Charmed pseudoscalar-meson masses in broken SU(4) × SU(4) symmetry*

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In the broken SU(4) × SU(4) model the possibility that the charmed pseudoscalar mesons can acquire heavy masses compared to the known particles is investigated. Solution for a symmetry-breaking parameter crucial to the model is obtained. It is shown that the masses of the charmed pseudoscalar mesons are of the order of 1 GeV if both SU(3) and SU(2) × SU(2) are good symmetries of the Hamiltonian.

Progress toward unified gauge theories of leptons and hadrons during the last two years has renewed interest in SU(4)× SU(4) as an approximate symmetry of hadrons.[1,2] In a recent paper[3] spectral-function sum rules for scalar and pseudoscalar densities were studied within the framework of broken SU(4)× SU(4) symmetry. The constraints imposed on the symmetry-breaking parameters of the theory by the exact and other approximate sum rules, derived under the assumption of octet-type breaking of SU(3) symmetry at $q^2=0$ and $q^2=\infty$, were investigated. It was shown that the $q^2=0$ sum rule rules out any solution for the symmetry-breaking parameters where SU(2) × SU(2) is a good symmetry of the Hamiltonian of hadrons when the vacuum is approximately SU(3)-invariant. On the other hand, it was shown that the $q^2=\infty$ sum rules are consistent with a solution where SU(2)× SU(2) is a good symmetry of the Hamiltonian with the vacuum approximately SU(3)-invariant. In particular, the $q^2=\infty$ sum rules were shown to be consistent with the solution of Gell-Mann, Oakes, and Renner[4] (GMOR). The asymptotic sum rules also enabled us to derive a mass formula for the charmed pseudoscalar mesons in terms of a single symmetry-breaking parameter e (see below) The charmed pseudoscalar mesons acquire heavy masses (~5 GeV) *only* when the value of the parameter e is very close to -1.

Since such heavy particles are yet to be detected experimentally, the credibility of the SU(4) × SU(4) model depends crucially on the value of the parameter e. It is, therefore, a matter of considerable interest, from a theoretical point of view, to examine possible solutions for e to see if the value of this parameter can indeed be close to -1. Dittner, Eliezer, and Kuo have obtained a solution[5] for e using a lepton-hadron analogy. They find $e\simeq -0.99$. Apart from obtaining heavy masses (~5 GeV) for the charmed mesons, the other attractive feature of their solution is that both SU(3) and SU(2)×SU(2) are good symmetries of the Hamiltonian in their scheme. A solution for e has also been sought by Dicus and Mathur[6] from a study of the spectral-function sum rules for the current densities and the η-X mixing problem in chiral SU(4) theory. Assuming that SU(3)-breaking effects appear only through the η-X mixing angle, Dicus and Mathur have shown[6] that the value of e closest to -1 which is acceptable is $e \simeq -0.58$. This value of e yields masses of the order of 1 GeV for the charmed pseudoscalar mesons and is in conflict with the results of Dittner, Eliezer, and Kuo. An independent evaluation of e is, therefore, called for.

The purpose of the present paper is to seek an independent solution for the parameter e on the basis of the sum rules derived in Ref. 3. A new constraint on the parameter e is obtained by solving the exact and the asymptotic sum rules of Ref. 3 in the pole-dominance approximation for the spectral weights and by neglecting those terms which are of second order in SU(3) breaking. The result of our investigation is that parameter e must assume values in a remarkably narrow range, that is, $-0.58 \leq e \leq -0.53$. For values of e lying in this range we find that the charmed pseudoscalar mesons can acquire masses of the order of 1 GeV only.

In the model under consideration the Hamiltonian density is taken to be of the form

$$H(x)=H_0(x)+\epsilon_0 u^0(x)+\epsilon_8 u^8(x)+\epsilon_{15}u^{15}(x), \qquad (1)$$

where $H_0(x)$ is invariant under SU(4)× SU(4). The scalar densities $u^j(x)$ $(j=0,1,\ldots,15)$ together with the pseudoscalar densities $v^j(x)$ $(j=0,1,\ldots,15)$ transform according to the $(4,4^*)+(4^*,4)$ representation of chiral SU(4). In what follows we shall use the notation of Ref. 3 and transmit only those equations which are essential to the present development. As shown in Ref. 3, given Eq. (1), the following exact sum rules can be derived:

$$P_{33}=\eta\,\frac{1+b+f}{1+a+e}, \qquad (2a)$$

$$P_{44}=\eta\,\frac{1-\frac{1}{2}b+f}{1-\frac{1}{2}a+e}, \tag{2b}$$

$$P_{99}=\eta\,\frac{1+\frac{1}{2}b-f}{1+\frac{1}{2}a-e}, \tag{2c}$$

$$P_{13\,13}=\eta\,\frac{1-b-f}{1-a-e}, \tag{2d}$$

$$\sqrt{6}eP_{00}+aP_{08}+\sqrt{2}(1-2e)P_{0\,15}=\sqrt{6}\eta f, \tag{2e}$$

$$\sqrt{6}eP_{0\,15}+aP_{8\,15}+\sqrt{2}(1-2e)P_{15\,15}=\sqrt{2}\eta(1-2f), \tag{2f}$$

where

$$P_{jk}=\int\frac{dm^2}{m^2}\,\rho_{jk}(m^2,v),$$

with

$$\rho_{jk}(m^2,v)=(2\pi)^3\sum_n\langle 0\,|v^j(0)|\,n\,\rangle\times\langle n\,|v^k(0)\,|0\rangle\,\delta^4(p_n-p),$$

$$p^2+m^2=0,$$

and

$$a\equiv(\tfrac{2}{3})^{1/2}\frac{\epsilon_8}{\epsilon_0},\quad b\equiv(\tfrac{2}{3})^{1/2}\frac{\lambda_8}{\lambda_0},$$

$$e\equiv\frac{1}{\sqrt{3}}\frac{\epsilon_{15}}{\epsilon_0},\quad f\equiv\frac{1}{\sqrt{3}}\frac{\lambda_{15}}{\lambda_0}, \tag{3}$$

$$\eta\equiv-\frac{\lambda_0}{\epsilon_0},$$

with

$$\lambda_m=\langle 0|u^m(0)|0\rangle,\quad m=0,8,15\,.$$

Note that Eq. (2f) is an additional relation, not displayed in Ref. 3, among the six quantities P_{jk} $(j,k=0,8,15)$. Only five of these six relations among P_{jk} $(j,k=0,8,15)$ are linearly independent, however.

We generate an extra relation valid to first order in SU(3) breaking in the pole-dominance approximation. To do so, we notice that to first order in SU(3) the spectral weights in P_{00}, $P_{0\,15}$, and $P_{15\,15}$ get contributions from the X meson alone (we ignore the E meson in the present discussion; its spin assignment is not yet experimentally conclusive). This immediately implies the relation

$$P_{00}P_{15\,15}=(P_{0\,15})^2\,. \tag{4}$$

Consistent with the approximation under which Eq. (4) is valid, we neglect $a\,P_{08}$ and $a\,P_{8\,15}$ in Eqs. (2e) and (2f) which are of second order in SU(3) breaking. Thus, to first order in SU(3) breaking we have the following two relations, in addition to Eq. (4), between P_{00}, $P_{0\,15}$, and $P_{15\,15}$:

$$\sqrt{6}\,eP_{00}+\sqrt{2}\,(1-2e)P_{0\,15}=\sqrt{6}\,\eta f\,, \tag{5a}$$

$$\sqrt{6}\,eP_{0\,15}+\sqrt{2}\,(1-2e)P_{15\,15}=\sqrt{2}\,\eta(1-2f)\,. \tag{5b}$$

Solving Eqs. (4), (5a), and (5b), we get

$$P_{00}=3\eta\,\frac{f^2}{(1-2e-2f+7ef)}, \tag{6a}$$

$$P_{0\,15}=\sqrt{3}\,\eta\,\frac{f(1-2f)}{(1-2e-2f+7ef)}, \tag{6b}$$

$$P_{15\,15}=\eta\,\frac{(1-2f)^2}{(1-2e-2f+7ef)}\,. \tag{6c}$$

In order to constrain the parameter e in terms of known masses and decay constants we next appeal to the asymptotic sum rules derived in Ref. 3. The assumption of octet-type breaking of SU(3) at $q^2=\infty$ for the two-point functions of pseudoscalar density gives the following sum rules[3]:

$$R_{33}-R_{44}-R_{99}+R_{13\,13}=0\,, \tag{7a}$$

$$2R_{44}-5R_{33}+9R_{99}-6R_{15\,15}=0\,, \tag{7b}$$

where

$$R_{jk}\equiv\int dm^2\rho_{jk}(m^2,v)\,.$$

It is convenient to replace the parameters a, b, and η by α, β, and γ, where

$$\alpha=\frac{a}{1+e},\quad \beta=\frac{b}{1+f},\quad \gamma=\eta\,\frac{1+f}{1+e}\,. \tag{8}$$

In what follows we shall assume that the GMOR mass formula holds, which in our notation reads[3]

$$m_\pi^{\,2}=(1+\alpha)m_8^{\,2}\,, \tag{9a}$$

$$m_K^{\,2}=(1-\tfrac{1}{2}\alpha)m_8^{\,2}\,, \tag{9b}$$

$$m_\eta^{\,2}=(1-\alpha)m_8^{\,2}\,, \tag{9c}$$

$$m_9^{\,2}=\left(1+\frac{\alpha}{2}\,\frac{1+e}{1-e}\right)m_3^{\,2}\,, \tag{9d}$$

$$m_{13}^{\,2}=\left(1-\frac{\alpha}{2}\,\frac{1+e}{1-e}\right)m_3^{\,2}\,, \tag{9e}$$

where m_8 and m_3 are the masses of the uncharmed octet and the charmed triplet of pseudoscalar mesons, respectively, in the SU(3) limit. The sum rule (7a) gives the following mass formula:

$$m_3^{\,2}=\left(\frac{1-e}{1+e}\right)\frac{(m_\pi^{\,2}+2m_K^{\,2})}{3}\,. \tag{10}$$

Before we proceed to investigate the consequences of the sum rule (7b), we first estimate the parameter β which measures the invariance of the vacuum under SU(3). β can be expressed in terms of F_π and F_K, the pion and the kaon decay constants, if we use pole dominance on the sum rules (2a), (2b) and use Eqs. (8), (9a), and (9b). We get

$$\beta = \frac{2(1 - F_K^2/F_\pi^2)}{(2F_K^2/F_\pi^2 + 1)} \ . \tag{11}$$

Experimentally, $F_K/F_\pi f_+(0) = 1.28$; since $f_+(0)$ is expected to be close to unity,[7] β must be small. The value $F_K/F_\pi = 1.13$ gives $\beta = -0.15$. An accurate numerical estimate of the parameter β is not important for the purposes of the present investigation, we therefore use $\beta = -0.15$ in what follows.

If we dominate the spectral weights in Eqs. (2a)–(2c), (6c), and (7b) by single-particle states and use Eqs. (8), (9a), (9b), (9d), and (10), we get

$$e = \frac{4(1-2f)^2(3M^2 - 2m^2 - 1) + \beta(1+f)(1-2f)(2m^2+1)}{(2m^2+1)(7f-2)[4(1-2f) - \beta(1+f)] - 12M^2(1-2f)^2} \ , \tag{12}$$

where

$$M = m_X/m_\pi$$

and

$$m = m_K/m_\pi \ .$$

Equation (12) is the key relation which can be plotted in the (e,f) plane to see if the allowed value of e can indeed be close to -1. The allowed domains of the parameters e and f when α and β lie between -1 and 0 have been studied previously[6] by Dicus and Mathur from the positivity requirements of the spectral weights. These allowed domains are shown in Fig. 1, when $\alpha = -0.89$, $\beta = -0.15$, by the shaded regions in the (e,f) plane. We have also plotted the curve given by Eq. (12) in Fig. 1. We find[8] that the constraint of Eq. (12) requires the parameter e to lie between -0.58 and -0.53 in the allowed region. Thus, e is determined to within a remarkably narrow range of values. The smallest allowed value of e is $e = -0.58$. The masses of the charmed pseudoscalar mesons can be calculated from Eqs. (9d), (9e), and (10) for this value of e. We get $m_9 \simeq 742$ MeV and $m_{13} \simeq 882$ MeV.

The allowed range of the parameter e is quite insensitive to the value of the parameter β. For $-0.3 \leq \beta \leq 0$ we get $-0.60 \leq e \leq -0.53$. The charmed pseudoscalar mesons thus acquire masses of the order of 1 GeV only. Our results are in good agreement with that of Dicus and Mathur[6] and do not confirm the results[5] of Dittner, Eliezer, and Kuo, who obtain masses of the order of 5 GeV for the charmed mesons.

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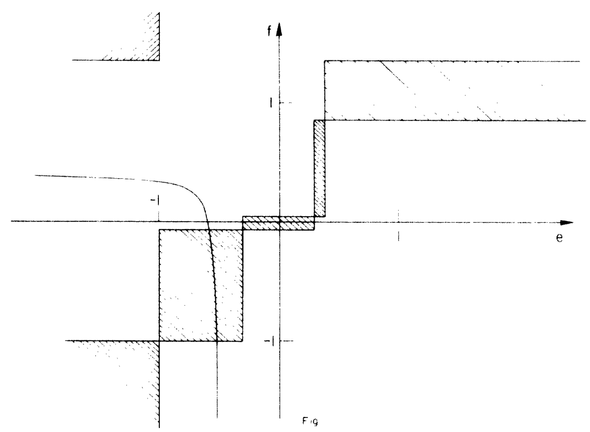

FIG. 1. The allowed domains of the parameters e and f are indicated by the shaded regions when $\alpha = -0.89$ and $\beta = -0.15$. The plotted curve is the constraint imposed by Eq. (12) on the physical values of the parameters e and f.

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