Generalized current algebra including first- and second-class currents

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The total weak $\Delta S = 0$ hadronic current is usually split into CP-odd and CP-even, or first- and second-class currents. It is postulated that these currents obey an $(SU(2)\otimes SU(2))\otimes(SU(2)\otimes SU(2))$ algebra. This implies a set of Adler-Weisberger relations which, in particular, indicate the coupling of weak leptonic $\eta\to\pi l\nu$ decay as being of order one. It is possible to consider the strong interactions approximately invariant under the group generated by all these currents by introducing an additional set of scalar Goldstone bosons as well as C-odd pseudoscalar mesons. Since the latter seem not to exist, one has an alternative realization by CP doublets. As a special example, a generalized σ model is discussed. Further consequences are partial conservation of the second-class vector current and Goldberger-Treiman-type equations. Finally, the extension of the full algebra to leptons is discussed, which suggests the interesting fact that electrons and muons are CP partners of each other and the separate conservation of electron and muon numbers is a consequence of CP conservation.

I. INTRODUCTION

In the last three years a great search for so-called second-class currents[1] has been conducted. The experiments performed by Wilkinson and collaborators[2,3] tested in detail the mirror asymmetry in nuclear β decay. The main contribution to the mirror asymmetry is expected from a so-called pseudotensor term in the axial-vector-current matrix element of the decaying nucleon. The situation is of course complicated by nuclear effects, but the result of the analysis gave the pseudotensor term compatible with zero.[4]

Another sensitive test of a pseudotensor term would be the spin-correlation coefficients in the leptonic decay of polarized hyperons. From the analysis of the electron and neutrino asymmetry in the decays $\Lambda\to pe\nu$ (Ref. 5) and $\Sigma^-\to ne\nu$ (Ref. 6) one can derive definitely nonzero values of the pseudotensor term.[7,8] These values are larger than expected from a first-order symmetry-breaking calculation based on dispersion theory.[9] If one is willing to accept this analysis, one can conclude the presence of a second-class current (SCC) contribution.

The most sensitive test of a SCC would be a measurement of the leptonic decay $\eta\to\pi e\nu$, as discussed in detail by Singer.[10] Since the transition from η to π is a pure vector SCC, the value of the decay constant is not determined by conservation of vector current (CVC). In particular, the branching ratio of this decay via SCC (assuming the strength of the SCC to be as large as the usual first-class current) to the decay by electromagnetic corrections via $\eta\to(\pi^0)\to\pi e\nu$ is predicted to be 10^4, so one expects a definitive result.

From the theoretical point of view, there is no reason why such currents should not be there. Lipkin[11] has given a detailed mechanism of mesonic contributions to β decay which are pure SCC. This ansatz of Lipkin has been generalized by Pietschmann and Rupertsberger[12] to the full octet current:

$$A_\mu^{i(\mathrm{SC})}(x)=gid_{ikl}\pi^k(x)\rho_\mu^l(x)\quad(i=1,\ldots,8)\,,\tag{1}$$

where $\pi^k(x)$ $(k=0,1,\ldots,8)$ is the pseudoscalar-meson nonet, $\rho_\mu^l(x)$ is the vector-meson nonet, g is some coupling constant, and $d_{ik0}=(\frac{2}{3})^{1/2}\delta_{ik}$. We can simply generalize this formula for the SC-vector current:

$$V_\mu^{i(\mathrm{SC})}(x)=g'id_{ikl}\pi^k(x)a_\mu^l(x)\quad(i=1,\ldots,8)\,,\tag{2}$$

where a_μ^i is the nonet of axial-vector mesons. From formula (1) or, better, from the $(\omega\pi)$ contribution, the pseudotensor term for nuclear β decay has been calculated[4] and found to be of the same order as the other "first-class" form factors.

The usual $SU(3)\otimes SU(3)$ algebra of currents proposed by Gell-Mann[13] ignores the SCC completely. The charges generating the algebra are integrals of the first-class currents, because only the first-class current has the right behavior under charge conjugation to be combined with the well-known electromagnetic current to a multiplet (CVC).[14] If we accept the SCC as being present—and that is our point of view—we have to worry about putting it into this scheme. Oehme[15] discussed this problem without enlarging the charge algebra, by postulating that the SCC have no charge. If one restricts oneself to the realization of currents by quarks, the above statement is true because the only SCC built of quarks contains derivatives, i.e.,

$$\begin{aligned}V_\mu^{i(\mathrm{SC})}&=-i\partial_\mu(\bar\psi\tfrac{1}{2}\lambda_i\psi)\,,\\A_\mu^{i(\mathrm{SC})}&=-i\partial_\nu(\bar\psi i\sigma_{\mu\nu}\gamma_5\tfrac{1}{2}\lambda_i\psi)\,,\end{aligned}\tag{3}$$

where $\psi(x)$ is the quark field. However, this is not true for mesonic currents as in Eqs. (1) and (2). But if the charges of the SCC are not zero, do they satisfy some commutation relations? If so, is there any meaning of this higher algebra of currents as generating a fundamental symmetry of our hadronic world?

It is the purpose of this work to discuss such questions, irrespective of whether or not one finds pseudotensor terms. We *assume* that the second-class vector and axial-vector currents satisfy an algebra similar to that of the usual currents. First a definition of the Hermitian charges of the SCC and their transformation under C, P, and T must be given, and then we define an $(\mathrm{SU}(3)\otimes\mathrm{SU}(3))\otimes(\mathrm{SU}(3)\otimes\mathrm{SU}(3))$ algebra of all charges. A generalization of G parity to strangeness-changing weak currents was proposed by Wolfenstein,[16] but because of the large breaking effects in SU(3) and because a higher number of nonexisting mesons would be involved, we restrict ourselves in this paper to $(\mathrm{SU}(2)\otimes\mathrm{SU}(2))\otimes(\mathrm{SU}(2)\otimes\mathrm{SU}(2))$, which we write as $[\mathrm{SU}(2)]^4$ for shorthand.

The idea of constructing an $[\mathrm{SU}(2)]^4$ algebra of weak currents is a straightforward generalization of a similar idea proposed by Maiani,[17] who wanted to include CP-violating effects in chiral symmetry. (The idea of SCC being responsible for CP violation has been proposed by Cabibbo.[18]) Maiani had already defined the commutator between two axial-vector SCC's to give back the usual vector current, but avoided introducing a second-class vector current and therefore enlarging the group. Nevertheless, he derived Weinberg sum rules for the spectral functions and, by saturating them with the only known $B(1235)$ resonance, obtained the relation $\gamma_B=\gamma_\rho$, which we also find in Sec. V. Obviously, the immediate consequence of such a current algebra is a set of Adler-Weisberger relations, which can be further improved if one uses partial conservation of vector current (PCVC).[19] This PCVC or, better, PCSCVC (partially conserved second-class vector current) concept states that the nonconserved part of the vector current, which is the SCC, is dominated by the scalar meson π_N (or δ), which has been found to be a 950-MeV $(\eta\pi)$ resonance.

One can go one step further and ask if there is an approximate symmetry of strong interactions corresponding to this enlarged current algebra, as is the case in chiral $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$. Since there is no larger multiplet structure in strong interactions, we expect such a symmetry to be realized, if at all, through Nambu-Goldstone bosons.[20] In addition to the massless pion, we need two other bosons, namely a scalar meson with $I^GJ^P=1^-0^+$ and a pseudoscalar meson with odd C parity, $I^GJ^P=1^+0^-$. In the first case, the above-mentioned $\pi_N(950)$ is a candidate, whereas in the second case the meson has not been found but can be represented by a $(\pi\pi_N)$ bound state. As has been discussed in detail by Dashen[21] and Dashen and Weinstein,[22] in the case of chiral symmetry such a scheme automatically gives PCAC and, in our case, also PCSCVC, which show the behavior of matrix elements in the symmetry limit. Since the third meson is not found experimentally, we have no PCAC relation in this case. From such a scheme, we can derive a set of Goldberger-Treiman-type equations which, together with our current algebra, give some predictions about form factors. Of course, such a symmetry, if it exists, is badly broken because we should have to deal with a massless scalar meson in the symmetry limit, whereas the mass is actually about 950 MeV. One may ask if such a badly broken symmetry is useful at all. In fact, it turns out that most of the results follow from current algebra plus PCSCVC alone. The reason for discussing this scheme is, therefore, more or less an aesthetic one. The beauty of the scheme of chiral symmetry in strong interactions lies in the fact that the charges of the weak currents are the generators of a symmetry of the Hamiltonian. Therefore, the weak currents act as "probes" of strong interaction. In our extended current algebra, where SCC's have the same "privileges" as first-class currents, we must discuss the possibility that they are also "probes" of the strong Hamiltonian. In addition, we automatically get a verification of PCSCVC from $[\mathrm{SU}(2)]^4$ symmetry of strong interactions.

The last problem with which we have to deal is the implication of our current algebra for leptonic currents. As can be shown, the $[\mathrm{SU}(2)]^4$ algebra of currents satisfies the concept of hadron-lepton universality, as stated by Gell-Mann and Ne'eman.[23] On the other hand, there is no direct analog of the SCC in the case of leptons. One way out, which is highly speculative, is to assign a negative CP value to the electron-muon system. In this case the lepton-number-violating currents $\bar\psi_{(e)}\Gamma_\mu\psi_{(\mu)}$ would have even CP and, therefore, the same transformation properties as SCC's. Although we have no evidence for this hypothesis, it would explain why we really *need* two different types of leptons.

The plan of this paper is as follows. In Sec. II we briefly repeat the definitions of first- and second-class currents and define their algebra. In Sec. III we discuss a generalized σ model of $[\mathrm{SU}(2)]^4$ as an example of a higher symmetry in strong interactions. Section IV deals with the possibility of having the strong interactions approximately invariant under this group. In Sec. V we give applications of both the current algebra and the higher symmetry to derive sum rules. Finally, in Sec. VI, the problems of leptons and weak interactions are discussed.

II. THE $[SU(2)]^4$ ALGEBRA OF WEAK CURRENTS

In this section we first repeat the definitions of first- and second-class Hermitian and anti-Hermitian weak currents. For simplicity, we are dealing only with $\Delta S = 0$ currents and therefore we need only SU(2).

The Hamiltonian for semileptonic weak interactions can be written as

$$\mathcal{H}_W = \frac{G}{\sqrt{2}} (J^+_\lambda l^{\lambda -} + \text{H.c.}) , \tag{4}$$

where the structure of the lepton current $l^\lambda = \bar{\psi}_l \gamma^\lambda (1+\gamma_5)\psi_{\nu_l}$ is well known. We have chosen the conventional current × current interaction; nothing in our discussion changes if we assume the weak interaction to be mediated by an intermediate boson. The hadronic current J^+_λ and its Hermitian conjugate, $J^-_\lambda \equiv (J^+_\lambda)^\dagger$, can be split into the following parts:

(i) vector and axial-vector currents, corresponding to $PJ_\lambda(x, t)P^{-1} = \pm\epsilon(\lambda)J_\lambda(-x, t)$, where $\epsilon(\lambda) \equiv g_{\lambda\lambda}$;

(ii) regular and irregular currents,[18] whether or not the relation

$$CPJ_\lambda(x, t)(CP)^{-1} = \mp\epsilon(\lambda)J^\dagger_\lambda(-x, t) \tag{5}$$

holds with the plus or minus sign; according to CPT, this definition is equivalent to

$$TJ_\lambda(x, t)T^{-1} = \pm\epsilon(\lambda)J_\lambda(x, -t) ; \tag{6}$$

(iii) first- and second-class currents, corresponding to $GPJ_\mu(GP)^{-1} = \pm\epsilon(\mu)J_\mu$, where $G = C\exp(i\pi I_2)$;

(iv) Hermitian or anti-Hermitian currents, whether or not the neutral current

$$J^0_\lambda = -\tfrac{1}{2}[I_-, J^+_\lambda] \tag{7}$$

is equal to its Hermitian conjugate

$$J^{0\dagger}_\lambda = \tfrac{1}{2}[I_+, J^-_\lambda] \tag{8}$$

with a plus or minus sign.

The definitions (iii) and (iv) both use isospin as exact symmetry and are not independent of each other. In fact, any regular first-class current is always generated from a Hermitian neutral current operator, and a second-class current from an anti-Hermitian one. For irregular currents the opposite is true. In general, the full hadronic current is assumed to be a sum of all possible contributions. If we know the behavior of the current under CP, then the splitting into Hermitian and anti-Hermitian parts is unique. We therefore write our current as the sum of two parts

$$J^0_\lambda = j^0_\lambda + ik^0_\lambda , \tag{9}$$

where both j^0_λ and k^0_λ are Hermitian operators. The weak currents J^+_λ and J^-_λ are defined through

$$\begin{aligned} J^+_\lambda &= -[I_+, j^0_\lambda + ik^0_\lambda] , \\ J^-_\lambda &= [I_-, j^0_\lambda - ik^0_\lambda] , \end{aligned} \tag{10}$$

so that both parts, j^0_λ and k^0_λ, can be measured in principle.

In the future we shall restrict ourselves to regular currents, since CP-violating effects seem to be coupled only to $\Delta S = 2$ transitions,[25] which do not appear in our game. In this case j_λ will be pure first-class and k_λ will be second-class. This can easily be seen from the definition

$$\begin{aligned} CPj^0_\lambda(CP)^{-1} &= -\epsilon(\lambda)j^0_\lambda , \\ CPk^0_\lambda(CP)^{-1} &= +\epsilon(\lambda)k^0_\lambda . \end{aligned} \tag{11}$$

Since both $(j^+_\lambda, j^0_\lambda, j^-_\lambda)$ and $(k^+_\lambda, k^0_\lambda, k^-_\lambda)$ form an isospin triplet, their behavior under rotations in isospace is the same, and, therefore, their G parities are opposite. The fact that k_λ has opposite CP need not give rise to any CP-violating effects. Actually, one measures matrix elements of $V^{(\text{SC})}_\lambda = ik_\lambda$, which must fulfill certain reality conditions.

It is now convenient to introduce Cartesian coordinates in isospin space and separate the matrix elements into vector and axial-vector currents:

$$j^i_\lambda = v^i_\lambda + a^i_\lambda , \quad k^i_\lambda = k^i_\lambda + k^i_{\lambda 5} \quad (i = 1, 2, 3) , \tag{12}$$

where all the currents are now Hermitian. [In Eq. (12) we used the expression k_λ twice, for the whole current and for its vector part; henceforth we shall use it solely for the vector part.]

If we wish to have commutation relations for the full weak currents, they must also include k and k_5. It is this feature which leads us to a higher current algebra. First, let us define the "charges":

$$\begin{aligned} Q^i &= \int d^3x\, v^i_0(x, t) , \\ Q^i_5(t) &= \int d^3x\, a^i_0(x, t) , \\ K^i(t) &= \int d^3x\, k^i_0(x, t) , \\ K^i_5(t) &= \int d^3x\, k^i_{05}(x, t) . \end{aligned} \tag{13}$$

Since the SCC's as well as the axial-vector current a^i_μ are generally not conserved, the charges are not constants of motion. Now, according to the well-known current algebra of Gell-Mann, we have the equal-time commutators (ETC)

$$[Q^i, Q^k] = i\epsilon_{ikl} Q^l \ ,$$
$$[Q^i, Q_5^k(t)] = i\epsilon_{ikl} Q_5^l(t) \ , \qquad (14)$$
$$[Q_5^i, (t), Q_5^k(t)] = i\epsilon_{ikl} Q^l \ .$$

Moreover, according to CVC, $Q_i \equiv I_i$ and Q_{i5} are the generators of SU(2)⊗SU(2), under which the strong interactions are, at least approximately, invariant. For the K^i we get

$$[Q^i, K^k(t)] = i\epsilon_{ikl} K^l(t) \ ,$$
$$[Q^i, K_5^k(t)] = i\epsilon_{ikl} K^l(t) \ ,$$
$$[Q_5^i(t), K^k(t)] = i\epsilon_{ikl} K_5^l(t) \ , \qquad (15)$$
$$[Q_5^i(t), K_5^k(t)] = i\epsilon_{ikl} K^l(t) \ .$$

The first two equations follow automatically from our definition of the SCC, whereas the last two are not trivial. They state that in the chiral limit K^i and K_5^i transform as a $\mathfrak{D}(1, 0) \oplus \mathfrak{D}(0, 1)$ representation of SU(2) ⊗ SU(2).

We close our algebra by *defining*

$$[K^i(t), K^k(t)] = i\epsilon_{ikl} Q^l \ ,$$
$$[K^i(t), K_5^k(t)] = i\epsilon_{ikl} Q_5^l(t) \ , \qquad (16)$$
$$[K_5^i(t), K_5^k(t)] = i\epsilon_{ikl} Q^l \ .$$

Equations (14)–(16) define the algebra of (SU(2)⊗SU(2))⊗ (SU(2)⊗ SU(2)). This can be seen by constructing the quantities

$$M^{i\pm} = \tfrac{1}{4}[Q^i + Q_5^i \pm (K^i + K^{i5})] \ ,$$
$$N^{i\pm} = \tfrac{1}{4}[Q^i - Q_5^i \pm (K^i - K^{i5})] \ . \qquad (17)$$

Each of these quantities forms an SU(2) algebra, and they commute with each other. They are related by parity and G parity:

$$PM^{i\pm}P^{-1} = N^{i\pm}, \quad GM^{i\pm}G^{-1} = N^{i\mp} \ , \qquad (18)$$

which follow from the well-known P and G properties of the currents. According to Eq. (17) we label our representation by $\mathfrak{D}(j, j', j'', j''')$. Representations which contain multiplets including P and G must be even under the exchange of the operations defined in (18).

To get an idea of what these K operators are, we look for the matrix element of K^3 between states of the same isospin multiplet:

$$\langle a|K^3(t)|a\rangle = \int d^3x \langle a|k_0{}^3(x, t)|a\rangle \ .$$

Since K^3 is a scalar operator, the dynamics of the matrix element simply give $(2\pi)^3 2E\delta^3(\vec{p}\,' - \vec{p})$ times a number, which depends only on the isospin. From the T-abnormal behavior of K^3 we find that this number is pure imaginary; from Hermiticity it follows that it is real; therefore it is zero. Using invariance under isospin rotations we finally have

$$\langle a|K^i|a'\rangle = 0 \ , \qquad (19)$$

which holds for *any* isospin multiplet. Similar considerations, using the current, can be shown for the axial SCC. It is this feature which makes it so difficult to measure form factors of SCC's; within transitions of an isomultiplet, they are never of the "charge" type but always of the "moment" type, and their contribution vanishes for zero momentum transfer.[26] In this connection, it should be noted that the neutral SCC $k_\mu^3(x, t)$, which is even under charge conjugation,

$$Ck_\mu^3(x, t)C^{-1} = + k_\mu^3(x, t) \ , \qquad (20)$$

has properties similar to the "anomalous" electromagnetic current introduced by Bernstein, Feinberg, and Lee.[27] They use two different C operators in their discussion, C_{strong} and C_γ, and, obviously our C operator corresponds to C_{strong}.[28]

From the charge algebra in Eqs. (14)–(16) it is very easy to use the well-known highly developed machinery of current algebra[29,30] to derive sum rules, low-energy theorems, or mass relations. This will be done in Sec. V. First we shall look for a field-theoretic model where we can explicitly verify the extended current algebra, and discuss some possible connections with strong interactions.

III. GENERALIZED σ MODEL

In the usual quark model the full symmetry of Eqs. (14)–(16) cannot be realized because, in taking the only SCC's which can be constructed from them,

$$ik_\mu^i(x) = -i\partial_\mu[\bar{q}(x)\tfrac{1}{2}\tau_i q(x)] \ ,$$
$$ik_{\mu 5}^i(x) = -i\partial_\nu[\bar{q}(x) i\sigma_{\mu\nu}\gamma_5 \tfrac{1}{2}\tau_i q(x)] \ , \qquad (21)$$

we get zero charges and therefore our algebra is wrong. This is a fact we have already mentioned in Sec. I. We could repair this, of course, by introducing another set of quarks, which have opposite CP with respect to the usual quarks. Instead of doing this, we study the features of $[\mathrm{SU}(2)]^4$ in a linear σ model[31] which has been useful in discussing chiral symmetries and their breaking mechanism.

We start with a set of eight mesons, transforming as a $\mathfrak{D}(\tfrac{1}{2}, 0, 0, \tfrac{1}{2}) \oplus \mathfrak{D}(0, \tfrac{1}{2}, \tfrac{1}{2}, 0)$ of $[\mathrm{SU}(2)]^4$. These are the pion $\pi_i(x)$, the sigma $\sigma(x)$, a scalar pion $s_i(x)$ with $I^G=1^-$, $J^P=0^+$, and the pseudoscalar singlet $\eta(x)$. We define the (infinitesimal) transformations (for details, see Ref. 32) as follows:

$$\begin{aligned}
\delta\pi_i &= -i\alpha_k[Q^k, \pi_i] \\
&= -\epsilon_{ikl}\alpha_k\pi_l \ , \\
\delta s_i &= -\epsilon_{ikl}\alpha_k s_l \ , \quad \delta\sigma=0, \quad \delta\eta=0 \ ; \\
\delta^5\pi_i &= -i\alpha_k'[Q_5^k, \pi_i] = \alpha_i'\sigma, \quad \delta^5 s_i = \alpha_i'\eta \ , \\
\delta^5\sigma &= -\alpha_k'\pi_k, \quad \delta^5\eta = -\alpha_k' s_k \ ; \\
\delta'\pi_i &\equiv -i\beta_k[K^k, \pi_i] = \beta_i\eta, \quad \delta' s_i = \beta_i\sigma \ , \\
\delta'\sigma &= -\beta_k s_k, \quad \delta'\eta = -\beta_k\pi_k \ ; \\
\delta'^5\pi_i &\equiv -i\beta_k'[K_5^k, \pi_i] = -\epsilon_{ikl}\beta_k' s_l \ , \\
\delta'^5 s_i &= -\epsilon_{ikl}\beta_k'\pi_l, \quad \delta'^5\sigma=0, \quad \delta'^5\eta=0 \ .
\end{aligned} \tag{22}$$

A Lagrangian which is invariant under these transformations is

$$\mathcal{L} = \tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\eta)^2 + (\partial_\mu\pi_i)^2 + (\partial_\mu s_i)^2] - \lambda^2(\sigma^2+\eta^2+\pi_i^{\,2}+s_i^{\,2}-c)^2 - a(\sigma\eta+\pi_i s_i)^2 \ . \tag{23}$$

The corresponding Noether currents can easily be found to be

$$\begin{aligned}
v_\mu^i &= \epsilon_{ikl}(\pi_k\partial_\mu\pi_l + s_k\partial_\mu s_l) \ , \\
a_\mu^i &= \pi_i\partial_\mu\sigma - \sigma\partial_\mu\pi_i + s_i\partial_\mu\eta - \eta\partial_\mu s_i \ , \\
k_\mu^i &= \pi_i\partial_\mu\eta - \eta\partial_\mu\pi_i + s_i\partial_\mu\sigma - \sigma\partial_\mu s_i \ , \\
k_{\mu 5}^i &= \epsilon_{ikl}(\pi_k\partial_\mu s_l + s_k\partial_\mu\pi_l) \ .
\end{aligned} \tag{24}$$

The currents are all conserved and, by the known transformation laws of the fields, have the right P and G properties. One can easily verify that they fulfill the extended current algebra of Eqs. (14)–(16) by using the canonical commutation rules of the fields.

The Langrangian is analogous to that used in chiral symmetries, apart from the term $a(\sigma\eta+\pi_i s_i)^2$, whose significance will soon be clear. The features of the Lagrangian, when one treats it as a function of classical fields, are well known and are not changed by this term. For $c<0$, we have the state with lowest energy for $\sigma=\eta=\pi_i=s_i=0$, which all have a common $(\text{mass})^2=-4c\lambda^2$. The interesting case is $c>0$, where we find the minimum of the potential energy for $\sigma=-\sqrt{c}$, $\pi_i=s_i=\eta=0$, and introduce a new field $\sigma'=\sigma+\sqrt{c}$. In this case, the masses of the π_i and s_i become zero, while we have $m_{\sigma'}{}^2=8\lambda^2 c$ and $m_\eta{}^2=2ac$. This is well known as the Goldstone solution. There are no $[\mathrm{SU}(2)]^4$ multiplets, but there still are $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ multiplets plus π_i and s_i as Goldstone bosons. The $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ which is left in this case is not the usual chiral symmetry but a symmetry which connects states with different CP. This can be seen by operating with our generators on the vacuum:

$$\begin{aligned}
Q^i|0\rangle &= |0\rangle \ , \\
Q_5^i|0\rangle &= |\pi_i, q_\mu=0\rangle \ , \\
K^i|0\rangle &= |s_i, q_\mu=0\rangle \ , \\
K_5^i|0\rangle &= |0'\rangle \ .
\end{aligned} \tag{25}$$

This means that the vacuum splits up into two parts which have opposite parity but the same C parity. Therefore, all particles must appear as CP doublets; a fact which has already been discussed in the literature in connection with γ_5 invariance.[33] In our case, π_i, s_i and σ, η are such doublets. If we are to couple our mesons to quarks, we shall also need a CP doublet, as stated at the beginning of this section. For nucleons, the CP partner can be found in the following way. Assuming the baryons are made up of three quarks, the usual nucleon is a $^2S_{1/2}$ state.[34] Its partner would be a $^4P_{1/2}$ state, which has opposite CP with respect to it. [One candidate may be the $N^*(1700)$ $I=\frac{1}{2}$, $J^P=\frac{1}{2}^-$ resonance.[35]]

We assume that the appearance of CP doublets is a special feature of our σ model, which should be modified for the real world. This we discuss in Sec. IV. Let us first come back to the other features of our Lagrangian. The term $a(\sigma\eta+\pi_i s_i)^2$ has now shown its significance: It is necessary for giving η a mass. Since η is not a Goldstone boson (at least not in the $[\mathrm{SU}(2)]^4$ model), there is no reason why it should not have a mass. If one is to avoid destroying the symmetry, this mass can only be introduced through such a term or, generally, through a function $F((\sigma\eta+\pi_i s_i)^2)$.

Let us now introduce symmetry breaking. The conventional term $\mathcal{L}_{s3}=f_\pi m_\pi{}^2\sigma$ breaks chiral invariance and automatically produces PCAC and PCSCVC, but the $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ of first- and second-class currents remains unchanged. Since we have no *a priori* reason to break this symmetry in a specific way, we simply introduce a mass term $-\frac{1}{2}\mu^2(\eta^2+s_i^{\,2})$, which gives the s_i particle a mass different from the π_i. Explicitly, we have

$$\mathcal{L} = \mathcal{L}^{\text{inv}} - \tfrac{1}{2}\mu^2(\eta^2+s_i^{\,2}) + f_\pi m_\pi{}^2\sigma \ . \tag{26}$$

In the customary way, we define $\sigma'=\sigma-\langle\sigma\rangle_0$, to eliminate the "tadpole graphs,"[36] which means all terms linear in σ' must vanish. With this, all parameters λ^2, c, μ^2, a can be expressed in terms of the physical masses $m_\sigma{}^2, m_\pi{}^2, m_s{}^2, m_\eta{}^2$, yielding

$$\lambda^2=\frac{1}{8f_\pi^2}(m_\sigma^2-m_\pi^2)\ ,$$
$$c=f_\pi^2\frac{m_\sigma^2-3m_\pi^2}{m_\sigma^2-m_\pi^2}\ ,$$
$$\langle\sigma\rangle_0=f_\pi,\quad \mu^2=(m_s^2-m_\pi^2)\ , \tag{27}$$
$$a=\frac{2}{f_\pi^2}(m_\eta^2-m_s^2)\ .$$

Furthermore, we have the currents

$$\partial^\mu v_\mu^i=0\ ,$$
$$\partial^\mu a_\mu^i=f_\pi m_\pi^2\pi_i\ ,$$
$$\partial^\mu k_\mu^i=f_\pi m_s^2 s_i+(m_s^2-m_\pi^2)(s_i\sigma'-\pi_i\eta)\ , \tag{28}$$
$$\partial^\mu k_{\mu 5}^i=(m_s^2-m_\pi^2)\epsilon_{ikl}s_k\pi_l\ .$$

The two parameters characterizing the symmetry breaking are $\epsilon\cong m_\pi^2$, which is responsible for the chiral breaking, and $\delta\cong(m_s^2-m_\pi^2)$, which is responsible for the breaking within the *CP* doublets. PCAC is automatically implied, but we get corrections to PCSCVC due to a more complicated operator which has no simple interpretation. For $k_{\mu 5}^i$ we can find a PCAC relation if there exists a state $\xi_i=\epsilon_{ikl}s_k\pi_l$, which is a bound state dominating the $(s\pi)$ system. Such a state with quantum numbers $I^G=1^+$, $J^P=0^-$, $C_n=-1$ is a "forbidden *C* state"; it cannot be constructed of two quarks or couple to the $N\bar{N}$ system. Decays are only possible into $\eta\pi\pi$, $\rho\rho$, etc.[32] Until now there have been no significant experimental data.

It is very straightforward to derive a nonlinear σ model of $[\mathrm{SU}(2)]^4$ by keeping only π_i and s_i fields. But since there is no new physics contained in such a model, there is no need to discuss it here.

IV. REALIZATION OF THE FULL SYMMETRY IN STRONG INTERACTIONS

Although we need not take the σ model too seriously, it gives a useful picture of the possible realization of a higher symmetry in actual nature. As is well known, a symmetry can be realized either through particle multiplets or through Goldstone bosons, in which case one refers to them as "spontaneously broken" symmetries. Obviously this will be the case in our model.

Let us imagine that we can write the total Hamiltonian in the following form:

$$H=H_0+\epsilon H_1+\delta H_2\ . \tag{29}$$

H_0 is *assumed* to be invariant under a group $\mathcal{G}'=[\mathrm{SU}(2)]^4$, generated by a set of charges $\{Q^{i\prime}, Q_5^{i\prime}, K^{i\prime}, K_5^{i\prime}\}$, which have the same transformation laws under *C* and *P* as the corresponding weak "charges" in Sec. II. We divide this group into three subgroups: the chiral group $\mathcal{G}_1'$ generated by $\{Q^{i\prime}, Q_5^{i\prime}\}$, the $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ subgroup $\mathcal{G}_2'$ generated by $\{Q^{i\prime}, K^{i\prime}\}$, and the $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ subgroup $\mathcal{G}_3'$ generated by $\{Q^{i\prime}, K_5^{i\prime}\}$. Suppose further that H_1 is invariant only under $\mathcal{G}_3'$, whereas H_2 is invariant only under the chiral $\mathcal{G}_1'$. With this, the full Hamiltonian is invariant only under an SU(2) group generated by $\{Q^{i\prime}\}$, which we *define* to be the isospin group.

The problem is that such a decomposition is physically useful only if both ϵ and δ are suitably small, so that one can use perturbation theory around $\epsilon=\delta=0$. In a recent work on chiral perturbation theory by Langacker and Pagels,[37] it was shown that the actual dimensionless parameter one uses is $m_\pi^2/(32\pi^2 f_\pi^2)\sim 0.006$. We therefore expect the strength of the perturbation δH_2 to be characterized by $m_s^2/32\pi^2 f_s^2\cong 0.29$. Here, m_s and f_s are mass and decay constants of the corresponding scalar particle, which we have chosen to be $m_s=m_{\pi_N}=975$ MeV, $f_s\sim f_\pi$. Therefore $[\mathrm{SU}(2)]^4$ is even worse than chiral $\mathrm{SU}(3)\otimes\mathrm{SU}(3)$ and we can expect only qualitative but not quantitative features by going to the symmetry limit. Nevertheless, let us be optimistic and discuss an ideal world with $\epsilon=\delta=0$.

The discussion of this case is completely analogous to that of chiral symmetries. The Hamiltonian is invariant under the full group $\mathcal{G}'$, whereas the vacuum is invariant only under the SU(2) subgroup. Therefore, we have

$$Q_5^{i\prime}|0\rangle=|\pi_{i'}(q_\mu=0)\rangle\ ,$$
$$K^{i\prime}|0\rangle=|s_{i'}(q_\mu=0)\rangle\ , \tag{30}$$
$$K^{i\prime}|0\rangle=|\xi_{i'}(q_\mu=0)\rangle\ .$$

According to the transformation laws of the generators, π_i has $I^GJ^P=1^-,0^-$, s_i has $I^GJ^P=1^-0^+$, and ξ_i has $I^GJ^P=1^+0^-$. From these quantum numbers we can see that one of these states, for example ξ_i, can be regarded as a system of the other two: $\xi_i=\epsilon_{ikl}\pi_k s_l$. Since all three states have mass zero, it is completely academic to discuss which one is the more fundamental. The question of which of them are "elementary" particles can be answered only in the case of broken symmetries, where they pick up different masses.

Before going into symmetry breaking, we should like to make one more statement. We assume that the generators of our group $\mathcal{G}'$, $\{Q^{i\prime}, Q_5^{i\prime}, K^{i\prime}, K_5^{i\prime}\}$, are identical to the charges of the weak first- and second-class currents $\{Q^i, Q_5^i, K^i, K_5^i\}$. For the isospin generators Q^i this is a well-known fact.[13] This statement is not trivial, because the generators generally should only be related by some unitary transformation.[38] Since we have no idea of how this transformation should look, although there are several models in the literature,[39] we shall restrict ourselves to the simpler case.

Let us now introduce symmetry breaking. It is well known that the Goldstone bosons pick up a mass. In addition, the theory will now satisfy the usual kind of PCAC-like relations, which show the behavior of the matrix elements in the limit $\epsilon,\delta=0$. It has been shown[21] that these PCAC relations are independent of the form of the symmetry-breaking interaction. Therefore, we get

$$\begin{aligned} &\langle \pi_i|\partial^\mu a_\mu^k(0)|0\rangle = m_\pi^2 f_\pi \delta_{ik} , \\ &\langle s_i|\partial^\mu k_\mu^k(0)|0\rangle = m_s^2 f_s \delta_{ik} , \\ &\langle \xi_i|\partial^\mu k_{\mu 5}^k(0)|0\rangle = m_\xi^2 f_\xi \delta_{ik} . \end{aligned} \tag{31}$$

Going back to our Hamiltonian we see that $m_\pi^2 = O(\epsilon)$, $m_\xi^2 = O(\delta)$, whereas $m_s^2 = O(\delta, \epsilon)$. Let us discuss in detail the consequences on different processes.

The idea that the nonconserved part of the vector current is dominated by a scalar meson (PCVC) is discussed extensively in the literature,[19] mainly in connection with electromagnetic corrections. Eliezer and Singer[40] first used this concept with an independent second-class vector current (PCSCVC) and derived a series of Goldberger-Treiman relations for baryon matrix elements. We briefly discuss PCSCVC in connection with our symmetry.

For the SCC, assuming CP conservation, we get $(v_\mu^{\mathrm{II}} \equiv ik_\mu)$

$$\langle s_i(q)|v_\mu^{\mathrm{II},l}|0\rangle = f_s q_\mu \delta_{il}$$

and (32)

$$\langle s_i(q)|\partial^\mu v_\mu^{\mathrm{II},l}|0\rangle = i f_s m_s^2 \delta_{il} .$$

(Note that due to the C properties of the SCC, the decay constant of s^+ is $\sqrt{2} f_s$, whereas that of s^- is $-\sqrt{2} f_s$.)

For nucleons, neglecting electromagnetic effects, we have

$$\begin{aligned} &\langle N(p')|v_\mu^{\mathrm{II},i}(0)|N(p)\rangle = \bar{u}(p') q_\mu \tfrac{1}{2}\tau_i u(p) F_3(q^2) , \\ &\langle N(p')|\partial^\mu v_\mu^{\mathrm{II},i}(0)|N(p)\rangle = i\bar{u}(p')\tfrac{1}{2}\tau_i u(p) d(q^2) , \end{aligned} \tag{33}$$

where $d(q^2) = q^2 F_3(q^2)$, $q_\mu = p'_\mu - p_\mu$. Using the same procedure as Dashen and Weinstein,[22] we write a dispersion relation for F_3:

$$F_3(q^2) = \frac{f_s g_{sNN}}{m_s^2 - q^2} + \int \frac{da^2 \rho(a^2)}{a^2 - q^2} ; \tag{34}$$

similarly for $d(q^2)$, using Eq. (32), we write

$$d(q^2) = \frac{f_s g_{sNN} m_s^2}{m_s^2 - q^2} + \bar{d}(q^2) . \tag{35}$$

Therefore, we have

$$\begin{aligned} q^2 F_3(q^2) &= \frac{f_s g_{sNN} q^2}{m_s^2 - q^2} + q^2 \int \frac{da^2 \rho(a^2)}{a^2 - q^2} \\ &= \frac{f_s g_{sNN} m_s^2}{m_s^2 - q^2} + \bar{d}(q^2) \end{aligned} \tag{36}$$

or

$$f_s g_{sNN} = -\bar{d}(q^2) + q^2 \int \frac{da^2 \rho(a^2)}{a^2 - q^2} , \tag{37}$$

irrespective of the value of m_s. The main point is that $\bar{d}(q^2)$, as a matrix element of a divergence, is of the order of (δ, ϵ) of our symmetry-breaking Hamiltonian. Letting $q^2 \to 0$, we see that

$$f_s g_{sNN} = O(\delta, \epsilon) . \tag{38}$$

Since f_s is a fundamental constant, independent of whether or not the symmetry is broken, we conclude that the coupling of the scalar meson to nucleons is of the order of the breaking of $[\mathrm{SU}(2)]^4$ (which may be large). Similar considerations with matrix elements of other baryons as well as mesons show that $g_{s\alpha\alpha'} = O(\delta, \epsilon)$ if α, α' belong to members of the same isomultiplet. This is a direct consequence of the fact, mentioned in Sec. II, that the matrix elements of SCC's between isomultiplets are always of the "derivative" type, proportional to the momentum transfer. Eliezer and Singer,[40] who use exact PCSCVC and hence $\delta = \epsilon = 0$, obtain the result that either f_s or $g_{s\alpha\alpha'}$ are of the order of the electromagnetic mass differences, which we have ignored. Therefore, calculating $g_{sK\bar{K}}$ from the decay width $\Gamma(\pi_N(1016\text{ MeV}) \to K\bar{K})$, they get a very small value, $f_s = 2.5$ MeV, which is not compatible with our scheme, since we expect something like $f_s \sim f_\pi$ (this, in fact, is shown in Sec. V). On the other hand, the value of $g_{\pi_N K\bar{K}}^2/4\pi = 0.41\text{ GeV}^2$ shows how far we are in reality from the symmetry limit.

Another interesting case is the leptonic $\eta \to \pi l \nu$ decay.[10] We have

$$\langle \eta(p')|v_\mu^{\mathrm{II},i}(0)|\pi_k(p)\rangle = \delta_{ik}[f_+(p'+p)_\mu + f_-(p'-p)_\mu] . \tag{39}$$

Similar considerations lead to the Goldberger-Treiman relation

$$\begin{aligned} f_+(0)(m_\eta^2 - m_\pi^2) - f_s g_{s\eta\pi} &= \bar{d}(0) \\ &= O(\delta, \epsilon) . \end{aligned} \tag{40}$$

In this case, as we have seen from the σ model, since η is not a Goldstone boson for $[\mathrm{SU}(2)]^4$, there is no reason for m_η^2 to vanish in the symmetry limit. [It should be noted that η must be replaced by the unitary singlet in the case of SU(3).] Therefore, we find in this case that $g_{s\pi\eta}$ is finite even in the symmetry limit.

We have seen from Eq. (32) that we can give a verification of PCSCVC and derive Goldberger-Treiman relations if we are willing to identify the scalar Goldstone boson s_i by the $\pi_N(975)$ $(\eta\pi)$ resonance. If we wish to do the same for the axial-vector SCC, we will have difficulties because no particle with the quantum numbers of the ξ_i state, $I^G J^P = 1^+ 0^-$, $C_n = -1$, has been seen and therefore we cannot write a PCAC relation. There are several possible explanations for this: Since the ξ_i cannot be constructed of two quarks it may show up with very high mass as a $qq\bar{q}\bar{q}$ excitation, or it may actually have a very low mass and therefore lie beyond the 4π threshold, or this Goldstone boson may not show up as a particle at all, but as a Regge trajectory[21] whose intercept with $\alpha = 0$ lies in negative $(\text{mass})^2$ values. None of these possibilities is very convincing and so we are faced with an open problem.

Another way out of this difficulty is the assumption that the vacuum possesses a higher symmetry than SU(2), i.e., it is invariant under the group $\mathcal{G}_3'$ generated by $\{Q^{i'}, K_5^i\}$. This is essentially the case we had in the σ model. In this instance, however, one still has the Goldstone bosons π_i and s_i, which give rise to PCAC and PCSCVC if symmetry breaking is turned on, but the existence of a ξ_i state is not necessary. Instead, all particles must appear in $SU(2)\otimes SU(2)$ multiplets, which mix states with different CP transformations. For the nonstrange mesons, we can find a series of examples: π and $\pi_N(975)$, ρ and $B(1235)$, and $A_2(1310)$ and $A_3(1640)$, which transform as a $(1,0)\oplus(0,1)$ representation. The partner of the $A_1(1070)$ axial-vector meson is found to be the $\omega(784)$ singlet; they form a $(\frac{1}{2},\frac{1}{2})$ representation. Furthermore, there are some singlets, such as η, which have as a partner the vacuum (or the σ, if it exists). For nucleons we have, as already mentioned, N and $N^*(1700)$ as a member of a $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ representation.

In this section we have seen that a realization of $[SU(2)]^4$ in strong interactions is possible if we are willing to accept the following:

(i) the $\pi_N(975)$ in addition to the pion as would-be Goldstone bosons;

(ii) some reason for the absence of the ξ_i bosons in real nature, for instance, the existence of a corresponding Regge trajectory; or as an alternative

(iii) the existence of CP-doubled multiplets.

As a result we get a justification of PCSCVC and of some Goldberger-Treiman relations. As we have seen, the symmetry-breaking effects are quite large, so we cannot expect very detailed quantitiative results. Nevertheless, in the author's personal opinion, we have learned a new way of thinking about strong interactions.

V. APPLICATIONS

We we shall see, most of our applications use only the current algebra and PCSCVC, and not explicitly the symmetry of strong interactions. On the other hand, PCSCVC can be understood only within the framework of this symmetry, as seen in Sec. IV.

We begin with the commutator

$$[K^+, K^-] = 2Q^3, \quad K^\pm \equiv K^1 \pm iK^2$$

which we sandwich between pions. After separating the η contribution and making use of the infinite-momentum limit, we get the analog of the Adler-Weisberger relation,

$$2 = |\sqrt{2}\, f_1^{\eta\pi}(0)|^2 + \frac{1}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^2}\,\mathrm{Im}\,W^-(\nu,0)\,, \tag{41}$$

where

$$\begin{aligned}\mathrm{Im}\,W(\nu,q^2) = \tfrac{1}{2}\sum_n (2\pi)^4 &\\ \times[\delta^4(p+q-p_n)|\langle n|\partial^\mu k_\mu^-|\pi\rangle|^2 &\\ -\delta^4(p-q-p_n)|\langle n|\partial^\mu k_\mu^+|\pi\rangle|^2]\,. &\end{aligned}$$

Since there is no coupling of the $I=0$ t channel to the antisymmetric part of the amplitude, we have the asymptotic behavior $\mathrm{Im}\,W^-(\nu,0) \sim \nu^{\alpha_1}$, with $\alpha_1(0) < 1$, and therefore the sum rule converges. If we now use PCSCVC, which means in this case

$$\langle n|\partial^\mu k_\mu^-|\pi^+\rangle = \frac{i f_s m_s^2}{m_s^2 - q^2}\langle n|J_s^-|\pi^+\rangle\ , \tag{42}$$

where J_s is the source of the s meson, we immediately get the familiar form[41]

$$\begin{aligned}1 = (f_1^{\eta\pi})^2 + \frac{f_s^2}{2\pi}\int_{(4m_\pi)}^{\infty}\frac{ds}{s-m_\pi^2}[\sigma^{\text{tot}}_{s^-\pi^+}(s,0) &\\ -\sigma^{\text{tot}}_{s^+\pi^-}(s,0)]\ , &\end{aligned} \tag{43}$$

which is more or less of academic interest because the cross section of zero-mass π_N particles on pions is not known.

It should be noted that Eq. (41) is the only one which affords the possibility of testing our current algebra experimentally. If it should follow from experiment that the $\eta \to \pi l\nu$ decay proceeds only via electromagnetic weak interactions, then $f_+^{\eta\pi}(0)$ would be of the order 10^{-2}.[10] In this case it would be very difficult to verify Eq. (41) because we expect the main contribution of the dispersion integral to be concentrated in the η pole. Such an experiment would definitely disprove the validity of our charge algebra. It should, however, be mentioned that in the SU(3) generalization of this scheme the decay of the octet part of the η into π is still suppressed. Therefore, the η should be replaced by a certain mixture of η and $X^0(958)$, which corresponds to the SU(3) singlet. In order to compare this result with some numbers, we first write down the conventional Adler-Weisberger sum rule as a low-energy theorem for scattering of pions on any external target[42]:

$$\frac{1}{\pi}\int_0^\infty \frac{d\nu}{\nu^2}\,\mathrm{Im}f^{(s)1}_{\lambda0,\lambda0}(\nu,0)=\frac{1}{f_\pi^{\,2}}\langle I_3\rangle \ , \tag{44}$$

where $f^{(s)}_{\lambda0,\lambda0}$ is the helicity amplitude in the s channel for pion-X (helicity λ) scattering, and 1 stands for the $I=1$ part in the t channel. Such an amplitude behaves like $\sim\nu^{\alpha_1}$, $\alpha_1<1$; therefore the sum rule converges. $\langle I_3\rangle$ is the isospin value of the target. Similarly, we get from our current algebra

$$\frac{1}{\pi}\int_0^\infty \frac{d\nu}{\nu^2}\,\mathrm{Im}g^{(s)1}_{\lambda0,\lambda0}(\nu,0)=\frac{1}{f_s^{\,2}}\langle I_3\rangle \ , \tag{45}$$

where $g^{(s)}_{\lambda0,\lambda0}$ is the helicity amplitude for scattering of $s(\pi_N)$ particles on an external target. Now take Eq. (44) and use π_N particles as a target, and take Eq. (45) for external pions: The former describes $\pi\pi_N$ scattering and the latter $\pi_N\pi$ scattering, which is, of course, the same thing. Therefore, the left-hand sides of the equations are equal and we have the important result

$$|f_s|=|f_\pi| \ . \tag{46}$$

This result is a consequence of the extended current algebra, PCAC, PCSCVC, and some assumptions about the asymptotic behavior of scattering amplitudes.

In order to calculate both sides of Eq. (41) we make use of the Goldberger-Treiman relation:

$$f_+^{\eta\pi}(0)=\frac{f_s g_{s\eta\pi}}{m_s^{\,2}-m_\pi^{\,2}} \ . \tag{47}$$

The coupling constant $g_{s\eta\pi}$ can be determined from the known width of $\pi_N(975)\to\eta\pi$,

$$\Gamma_{\pi_N\to\eta\pi}=\frac{g_{s\eta\pi}^{\,2}}{4\pi}\,\frac{p_{\mathrm{c.m.}}}{2m_s^{\,2}}\cong 60\ \mathrm{MeV} \ , \tag{48}$$

yielding $g_{s\eta\pi}\sim 2.2$ GeV and, therefore, with $f_s\sim 94$ MeV,

$$f_+^{\eta\pi}(0)\sim 0.735 \ . \tag{49}$$

We approximate the contribution of the continuum part by the $\eta'(X^0)$ resonance, so we have

$$1=f_+^{\eta\pi}(0)^2+f_+^{X^0\pi}(0)^2+\text{higher terms} \ . \tag{50}$$

For $f_+^{X^0\pi}$ we have a similar relation:

$$f_+^{X^0\pi}=\frac{f_s g_{X^0s\pi}}{m_{X^0}^{\,2}-m_\pi^{\,2}} \ . \tag{51}$$

An upper limit for $g_{X^0s\pi}$ can be found, assuming the decay $X^0\to\eta\pi\pi$ to be dominated by the π_N pole in the $\eta\pi$ system, which yields for the matrix element

$$\langle\eta\pi\pi|T|X^0\rangle=\frac{g_{X^0s\pi}g_{s\pi\eta}}{m_s^{\,2}-(p_\eta+p_\pi)^2} \ . \tag{52}$$

From the known width $\Gamma_{X^0\to\eta\pi\pi}<2.7$ MeV we obtain $g_{X^0s\pi}<3.6$ GeV and, therefore, $f_+^{X^0\pi}(0)\lesssim 0.38$. Thus, the squares on the right-hand side of Eq. (50) add up to a total of about 0.7, which is not so bad if one recognizes the fact that we have used exact PCSCVC. At least we are in the right order of magnitude.

Since we have no PCAC for the axial-vector SCC, it is of no use to discuss the analog of the Adler-Weisberger relation for the K_5' commutators. To obtain more results we have to extend our charge algebra of $[\mathrm{SU}(2)]^4$ to the current components k_μ^i, $k_{\mu5}^i$ themselves, as can easily be done. In this case we can write down sum rules for the k and k_5 propagators analogous to those obtained by Weinberg[43] for the axial-vector and vector currents:

$$\begin{aligned}&\int\frac{\rho_k^{(1)}(a^2)}{a^2}\,da^2+\int\rho_k^{(0)}(a^2)da^2=\int\frac{\rho_V^{(1)}(a^2)}{a^2}\,da^2 \ ,\\&\int\frac{\rho_{k5}^{(1)}(a^2)}{a^2}\,da^2+\int\rho_{k5}^{(0)}(a^2)da^2=\int\frac{\rho_V^{(1)}(a^2)}{a^2}\,da^2 \ ,\end{aligned}\tag{53}$$

and

$$\begin{aligned}\int\rho_k^{(1)}(a^2)da^2&=\int\rho_{k5}^{(1)}(a^2)da^2\\&=\int\rho_V^{(1)}(a^2)da^2 \ .\end{aligned}\tag{54}$$

These sum rules can be dominated partly by the ρ and B mesons, as well as the π_N meson, whereas we have no vector meson for the vector SCC (a $I^GJ^P=1^-1^-$ state cannot be created by quark-antiquark), and no pseudoscalar meson for the spin-zero part of the axial-vector SCC. Therefore, the only direct result we get is

$$\gamma_B=\gamma_\rho \ , \tag{55}$$

which has already been derived in the same way by Maiani.[17]

We have presented a few results which depend only on the $[\mathrm{SU}(2)]^4$ algebra of charges and PCSCVC. As already pointed out, the most sensitive test would be a measurement of $f_+^{\eta\pi}$. A set of relations between masses and coupling constants can be obtained by saturating Eq. (45) with lowlying meson states and improving the scheme proposed by Gilman and Harari.[42] Further information can be obtained by extending the current algebra for the current divergences, but in this case the behavior of the strong Hamiltonian in Eq. (29) definitely comes into the game.

VI. LEPTONS AND WEAK INTERACTIONS

Let us look at the implications for leptons of our $[SU(2)]^4$. Gell-Mann and Ne'eman[23] formulated the concept of hadron-lepton universality, which means the following: The lepton currents l_μ^+ and l_μ^- participating in the weak interactions defined through

$$\begin{aligned} l_\lambda^+ &= \bar\psi_{\nu_e}\Gamma_\lambda\psi_e + \bar\psi_{\nu_\mu}\Gamma_\lambda\psi_\mu, \\ l_\lambda^- &= \bar\psi_e\Gamma_\lambda\psi_{\nu_e} + \bar\psi_\mu\Gamma_\lambda\psi_{\nu_\mu}, \\ \Gamma_\lambda &\equiv \tfrac{1}{2}\gamma_\lambda(1+\gamma_5), \quad l_\lambda^+ = (l_\lambda^-)^\dagger, \end{aligned} \tag{56}$$

fulfill the following algebra:

$$\begin{aligned} [W_l^+, W_l^-] &= 2W_l^0, \\ [W_l^0, W_l^\pm] &= \pm W_l^\pm, \end{aligned} \tag{57}$$

where

$$W_l^\pm = \int d^3x\, l_0^\pm(x)$$

and

$$W_l^0 = \int d^3x\, \tfrac{1}{2}(\bar\psi_{\nu_e}\Gamma_0\psi_{\nu_e} - \bar\psi_e\Gamma_0\psi_e + \bar\psi_{\nu_\mu}\Gamma_0\psi_{\nu_\mu} - \bar\psi_\mu\Gamma_0\psi_\mu).$$

The statement of lepton-hadron universality means that any hadronic current coupled to the leptons by weak interactions must fulfill the same algebra, i.e.,

$$\begin{aligned} W_h^\pm &= \int d^3x\, J_0^\pm(x), \\ [W_h^+, W_h^-] &= 2W_h^0, \quad [W_h^0, W_h^\pm] = \pm W_h^\pm. \end{aligned} \tag{58}$$

If we write down our total weak hadronic current,

$$\begin{aligned} J_\lambda^+ &= J_\lambda^{\mathrm{I}+} + J_\lambda^{\mathrm{II}+} \\ &= v_\lambda + a_\lambda + ik_\lambda^+ + ik_{\lambda 5}^+, \\ J_\lambda^- &= v_\lambda^- + a_\lambda^- - ik_\lambda^- - ik_{\lambda 5}^-, \end{aligned} \tag{59}$$

we see that the algebra of (48) is fulfilled by the "charges"

$$\begin{aligned} W_h^+ &= \frac{1}{2\sqrt{2}}(Q^+ + Q_5^+ + iK^+ + iK_5^+), \quad W_h^- = (W_h^+)^\dagger, \\ W_h^3 &= \tfrac{1}{2}(Q^3 + Q_5^3), \end{aligned} \tag{60}$$

which is consistent with lepton-hadron universality. There exists, however, an alternative possibility for the total hadronic current:

$$J_\lambda^+ = v_\lambda^+ + a_\lambda^+ + k_\lambda^+ + k_{\lambda 5}^+, \tag{61}$$

which realizes the algebra by means of

$$\begin{aligned} W_h^+ &= \tfrac{1}{4}(Q^+ + Q_5^+ + K^+ + K_5^+), \quad W_h^- = (W_h^+)^\dagger, \\ W_h^3 &= \tfrac{1}{4}(Q^3 + Q_5^3 + K^3 + K_5^3). \end{aligned} \tag{62}$$

Owing to the different CP properties of first- and second-class currents, in the second case we get a Lagrangian for semileptonic weak interactions which produces CP-violating effects proportional to the interference terms of first- and second-class form factors. This is essentially the original idea of Cabibbo.[18] The CP violation introduced in this way is maximal, whereas a weak current of the form $j_\lambda + e^{i\phi}k_\lambda$ will in general *not* fulfill the algebra of Eq. (58) and, therefore, will violate lepton-hadron universality. Thus, we have two possibilities for the weak interaction compatible with $[SU(2)]^4$ and lepton-hadron universality: CP-violation effects are either zero or maximal.

As has been discussed in the literature,[44] the general hadronic current, including $\Delta S = 1$ transitions, can be obtained by a rotation $U = e^{2i\theta F_7}$ about the seventh axis of SU(3) from the $\Delta S = 0$ current. From universality, it follows that the Cabibbo angle θ must be the same for first- and second-class currents.

One can go one step further and ask if the lepton-hadron universality goes so far that there exist analogs of the SCC in the leptonic case. Since G parity has no meaning for leptons, we may formulate the question in a different way: Do there exist CP-even Hermitian leptonic currents? Such a direct analogy between hadrons and leptons would be extremely important if we wished to generalize the weak interactions in the sense of unified theories.[45]

Answering this question in a positive way, we are in the same situation as described at the beginning of Sec. III, as we wanted to construct SCC's from quarks. Derivative coupling terms would be incompatible with the pointlike structure of leptons. Therefore, we are faced with the fact that *two different sets of leptons should exist* which are CP partners of each other. Calling these sets (l, ν_l) and $(l', \nu_{l'})$, we have the currents

$$\begin{aligned} l_\lambda^+ &= \bar\psi_l\Gamma_\lambda\psi_{\nu_l} + \bar\psi_{l'}\Gamma_\lambda\psi_{\nu_{l'}}, \\ CP\,l_\lambda^+(CP)^{-1} &= -\epsilon(\lambda)(l_\lambda^+)^\dagger \end{aligned}$$

and

$$\begin{aligned} m_\lambda^+ &= \bar\psi_l\Gamma_\lambda\psi_{\nu_{l'}} + \bar\psi_{l'}\Gamma_\lambda\psi_{\nu_l}, \\ CP\,m_\lambda^+(CP)^{-1} &= +\epsilon(\lambda)(m_\lambda^+)^\dagger. \end{aligned} \tag{63}$$

Although this suggestion is highly speculative, we should like to follow it one step further and discuss the nature of sets l and l': Either they are the usual electron and muon plus some heavy leptons, or they are the electron and the muon themselves. Concerning the former, all heavy leptons discussed in the literature[46] have the same CP properties as the lepton itself. Concerning the latter we are faced with the fact that the separate conservation of electronic and muonic lepton number is related to CP conservation.[47] This possibility would also have the very attractive feature of explaining *why* we have an electron and a muon: They are CP partners of each other, in the same sense as we have already discussed in Sec. IV.

We should like to stress the fact that the necessary existence of two sets of leptons, which are CP partners to each other (probably electrons and muons), is a consequence of (i) the existence of an algebra which contains CP-even and CP-odd hadronic currents; (ii) extended hadron-lepton universality in the sense that such CP-even Hermitian currents must exist for leptons also; and (iii) the fact that leptons have a pointlike structure.

If we are willing to accept this hypothesis, then we are left with a leptonic $SU(2)\otimes SU(2)$ generated by the currents l_λ and m_λ. Since both currents contain vector and axial-vector terms, we have a complete analogy to the $[SU(2)]^4$ symmetry of the hadronic case.

Before closing this section we should discuss the weak-interaction Lagrangian. The form

$$\mathfrak{L}_W = \frac{G}{\sqrt{2}}(J_\lambda + K_\lambda)(l^\lambda + m^\lambda) \tag{64}$$

(for simplicity, we have $J_\lambda = v_\lambda + a_\lambda$, $K_\lambda = k_\lambda + k_{5\lambda}$) is definitely excluded by experiment: It would couple the currents l^λ and m^λ with the same strength to the first-class current J_λ and therefore give lepton-number-violating effects. We can avoid this by introducing some angle ψ between the currents l_λ and m_λ:

$$\mathfrak{L} = \frac{G}{\sqrt{2}}(J_\lambda + K_\lambda)(l^\lambda \cos\psi + m^\lambda \sin\psi). \tag{65}$$

This Lagrangian, without the SCC K_λ, has been studied in detail by Iro,[48] and the value of $\sin\psi$ has been related to the measured CP-violating effects.

A third possibility would be the Lagrangian

$$\mathfrak{L} = \frac{G'}{\sqrt{2}}(J_\lambda l^\lambda + K_\lambda m^\lambda), \tag{66}$$

which is interesting because it gives rise to lepton-number violation without CP violation, but it is suppressed because the lepton-number-violating current couples to the SCC only. On the other hand, generalizing it to pure leptonic processes yields

$$\mathfrak{L} = \frac{G'}{2\sqrt{2}}(l_\lambda l^\lambda + m_\lambda m^\lambda). \tag{67}$$

This predicts for the muon decay an equal amount of $(\nu_\mu, \bar{\nu}_e)$ as well as $(\nu_e \bar{\nu}_\mu)$ pairs in the final state, which is probably ruled out by experiment.[49] In any case, we think that the final form of the weak Lagrangian containing CP-even currents may probably be given in the framework of unified theories.[50]

VII. CONCLUDING REMARKS

As we have seen, the problem of putting the SCC into a group-theoretic framework can be simply solved by doubling the commutation relations, not only with respect to parity, as has been done in chiral symmetries, but also with respect to charge conjugation. The applications do not give any contradictions; rather, the problem is one of obtaining practical results. Nevertheless, the test of the $\eta \to \pi e \nu$ decay will be crucial for our hypothesis. A further consequence of our current algebra is that the SCC always keeps the canonical dimension.

What we wish to suggest with this work is that nobody should be unduly surprised if SCC's sometimes show up in experiments. They can be treated in the same way as the usual currents. The question of whether they also act as "probes" of the strong interactions due to an $[SU(2)]^4$ symmetry of nature is a little more delicate, however. If this were actually the case, we would not expect any practical use of the symmetry, because of the large breaking effects. But we have some reason for using PCSCVC, and there is also the interesting possibility that particles may appear in CP-coupled multiplets. Using an extended version of hadron-lepton universality, we are faced with the same effect in the lepton case, which would either give a possible answer to the electron-muon problem or postulate a set of heavy leptons.

Naturally we shall later have to generalize the concept to SU(3). Before doing so, it is extremely important to increase the knowledge of first-class currents, including SU(3)-breaking effects. The first promising result has been the calculation of the pseudotensor coupling constant in hyperon decays.[9] Apart from this the generalization is straightforward. As Goldstone bosons we need the pseudoscalar octet (π, K, η), a scalar octet (π_N, κ, σ), and an octet of C-odd ξ particles. A search for such C-odd mesons would also be interesting for astrophysical reasons.[51] It should be noted that in generalizing our σ model to SU(3) we need four nonets of bosons transforming like a $(\bar{3},1,1,3) \oplus (3,1,1,\bar{3}) \oplus (1,\bar{3},3,1) \oplus (1,3,\bar{3},1)$ representation of $[SU(3)]^4$, which also means scalar mesons with odd C parity, or so-called C mesons.[52] Johnson and Low,[53] who calculated current commutators in higher-order perturbation theory of a quark model, mentioned that C breakdown appears only in very high orders but is possible. This may be a hint that such mesons have a very large mass.

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