction with respect to which the transversal and longitudinal components of $\vec{q}$ are defined given by the proton momentum. Also,

$$s\equiv(p+p')^2;\quad \nu\equiv p\cdot q;\quad \nu'\equiv p'\cdot q;$$

$$p^2=p'^2=1;\quad q^{+2}=q^{-2}=0 .$$

To first order in electromagnetism the center-of-mass triple-differential cross section for the scattering in which the final "anything" state is not observed can be written as[5]

$$\frac{d^2\sigma^r}{dq^2 d\nu\, d\nu'} = \frac{4\alpha^2}{3\pi^2}\frac{1}{[s(s-4)]^{1/2}}\frac{1}{q^2 s} \times W^{\mu\nu}(q^2,\nu,\nu';s)\epsilon^r_\mu(q)\epsilon^r_\nu(q). \qquad (2.4)$$

Here $\epsilon^r_\mu(q)$ describes the polarization $r=T_1, T_2$ (transversal) or L (longitudinal) of the μ pair and (see Fig. 2)

$$W^{\mu\nu}(q^2,\nu,\nu';s)\equiv\int d^4x\, e^{iq\cdot x} \times {}_{\text{in}}\langle p,p'|j^\mu(x)j^\nu(0)|p,p'\rangle^c_{\text{in}}, \qquad (2.5)$$

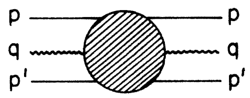

FIG. 2. The forward three-to-three amplitude.

where $j^\mu(x)$ is the electromagnetic current. A spin average is, here and everywhere, understood and only the connected part of the matrix element occurs. Notice also that because two particles are present in the initial state, we must, as indicated in (2.5), distinguish between (in) and (out) states.

Using

$$\sum_r \epsilon_\mu^r(q)\epsilon_\nu^r(q) = -g_{\mu\nu} + q_\mu q_\nu/q^2$$

and the current conservation condition

$$q_\mu q_\nu W^{\mu\nu}(q^2, \nu, \nu'; s) = 0\,,$$

we obtain for the unpolarized triple-differential cross section

$$\frac{d^3\sigma}{dq^2 d\nu\, d\nu'} = -\frac{4\alpha^2}{3\pi^2}\frac{1}{[s(s-4)]^{1/2}}\frac{1}{q^2 s} W_\mu^\mu(q^2, \nu, \nu'; s) \tag{2.6}$$

The physically accessible phase-space region for the process in the center-of-mass frame at fixed q^2, R_{q^2}, follows from

$$(p+p'-q)^2 \geq 4; \quad q_\perp^2 \geq 0, \tag{2.7}$$

and the fact that for large values of s the transverse momentum $\vec{q}_\perp$ satisfies

$$4\nu\nu' = (q^2 + q_\perp^2)s\,. \tag{2.8}$$

In fact, neglecting the proton mass squared compared to the s-channel energy, we get (see Fig. 3)

$$\nu + \nu' \leq \tfrac{1}{2}(s+q^2), \quad \nu\nu' \geq \tfrac{1}{4}(q^2 s)\,, \tag{2.9}$$

and, therefore, both ν and ν' are allowed to run only over the finite interval $(\frac{1}{2}q^2; \frac{1}{2}s)$ for any given set $(q^2; s)$. Notice, parenthetically, that Eq. (2.8) obtains quite immediately from the definition of ν and ν'. Specifically, in the center-of-mass system,

$$\nu = (\tfrac{1}{2}s)^{1/2}(q_0 - q_\parallel)\,,$$
$$\nu' = (\tfrac{1}{2}s)^{1/2}(q_0 + q_\parallel)$$

when s becomes very large, and therefore

$$4\nu\nu' = s(q_0^2 - q_\parallel^2) = s(q^2 + q_\perp^2)\,.$$

From (2.8) and (2.9) it also follows that

$$0 \leq q_\perp^2 \leq 2\nu(1 - q^2/2\nu)(1 - 2\nu/s) \tag{2.10}$$

for some given set (q^2, ν, s). Alternatively,

$$0 \leq q_\perp^2 \leq 2\nu'(1 - q^2/2\nu')(1 - 2\nu'/s)\,. \tag{2.10$'$}$$

The invariant-mass differential cross section $d\sigma/dq^2$ is then given by

$$\frac{d\sigma}{dq^2} = \int_{R_{q^2}} d\nu\, d\nu' \frac{d^3\sigma}{dq^2 d\nu\, d\nu'}\,. \tag{2.11}$$

We shall find it useful sometimes to consider $q_\perp^2$ and one of the subenergies, say, ν', as independent variables rather than ν and ν'. Under these conditions the phase space R_{q^2} reads (Fig. 4)

$$0 \leq q_\perp^2 \leq (s-q^2)^2/4s\,,$$
$$\tfrac{1}{2}(s+q^2) - \{[\tfrac{1}{2}(s-q^2)]^2 - q_\perp^2 s\}^{1/2} \leq 2\nu' \leq \tfrac{1}{2}(s+q^2) + \{[\tfrac{1}{2}(s-q^2)]^2 - q_\perp^2 s\}^{1/2}\,. \tag{2.12}$$

Naturally, the corresponding $(2\nu, q_\perp^2)$ phase space is obtained by simply replacing ν' with ν.

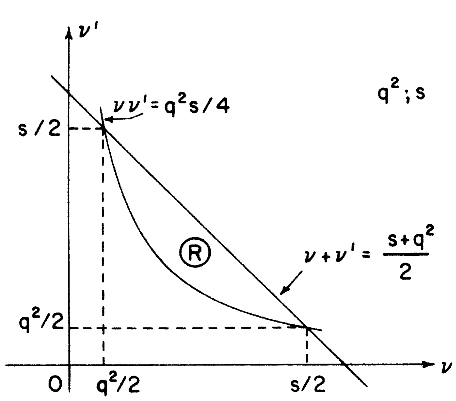

FIG. 3. The physical region (R) in the ν-ν' plane.

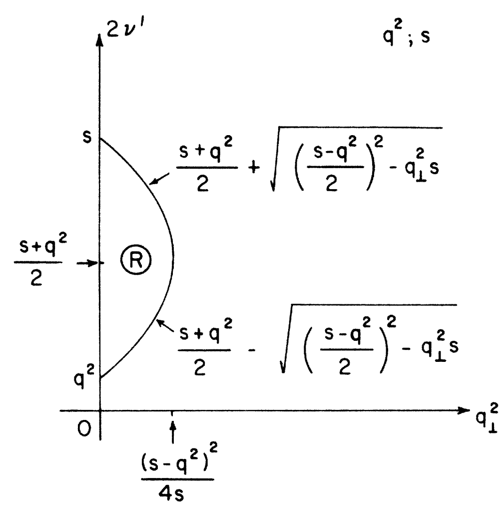

FIG. 4. The physical region (R) in the $q_\perp^2$-ν' plane.

The other two relevant cross sections, $d\sigma/dq_{\parallel}$ and $d\sigma/d(\cos\theta)$, where $\theta \equiv \tan^{-1}(|q_\perp|/q_\parallel)$, are best evaluated in the laboratory frame of p. Here

$$\nu \equiv p\cdot q = q_0 = (q^2+q_\parallel{}^2+q_\perp{}^2)^{1/2}$$

and, since experimentally $q_\parallel{}^2 \gg q^2+q_\perp{}^2$, we use

$$\nu = q_\parallel, \quad \nu' = (q^2+q_\parallel{}^2\tan^2\theta)s/4q_\parallel, \tag{2.13}$$

to translate (2.6) from the center-of-mass frame with the result that

$$\frac{d^3\sigma}{dq^2dq_\parallel d(\cos\theta)} = -\frac{2\alpha^2}{3\pi^2}\frac{1}{[s(s-4)]^{1/2}}\frac{q_\parallel}{q^2 s\cos^2\theta} \times W^\mu_\mu(q^2, q_\parallel, \cos\theta; s). \tag{2.14}$$

Hence,

$$\frac{d\sigma}{dq_\parallel} = \int_{R_{q_\parallel}} dq^2\, d(\cos\theta)\,\frac{d^3\sigma}{dq^2dq_\parallel d(\cos\theta)}, \tag{2.15}$$

and similarly

$$\frac{d\sigma}{d(\cos\theta)} = \int_{R_{\cos\theta}} dq^2 dq_\parallel\,\frac{d^3\sigma}{dq^2dq_\parallel d(\cos\theta)}, \tag{2.16}$$

where a simple calculation gives

$$R_{q_\parallel}: \begin{cases} q^2{}_{\min} \le q^2 \le 2q_\parallel, \\ [1+(4/2q_\parallel)(1-2q_\parallel/s)(1-q^2/2q_\parallel)]^{-1/2} \le \cos\theta \le 1, \end{cases} \tag{2.17}$$

$$R_{\cos\theta}: \begin{cases} q^2{}_{\min} \le q^2 \le q^2{}_{\max}, \\ 2q_\parallel^- \le 2q_\parallel \le 2q_\parallel^+, \end{cases}$$

for $q^2{}_{\min}$ ($q^2{}_{\max}$) the minimal (maximal) value of q^2 experimentally achieved, and

$$2q_\parallel^\pm \equiv \frac{1+q^2/s \pm [(1-q^2/s)^2 - 4q^2\tan^2\theta]^{1/2}}{2(\frac{1}{4}\tan^2\theta + 1/s)}. \tag{2.18}$$

C. Model calculations

In this subsection, we will review the discussion of the scalar amplitude

$$W(q^2, s, \nu, \nu') \equiv \int d^4x\, e^{iq\cdot x} \times {}_{\rm in}\langle p, p'|j(x)j(0)|p, p'\rangle_{\rm in}$$

given in Ref. 7 in the nonperturbative parton model.[17] The contribution W_a to W from the "annihilation" diagram [Fig. 5(a)] is[7,10]

$$W_a(q) \underset{A}{\sim} \int d\omega \int d\omega' F(\omega)F(\omega')\,\delta^4(q-\omega p - \omega' p'), \tag{2.19}$$

and this gives the scaling behavior

$$W_a(q^2, s) \equiv s\int d^4k\,\delta(k^2-q^2)W_a(k) \underset{A}{\sim} F(\tau), \quad \tau = q^2/s \tag{2.20}$$

with

$$F(\tau) = \int d\omega\, d\omega'\, F(\omega)F(\omega')\delta(\omega\omega'-\tau). \tag{2.21}$$

This is precisely the parton-model result[11] for the scalar case. The contribution W_b to W from the "bremsstrahlung" diagram [Fig. 5(b)] is[7]

$$W_b(q) \underset{A}{\sim} s^{1+\alpha}\int d\omega\, d\omega'\, B(\omega\omega' s) \times \delta(q^2 - 2\omega\nu - 2\omega'\nu' + \omega\omega' s) \times \theta(\omega\omega' - q^2/s). \tag{2.22}$$

This is a special case of the general LC amplitude we will study in detail in the following sections. We note only that a comparison of (2.19) and (2.22) is a dynamical question—either may, in principle, dominate.

Unlike the application of this model to deep-inelastic scattering,[17] it is not possible in this case to exclude the relevance of other contributions

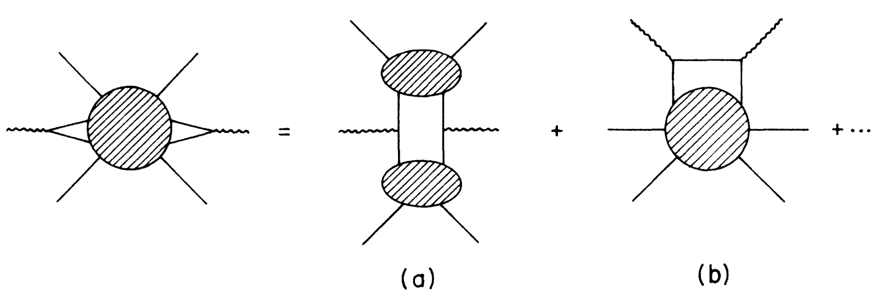

FIG. 5. Contributions to the μ-pair amplitude. Graph (a) corresponds to parton-antiparton annihilation and graph (b) corresponds to parton bremsstrahlung.

without further strong and as yet untested assumptions.[7] For example, form-factor corrections to the electromagnetic vertices in Fig. 5(a) may be important here because the parton lines are nearly on-shell, whereas they are far off-shell in the corresponding deep-inelastic case.[17] In view of the lack of control over these other diagrams, and also because of weaknesses of the model itself,[7] a model-independent approach is desirable. Our attempts in this direction will be the subject of the following sections.

III. SCALING AND REGGE LIMITS

We shall now consider the amplitude $W^{\mu\nu}(q^2, \nu, \nu'; s)$ and analyze its behavior in the appropriate scaling and Regge limits. In order to render the analysis to follow as simple as possible, we temporarily remove the clouding complication due to the spin carried by the current $j^\mu(x)$. In a later chapter spinology modifications will be taken into account. We shall, therefore, study the amplitude

$$W(q^2, \nu, \nu'; s) \equiv \int d^4x\, e^{-iq\cdot x} \times {}_{\rm in}\langle p, p'|\, j(x) j(0)\,|p, p'\rangle_{\rm in}\,, \tag{3.1}$$

with $j(x)$ a scalar current of dimension 2.

A. Regge limits

We recall in this subsection the main results of a multi-Regge analysis of amplitudes like $W(q^2, \nu, \nu'; s)$.[18] Three different Regge limits are defined as follows:

(1) The pionization Regge limit (P) invariantly characterized as

$$\nu, \nu', s \to \infty, \quad q^2, \nu\nu'/s \text{ fixed},$$

corresponds to the kinematical situation in which the photon momentum q remains finite in the (p, p') center-of-mass system, while both p and p' become large. [Fig. 6(b).]

(2) The ν fragmentation Regge limit (F) invariantly characterized as

$$\nu, s \to \infty, \quad q^2, \nu', \nu/s \text{ fixed},$$

corresponds to the kinematical situation in which the photon momentum q remains finite in the rest system of p' while p becomes large. [Fig. 6(c).]

(3) The ν' fragmentation Regge limit (F') simply follows from F upon interchanging ν with ν'. [Fig. 6(a).]

With these definitions, the Regge limits are (see Fig. 5)

$$W(q^2, \nu, \nu'; s) \underset{P}{\sim} s^\alpha \beta(\nu\nu'/q^2 s; q^2), \tag{3.2}$$

$$W(q^2, \nu, \nu'; s) \underset{F}{\sim} s^\alpha \beta(\nu'; \nu/s; q^2). \tag{3.3}$$

Here the power α is expected to be the same as occurs in the two-body process and, hence, is the forward intercept of the relevant Regge trajectory, presumably the Pomeron. Notice the scaling behavior displayed in (3.2) and (3.3), according to which the residue functions β depend only on the fixed ratios $\nu\nu'/q^2 s$ and ν/s, respectively. It is this scaling property that will presently allow us to make the statement of commutativity between the LC-dominated Bjorken limit and the Regge limit a very simple one.

B. Scaling limit

We consider next the scaling limit (A) for the amplitude $W(q^2, \nu, \nu'; s)$ defined to be the following generalized Bjorken limit:

$$q^2, \nu, \nu'; s \to \infty, \quad q^2/2\nu,\ q^2/2\nu',\ q^2/s \text{ fixed}. \tag{3.4}$$

We expect this limit to be LC-dominated,[9] i.e., we expect that only the region near and on the LC $x^2 = 0$ should contribute to (3.1) in the limit (3.4). We now show explicitly that this is indeed the case if Regge behavior for $W(q^2, \nu, \nu'; s)$ at large values of ν, ν', s and fixed q^2 is assumed.

Consider the following representation for $W(q^2, \nu, \nu'; s)$:

$$W(q^2, \nu, \nu'; s) = |J|^{-1} \int d^4x\, \mathfrak{D}_R(q^2, \nu, \nu'; x) \times {}_{\rm in}\langle p, p'|\, j(x) j(0)\,|p, p'\rangle_{\rm in}\,, \tag{3.5}$$

where

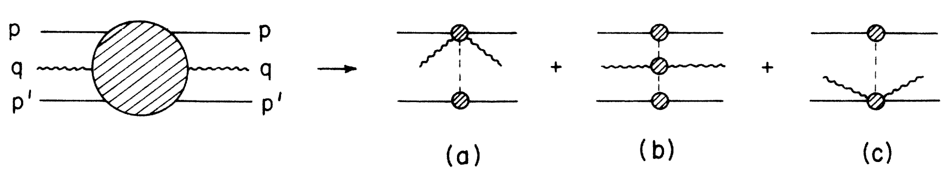

FIG. 6. Regge-pole contributions to the three-to-three amplitude. (a) and (c) correspond to fragmentation and (b) corresponds to pionization.

$$\mathfrak{D}_R(q^2,\nu,\nu';x)\equiv\int\frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}e^{i(\alpha\nu+\alpha'\nu')}\times\Delta_R^+((x-\alpha p-\alpha' p')^2;q^2)\,,$$

$$\Delta_R^+(\xi^2;q^2)\equiv\int_R d^4k\,e^{ik\xi}\theta(k_0)\,\delta(k^2-q^2)\,,$$

and

$$J(q^2,\nu,\nu')\equiv\left.\frac{\partial(k^2:kp,kp')}{\partial(k_0,k_\perp,k_\parallel)}\right|_{k^2=q^2;kp=\nu;kp'=\nu'},$$

with the region of integration in $\Delta_R^+(\xi^2;q^2)$ specified by

$$R:\ 0\le|\vec{k}|\le\frac{s-q^2}{2\sqrt{s}}\,. \tag{3.6}$$

Now let q^2 and s approach infinity with fixed ratio q^2/s. In this limit R becomes all space and

$$\Delta_R^+(\xi^2;q^2)\to\Delta^+(\xi^2;q^2)\,,$$

the ordinary free-field Wightman function. Correspondingly, the $\mathfrak{D}_R(q^2,\nu,\nu';x)$ function becomes

$$\int\frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}e^{i(\alpha\nu+\alpha'\nu')}\Delta^+((x-\alpha p-\alpha' p')^2;q^2)\,, \tag{3.7}$$

with support in

$$|(x-\alpha p-\alpha' p')^2|\le1/q^2\,.$$

If ν and ν' become large, the support of (3.7) is modified by

$$\alpha\le1/\nu,\quad\alpha'\le1/\nu'\,,$$

and reads

$$|x^2-2px/\nu-2p'x/\nu'+s/\nu\nu'|\le1/q^2\,.$$

It therefore follows that the LC dominates the x integration in (3.1) for

$$q^2,\nu,\nu';s\to\infty;\quad s/\nu\nu'\to0,\quad q^2/s\text{ fixed.}$$

The desired result now follows, provided (px) and $(p'x)\ll\nu,\nu'$, from the simple observation that $s/\nu\nu'$ approaches zero in the scaling limit (3.4).

The contribution from the potentially dangerous region $(p\cdot x)\sim O(\nu)$, $(p'\cdot x)\sim O(\nu')$ can, however, be analyzed by noting that for q^2 fixed and ν,ν',s large (i.e., the Regge region), the amplitude $W(q^2,\nu,\nu';s)$ receives a contribution from the x-space region: $|x|^2\ge0$, and $(p\cdot x)\sim O(\nu)$, $(p'\cdot x)\sim O(\nu')$. By requiring W to have Regge behavior we can obtain, by a simple Fourier transformation, the behavior of W in the relevant region of configuration space. It is then straightforward to show that the contribution from this region is exponentially falling for $q^2\to\infty$.

Thus we see that Regge behavior in the fixed-q^2 limit indeed ensures LC dominance for the μ-pair production processes. This Regge "smoothness" turns out to be necessary to derive LC dominance in the Bjorken limit for a general process. We therefore expect that the region of configuration space which controls the behavior of W in the A limit is the LC:

$$|x^2|\le\frac{1}{|q^2|}\,.$$

As mentioned in Sec. I, this does not automatically imply that the leading LC singularity dominates the A limit of $W(q^2,\nu,\nu';s)$. To see why, assume for the moment canonical singularities so that

$$\langle p,p'|j(x)j(0)|p,p'\rangle\underset{x^2\to0}{\sim}\frac{1}{x^2}f(x\cdot p,x\cdot p',s)+g(x\cdot p,x\cdot p',s)+\cdots\,, \tag{3.8}$$

where we have exhibited the leading and first nonleading contributions. Because of (1.4), the contribution of f will be a power of q^2 greater than the contribution of g. If, however, the large-s behavior of g is greater than that of f by a power (or more), then the contribution of g will be the same as (or greater than) that of f since q^2/s is fixed in the A limit. For example, if $f\sim s^\alpha$ but $g\sim s^{\alpha+h}$, then the contribution of f is $(1/q^2)s^\alpha$ and that of g is $(1/q^4)s^{\alpha+h}$ which dominates if $h>1$. Another way of posing the problem is to note that if the difference of dimension of f and g is contained in an (internal) mass factor m^2, then f will dominate $[f\sim(1/q^2)s^\alpha,\ g\sim(1/q^4)s^\alpha m^2]$, but if the difference comes from a factor of s, then the contribution of f and of g will be comparable $[f\sim(1/q^2)s^\alpha,\ g\sim(1/q^4)s^{\alpha+1}]$. In other words, if the x^2 dependence of $\langle p,p'|j(x)j(0)|p,p'\rangle\equiv M$ is scaled by m^2 $[M\sim M(x^2m^2)]$, then the leading LC singularities will dominate, whereas if it is scaled by s $[M\sim M(x^2s)]$, then nonleading LC singularities will also be important. Of course, scaling by factors like s^2/m^2 would make things even worse.

It thus becomes a dynamical question whether or not the leading LC singularity dominates. It turns out that in all presently known models (multi-peripheral,[10] parton,[11] Feynman diagrams,[19] non-perturbative parton[7]) the above possibilities for ruining *leading* LC dominance do *not* occur. (Actually, since in the parton model the only configurations [Fig. 5(b)] considered *exclude* those which contribute to the *leading* LC singularity, there it is the *second leading* contribution $g(\lambda,\lambda',s)$ which dominates.)

More generally, any uniform bound on the large-s behavior at fixed q^2, such as that provided by Regge theory, is sufficient to ensure leading LC dominance. Strictly speaking, such a bound would only be relevant in the Regge limit $s\to\infty$ with q^2 fixed, but commutativity relations of the type discussed in Secs. I and II make these bounds relevant in the A limit as well. This will be discussed in Sec. IIIC. Furthermore, in the LC treatment the large-q^2 and large-s dependencies are effectively decoupled and only the behavior of the five-point functions $\langle p, p'|O_{\alpha_1}\cdots_{\alpha_n}(0)|p,p'\rangle$ in the R limit is relevant, provided the sum over n is sufficiently well convergent. This will be seen in Sec. IV.

The final result of this discussion is that we can now replace $j(x)j(0)$ in (3.1) with the LC expansion

$$j(x)j(0)\underset{x^2\to 0}{\sim}(x^2-i\epsilon x_0)^{-1}\sum_n x^{\alpha_1}x^{\alpha_2}\cdots x^{\alpha_n}\times O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)+\text{LS}, \tag{3.9}$$

and obtain

$$W(q^2,\nu,\nu';s)\underset{A}{\sim}\int d^4x\, e^{-iq\cdot x}(x^2-i\epsilon x_0)^{-1}\times f(p\cdot x;p'\cdot x;s), \tag{3.10}$$

where

$$f(p\cdot x,p'\cdot x;s)=\sum_{n=0}^{\infty}\sum_{i=0}^{n}f_{ni}(s)(p\cdot x)^i(p'\cdot x)^{n-i}. \tag{3.11}$$

C. Commutativity

The LC-dominated A limit

$$W(q^2,s,\nu,\nu')\underset{A}{\sim}W_A(q^2,s,\nu,\nu'),$$

and the pionization limit (3.2)

$$W(q^2,s,\nu,\nu')\underset{P}{\sim}s^{\alpha}\beta\left(\frac{\nu\nu'}{sq^2},q^2\right),$$

are related by a commutativity relation of the sort (2.3) valid in deep-inelastic scattering. The appropriate relation is

$$\lim_{\substack{q^2\to\text{fixed}\\ \eta\,\text{fixed}}} s^{\alpha}\beta(\eta,q^2)=\lim_{\substack{s/q^2,\,\nu/q^2,\,\nu'/q^2\to\infty\\ \eta\ \text{fixed}}} W_A(q^2,s,\nu,\nu'), \tag{3.12}$$

where we have introduced the scaling variable

$$\eta=\frac{\nu\nu'}{sq^2}. \tag{3.13}$$

Equation (3.12) can be derived from an integral representation for W just as (2.3) can be derived from an integral representation for the deep-inelastic amplitude.[7]

IV. FIVE-POINT FUNCTION ANALYSIS

A. Light-cone dominance

To arrive at (3.10), only the leading LC singularity in the expression (3.9) has been kept. As we showed in Sec. IIIB, the dominance of this leading LC contribution is a dynamical question. We will see here how uniform Regge behavior at the five-point function level answers this question in the affirmative.

To render these purely hadronic arguments useful in establishing the validity of (3.10), we now make the assumption that the power series (3.11) is uniformly convergent in s so that the limit $s\to\infty$ can be taken termwise. We can then concentrate our attention only on the matrix elements

$${}_{\text{in}}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}$$

and analyze their behavior in the relevant Regge limit. Notice, however, that the asymptotic behavior under consideration for the (in-in) amplitudes follows from the behavior of the corresponding (in-out) production amplitudes

$${}_{\text{out}}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}$$

in the same limit.

In fact,

$$\begin{aligned}{}_{\text{in}}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}&={}_{\text{in}}\langle p'|b_{\text{in}}(p)O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}\\&={}_{\text{out}}\langle p'|b_{\text{in}}(\vec{p})O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}\\&={}_{\text{out}}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)|p,p'\rangle_{\text{in}}\\&\quad+{}_{\text{out}}\langle p'|[b_{\text{in}}(p)O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)-b_{\text{out}}(\vec{p})O^{(n)}_{\alpha_1\alpha_2}\cdots_{\alpha_n}(0)]|p,p'\rangle_{\text{in}},\end{aligned}$$

and standard reduction techniques give

$$ {}_{\rm in}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots{}_{\alpha_n}(0)|p,p'\rangle_{\rm in} = {}_{\rm out}\langle p,p'|O^{(n)}_{\alpha_1\alpha_2}\cdots{}_{\alpha_n}(0)|p,p'\rangle_{\rm in} - i\int d^4x\,\bar U(x)\vec{\mathfrak{D}}_x\,{}_{\rm out}\langle p'|\Psi(x)O^{(n)}_{\alpha_1\alpha_2}\cdots{}_{\alpha_n}(0)|p,p'\rangle_{\rm in}\,. \quad (4.1) $$

The content of (4.1) is simply that the (in-in) matrix elements of the LC operators $O^{(n)}_{\alpha_1\alpha_2}\cdots{}_{\alpha_n}(0)$ differ from the (in-out) ones only by an s-channel absorptive part. Therefore, the (in-in) amplitudes will have the asymptotic behavior of the corresponding (in-out) production amplitudes.

The uniform Regge bound, seen earlier to be sufficient in establishing leading LC dominance, is now simply provided by the assumption that the (in-out) production amplitudes have the pure Regge-pole behavior for large values of s.

We shall, therefore, take it that under the quite general circumstances discussed above, (3.10) is valid as written and proceed to use our Regge assumption in determining the high-s behavior of the five-point function $f(p\cdot x, p'\cdot x; s)$. Further discussion will be given in Sec. X.

B. High-energy behavior

It follows from multi-Regge theory that the asymptotic behavior is

$$ f_{\rm in}(s) \underset{s\to\infty}{\sim} s^{\alpha}(\ln s)\int_0^1 da\,\sigma_{\rm in}(a)\left(\frac{1}{s^a}\right)^i\left(\frac{1}{s^{1-a}}\right)^{n-i}, \quad (4.2) $$

where α is the $t=0$ intercept of the leading exchanged Regge pole in the configuration of Fig. 7, presumably the Pomeron with $\alpha\cong1$. The result (4.2) can be visualized in the usual helicity formalism if we give to $O^{(n)}_{\alpha_1}\cdots{}_{\alpha_n}(0)$ a small four-momentum k_μ to be taken to zero at the end. The integration over a in (4.2) then corresponds to the various ways of having the subenergies $E=p\cdot k/(k)^{1/2}\sim s^a$ and $E'=p'\cdot k/(k)^{1/2}\sim s^{1-a}$ comprise the energy $s\sim EE'$, and powers i and $n-i$ correspond to the usual helicity-flip factors in (3.11).

A significant restriction is placed on (4.2) by the requirement of hadronic scaling: $\sigma_{\rm in}(a)$ is independent of a for a away from the end points 0 and 1. These central a values corresponds to "pionization" for the five-point function [Fig. 7(b)]. A strong a dependence of $\sigma_{\rm in}(a)$ can, however, be present near the end points, corresponding to fragmentation [Figs. 7(a), 7(c)]. Numerically, the pionization piece is from a's satisfying $\epsilon(s)\le a\le 1-\epsilon(s)$, with $\epsilon(s)$ such that $s^{\epsilon(s)}=N\simeq 2\ \text{GeV}^2$ is the energy at which Regge behavior is expected. Thus $\epsilon(s)=\ln N/\ln s$. The fragmentation piece comes from $0\le a\le\epsilon(s)$ and $1-\epsilon(s)\le a\le 1$.

Another way of understanding (4.2) is to consider the five-point function

$$ {}_{\rm in}\langle p,p'|O_{\alpha_1}\cdots{}_{\alpha_n}(0)|p,p'\rangle_{\rm in} = \sum_{i=1}^n f_{\rm in}(s)p_{\alpha_1}\cdots p_{\alpha_i}p'_{\alpha_{i+1}}\cdots p'_{\alpha_n} + (g_{\alpha\beta}\ \text{terms}) $$

as being constructed out of a six-point function

$$ {}_{\rm in}\langle p,p'|\,\phi(0)\,\partial_{\alpha_1}\cdots\partial_{\alpha_n}\phi(y)|p,p'\rangle_{\rm in} $$

by bringing together the two external ϕ legs (this corresponds to setting $y\to0$). If this six-point function has the usual multi-Regge behavior (pionization and fragmentation) then (4.2) will result [with $\sigma_{\rm in}(a)$ independent of a in the pionization region].

We should mention that it is not really necessary for us to use this scaling at the five-point function level. We could allow $\sigma_{\rm in}(a)$ to be arbitrary now and in our final expression for W_A invoke commutativity with the six-point P and F limits to conclude from the six-point scaling behavior the same a independence.[6]

V. MOMENTUM-SPACE REPRESENTATION

Now that the large-s behavior of the five-point function $f(p\cdot x, p'\cdot x; s)$ has been ascertained, we shall proceed with the LC analysis of the amplitude $W(q^2,\nu,\nu';s)$. Combining (4.2) and (3.11) gives

$$ f(p\cdot x, p'\cdot x; s) \underset{s\to\infty}{\sim} s^{\alpha(0)}\int_0^1 da\,F\left(\frac{p\cdot x}{s^a};\frac{p'\cdot x}{s^{1-a}}\right), \quad (5.1) $$

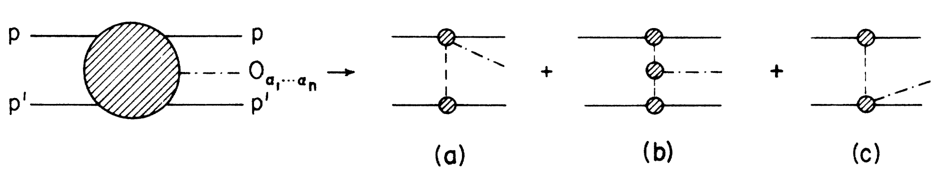

FIG. 7. Regge-pole contributions to the two-to-three amplitude. (a) and (c) correspond to fragmentation and (b) corresponds to pionization.

where

$$F(\zeta,\zeta')\equiv\sum_n\sum_{m=0}^{n}\sigma_{nm}\zeta^m\zeta'^{n-m}. \tag{5.2}$$

Equation (3.10) therefore becomes

$$W(q^2,\nu,\nu';s)\underset{A}{\sim}s^{\alpha(0)}\int_0^1 da\int d^4x\, e^{iq\cdot x}(x^2-i\epsilon x_0)^{-1}\times F\left(\frac{p\cdot x}{s^a};\frac{p'\cdot x}{s^{1-a}}\right). \tag{5.3}$$

We introduce next the double Fourier transform of $F(p\cdot x/s^a; p'\cdot x/s^{1-a})$ according to

$$F\left(\frac{p\cdot x}{s^a};\frac{p'\cdot x}{s^{1-a}}\right)=\int\frac{d\sigma}{2\pi}\frac{d\sigma'}{2\pi}e^{i[(\sigma p\cdot x)/s^a+(\sigma' p'\cdot x)/s^{1-a}]}\times\Psi(\sigma,\sigma') \tag{5.4}$$

and change variables to

$$\alpha\equiv\sigma/s^a,\quad \alpha'\equiv\sigma'/s^{1-a}:$$

$$F\left(\frac{p\cdot x}{s^a};\frac{p'\cdot x}{s^{1-a}}\right)=\int\frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}s^a s^{1-a}e^{i(\alpha p+\alpha' p')x}\times\Psi(\alpha s^a;\alpha' s^{1-a}). \tag{5.5}$$

Invariance under the interchange of p and p' gives

$$\Psi(\alpha s^a;\alpha' s^{1-a})=\Psi(\alpha' s^{1-a};\alpha s^a). \tag{5.6}$$

Now that the x dependence of F has been fully disentangled, we can proceed with the x integration in (5.3) and obtain

$$W(q^2,\nu,\nu';s)\underset{A}{\sim}s^{\alpha(0)}\int_0^1 da\int\frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}s^a s^{1-a}\times\Psi(\alpha s^a;\alpha' s^{1-a})\times\theta(-Q_0)\,\delta(Q^2), \tag{5.7}$$

where

$$Q\equiv q-\alpha p-\alpha' p'. \tag{5.8}$$

We want to express the support properties embodied in the $\theta(-Q_0)\,\delta(Q^2)$ product in a more convenient form. To do so, we use the Sudakov[20] parametrization

$$q=\lambda p+\lambda' p'+t,$$

where $p\cdot t=p'\cdot t=0$; $t^2=-q_\perp{}^2$. Then,

$$Q=(\lambda-\alpha)p+(\lambda'-\alpha')p'+t,$$

and the δ function condition requires

$$(\lambda-\alpha)(\lambda'-\alpha')=q_\perp{}^2/s, \tag{5.9}$$

if one ignores the mass terms. Combining (5.9) with

$$Q_0=(\lambda-\alpha)p_0+(\lambda'-\alpha')p_0'\leqslant 0$$

gives the inequalities

$$\alpha\geqslant\lambda,\quad \alpha'\geqslant\lambda'.$$

Hence

$$\alpha\alpha'\geqslant\lambda\lambda'=\frac{q^2+q_\perp{}^2}{s},$$

and so

$$\alpha\alpha'\geqslant q^2/s, \tag{5.10}$$

and $\theta(-Q_0)$ can be replaced by $\theta(\alpha\alpha'-q^2/s)$. Note that

$$\lambda+\lambda'=2(\nu+\nu')/s\leqslant 1+q^2/s$$

imposes no restrictions on the α,α' integration as $\alpha+\alpha'\geqslant\lambda+\lambda'$. We finally have

$$W(q^2,\nu,\nu';s)\underset{A}{\sim}s^{\alpha(0)}\int_0^1 da\int\frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}s^a s^{1-a}\Psi(\alpha s^a;\alpha' s^{1-a})\theta(\alpha\alpha'-q^2/s)\delta(q^2-2\alpha\nu-2\alpha'\nu'+s\alpha\alpha'). \tag{5.11}$$

Here the α,α' integration extends only over the square

$$S:\ |\alpha|\leqslant 1;\quad |\alpha'|\leqslant 1.$$

To see this, first notice that the spectral properties of the LC amplitude require the Fourier transform of $f(p\cdot x, p'\cdot x; s)$ to vanish outside S. Then, using the entirety of $f(p\cdot x, p'\cdot x; s)$ one can show[14] that, apart from nonleading terms in $1/s$, $s\Psi(\alpha s^a;\alpha' s^{1-a})$ is the Fourier transform of $F((p\cdot x)/s^a;(p'\cdot x)/s^{1-a})$ over the same S.

A most important consequence of (4.2) and the support properties discussed above, is that $\Psi(\alpha s^a;\alpha' s^{1-a})$ is a function of fast decrease in its variables. In fact, the entirety of F, as displayed in (5.2), implies the existence of all integrals

$$\int d\beta\, d\beta'\,\Psi(\beta,\beta')\beta^n\beta'^m$$

over the domain

$$|\beta|\leqslant s^a;\quad |\beta'|\leqslant s^{1-a},$$

which becomes unbounded for $s\to\infty$, thus requiring $\Psi(\beta,\beta')$ to vanish faster than any power when

$\beta, \beta' \to \infty$.

In view of the strength of this result, we should discuss its origin and its limitations more amply. The fast decrease of $\Psi(\beta, \beta')$ is a consequence of the combined assumptions of Regge behavior for the five-point functions and spectral constraints on $W(q^2, \nu, \nu'; s)$, provided the assumption is made that (3.11) is uniformly convergent for $s \to \infty$.

Although a natural assumption, the extent of its validity in general is not known. It is, however, true in the model calculation of Sec. II C, and our treatment will fully characterize the LC behavior of the class of models which show such a uniformity.

VI. PIONIZATION AND FRAGMENTATION

A. Decomposition

Equation (5.11), when supplemented with the fast-decrease property of $\Psi(\alpha s^a; \alpha' s^{1-a})$ as discussed above, is our main result. Obviously, as it stands, (5.11) is of no use in explicitly evaluating $W(q^2; \nu, \nu'; s)$ as long as $\Psi(\alpha s^a; \alpha' s^{1-a})$ is not known. Nevertheless, assuming a certain amount of smoothness, we can still extract from (5.11) substantial information about the amplitude, as we now proceed to show. Since we shall repeatedly invoke the mean-value theorem throughout this section, the assumption will be made that $\Psi(\alpha s^a; \alpha' s^{1-a})$ does not change sign within the rerelevant domains of integration. In support of this assumption we recall the observed precocious onset of light-cone dominance, according to which $\Psi(\alpha s^a; \alpha' s^{1-a})$ should already have achieved its nonoscillating asymptotic behavior when the arguments span the integration ranges indicated in (5.11).

To continue our discussion, we shall find it convenient to decompose the amplitude into two pieces by cutting the a integration in (5.11) at $\epsilon(s)$ and $1-\epsilon(s)$, where $s^{\epsilon(s)} \equiv N$ is the energy at which Regge behavior is expected. The contribution to $W(q^2, \nu, \nu'; s)$ from all a's satisfying

$$\epsilon(s) \le a \le 1-\epsilon(s)$$

will be called the "pionization" contribution as it will become apparent that this region of integration gives the dominant piece of $W(q^2, \nu, \nu'; s)$ when $\nu \sim \nu'$. Correspondingly, the integration regions

$$0 \le a \le \epsilon(s), \quad 1-\epsilon(s) \le a \le 1$$

give the dominant contribution when $\nu \gg \nu'$ or $\nu' \gg \nu$, respectively, and therefore the end-point contributions to the amplitude shall be called "fragmentation" contributions.

B. Pionization contribution

We begin by analyzing the pionization contribution to the amplitude. Performing the α' integration in (5.11), we display the symmetry properties of $\Psi(\alpha s^a, \alpha' s^{1-a})$ in the general form

$$\Psi\left(\alpha s^a; \frac{2\nu}{s} \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} s^{1-a}\right) = \Psi(\sigma, \pi^{1/2}),$$

where

$$\sigma \equiv \frac{1}{2}\left(\alpha s^a + \frac{2\nu}{s} \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} s^{1-a}\right) \tag{6.1}$$

and

$$\pi \equiv 2\nu \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} \alpha . \tag{6.2}$$

The intervals over which $\Psi(\alpha s^a; \alpha' s^{1-a})$ is a monotonic function in the variable a, important in estimating the a integral via the mean-value theorem, follow from the observation that π is independent of a. In fact,

$$d\pi/da = 0$$

implies

$$d\Psi/da = (\partial\Psi/\partial\sigma) d\sigma/da .$$

Therefore, $d\Psi/da$ vanishes for

$$a = \bar{a} \equiv \frac{1}{2\ln s} \ln\left(2\nu \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} \frac{1}{\alpha}\right), \tag{6.3}$$

is positive for $a < \bar{a}$ [because $\partial\Psi/\partial\sigma < 0$ for Ψ a rapidly decreasing function in σ], and remains negative for $a > \bar{a}$. It is not difficult to show that

$$\epsilon \le \bar{a} \le 1-\epsilon$$

for all values of α within the α-integration range and for all physically accessible values of (ν, ν'). We can therefore split the pionization a integral into two parts according to

$$\int_\epsilon^{1-\epsilon} da = \int_\epsilon^{\bar{a}} da + \int_{\bar{a}}^{1-\epsilon} da,$$

and use the mean-value theorem for monotonic functions to obtain

$$W_P = s^{1+\alpha(0)} \int_{(2\nu'/s)(1-q^2/2\nu')/(1-2\nu/s)}^{1} d\alpha \, \frac{\lambda_2 - \lambda_1}{\alpha - 2\nu'/s} \, \Psi\left[\alpha s^{\bar{a}} + \frac{2\nu}{s} \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} s^{1-\bar{a}}; \left(2\nu \frac{\alpha - q^2/2\nu}{\alpha - 2\nu'/s} \alpha\right)^{1/2}\right], \tag{6.4}$$

where

$$\epsilon \leq \lambda_1 \leq \bar{a}; \quad \bar{a} \leq \lambda_2 \leq 1-\epsilon .$$

Putting in the known value of $\bar{a}$, we obtain

$$W_P = s^{1+\alpha(0)} \int_{(2\nu'/s)(1-q^2/2\nu')/(1-2\nu/s)}^{1} d\alpha \, \frac{\lambda_2-\lambda_1}{\alpha-2\nu'/s} \, \Psi\left[\left(2\nu \frac{\alpha-q^2/2\nu}{\alpha-2\nu'/s}\, \alpha\right)^{1/2}; \left(2\nu \frac{\alpha-q^2/2\nu}{\alpha-2\nu'/s}\, \alpha\right)^{1/2}\right]. \tag{6.5}$$

The same technique can be applied to evaluating the α integral. The possibility of doing so follows from the observation that σ and $\pi^{1/2}$ are now equal. We then have

$$\frac{d\Psi}{d\alpha} = \left(\frac{\partial\Psi}{\partial\sigma} + \frac{\partial\Psi}{\partial\pi^{1/2}}\right)\frac{d\pi^{1/2}}{d\alpha}$$

and $d\Psi/d\alpha$ vanishes for

$$\alpha = \bar{\alpha} \equiv \frac{2\nu'}{s}\left[1+\left(\frac{q_\perp^2}{q^2+q_\perp^2}\right)^{1/2}\right], \tag{6.6}$$

having the opposite sign of $d\pi^{1/2}/d\alpha$ for α above or below $\bar{\alpha}$ because

$$\frac{\partial\Psi}{\partial\sigma} + \frac{\partial\Psi}{\partial\pi^{1/2}} < 0 .$$

Unfortunately the regions of monotonicity implied by this argument are strongly dependent on (ν, ν'). A simple but lengthy analysis in the $(q_\perp^2, \nu')$ phase space (Fig. 8) yields the following behaviors:

$$W^P \sim s^{1+\alpha(0)} \, \Psi\left[\left(2\nu' + \frac{q_\perp^2}{1-(q^2+q_\perp^2)/2\nu'}\right)^{1/2}; \text{ same}\right]$$

for

$$\tfrac{1}{2}(s+q^2) - \{[\tfrac{1}{2}(s-q^2)]^2 - q_\perp^2 s\}^{1/2} \leq 2\nu' \leq q^2 + q_\perp^2 + [q_\perp^2(q^2+q_\perp^2)]^{1/2};$$

$$W^P \sim s^{1+\alpha(0)} \, \Psi[\{q^2+2q_\perp^2 + 2[q_\perp^2(q^2+q_\perp^2)]^{1/2}\}^{1/2}; \text{ same}]$$

for

$$q^2+q_\perp^2+[q_\perp^2(q^2+q_\perp^2)]^{1/2} \leq 2\nu' \leq \frac{s}{1+[q_\perp^2/(q^2+q_\perp^2)]^{1/2}};$$

and

$$W^P \sim s^{1+\alpha(0)} \, \Psi\left[\left(\frac{q^2+q_\perp^2}{2\nu'}\, s + \frac{q_\perp^2}{1-2\nu'/s}\right)^{1/2}; \text{ same}\right]$$

for

$$\frac{s}{1+[q_\perp^2/(q^2+q_\perp^2)]^{1/2}} \leq 2\nu' \leq \tfrac{1}{2}(s+q^2) + \{[\tfrac{1}{2}(s-q^2)]^2 - q_\perp^2 s\}^{1/2} . \tag{6.7}$$

For convenience, various multiplicative factors in (6.7) have been neglected as they are characteristic only of the no-spin model that we are investigating in this section.

Next we proceed to an evaluation of various cross sections based on the result we have obtained for the amplitude $W^P(q^2, \nu, \nu'; s)$. Consider first the pionization contribution to the invariant-mass differential cross section:

$$\left(\frac{d\sigma}{dq^2}\right)^P \sim \frac{1}{s}\int_0^{(s-q^2)^2/4s} dq_\perp^2 \int_{(s+q^2)/2-[(s-q^2)^2/4-q_\perp^2 s]^{1/2}}^{(s+q^2)/2+[(s-q^2)^2/4-q_\perp^2 s]^{1/2}} \frac{d(2\nu')}{2\nu'} \, W^P(q^2,\nu',q_\perp^2;s) . \tag{6.8}$$

For what follows, we shall find it convenient to write the above expression in the form

$$\left(\frac{d\sigma}{dq^2}\right)^P \sim \frac{1}{s}\int_{q^2}^{(s+q^2)/2} \frac{d(2\nu')}{2\nu'} \int_0^{2\nu'(1-q^2/2\nu')(1-2\nu'/s)} dq_\perp^2 W^P(q^2,\nu',q_\perp^2;s)$$
$$+ \frac{1}{s}\int_{(s+q^2)/2}^{s} \frac{d(2\nu')}{2\nu'} \int_0^{2\nu'(1-q^2/2\nu')(1-2\nu'/s)} dq_\perp^2 \, W^P(q^2,\nu',q_\perp^2;s), \tag{6.9}$$

and split both $q_\perp^2$ integrals into two pieces that integrate $W^P(q^2, \nu', q_\perp^2; s)$ over the regions in the $(2\nu'; q_\perp^2)$ plane which proved to be relevant in the previous discussion of W^P. Once again, for each region in turn we analyze the monotonicity of $W^P(q^2, \nu', q_\perp^2; s)$ in the variables $q_\perp^2$, $2\nu'$ successively and then use the mean-value theorem for monotonic functions to perform the integrations.

Thus with reference to Fig. 9, for

$$q^2 \leq 2\nu' \leq \tfrac{1}{2}(s+q^2),$$
$$0 \leq q_\perp^2 \leq \frac{(2\nu'-q^2)^2}{4\nu'-q^2},$$

the amplitude

$$W^P \sim s^{1+\alpha(0)} \, \Psi[\{q^2+2q_\perp^2 + 2[q_\perp^2(q^2+q_\perp^2)]^{1/2}\}^{1/2}; \text{ same}]$$

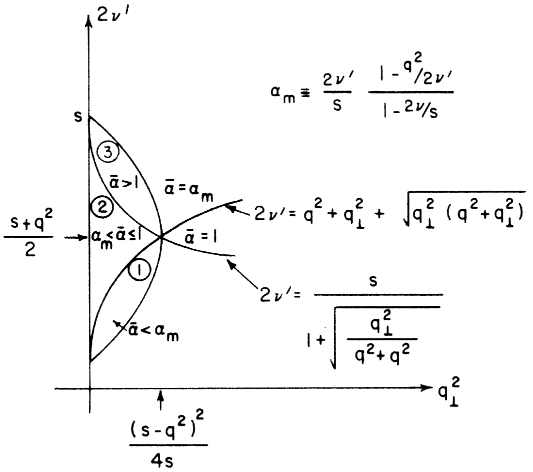

FIG. 8. Relation between $q_\perp{}^2$-ν' phase space and α space.

is evidently a monotonically decreasing function of $q_\perp{}^2$ and, therefore, the mean-value theorem for the $q_\perp{}^2$ integration gives essentially

$$s^{\alpha(0)}\,\Psi(\sqrt{q^2};\ \sqrt{q^2})\int_{q^2}^{(s+q^2)/2}\frac{d(2\nu')}{2\nu'}\ \frac{(2\nu'-q^2)^2}{4\nu'-q^2}\ .$$

Now the $2\nu'$ integral is trivial and the corresponding piece of the cross section must have the behavior of

$$s^{1+\alpha(0)}\,\Psi(\sqrt{q^2},\sqrt{q^2})\,. \tag{6.10}$$

Next, for

$$q^2 \le 2\nu' \le \tfrac{1}{2}(s+q^2)\,,$$

$$\frac{(2\nu'-q^2)^2}{4\nu'-q^2} \le q_\perp{}^2 \le 2\nu'\left(1-\frac{q^2}{2\nu'}\right)\left(1-\frac{2\nu'}{s}\right),$$

the amplitude

$$W^P \sim s^{1+\alpha(0)}\,\Psi\left[\left(2\nu'+\frac{q_\perp{}^2}{1-(q^2+q_\perp{}^2)/2\nu'}\right)^{1/2};\ \text{same}\right]$$

is a monotonically decreasing function of $q_\perp{}^2$, so that the $q_\perp{}^2$ integration is easily done to give

$$s^{\alpha(0)}\int_{q^2}^{(s+q^2)/2} d(2\nu')\Psi[(4\nu'-q^2)^{1/2};\ \text{same}]$$
$$\times(1-q^2/2\nu')(1-2\nu'/s)\,.$$

Hence, after having performed the $2\nu'$ integral, this piece of the cross section must again have the behavior of (6.10). Similar arguments show that the remaining pieces integrate to the same form that the pieces already discussed have. Therefore, we can conclude that apart from inessential factors,

$$\left(\frac{d\sigma}{dq^2}\right)^P \sim s^{1+\alpha(0)}\Psi(\sqrt{q^2};\ \sqrt{q^2})\,. \tag{6.11}$$

Similarly, we find

$$\left(\frac{d\sigma}{dq_\perp{}^2}\right)^P \sim \Psi(\sqrt{q_\perp{}^2};\ \sqrt{q_\perp{}^2})\,. \tag{6.12}$$

C. Fragmentation contribution

The technique developed above for analyzing the pionization contribution fails in general in the fragmentation regions $a=0$, 1 as there is no a integration to provide for the key property whereby σ and $\pi^{1/2}$ are equal. Correspondingly, less general statements can be made about the fragmentation amplitude $W^F(q^2, \nu', q^2; s)$. We can still evaluate the amplitude when either of $\partial\Psi/\partial\sigma \gg \partial\Psi/2\pi^{1/2}$, $\partial\Psi/\partial\pi^{1/2} \gg \partial\Psi/\partial\sigma$ holds and then

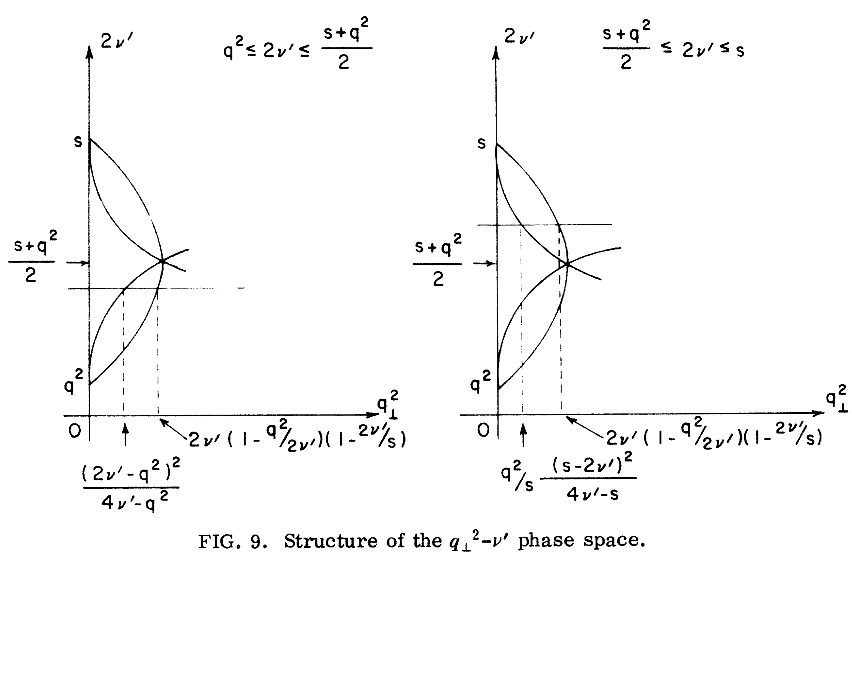

FIG. 9. Structure of the $q_\perp{}^2$-ν' phase space.

proceed to a determination of $(d\sigma/dq^2)^F$. We find for each of these cases that

$$\left(\frac{d\sigma}{dq^2}\right)^F \sim s^{1+\alpha(0)}\Psi(q^2;\sqrt{q^2}). \tag{6.13}$$

We now show that although one cannot establish this result in general by integrating the fragmentation amplitude, (6.13) is nevertheless generally true. In fact, if one imagines having performed all integrations by the mean-value theorem, one expects

$$\left(\frac{d\sigma}{dq^2}\right)^F \sim \Psi[\sigma(\overline{\alpha}),\ \pi^{1/2}(\overline{\alpha})],$$

where $\overline{\alpha}$ is the α integration take-out point evaluated at $2\nu' = 2\overline{\nu}'$ and $q^2 = \overline{q}^2$, the corresponding $2\nu'$ and q^2 integration take-out points. Because $d\pi/da = 0$, we must have

$$\pi(\overline{\alpha}) = q^2,$$

and therefore

$$\overline{\alpha}\frac{\overline{\alpha}-\overline{\omega}}{\overline{\alpha}-\overline{\xi}'} = \overline{\omega}, \tag{6.14}$$

where $\omega \equiv q^2/2\nu$; $\xi' \equiv 2\nu'/s$. It follows from (6.14), however, that an $\overline{\alpha}$ exists if and only if $\overline{\omega} = \overline{\xi}'$, i.e., if $\overline{q}^2 = 0$. Then

$$\overline{\alpha} = \overline{\xi}', \quad \overline{\alpha}' = \overline{\xi},$$

providing for the same variable

$$\sigma(\overline{\alpha}) = \overline{\xi}' + s\overline{\xi}.$$

Notice, that while the condition (6.14) has fixed the q^2 take-out point, it has left $2\overline{\nu}'$ and, correspondingly, $2\overline{\nu}$ free. We shall now fix them by the requirement that $\overline{\nu}$ be minimal. It is then easy to see that

$$2\overline{\nu}' = s, \quad 2\overline{\nu} = q^2,$$

with

$$\sigma(\overline{\alpha}) = q^2 + O(q^2 s),$$

and (6.13) follows. We also find, that

$$\left(\frac{d\sigma}{dq_\perp{}^2}\right)^F \sim \Psi(q_\perp{}^2;\sqrt{q_\perp{}^2}). \tag{6.15}$$

D. General properties

Notice that for small values of q^2 the fragmentation contribution to $d\sigma/dq^2$ is more important than the pionization contribution, while for large q^2 the importance of the two contributions gets interchanged. Consequently, as long as the sum variable σ in $\Psi(\sigma;\pi^{1/2})$ is at least as important as the product variable π, we can choose the mixing factors P and F in

$$\left(\frac{d\sigma}{dq^2}\right) \sim s^{1+\alpha(0)}[P\Psi(\sqrt{q^2};\sqrt{q^2}) + F\Psi(q^2;\sqrt{q^2})] \tag{6.16}$$

such that the experimentally observed[4] shoulder in the invariant-mass differential cross section at $\sqrt{q^2} \sim 3.1$ GeV/c^2 is reproduced by our theory. Naturally, the values of P and F required here cannot be obtained unless some choice for $\Psi(\sigma;\pi^{1/2})$ is made (Sec. VIII), but it should already be evident that F is much larger than P, perhaps with a ratio of as much as 10^2–10^3.

We finally quote our result for the transverse momentum differential cross section:

$$\frac{d\sigma}{dq_\perp{}^2} \sim P'\Psi(\sqrt{q_\perp{}^2};\sqrt{q_\perp{}^2}) + F'\Psi(q_\perp{}^2;\sqrt{q_\perp{}^2}), \tag{6.17}$$

with $P' = \text{const}\times P$ and $F' = \text{const}\times F$. Since $q_\perp{}^2$ never gets large, the shoulder feature of (6.16) is unimportant here. In fact, for $F' \gg P'$, we expect to obtain a good representation of the data by simply using

$$\frac{d\sigma}{dq_\perp{}^2} \sim \Psi(q_\perp{}^2;\sqrt{q_\perp{}^2}). \tag{6.18}$$

VII. ANALYSIS WITH VECTOR CURRENTS

The analysis of Sec. VI shows that the LC theory of the μ-pair production process, when supplemented with multi-Regge arguments, can account for the main features of the experimental data. Thus, a shoulder in the invariant-mass differential cross section $d\sigma/dq^2$ can be provided by the change in slope from the fragmentation to the pionization regions, while the transverse-momentum differential cross section $d\sigma/dq_\perp{}^2$ in the laboratory system is predicted to have the behavior of a function of fast decrease in $q_\perp{}^2$.

We now want to proceed to a more quantitative phenomenological stage of the theory and eventually show that our expressions can nicely fit the data. To that end, we first determine the form of the triple-differential cross section in the presence of spin. The LC expansion to be inserted in (2.15) is now given by[3]

$$j^\mu(x)j^\nu(0) \underset{x^2\to 0}{\sim} -(g^{\alpha\nu}g^{\beta\mu}\Box + g^{\mu\nu}\partial^\alpha\partial^\beta - g^{\alpha\mu}\partial^\beta\partial^\nu - g^{\alpha\nu}\partial^\beta\partial^\mu)\,O_{\alpha\beta}(x), \tag{7.1}$$

where

$$O_{\alpha\beta}(x) \equiv \ln(x^2 - i\epsilon x_0)\sum_n x^{\alpha_1}x^{\alpha_2}\cdots x^{\alpha_n} \times O^{(n)}_{\alpha\beta,\alpha_1\alpha_2\cdots\alpha_n}(0),$$
$$\dim O^{(n)} = n+2, \tag{7.2}$$

and

$$O^\alpha_\alpha(x) = 0. \tag{7.3}$$

The five-point function now has the form

$$ {}_{\rm in}\langle p,p'|O_{\alpha\beta}(x)|p,p'\rangle_{\rm in}\equiv \ln(x^2-i\epsilon x_0)\,[g_{\alpha\beta}f_0(p\cdot x,p'\cdot x;s)+p_\alpha p_\beta f_1(p\cdot x,p'\cdot x;s) + \tfrac{1}{2}(p_\alpha p'_\beta+p'_\alpha p_\beta)f_2(p\cdot x,p'\cdot x;s)+p'_\alpha p'_\beta f_3(p\cdot x,p'\cdot x;s)], \tag{7.4}$$

and the trace condition (7.3) implies

$$4f_0+f_2=0\,. \tag{7.5}$$

An analysis similar to the one given in Sec. IIIB and Sec. IV gives for the scalar amplitudes $f_i(p\cdot x,p'\cdot x;s)$ the forms

$$f_i(p\cdot x,p'\cdot x;s)=s^{\alpha(0)}\int_0^1 da\,h_i(s;a)\int_S \frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}\,s^a s^{1-a}\Psi_i(\alpha s^a;\alpha' s^{1-a})e^{i(\alpha p+\alpha' p')x}, \tag{7.6}$$

where all $\Psi_i(\alpha s^a;\alpha' s^{1-a})$ are rapidly decreasing functions of their variables, and where

$$h_0(s;a)\equiv 1;\quad h_1(s;a)\equiv s^{-2a};\qquad h_2(s;a)\equiv s^{-1};\quad h_3(s;a)\equiv s^{-2(1-a)}\,. \tag{7.7}$$

We then find for the trace of $W^{\mu\nu}(q^2,\nu,\nu';s)$

$$W^\mu_\mu \underset{A}{\sim} \sum_{i=0}^{3} W_i\,, \tag{7.8}$$

where

$$W_i\equiv s^{\alpha(0)}\int_0^1 da\,h_i(s;a)\int_S \frac{d\alpha}{2\pi}\frac{d\alpha'}{2\pi}\,s^a s^{1-a} \times\Psi_i(\alpha s^a;\alpha's^{1-a})w_i\delta'_+(Q^2), \tag{7.9}$$

with[21]

$$Q\equiv q-\alpha p-\alpha' p'\,,\qquad \delta'_+(Q^2)\equiv\frac{d}{dQ^2}[\theta(-Q_0)\delta(Q^2)]\,, \tag{7.10}$$

and

$$w_0\equiv 2Q^2;\quad w_1\equiv 2(p\cdot Q)^2;\qquad w_2\equiv 2(p\cdot Q)(p'\cdot Q);\quad w_3\equiv 2(p'\cdot Q)^2\,.$$

Notice that here we have kept only the leading contribution of the leading LC piece and dropped terms of the type

$$\ln(x^2-i\epsilon x_0)\partial^\mu\partial^\nu f_0(p\cdot x,p'\cdot x;s)\,,$$
$$\partial^\mu f_0(p\cdot x,p'\cdot x;s)\partial^\nu \ln(x^2-i\epsilon x_0)\,,$$
$$g^{\mu\nu}f_0(p\cdot x,p'\cdot x;s)\Box\ln(x^2-i\epsilon x_0)\,,$$

and so on. In fact, these terms were introduced in (7.1) only to make the expansion manifestly current-conserving and they correspond to nonleading contributions.

VIII. PHENOMENOLOGICAL MODEL

A. Model

We are now ready to choose some reasonable model for the $\Psi_i(\alpha s^a;\alpha's^{1-a})$. Specifically we take the following:

(1) All Ψ_i's are only functions of the sum variable $\sigma\equiv\frac{1}{2}(\alpha s^a+\alpha's^{1-a})$.

(2) In the pionization region ($\epsilon\le a\le 1-\epsilon$) $\Psi_i(\alpha s^a;\alpha's^{1-a})\sim e^{-h\alpha s^a}e^{-h\alpha's^{1-a}}$.

(3) In the fragmentation region ($0\le a\le\epsilon$) $\Psi_i(\alpha s^a;\alpha's^{1-a})\sim e^{-k\alpha's}(1-\alpha)^n$.

(4) In the fragmentation region ($1-\epsilon\le a\le 1$) $\Psi_i(\alpha s^a;\alpha's^{1-a})\sim e^{-k\alpha s}(1-\alpha')^n$.

The requirement (1) can be made transparent if one recalls the discussion at the end of Sec. VIC. In fact, we have noticed there that only the sum variable dependence has the virtue of providing the shoulder structure of the experimental data on $d\sigma/dq^2$. It is, then, only a matter of convenience to drop the dependence on $\pi\equiv\alpha\alpha's$. Naturally the choice of exponential falloff for the $\Psi_i(\alpha s^a;\alpha's^{1-a})$, as displayed in (2)–(4), is by no means the only one possible, although it is no doubt a reasonable one. Nevertheless, we have made this particular choice on the grounds that we expect to fit the data on $d\sigma/dq_\perp{}^2$, known to have exponential behavior,[4] by (6.18) alone. Finally, we have incorporated the SLAC-MIT-like threshold behavior[8] we expect for $\Psi_i(\alpha s^a;\alpha's^{1-a})$ in the fragmentation region when either α or α' are near 1.

B. General results

We shall now explicitly evaluate the triple-differential cross section $W^\mu_\mu(q^2,\nu,\nu';s)$, in terms of the five phenomenological parameters, later to be fitted to the data, P, h, F, k, and n:

(1) Consider the expression for W_2^P. With $h_2(s;a)=s^{-1}$, we have

$$W_2^P=\frac{A_2 s^3}{8\pi^2}\int_\epsilon^{1-\epsilon}da\int_S d\alpha\,d\alpha'\,e^{-h\alpha s^a}e^{-h\alpha's^{1-a}} \times\left(\frac{2\nu}{s}-\alpha'\right)\left(\frac{2\nu'}{s}-\alpha\right)\delta'_+(Q^2). \tag{8.1}$$

The integration over a can now be done rather easily to obtain

$$W_2^P = \frac{A_2 s^3}{4\pi^2} \frac{d}{dq^2} \int_S d\alpha\, d\alpha' K_0(2h(\alpha\alpha' s)^{1/2}) \times \left(\frac{2\nu}{s} - \alpha'\right)\left(\frac{2\nu'}{s} - \alpha\right)\theta\left(\alpha\alpha' - \frac{q^2}{s}\right) \times \delta(q^2 - 2\alpha\nu - 2\alpha'\nu' + s\alpha\alpha'), \tag{8.2}$$

where d/dQ^2 has been taken outside the α, α' integral as a derivative with respect to q^2 at fixed (ν, ν', s), and where the $\theta\delta$ product has been rearranged in accordance with Sec. V. We shall find it convenient to write

$$W_2^P \equiv \frac{A_2 s}{4\pi^2} \frac{dJ}{dq^2}, \tag{8.3}$$

where

$$J \equiv s^2 \int_S d\alpha\, d\alpha' K_0(2h(\alpha\alpha' s)^{1/2})\left(\frac{2\nu}{s} - \alpha'\right)\left(\frac{2\nu'}{s} - \alpha\right) \times \theta\left(\alpha\alpha' - \frac{q^2}{s}\right)\delta(q^2 - 2\alpha\nu - 2\alpha'\nu' + s\alpha\alpha'). \tag{8.4}$$

Introducing

$$\zeta \equiv \alpha\nu + \alpha'\nu', \quad \eta \equiv \alpha\alpha' s,$$

and performing the ζ integration with the δ function, we can rewrite J as

$$J = 2q_\perp^2 \int_0^{\eta_0 - \bar\eta_0} dx \frac{K_0(2h(x^2 + \eta_0)^{1/2})}{(x^2 + \Delta)^{1/2}}, \tag{8.5}$$

where

$$\Delta \equiv 4[q_\perp^2(q^2 + q_\perp^2)]^{1/2} \tag{8.6}$$

and

$$\eta_0 \equiv q^2 + 2q_\perp^2 + 2[q_\perp^2(q^2 + q_\perp^2)]^{1/2}, \qquad \bar\eta_0 \equiv 2\nu \frac{1 - q^2/2\nu}{1 - 2\nu'/s}. \tag{8.7}$$

Notice that in the A limit $\eta_0 \to \infty$ as q^2 and therefore J is not very sensitive to the value of the upper limit $(\eta_0 - \bar\eta_0)$. Correspondingly, we can take

$$J = 2q_\perp^2 \int_0^\infty dx \frac{K_0(2h(x^2 + \eta_0)^{1/2})}{(x^2 + \Delta)^{1/2}}. \tag{8.8}$$

Now the x integration can be performed to give

$$J = q_\perp^2 K_0(h[\eta_0^{1/2} + (\eta_0 - \Delta)^{1/2}]) \times K_0(h[\eta_0^{1/2} - (\eta_0 - \Delta)^{1/2}]). \tag{8.9}$$

Similar methods can be employed to reduce W_0^P, W_1^P and W_3^P to quadratures. We give here only the result:

$$W_0^P = \frac{A_0 s}{4\pi^2} R_0, \quad W_1^P = W_3^P \equiv \frac{A_3 s}{4\pi^2}\frac{dL}{dq^2}, \tag{8.10}$$

where

$$L \equiv \frac{1}{h\eta_0^{1/2}}(q_\perp^2 + \tfrac{1}{4}\Delta)K_1(2h\eta_0^{1/2}) + \frac{1}{2h^2}K_0(2h\eta_0^{1/2}) + [4(q_\perp^2)^2 + \tfrac{1}{8}\Delta^2]\frac{1}{4h^2}\frac{d^2R_0}{d\eta_0^2} + \frac{q_\perp^2\Delta}{2h^2}\frac{d^2R_1}{d\eta_0} - \frac{\Delta}{4h^2}\frac{dR_1}{d\eta_0} \tag{8.11}$$

and

$$R_\mu \equiv K_\mu(h[\eta_0^{1/2} + (\eta_0 - \Delta)^{1/2}]) \times K_\mu(h[\eta_0^{1/2} - (\eta_0 - \Delta)^{1/2}]). \tag{8.12}$$

Notice that, while exact when $(\eta_0 - \bar\eta_0) \to \infty$, the above expressions for the W_i^P are unnecessarily complicated to work with. However, since $(q_\perp^2/q^2)^{1/2}$ is small in the A limit, it will be sufficient to keep W_i^P only to first order in $(q_\perp^2/q^2)^{1/2}$ and thereby simplify the calculation of the cross sections. To leading order in $1/q^2$ one can then show, after some lengthy algebra, that

$$W_1^P = W_3^P \sim \frac{A_3 s}{4\pi^2}\tfrac{3}{2}K_0(h[\eta_0^{1/2} + (\eta_0 - \Delta)^{1/2}]), \tag{8.13}$$

while W_0^P and W_2^P have the logarithmic singularity of R_0 when $q_\perp^2 \to 0$:

$$W_0^P \sim \frac{4A_0 s}{4\pi^2}R_0, \qquad W_2^P \sim \frac{A_2 s}{4\pi^2}[\tfrac{1}{2}K_0(h[\eta_0^{1/2} + (\eta_0 - \Delta)^{1/2}]) - R_0]. \tag{8.14}$$

The apparent singularity at $q_\perp^2 \to 0$ is not really present if one recalls (7.5),

$$4A_0 + A_2 = 0.$$

The singularity in W_0 now exactly cancels the one in W_2 and we have

$$(W_\mu^\mu)^P \sim AsK_0(h[\eta_0^{1/2} + (\eta_0 - \Delta)^{1/2}]), \tag{8.15}$$

where $A \equiv 1/4\pi^2(\frac{1}{2}A_0 + \frac{3}{2}A_3)$. This is our main result for the pionization contribution.

(2) We next discuss the fragmentation contribution to $W_\mu^\mu(q^2, \nu, \nu'; s)$. For $a \sim 0$, only W_1 contributes sensibly, as dictated by the helicity factors $h_i(s; a)$, while for $a \sim 1$ only W_3 contributes. Consider then

$$W_1^F = \frac{Bs^4}{8\pi^2}\frac{dI}{dq^2},$$

where

$$I \equiv \int_S d\alpha\, d\alpha' e^{-k\alpha' s}\left(\frac{2\nu}{s} - \alpha'\right)^2(1 - \alpha)^n\theta\left(\alpha\alpha' - \frac{q^2}{s}\right) \times \delta(q^2 - 2\alpha\nu - 2\alpha'\nu' + s\alpha\alpha'). \tag{8.17}$$

Changing variables to

$$x \equiv \alpha' - 2\nu/s\,, \quad y \equiv \alpha - 2\nu'/s\,,$$

and performing the y integration with the δ function, we obtain

$$I = \frac{1}{s} e^{-k2\nu} \int_{(q_\perp^2/s)/(1-2\nu'/s)}^{1-2\nu/s} dx\, e^{-ksx} x \left(1 - \frac{2\nu'}{s} - \frac{q_\perp^2}{sx}\right)^n . \tag{8.18}$$

Define next

$$l \equiv \frac{q_\perp^2/s}{1-2\nu'/s}\,, \quad u \equiv 1-2\nu/s\,, \quad \rho \equiv 1-2\nu'/s\,, \tag{8.19}$$

and

$$N(l, u; \lambda) \equiv \int_l^u dx\, e^{-ksx} e^{-\lambda q_\perp^2/sx} x\,.$$

Notice that $N(l, u; \lambda)$ has been chosen such that

$$I = \frac{1}{s} e^{-k2\nu} \sum_{j=0}^{n} \binom{n}{j} \rho^{n-j} \left[\frac{\partial^j}{\partial \lambda^j} N(l, u; \lambda)\right]_{\lambda=0}, \tag{8.20}$$

thus reducing I to an evaluation of $N(l, u; \lambda)$. $N(l, u; \lambda)$ itself can be integrated rather straightforwardly by use of Laplace-transform methods. When this is done, (8.20) gives for I

$$\begin{aligned} I = -(1/ks^3)e^{-ks}\Bigg[&\sum_{j=0}^{n} (-1)^j \binom{n}{j} \rho^{n-j} \left(\frac{q_\perp^2}{su}\right)^j (ksu)^{j+2}\, \Psi(j+1\,|\,j+1\,|\,ksu) \\ &+ 2\sum_{j=0}^{n} (-1)^j \binom{n}{j} \rho^{n-j} \left(\frac{q_\perp^2}{su}\right)^j (ksu)^{j+1}\, \Psi(j+1\,|\,j\,|\,ksu) \\ &+ 2\sum_{j=0}^{n} (-1)^j \binom{n}{j} \rho^{n-j} \left(\frac{q_\perp^2}{su}\right)^j (ksu)^{j}\, \Psi(j+1\,|\,j-1\,|\,ksu)\Bigg] \\ + (1/ks^3)e^{-k2\nu}e^{-ksl}\rho^n \Bigg[&\sum_{j=0}^{n} (-1)^j \binom{n}{j} (ksl)^{j+2}\, \Psi(j+1\,|\,j+1\,|\,ksl) + 2\sum_{j=0}^{n} (-1)^j \binom{n}{j} (ksl)^{j+1}\, \Psi(j+1\,|\,j\,|\,ksl) \\ &+ 2\sum_{j=0}^{n} (-1)^j \binom{n}{j} (ksl)^{j}\, \Psi(j+1\,|\,j-1\,|\,ksl)\Bigg]. \end{aligned} \tag{8.21}$$

Here $\Psi(a|\,c\,|\,x)$ stands for the Tricomi confluent hypergeometric function.[22] It is again useful to simplify the rather complicated, but exact, expression for W_1^F. To this end, we notice that according to our general discussion of Sec. VI D, the fragmentation piece is expected to contribute importantly over the pionization one only at the small-q^2 end of the dimuon mass spectrum. For these values of q^2, the momentum cut present experimentally ($2q_\parallel > 24$) restricts 2ν to values much larger than q^2 and therefore $q^2/2\nu \ll 1$. Then $sl \sim q_\perp^2$ and for small $q_\perp^2$ it will suffice to keep W_1^F only in the limit $ksl \to 0$. Note that the last assumption is more restrictive than the $(q_\perp^2/q^2)^{1/2}$-small assumption needed in estimating the pionization contribution. We are, nevertheless, willing to make it both on the grounds that the angle cut present in the experiment ($\cos^{-1}\theta \geq 0.998$) does not allow $q_\perp^2$ to get very large and that the rapid falloff displayed by $d\sigma/dq_\perp^2$ with $q_\perp^2$ favors events with $q_\perp^2$ near zero. Finally, the upper-limit contribution becomes relatively important only for $2\nu \sim s$ and, hence, we can also take ksu to be small.

Under these conditions, and using the small-x behavior of the Tricomi functions $\Psi(a|c|\,x)$,[22] we find

$$W_1^F \sim Bs\left(1 - \frac{2\nu'}{s}\right)^{n-1} \times \left\{e^{-ks} - \exp\left[-k\left(2\nu + \frac{q_\perp^2}{1-2\nu'/s}\right)\right]\right\}$$

and

$$W_3^F \sim Bs\left(1 - \frac{2\nu}{s}\right)^{n-1} \times \left\{e^{-ks} - \exp\left[-k\left(2\nu' + \frac{q_\perp^2}{1-2\nu/s}\right)\right]\right\}, \tag{8.22}$$

where

$$B \equiv nB_1/8\pi^2 k$$

and

$$(W_\mu^\mu)^F = W_1^F + W_3^F\,. \tag{8.23}$$

This is our main result for the fragmentation contribution to $W_\mu^\mu(q^2, \nu, \nu'; s)$.

C. Experimental restrictions

We are now in a position to integrate the triple-differential cross section

$$\frac{d^3\sigma}{dq^2 dq_\parallel dq_\perp^2} = -\frac{\alpha^2}{3\pi^2}\frac{1}{q^2 s}\frac{1}{q_\parallel}(W_\mu^\mu)^{\rm mod}(q^2, q_\parallel, q_\perp^2; s)$$

over the experimentally accessible phase space, and obtain the various measured cross sections.

The experimental phase space associated with these cross sections obtains by implementing the experimental cuts,[4]

$$\theta \leq \theta_M, \quad q_\parallel > Q_\parallel,$$

for $\theta_M^2 = 4\times 10^{-3}$, $Q_\parallel = 12$ GeV/c over the kinematically accessible phase space at fixed q^2, $q_\parallel$, or $\cos\theta$, correspondingly (Sec. II B).

Consider first $d\sigma/dq^2$. At fixed q^2, kinematics alone requires

$$q^2 \leq 2q_\parallel \leq s,$$

$$0 \leq q_\perp^2 \leq 2q_\parallel\left(1-\frac{2q_\parallel}{s}\right)\left(1-\frac{q^2}{2q_\parallel}\right).$$

The angle cut, now given by the constraint

$$q_\perp^2 \leq (2q_\parallel)^2 \tfrac{1}{4}\theta_M^2,$$

intersects the boundary of the kinematical phase space at

$$2q_\parallel = \frac{1+q^2/s+[(1-q^2/s)^2-q^2\theta_M^2]^{1/2}}{2(\frac{1}{4}\theta_M^2+1/s)},$$

$$q_\perp^2 = (2q_\parallel)^2\tfrac{1}{4}\theta_M^2,$$

the corresponding minus solution having been ruled out by the momentum-cut contraint.

Notice also that since $(1-q^2/s)^2 - q^2\theta_M^2 \geq 0$ for all $q^2 \leq 36.7$ and since the intersection coordinate $2q_\parallel$ is always above the cut, the experimental-angle cut remains effective for the entire measured range of dimuon invariant-mass-squared $1.1 \leq q^2 \leq 24$.

Finally, the two experimental cuts intersect at the q^2-independent point $(2Q_\parallel; (2Q_\parallel)^2\frac{1}{4}\theta_M^2)$ implying that the $2q_\parallel$ range gets affected only by the momentum cut below $q_\perp^2 = (2Q_\parallel)^2\frac{1}{4}\theta_M^2$ and only by the angle cut above that. With all this information incorporated, the fixed-q^2 experimental phase space becomes (Fig. 10)

$$L_{q^2}: \begin{cases} 0 \leq q_\perp^2 \leq (2Q_\parallel)^2\frac{1}{4}\theta_M^2, \quad 2Q_\parallel \leq 2q_\parallel \leq \frac{1}{2}(s+q^2)+\{[\frac{1}{2}(s-q^2)]^2-q_\perp^2 s\}^{1/2}, \\ (2Q_\parallel)^2\frac{1}{4}\theta_M^2 \leq q_\perp^2 \leq \dfrac{\theta_M^2}{8(\frac{1}{4}\theta_M^2+1/s)^2}\left\{1+\left(\dfrac{q^2}{s}\right)^2 - q^2\dfrac{\theta_M^2}{2}+\left(1+\dfrac{q^2}{s}\right)\left[\left(1-\dfrac{q^2}{s}\right)^2-q^2\theta_M^2\right]^{1/2}\right\}, \\ \left(\dfrac{4q_\perp^2}{\theta_M^2}\right)^{1/2} \leq 2q_\parallel \leq \frac{1}{2}(s+q^2)+\{[\frac{1}{2}(s-q^2]^2-q_\perp^2 s\}^{1/2}. \end{cases}$$

Correspondingly, the cross section is

$$\frac{d\sigma}{dq^2} = -\frac{\alpha^2}{3\pi^2}\frac{1}{q^2 s}\int_{L_{q^2}} dq^2\frac{d(2q_\parallel)}{2q_\parallel}(W^\mu_\mu)^{\mathrm{mod}}(q^2, q_\parallel, q_\perp^2; s). \tag{8.24}$$

A simple calculation further gives (Figs. 11, 12)

$$L_{q_\parallel}: \begin{cases} 1.1 \leq q^2 \leq 2q_\parallel - \dfrac{(2q_\parallel)^2}{1-2q_\parallel/s}\dfrac{\theta_M^2}{4}, \quad 0 \leq q_\perp^2 \leq (2q_\parallel)^2\frac{1}{4}\theta_M^2, \\ 2q_\parallel - \dfrac{(2q_\parallel)^2}{1-2q_\parallel/s}\dfrac{\theta_M^2}{4} \leq q^2 \leq 2q_\parallel, \\ 0 \leq q_\perp^2 \leq 2q(1-2q_\parallel/s)(1-q^2/2q_\parallel), \end{cases}$$

and

$$L_{\cos\theta}: \begin{cases} 24 \leq 2q_\parallel \leq (2q_\parallel)^-, \quad 1.21 \leq q^2 \leq 2q_\parallel - \dfrac{(2q_\parallel)^2}{1-2q_\parallel/s}\dfrac{\tan^2\theta}{4}, \\ (2q_\parallel)^- \leq 2q_\parallel \leq (2q_\parallel)^+, \quad 1.21 \leq q^2 \leq 24, \\ (2q_\parallel)^+ \leq 2q_\parallel \leq s, \quad 1.21 \leq q^2 \leq 2q_\parallel - \dfrac{(2q_\parallel)^2}{1-2q_\parallel/s}\dfrac{\tan^2\theta}{4}, \end{cases}$$

where $(2q_\parallel)^\pm$ are given by

$$2q_\parallel = \frac{1+q^2/s \pm [(1-q^2/s)^2-4q^2\tan^2\theta]^{1/2}}{2(\frac{1}{4}\tan^2\theta+1/s)}$$

at $q^2 = 24$. Therefore,

$$\frac{d\sigma}{dq_{\parallel}} = -\frac{\alpha^2}{3\pi^2}\frac{1}{q_{\parallel}s}\int_{L_{q_{\parallel}}} dq_{\perp}^2 \frac{dq^2}{q^2}(W^{\mu}_{\mu})^{\rm mod}(q^2, q_{\parallel}, q_{\perp}^2; s),$$

$$\frac{d\sigma}{d(\cos\theta)} = -\frac{2\alpha^2}{3\pi^2}\frac{1}{s\cos^2\theta}\int_{L_{\cos\theta}} q_{\parallel}dq_{\parallel}\frac{dq^2}{q^2}(W^{\mu}_{\mu})^{\rm mod}(q^2, q_{\parallel}, \cos\theta, s),$$

and

$$\sigma(s) = \int_{1.21}^{24} dq^2 \frac{d\sigma}{dq^2}.$$

IX. COMPARISON WITH EXPERIMENT

We are now left with the problem of choosing the set of five parameters which our analysis cannot fix (A, h, B, k, and n) such that, upon numerical integration of (8.24), as close a fit as possible to the experimental data on $d\sigma/d\sqrt{q^2}$ is obtained. To accomplish this, a set of initial values for the free parameters was chosen in accordance with the following few simple observations:

(1) For $q^2 \gtrsim 10$, the fragmentation contribution, seen earlier to have the behavior of e^{-kq^2}, can be neglected compared to the pionization contribution $e^{-h(q^2)^{1/2}}$. Therefore, a two-parameter fit to the high-q^2 end of the data is sufficient to give

$$-(2\pi)^{1/2}A_0 = 2.5; \quad h_0 = 0.15\ (\text{GeV}/c)^{-1}. \tag{9.1}$$

(2) A simple calculation, in which complications due to the complexity of the experimental phase space are neglected, provides for $d\sigma^F/d(\cos\theta)$ the behavior

$$e^{-k(s^2/4s_0)(1-\cos^2\theta)}, \tag{9.2}$$

corresponding to an experimental decrease of $e^{-500(1-\cos\theta)}$, while $d\sigma^P/d(\cos\theta)$ turns out to be a constant. We therefore choose

$$k_0 = 1\ (\text{GeV}/c)^{-2}. \tag{9.3}$$

(3) A similar calculation for $d\sigma^F/dq_{\parallel}$ shows that a maximum exists at a $q_{\parallel}$ satisfying

$$2q_{\parallel}\,\text{Ei}(k\xi 1.21/2q_{\parallel}) = (s/n)(1-2q_{\parallel}/s)e^{-k\xi 1.21/2q_{\parallel}}. \tag{9.4}$$

Taking $2q_{\parallel} = 20$ GeV/c as the data indicate, a first estimate for n is given by

$$n_0 = 3. \tag{9.5}$$

This nicely corresponds to the threshold behavior of the SLAC data on deep-inelastic ep scattering in which one probes the LC behavior of the same product of two electromagnetic currents that one does in the μ-pair production experiments. The contribution from the pionization piece does not essentially change the value of n_0 since $d\sigma^P/dq_{\parallel}$ is only a slowly varying function of $2q_{\parallel}$. Now B_0 follows simply from normalization on $d\sigma/d(q^2)^{1/2}$.

Varying the free parameters around these initial

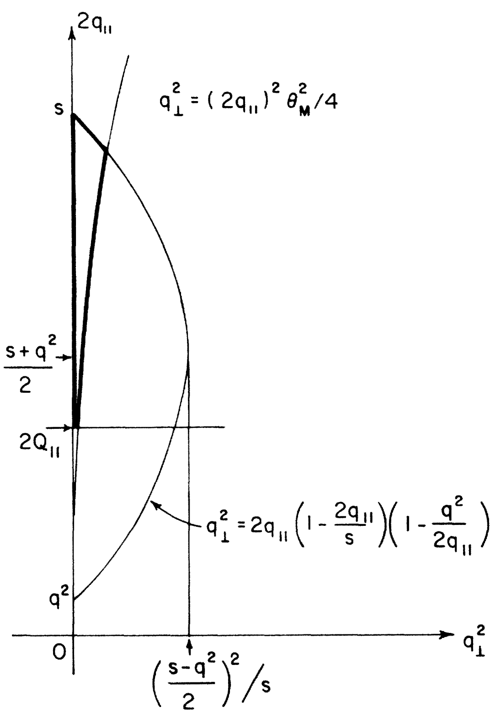

FIG. 10. The experimental $q_{\parallel}$-$q_{\perp}^2$ phase space.

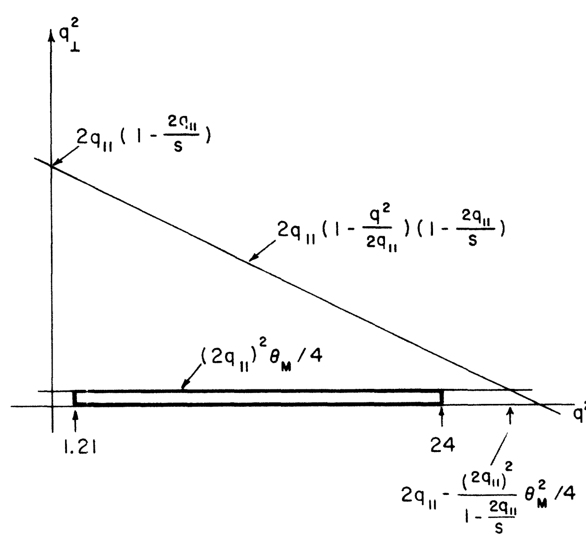

FIG. 11. The experimental $q_{\perp}^2$-q^2 phase space. ($q_{\perp}$ and q in GeV/c.)

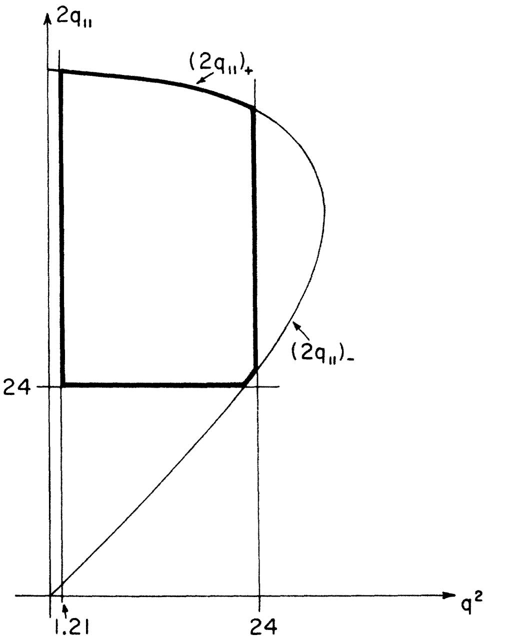

FIG. 12. The experimental $q_{\parallel}$-q^2 phase space. ($q_{\parallel}$ and q in GeV/c.)

values, the best arrangement is found to correspond to (See Figs. 13, 14, 15, 16)

$$-(2\pi)^{1/2}A=1.67;\quad h=0.1\ (\text{GeV}/c)^{-1};$$
$$B=10^4;\quad k=2.0\ (\text{GeV}/c)^{-2};\quad n=4. \tag{9.6}$$

It is perhaps worth noticing that the fit to the data breaks down for $q^2 \geq 19$ (GeV/c)^2. Such a breakdown is probably accounted for by the Fermi motion of the target proton in the U nucleus which has not been compensated for either in our calculation or in the experimental data.[23] As we do not know exactly how such a compensation should be handled quantitatively, we can say little about the improvement that such calculation could provide. It is nevertheless clear that at least qualitatively, the correction does go in the right direction.

Our hand-fitting could be improved somewhat by performing a least-squares fit to the data on $d\sigma/d(q^2)^{1/2}$, but since the error bars do not fully include all of the systematic errors encountered in the experiment, this is hardly warranted.

In any case, the smallness of the ratio $A/B \sim 10^{-4}$ is strongly indicative of the fact that the Pomeron-particle-Pomeron vertex contributing only to the pionization piece should be very small [Fig. 7(b)]. This must be the case if the Pomeron is an isolated pole at $J=1$.[13] Furthermore, the strong experimental drop of $d\sigma/dq_{\parallel}$ when $q_{\parallel}$ approaches its kinematical limit $\frac{1}{2}s$ is seen to be easily accounted for by the threshold behavior $(1-x)^n$, where our fit indicates $n=4$. Our numerical analysis shows, however, that a value of $n=3$, suggested by the SLAC-MIT data, still provides a satisfactory result for all cross sections. Finally, as mentioned earlier, the $d\sigma/d(\cos\theta)$ cross section we obtain is well represented by the rather slow experimental fall $e^{-(1-\cos)500}$. It is interesting to note that in these experimental conditions for a hadron one would have expected, rather, $e^{-(1-\cos\theta)2000}$.[11] This shows that the highly virtual photon is much "thinner" than a hadron.

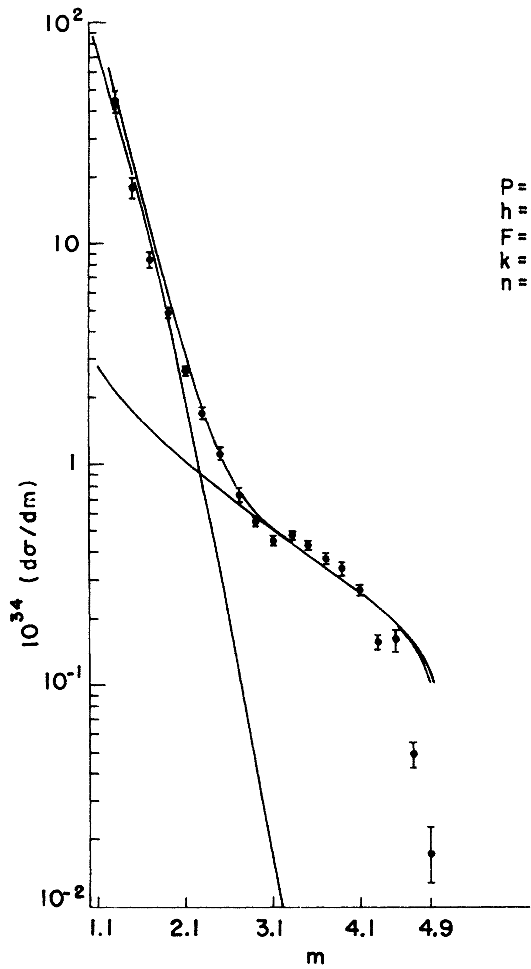

FIG. 13. The experimental results for $d\sigma/d\sqrt{q^2}$ (in cm^2 GeV) and the fit given by the parameters in Eq. (9.6). (m in GeV.)

X. DISCUSSION

We have seen how the combined assumptions of LCOPE's and (strongly convergent) multi-Regge theory (4.2) lead to strong restrictions on the form of the μ-pair production amplitude (1.1). These assumptions lead to the general form (1.7) with the spectral function $\Psi(\beta,\beta')$ a rapidly decreasing function of its arguments. This general form is

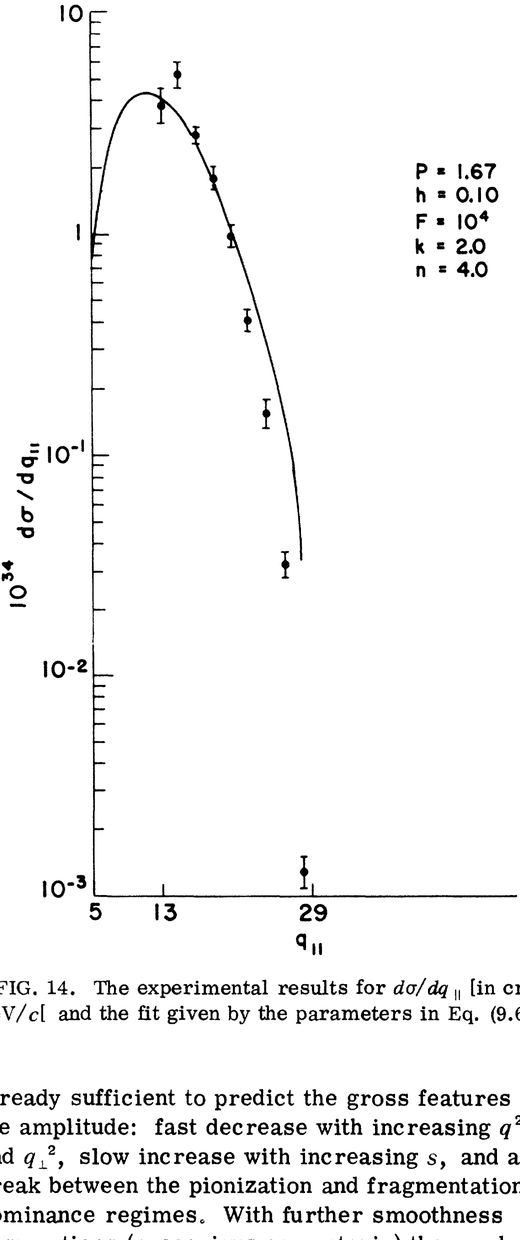

FIG. 14. The experimental results for $d\sigma/dq_{\parallel}$ [in cm^2/GeV/c[and the fit given by the parameters in Eq. (9.6).

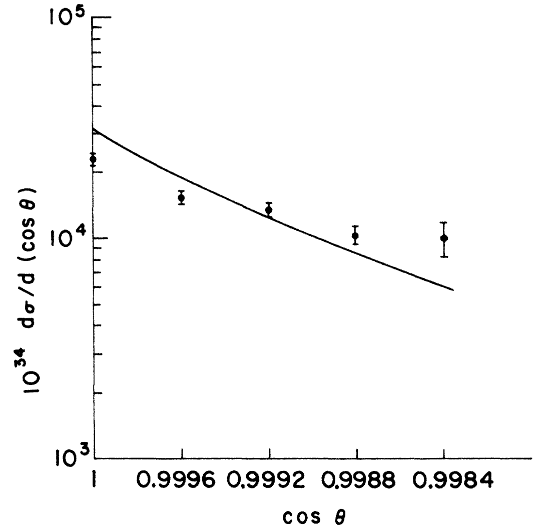

FIG. 15. The experimental results for $d\sigma/d\cos\theta$ (in cm^2) and the fit given by the parameters in Eq. (9.6).

already sufficient to predict the gross features of the amplitude: fast decrease with increasing q^2 and $q_{\perp}{}^2$, slow increase with increasing s, and a break between the pionization and fragmentation dominance regimes. With further smoothness assumptions (precocious asymptopia) the much more specific results of Sec. VI can be obtained. Finally, the five-parameter model introduced in Sec. VIII leads to precise expressions for the amplitudes which embody the general features of the exact expressions but which can be directly used in a phenomenological analysis.

To achieve these results, we had to invoke strong assumptions. Our general results are, however, rather insensitive to most of these assumptions. For example, we can inquire about the validity of our use of the leading LC singularity. Actually, the contribution of a nonleading singularity [e.g., $(x^2)^k \ln x^2$] will be more important than that of (3.8) if the associated spectral function $\Psi_k(\beta, \beta')$ falls (exponentially) sufficiently slowly. This is obviously no problem since inclusion of this, or any similar, term will not change the basic form of (1.7). We expect, however, on the basis of the general kinematical analysis and of the obtained Regge behavior [plus commutativity (3.12)], that the leading singularity will, in fact, dominate. One easily sees, moreover, that replacing $j(x)j(0)$ in (1.1) by $x^2 j(x)j(0)$ gives a less leading contribution provided $\Psi(\beta, \beta')$ falls slower than $\exp[-(\beta+\beta')^2]$.[24] It is interesting that the requirement of LC dominance can place such a strong restriction on Ψ.

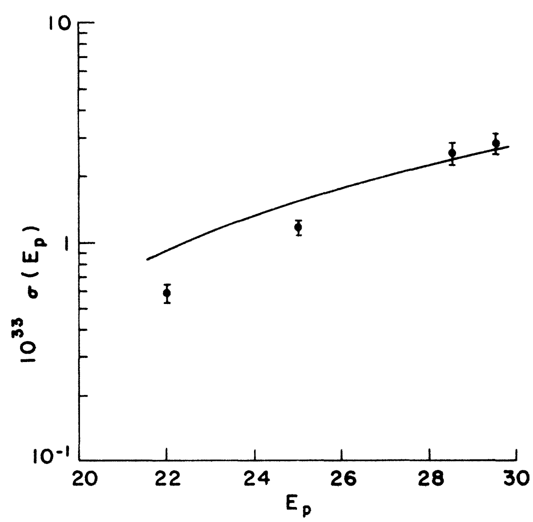

FIG. 16. The experimental results for $\sigma(E)$ (in cm^2) and the fit given by the parameters in Eq. (9.6). (E_p in GeV.)

For similar reasons, it should be clear that our use of canonical LC singularities is inessential. Allowing an arbitrary singularity would only change the form of our result by a polynomially bounded function. Conversely, this means that μ-pair production is not going to check the hypothesis of canonical dimensions.

The assumptions we employed are valid in a large number of models: Feynman diagrams,[19] multiperipheral,[10] partons,[11] nonperturbative parton.[7] Our model-independent treatment incorporates the general features of these models without the commitment to a specific model. This is desirable since none of the models is totally satisfactory from other points of view.

It is particularly important to compare our analysis with that of the parton model.[11] A detailed comparison has been given in Refs. 7 and 10 and the conclusions are as follows: The contribution (2.22) of the bremsstrahlung diagrams [Fig. 6(b)] has our general form (1.7) with spectral functions

$$\Psi_b(\beta, \beta') = B(\beta, \beta').$$

The contribution (2.20) of the annihilation diagrams [Fig. 6(a)] does not have the form (1.7) because these diagrams do not have leading Regge behavior at the five-point function level. They correspond to a Kronecker delta at $J=0$ in the complex J plane. If further diagrams do not cancel this effect, then (2.20) should be added on to (1.7). It is, however, quite possible that the effect will be canceled. For example, the form-factor corrections mentioned in Sec. II C will accomplish this if they are such that the total form factor is a rapidly decreasing function. Then the form-factor-corrected annihilation diagrams do have the form (1.7).[7] This must be the case since the form-factor-corrected annihilation diagrams *do* satisfy the five-point function Regge behavior. It should also be noted that, even without form-factor corrections, the parton-model result (2.20) is a special case of (1.7) if the fast decrease of Ψ is given up.[7]

We have seen how amplitudes of our (LC-dominated multi-Regge) form lead to very good agreement with the existing experimental data. This indicates that the parton-model Kronecker delta contribution is not present. Of course, the real test of our results and of the parton model will be the comparison with the future data. Preliminary experimental results already indicate that the parton-model expression for $d\sigma/dq^2$ does not fall fast enough at the larger q^2 values.[25]

An amusing consequence of our fit to the data is the small value ($\sim 10^{-4}$) obtained for the P/F ratio. This smallness of the Pomeron-particle-Pomeron ratio has been theoretically anticipated[13] and such matters have recently been the subject of considerable interest.[26] One may speculate that this provides an empirical indication of the decoupling theorem.

As we have said, the present data are not sufficient to determine the correctness of our assumptions or of other approaches such as the parton model. The future data should settle this question and should indicate the way in which leptonic and hadronic physics merge in nature.

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*This work supported in part by funds from the National Science Foundation.

†Present address: Center for Naval Analysis, 1401 Wilson Boulevard, Arlington, Virginia 22209.

‡Present address: Istituto di Fisica, Universitá di Roma, Roma, Italy.

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