PHYSICAL REVIEW D VOLUME 10, NUMBER 4 15 AUGUST 1974

Effective-Lagrangian formulation of generalized vector dominance

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(Received 29 October 1973)

The Lagrangian of Lee and Zumino is generalized to include several vector mesons. Their mutual interactions are chosen so as to guarantee a current-vector-fields proportionality relation analogous to the original current-field proportionality relation of Kroll, Lee, and Zumino. Implications of the new Lagrangian for the electromagnetic couplings of charged vector mesons are studied. It is shown, in particular, that the $\rho^{\pm}(770)$ can carry *any* magnetic moment in this scheme.

I. INTRODUCTION

The vector-dominance idea, namely the idea that the electromagnetic (em) field couples to the hadrons via the neutral vector mesons, found its Lagrangian formulation in the 1967 paper of Kroll, Lee, and Zumino,[1] where an extensive review and enumeration of previous formulations can also be found. In a further paper[2] Lee and Zumino utilized the proportionality of the em current and the neutral vector field, as formulated in Ref. 1, to derive the general structure of the hadronic part of the total Lagrangian and the explicit form of the em interaction.

Recently, new neutral vector mesons were indicated,[3] and it seems proper to consider their possible effects on the ideas and results of Refs. 1 and 2. For this purpose one can adopt two alternative attitudes: First, one can retain the Lagrangian of Ref. 2 and consider the higher vector mesons as further excitations of the fundamental vector field $\vec{\rho}_\mu$ or as continuum excitations of its propagator. This attitude was, for example, adopted by Shtokhamer and Singer in Sec. III of their paper[4] which investigated the continuum contributions to the em moments of charged vector mesons. An alternative approach to vector-meson proliferation is to look at the Lagrangians in Ref. 2 in the spirit of the "effective-Lagrangian" approach.[5] In this spirit the Lagrangians in Ref. 2 correspond to the situation where only one vector meson is involved, described by the field $\vec{\rho}_\mu$.[6] The presence of heavier vector mesons dictates, in this approach, the introduction of further vector fields into the Lagrangian. In particular, attention must be paid to possible interaction terms among the various vector fields. This brings in a qualitative change in the structure of the Lagrangian as compared with Ref. 2. Our task now is to introduce these new mutual interaction terms among the different vector fields in such a way as to guarantee some generalized form of vector dominance or of the field-current proportionality relation. It will be shown that this restricts the mutual interaction terms considerably but does not fix them completely, in contrast with the corresponding situation in Ref. 2. It will be further shown that the different possible mutual interactions of the vector mesons result also in different couplings of these mesons to the em field.

The organization of the paper is as follows: In Sec. II we review briefly Ref. 2 in the spirit of the effective-Lagrangian approach for the case of one vector meson. This enables us to introduce the concepts and notations used in Sec. III for a formulation of a two-vector-meson effective-Lagrangian model. Our conclusions will be summarized in Sec. IV.

II. EFFECTIVE-LAGRANGIAN FORMULATION OF VECTOR DOMINANCE–THE ONE-MESON CASE

It is not our purpose here to repeat the very clear exposition of Refs. 1 and 2, but only to recapitulate those aspects used afterwards in Sec. III, to stress the effective-Lagrangian attitude here adopted, and to introduce our notations and conventions. The field ρ_μ^i, where $i=1,2,3$ is the isospin index and $\mu=0,1,2,3$ is the Minkowski index, corresponds to the vector-meson triplet with mass m. Because of the field-current proportionality and since the charged ρ meson does interact with the em field, there must also exist a strong-interaction term between the neutral ρ meson and the charged ρ meson. Such a term should be proportional to $(G^1_{\mu\nu}\rho^2_\nu - G^2_{\mu\nu}\rho^1_\nu)\rho^3_\mu$, where $\vec{G}_{\mu\nu}=\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$. From isospin invariance this interaction term is proportional to $(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)\cdot\vec{\rho}_\mu$. Thus one assumes the strong-interaction Lagrangian of the ρ-meson system to be of the general form

$$\mathcal{L} = -\tfrac{1}{4}(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \tfrac{1}{2}g\,\vec{G}_{\mu\nu}\cdot(\vec{\rho}_\mu\times\vec{\rho}_\nu) + F(\vec{\rho}_\mu), \tag{2.1}$$

where F is assumed to depend only on $\vec{\rho}_\mu$ and to be invariant under isospin rotations. Because of the field-current proportionality and the current conservation, one has to fix F so as to guarantee $\partial_\mu \vec{\rho}_\mu = 0$. To this end recall that isospin invariance of $\mathcal{L}$ implies conservation of $\vec{S}_\mu = (\partial\mathcal{L}/\partial\vec{\rho}_{\nu,\mu}) \times \vec{\rho}_\nu$, where $\vec{\rho}_{\nu,\mu} \equiv \partial_\mu \vec{\rho}_\nu$. Explicit calculation of $\vec{S}_\mu$ utilizing (2.1) yields

$$\vec{S}_\mu = [-\vec{G}_{\mu\nu} + g(\vec{\rho}_\mu \times \vec{\rho}_\nu)] \times \vec{\rho}_\nu . \quad (2.2)$$

The equations of motion implied by (2.1) are

$$\partial_\nu \frac{\partial \mathcal{L}}{\partial \vec{\rho}_{\mu,\nu}} = m^2 \vec{\rho}_\mu - g(\vec{G}_{\mu\nu} \times \vec{\rho}_\nu) + \frac{\partial F}{\partial \vec{\rho}_\mu} . \quad (2.3)$$

Since $\partial\mathcal{L}/\partial\vec{\rho}_{\mu,\nu}$ is antisymmetric in μ and ν, one has

$$\partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial \vec{\rho}_{\mu,\nu}} = 0$$

and so

$$0 = m^2 \partial_\mu \vec{\rho}_\mu - g\,\partial_\mu (\vec{G}_{\mu\nu} \times \vec{\rho}_\nu) + \partial_\mu \frac{\partial F}{\partial \vec{\rho}_\mu} . \quad (2.4)$$

Substituting (2.2) into (2.4) and utilizing $\partial_\mu \vec{S}_\mu = 0$, one obtains

$$-m^2 \partial_\mu \vec{\rho}_\mu = \partial_\mu \left[\frac{\partial F}{\partial \vec{\rho}_\mu} - g^2 (\vec{\rho}_\mu \times \vec{\rho}_\nu) \times \vec{\rho}_\nu \right] , \quad (2.5)$$

so $\partial_\mu \vec{\rho}_\mu = 0$ will be satisfied if F is so chosen that

$$\frac{\partial F}{\partial \vec{\rho}_\mu} = g^2 (\vec{\rho}_\mu \times \vec{\rho}_\nu) \times \vec{\rho}_\nu . \quad (2.6)$$

This is achieved for

$$F = -\tfrac{1}{4} g^2 (\vec{\rho}_\mu \times \vec{\rho}_\nu)^2 . \quad (2.7)$$

To conclude: The Lagrangian for the vector field dictated by the necessity of having a $(G^1_{\mu\nu}\rho^2_\nu - G^2_{\mu\nu}\rho^1_\nu)\rho^2_\mu$ in it and being in conformity with $\partial_\mu \vec{\rho}_\mu = 0$ is given by

$$\mathcal{L} = -\tfrac{1}{4}(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2} m^2 (\vec{\rho}_\mu)^2 + \tfrac{1}{2} g\, \vec{G}_{\mu\nu} \cdot (\vec{\rho}_\mu \times \vec{\rho}_\nu) - \tfrac{1}{4} g^2 (\vec{\rho}_\mu \times \vec{\rho}_\nu)^2 . \quad (2.8)$$

Let us now turn to the introduction of em interaction terms into (2.8) which satisfy the field-current proportionality relation as well as the gauge-invariance constraints. This, according to Ref. 2, is achieved through the replacement

$$\rho^3_\mu \to \rho^3_\mu + (e/g) A_\mu \quad (2.9)$$

in all terms of (2.8) except in the mass term $\frac{1}{2} m^2 (\vec{\rho}_\mu)^2$. To show this, consider that piece $\mathcal{L}_3$ of the Lagrangian (2.8) containing ρ^3_μ and its derivatives,

$$\mathcal{L}_3 = -\tfrac{1}{4}(G^3_{\mu\nu})^2 + \tfrac{1}{2} m^2 (\rho^3_\mu)^2 + \tfrac{1}{2} g\, G^3_{\mu\nu} h^3_{\mu\nu} + \tfrac{1}{2} g\, G^2_{\mu\nu} h^2_{\mu\nu} + \tfrac{1}{2} g\, G^1_{\mu\nu} h^1_{\mu\nu} - \tfrac{1}{4} g^2 (h^1_{\mu\nu})^2 - \tfrac{1}{4} g^2 (h^2_{\mu\nu})^2 , \quad (2.10)$$

where $\vec{h}_{\mu\nu} \equiv \vec{\rho}_\mu \times \vec{\rho}_\nu$ and where $h^{1,2}_{\mu\nu}$ contain ρ^3_μ (linearly) and $h^3_{\mu\nu}$ does *not* contain ρ^3_μ or derivatives thereof. So we can write

$$\mathcal{L}_3 = -\tfrac{1}{4}(G^3_{\mu\nu})^2 + \tfrac{1}{2} g\, G^3_{\mu\nu} h^3_{\mu\nu} + \tfrac{1}{2} m^2 (\rho^3_\mu)^2 - J(\rho^3_\mu) \quad (2.11)$$

with

$$J(\rho^3_\mu) \equiv g J_\mu \rho^3_\mu + g^2 J_{\mu\nu} \rho^3_\mu \rho^3_\nu , \quad (2.12)$$

where J_μ and $J_{\mu\nu} = J_{\nu\mu}$ do not depend on ρ^3_μ or its derivatives. The equations of motion for ρ^3_μ obtained from (2.11) are

$$\partial_\nu (G^3_{\nu\mu} - g h^3_{\nu\mu}) + m^2 \rho^3_\mu = \frac{\partial J}{\partial \rho^3_\mu} = g J_\mu + 2g^2 J_{\mu\nu} \rho^3_\nu \equiv j^3_\mu . \quad (2.13)$$

Because of $\partial_\mu \rho^3_\mu = 0$ and the antisymmetry of $G^3_{\mu\nu}, h^3_{\mu\nu}$ we have $\partial_\mu j^3_\mu = 0$. Substituting (2.9) into (2.11) and (2.12), one obtains the following interaction between ρ_μ and A_μ:

$$\mathcal{L}_{\rho-A} = -\frac{1}{2} \frac{e}{g} G^3_{\mu\nu} F_{\mu\nu} + \frac{1}{2} \frac{e}{g} g F_{\mu\nu} h^3_{\mu\nu} - \frac{e}{g} A_\mu j^3_\mu + O(e^2) , \quad (2.14)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

The total Lagrangian is therefore

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + \mathcal{L}_{\rho-A} + \mathcal{L}_3 . \quad (2.15)$$

$\mathcal{L}$ is gauge-invariant since in $\mathcal{L}_{\rho-A}$, A_μ couples only to the conserved current j^3_μ. Moreover, from (2.15) one obtains the following equation of motion for $F_{\mu\nu}$:

$$\partial_\nu F_{\mu\nu} + \frac{e}{g} \partial_\nu G^3_{\mu\nu} - g \frac{e}{g} \partial_\nu h^3_{\mu\nu} = -\frac{e}{g} j^3_\mu . \quad (2.16)$$

Utilizing (2.13) and the antisymmetry of $F_{\mu\nu}$, one finally obtains

$$\partial_\nu F_{\nu\mu} = (e m^2/g) \rho^3_\mu , \quad (2.17)$$

i.e.,

$$j^{\rm em}_\mu = (e m^2/g) \rho^3_\mu \quad (2.18)$$

as expected.

Let us now consider the explicit form of that part of $\mathcal{L}_{\rho-A}$ which corresponds to a $\rho\rho\gamma$ vertex. This part is obtainable directly by substituting (2.9) into the third term of (2.8), and gives—after some straightforward algebra, partial integrations, and the use of $\partial_\mu \vec{\rho}_\mu = 0$—the following result:

$$\mathcal{L}_{\rho\rho\gamma} = e A_\mu (\rho^1_\nu \overleftrightarrow{\partial}_\mu \rho^2_\nu) + 2e F_{\mu\nu} \rho^1_\mu \rho^2_\nu . \quad (2.19)$$

It is easily seen by taking $\rho^\pm = (\rho_1 \pm i\rho_2)/\sqrt{2}$ that this vertex corresponds to $\mu_\rho = 2, \kappa_\rho = 0$ in the usual definition[7] of the em vertex of charged vector mesons:

$$\mathcal{L}_{\rho\rho\gamma} = e[A_\mu(\rho_\nu^\dagger \overleftrightarrow{\partial}_\mu \rho_\nu) + \mu_\rho F_{\mu\nu}\rho_\mu^\dagger \rho_\nu + \tfrac{1}{2}(\kappa_\rho/m^2)(\partial_\alpha F_{\mu\beta} + \partial_\beta F_{\mu\alpha})(\rho_\alpha^\dagger \overleftrightarrow{\partial}_\mu \rho_\beta)], \tag{2.20}$$

where μ_ρ is t e magnetic moment of the charged ρ meson in ρ magnetons and

$$\kappa_\rho = -\tfrac{1}{2}(Q_\rho + \mu_\rho - 1), \tag{2.21}$$

with Q_ρ the quadrupole moment of the ρ in units of $1/m^2$. We conclude therefore that the effective-Lagrangian formulation of vector dominance in the one-meson case not only determines the Lagrangian (2.8) but also the $\rho\rho\gamma$ vertex uniquely.

III. EFFECTIVE-LAGRANGIAN FORMULATION OF GENERALIZED VECTOR DOMINANCE–THE TWO-MESON CASE

In the spirit of the effective-Lagrangian approach,[5] one introduces two fields ρ_μ^i and $\rho_\mu'^i$, with $i = 1, 2, 3$, $\mu = 0, 1, 2, 3$ as before. The Lagrangian can now also contain in addition to the previously mentioned $(\vec{G}_{\mu\nu} \times \vec{\rho}_\nu)\cdot\vec{\rho}_\mu$ term, terms representing the interaction of the charged ρ' meson with the em field via a ρ_0 meson, i.e., terms of the form $(G_{\mu\nu}'^1\rho_\nu'^2 - G_{\mu\nu}'^2\rho_\nu'^1)\rho_\mu^3$, leading to the isospin-invariant interaction term $(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu)\cdot\vec{\rho}_\mu$. Other conceivable interaction terms are $(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu)\cdot\vec{\rho}'_\mu$, $(\vec{G}_{\mu\nu}\times\vec{\rho}'_\nu)\cdot\vec{\rho}'_\mu$, and $(\vec{G}'_{\mu\nu}\times\vec{\rho}_\nu)\cdot\vec{\rho}_\mu$. As in Sec. II we now assume a strong-interaction Lagrangian including these mixing terms and try to construct it in such a way as to guarantee

$$\partial_\mu(m^2\vec{\rho}_\mu + m'^2\vec{\rho}'_\mu) = 0. \tag{3.1}$$

This we do in the expectation that after the introduction of the em interaction it will turn out, as a generalization of (2.18), that

$$j_\mu^{\text{em}} = (e/g)(m^2\rho_\mu^3 + m'^2\rho_\mu'^3). \tag{3.2}$$

Equation (3.2) will now be considered as a generalized field-current proportionality relation or as a formal expression of generalized vector dominance.[8]

We shall begin with a strong-interaction Lagrangian containing only a restricted subset of the mixed interaction terms, namely, with the Lagrangian

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \tfrac{1}{2}g\vec{G}_{\mu\nu}\cdot(\vec{\rho}_\mu\times\vec{\rho}_\nu)\\ &+\tfrac{1}{2}\alpha g\vec{G}_{\mu\nu}\cdot(\vec{\rho}'_\mu\times\vec{\rho}'_\nu) + (\text{same with } \rho\leftrightarrow\rho',\ m\leftrightarrow m')\\ &-\tfrac{1}{2}\alpha\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu} + F(\vec{\rho}_\mu,\vec{\rho}'_\nu).\end{aligned} \tag{3.3}$$

The direct mixing term $\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu}$ in (3.3) is required, as will be shown immediately, for the achievement of (3.1).

In analogy with (2.2) we now construct

$$\begin{aligned}\vec{S}_\mu &= \frac{\partial\mathcal{L}}{\partial\vec{\rho}_{\nu,\mu}}\times\vec{\rho}_\nu\\ &= [-\vec{G}_{\mu\nu} - \alpha\vec{G}'_{\mu\nu} + g(\vec{\rho}_\mu\times\vec{\rho}_\nu) + \alpha g(\vec{\rho}'_\mu\times\vec{\rho}'_\nu)]\times\vec{\rho}_\nu\\ \vec{S}'_\mu &= \frac{\partial\mathcal{L}}{\partial\vec{\rho}'_{\nu,\mu}}\times\vec{\rho}'_\nu\\ &= [\text{same with } \rho\leftrightarrow\rho'].\end{aligned} \tag{3.4}$$

Isospin invariance of (3.3) now implies

$$\partial_\mu(\vec{S}_\mu + \vec{S}'_\mu) = 0. \tag{3.5}$$

The equations of motion implied by (3.3) are

$$\begin{aligned}\partial_\nu\left(\frac{\partial\mathcal{L}}{\partial\vec{\rho}_{\mu,\nu}}\right) &= m^2\vec{\rho}_\mu - g(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu) - \alpha g(\vec{G}'_{\mu\nu}\times\vec{\rho}_\nu) + \frac{\partial F}{\partial\vec{\rho}_\mu},\\ \partial_\nu\left(\frac{\partial\mathcal{L}}{\partial\vec{\rho}'_{\mu,\nu}}\right) &= m'^2\vec{\rho}'_\mu - g(\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu) - \alpha g(\vec{G}_{\mu\nu}\times\vec{\rho}'_\nu) + \frac{\partial F}{\partial\vec{\rho}'_\mu}.\end{aligned} \tag{3.6}$$

Since $\partial\mathcal{L}/\partial\vec{\rho}_{\nu,\mu}$ and $\partial\mathcal{L}/\partial\vec{\rho}'_{\nu,\mu}$ are antisymmetric in μ and ν, one has

$$\partial_\mu\partial_\nu\frac{\partial\mathcal{L}}{\partial\vec{\rho}_{\nu,\mu}} = \partial_\mu\partial_\nu\frac{\partial\mathcal{L}}{\partial\vec{\rho}'_{\nu,\mu}} = 0$$

and so

$$\begin{aligned}0 = &\partial_\mu(m^2\vec{\rho}_\mu + m'^2\vec{\rho}'_\mu)\\ &- g\partial_\mu[(\vec{G}_{\mu\nu}\times\vec{\rho}_\nu) + \alpha(\vec{G}'_{\mu\nu}\times\vec{\rho}_\nu) + (\vec{G}'_{\mu\nu}\times\vec{\rho}'_\nu) + \alpha(\vec{G}_{\mu\nu}\times\vec{\rho}'_\nu)]\\ &+\partial_\mu\left(\frac{\partial F}{\partial\vec{\rho}_\mu} + \frac{\partial F}{\partial\vec{\rho}'_\mu}\right).\end{aligned} \tag{3.7}$$

Substituting from (3.4) and (3.5) into (3.7), one obtains that (3.1) will be satisfied if F is so chosen that

$$\frac{\partial F}{\partial\vec{\rho}_\mu} + \frac{\partial F}{\partial\vec{\rho}'_\mu} = g^2[(\vec{\rho}_\mu\times\vec{\rho}_\nu) + \alpha(\vec{\rho}'_\mu\times\vec{\rho}'_\nu)]\times\vec{\rho}_\nu + [\text{same with } \rho\leftrightarrow\rho'], \tag{3.8}$$

i.e., for

$$F = -\tfrac{1}{4}g^2(\vec{\rho}_\mu\times\vec{\rho}_\nu)^2 - \tfrac{1}{2}\alpha g^2(\vec{\rho}_\mu\times\vec{\rho}_\nu)\cdot(\vec{\rho}'_\mu\times\vec{\rho}'_\nu) - \tfrac{1}{4}g^2(\vec{\rho}'_\mu\times\vec{\rho}'_\nu)^2. \tag{3.9}$$

The Lagrangian (3.3) and (3.9) satisfying (3.1) has, as it stands, an unpleasant property, namely the mixing term $\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu}$ in its kinetic-energy part. One has now to diagonalize the free part of the Lagrangian

$$\mathcal{L}_{\text{free}} = -\tfrac{1}{4}(\vec{G}_{\mu\nu})^2 - \tfrac{1}{2}\alpha\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu} - \tfrac{1}{4}(\vec{G}'_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \tfrac{1}{2}m'^2(\vec{\rho}'_\mu)^2 \tag{3.10}$$

by means of a linear transformation of the fields $\vec{\rho}_\mu$

and $\vec{\rho}'_\mu$. The appropriate transformation is

$$\varphi = \frac{1}{M} T \frac{1}{\sqrt{\lambda}} \varphi_t , \qquad (3.11)$$

where

$$\varphi = \begin{pmatrix} \vec{\rho}_\mu \\ \vec{\rho}'_\mu \end{pmatrix}, \quad \varphi_t = \begin{pmatrix} \vec{\rho}^a_\mu \\ \vec{\rho}^b_\mu \end{pmatrix}, \quad M = \begin{pmatrix} m & 0 \\ 0 & m' \end{pmatrix}, \quad TT^\dagger = 1$$

and where

$$\lambda = \begin{pmatrix} \lambda_a & 0 \\ 0 & \lambda_b \end{pmatrix} = T^\dagger K T . \qquad (3.12)$$

The matrix K represents the kinetic-energy part expressed in terms of the dilated field $\varphi_d = M\varphi$, i.e.,

$$K = \begin{pmatrix} \dfrac{1}{m^2} & \dfrac{\alpha}{mm'} \\ \dfrac{\alpha}{mm'} & \dfrac{1}{m'^2} \end{pmatrix}. \qquad (3.13)$$

The free Lagrangian in terms of the transformed fields $\vec{\rho}^a_\mu, \vec{\rho}^b_\mu$ has the standard form

$$\mathcal{L}_{\text{free}} = -\tfrac{1}{4}(\vec{G}^a_{\mu\nu})^2 + \tfrac{1}{2} m_a{}^2 (\vec{\rho}^a_\mu)^2 - \tfrac{1}{4}(\vec{G}^b_{\mu\nu})^2 + \tfrac{1}{2} m_b{}^2 (\vec{\rho}^b_\mu)^2 , \qquad (3.14)$$

with

$$m_a = (\lambda_a)^{-1/2}, \quad m_b = (\lambda_b)^{-1/2} . \qquad (3.15)$$

To obtain the matrices λ and T, we express K in terms of the Pauli and the unit matrices:

$$K = \frac{1}{2}\left(\frac{1}{m^2} + \frac{1}{m'^2}\right) I + \frac{1}{2}\left(\frac{1}{m^2} - \frac{1}{m'^2}\right)\sigma_3 + \frac{\alpha}{mm'}\sigma_1 . \qquad (3.16)$$

Utilizing the Pauli algebra, we can easily see that

$$T = \exp(i\tfrac{1}{2}\theta\sigma_2) = \cos\tfrac{1}{2}\theta + i \sin\tfrac{1}{2}\theta\, \sigma_2 , \qquad (3.17)$$

where

$$\theta = \arctan(\beta/\gamma)$$

and

$$\beta = \frac{\alpha}{mm'}, \quad \gamma = \frac{1}{2}\left(\frac{1}{m^2} - \frac{1}{m'^2}\right).$$

The resulting matrix λ is

$$\lambda = \frac{1}{2}\left(\frac{1}{m^2} + \frac{1}{m'^2}\right) I + (\beta^2 + \gamma^2)^{1/2}\sigma_3 . \qquad (3.18)$$

A particularly simple result is obtained for the special case $m = m'$. Here

$$\beta = \alpha/m^2, \quad \gamma = 0, \quad \theta = \tfrac{1}{2}\pi, \quad T = 2^{-1/2}(1 + i\sigma_2),$$
$$\lambda = \frac{1}{m^2} I + \frac{\alpha}{m^2}\sigma_3 = \frac{1}{m^2}\begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix}. \qquad (3.19)$$

With (3.10) transformed into (3.14) with the aid of (3.11), it is obvious that also the interaction in (3.3) will undergo a corresponding transformation. It is, however, not necessary for our purposes to exhibit the transformed interaction terms explicitly. Rather, we shall now introduce the em interaction and investigate its gauge invariance and the generalized vector-dominance relation (3.2) in terms of the original fields $\vec{\rho}_\mu$ and $\vec{\rho}'_\mu$.

The em interaction will, in analogy to Sec. II, be introduced through the replacement

$$\rho^3_\mu \to \rho^3_\mu + (e/g) A_\mu ,$$
$$\rho'^3_\mu \to \rho'^3_\mu + (e/g) A_\mu \qquad (3.20)$$

in all terms of (3.3) except the mass terms. To demonstrate the gauge invariance of the resulting em interaction, one again considers that part $\mathcal{L}_3$ of the Lagrangian (3.3) containing ρ^3_μ, ρ'^3_μ and their derivatives:

$$\mathcal{L}_3 = -\tfrac{1}{4}(G^3_{\mu\nu})^2 - \tfrac{1}{2}\alpha G^3_{\mu\nu} G'^3_{\mu\nu} - \tfrac{1}{4}(G'^3_{\mu\nu})^2 + \tfrac{1}{2} g G^3_{\mu\nu}(h^3_{\mu\nu} + \alpha h'^3_{\mu\nu}) + \tfrac{1}{2} g G'^3_{\mu\nu}(h'^3_{\mu\nu} + \alpha h^3_{\mu\nu}) + \tfrac{1}{2} m^2 (\rho^3_\mu)^2 + \tfrac{1}{2} m'^2 (\rho'^3_\mu)^2 + J(\rho^3_\mu, \rho'^3_\mu) , \qquad (3.21)$$

where

$$-J = \tfrac{1}{2} g (G^2_{\mu\nu} h^2_{\mu\nu} + G^1_{\mu\nu} h^1_{\mu\nu}) + \tfrac{1}{2} g (G'^2_{\mu\nu} h'^2_{\mu\nu} + G'^1_{\mu\nu} h'^1_{\mu\nu}) + \tfrac{1}{2}\alpha g (G^2_{\mu\nu} h'^2_{\mu\nu} + G^1_{\mu\nu} h'^1_{\mu\nu}) + \tfrac{1}{2}\alpha g (G'^2_{\mu\nu} h^2_{\mu\nu} + G'^1_{\mu\nu} h^1_{\mu\nu}) - \tfrac{1}{4} g^2 [(h^1_{\mu\nu})^2 + (h^2_{\mu\nu})^2 - 2\alpha (h^1_{\mu\nu} h'^1_{\mu\nu} + h^2_{\mu\nu} h'^2_{\mu\nu}) + (h'^1_{\mu\nu})^2 + (h'^2_{\mu\nu})^2] \qquad (3.22)$$

and

$$\vec{h}'_{\mu\nu} = \vec{\rho}'_\mu \times \vec{\rho}'_\nu . \qquad (3.23)$$

As before and because of its symmetry under $\rho \leftrightarrow \rho'$, J can be written in the form

$$J = g J_\mu (\rho^3_\mu + \rho'^3_\mu) + g^2 J_{\mu\nu} (\rho^3_\mu \rho^3_\nu + \rho'^3_\mu \rho'^3_\nu) + g^2 J'_{\mu\nu} (\rho^3_\mu \rho'^3_\nu + \rho'^3_\mu \rho^3_\nu) , \qquad (3.24)$$

where J_μ, $J_{\mu\nu} = J_{\nu\mu}$, $J'_{\mu\nu} = J'_{\nu\mu}$ do not depend on ρ^3_μ, ρ'^3_μ or their derivataves. The equations of motion obtained from (3.21) are

$$\partial_\nu (G^3_{\nu\mu} + \alpha G'^3_{\nu\mu} - g h^3_{\nu\mu} - \alpha g h'^3_{\nu\mu}) - m^2 \rho^3_\mu = \frac{\partial J}{\partial \rho^3_\mu} \equiv j^3_\mu ,$$
$$\partial_\nu (G'^3_{\nu\mu} + \alpha G^3_{\nu\mu} - g h'^3_{\nu\mu} - \alpha g h^3_{\nu\mu}) - m'^2 \rho'^3_\mu = \frac{\partial J}{\partial \rho'^3_\mu} \equiv j'^3_\mu . \qquad (3.25)$$

Because of the antisymmetry of $G^3_{\mu\nu}$, $G'^3_{\mu\nu}$, $h^3_{\mu\nu}$, $h'^3_{\mu\nu}$ and of (3.1), we obtain

$$\partial_\mu (j^3_\mu + j'^3_\mu) = 0 . \qquad (3.26)$$

Now the substitution of (3.20) into (3.3), except in the mass terms, gives the following interaction between ρ, ρ' and A:

$$\mathcal{L}_{\rho-A} = -\frac{1}{2}\frac{e}{g}F_{\mu\nu}[G^3_{\mu\nu} + \alpha(G'^3_{\mu\nu} + G^3_{\mu\nu}) + G'^3_{\mu\nu} - g(h^3_{\mu\nu} + h'^3_{\mu\nu}) - \alpha g(h^3_{\mu\nu} + h'^3_{\mu\nu})] - (e/g)A_\mu(j^3_\mu + j'^3_\mu) + O(e^2). \tag{3.27}$$

Again $\mathcal{L}_{\rho-A}$ is gauge-invariant since A_μ couples to a conserved current. It is also not difficult to see how, in analogy to the derivation of (2.18) in Sec. II, one derives its generalization (3.2) from the equations of motion for A_μ following from the Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 + \mathcal{L}_{\rho-A} + \mathcal{L}_3 \tag{3.28}$$

and from the equations of motion (3.25).

We now consider, as in Sec. II, that part of $\mathcal{L}_{\rho-A}$ which corresponds to a vector-meson–vector-meson–photon vertex. This part is obtainable directly by substituting (3.20) into the third and fourth terms in (3.3) and also into the corresponding $\rho \leftrightarrow \rho'$ terms there. After some straightforward algebra and partial integrations the following expression is obtained:

$$\begin{aligned}\mathcal{L}_{VV\gamma} = {} & eA_\mu\{(\rho^1_\nu \overleftrightarrow{\partial}_\mu \rho^2_\nu) + \alpha[(\rho'^1_\nu \overleftrightarrow{\partial}_\mu \rho^2_\nu) + (\rho^1_\nu \overleftrightarrow{\partial}_\mu \rho'^2_\nu)] + (\rho'^1_\nu \overleftrightarrow{\partial}_\mu \rho'^2_\nu)\} \\ & + eF_{\mu\nu}[(2+\alpha)\rho^1_\mu\rho^2_\nu + \alpha(\rho'^1_\mu\rho^2_\nu + \rho^1_\mu\rho'^2_\nu) + (2+\alpha)\rho'^1_\mu\rho'^2_\nu] \\ & + eA_\mu\{[(\partial_\nu\rho^1_\nu)\rho^2_\mu - (\partial_\nu\rho^2_\nu)\rho^1_\mu] + \alpha[(\partial_\nu\rho'^1_\nu)\rho^2_\mu - (\partial_\nu\rho'^2_\nu)\rho^1_\mu + (\partial_\nu\rho^1_\nu)\rho'^2_\mu - (\partial_\nu\rho^2_\nu)\rho'^1_\mu] + [(\partial_\nu\rho'^1_\nu)\rho'^2_\mu - (\partial_\nu\rho'^2_\nu)\rho'^1_\mu]\}.\end{aligned} \tag{3.29}$$

The term in the last curly bracket in (3.29) is not a genuine $VV\gamma$ term since the divergence terms $\partial_\nu\rho^i_\nu, \partial_\nu\rho'^i_\nu$, which vanish for $\alpha=0$, are expressible for $\alpha\neq 0$ through Eqs. (3.4)–(3.6) in terms of expressions which are at least bilinear in the vector fields. The first two terms in (3.29) when expressed in terms of $\vec{\rho}^{\,a}_\mu, \vec{\rho}^{\,b}_\mu$ in the special case $m = m'$ give the following $VV\gamma$ vertex:

$$\begin{aligned}\mathcal{L}_{VV\gamma} = {} & eA_\mu[(\rho^{a,1}_\nu \overleftrightarrow{\partial}_\mu \rho^{a,2}_\nu) + (\rho^{b,1}_\nu \overleftrightarrow{\partial}_\mu \rho^{b,2}_\nu)] \\ & + eF_{\mu\nu}\{2\rho^{a,1}_\mu\rho^{a,2}_\nu + [2/(1-\alpha)]\rho^{b,1}_\mu\rho^{b,2}_\nu\},\end{aligned} \tag{3.30}$$

According to (3.15) and (3.19), one has $m_a = m(1+\alpha)^{-1/2}$, $m_b = m(1-\alpha)^{-1/2}$; hence for $\alpha>0$, $m_a<m_b$, $\mu(\rho^a)=2$, $\mu(\rho^b)=2/(1-\alpha)>2$, i.e., the lighter vector meson has magnetic moment 2, while the heavier one carries a magnetic moment which is greater than 2. On the other hand, for $\alpha<0$, $m_a>m_b$, $\mu(\rho^a)=2$, $\mu(\rho^b)=2/(1+|\alpha|)<2$, i.e., the lighter vector meson carries, in this case, a magnetic moment smaller than 2.

The whole reasoning of this section can be repeated, with only minor modifications, for a hadronic Lagrangian containing a different subset of mixed interaction terms:

$$\begin{aligned}\mathcal{L}_b = {} & -\tfrac{1}{4}(1+\alpha)(\vec{G}_{\mu\nu})^2 + \tfrac{1}{2}m^2(\vec{\rho}_\mu)^2 + \tfrac{1}{2}g\vec{G}_{\mu\nu}\cdot(\vec{\rho}_\mu\times\vec{\rho}_\nu) \\ & + \tfrac{1}{2}\alpha g\vec{G}_{\mu\nu}[(\vec{\rho}_\mu\times\vec{\rho}'_\nu) - (\vec{\rho}'_\mu\times\vec{\rho}_\nu)] \\ & + (\text{same with } \rho\leftrightarrow\rho',\ m\leftrightarrow m') \\ & - \tfrac{1}{2}\alpha\vec{G}_{\mu\nu}\cdot\vec{G}'_{\mu\nu} + F_b(\vec{\rho}_\mu,\vec{\rho}'_\nu).\end{aligned} \tag{3.3b}$$

Again it can be shown that (3.3b) leads to (3.1), that electromagnetic interactions can be introduced in a gauge-invariant way through (3.20), and that the field-current proportionality relation (3.2) is also satisfied here.

The diagonalization matrices are now somewhat different since the kinetic energy matrix now has the form

$$K_b = \begin{pmatrix} (1+\alpha)\dfrac{1}{m^2} & \dfrac{\alpha}{mm'} \\ \dfrac{\alpha}{mm'} & (1+\alpha)\dfrac{1}{m'^2}\end{pmatrix}, \tag{3.13b}$$

so in this case $\beta=\alpha/mm'$ as before but

$$\gamma_b = \frac{1+\alpha}{2}\left(\frac{1}{m^2} - \frac{1}{m'^2}\right). \tag{3.17b}$$

The matrix λ now has the form

$$\lambda_b = \frac{1+\alpha}{2}\left(\frac{1}{m^2} + \frac{1}{m'^2}\right)I + (\beta^2+\gamma^2)^{1/2}\sigma_3. \tag{3.18b}$$

For the special case $m=m'$ we now have

$$\lambda_b = \frac{1}{m^2}\begin{pmatrix}1+2\alpha & 0 \\ 0 & 1\end{pmatrix}. \tag{3.19b}$$

The vector-meson–vector-meson–photon vertex resulting from substituting (3.20) in (3.3b) is now

$$\begin{aligned}\mathcal{L}_{b,VV\gamma} = {} & eA_\mu\{(1+\alpha)(\rho^1_\nu \overleftrightarrow{\partial}_\mu \rho^2_\nu) + \alpha[(\rho'^1_\nu \overleftrightarrow{\partial}_\mu \rho^2_\nu) + (\rho^1_\nu \overleftrightarrow{\partial}_\mu \rho'^2_\nu)] + (1+\alpha)(\rho'^1_\nu \overleftrightarrow{\partial}_\mu \rho'^2_\nu)\} \\ & + eF_{\mu\nu}[(2+\alpha)\rho^1_\mu\rho^2_\nu + 3\alpha(\rho'^1_\mu\rho^2_\nu + \rho^1_\mu\rho'^2_\nu) + (2+\alpha)\rho'^1_\mu\rho'^2_\nu].\end{aligned} \tag{3.29b}$$

In (3.29b) we have already omitted the divergence terms $(\partial_\nu \rho_\nu^1)\rho_\mu^2 A_\mu$, etc., which, as before, are not genuine $VV\gamma$ terms.

When expressed in terms of $\vec{\rho}_\mu^a$ and $\vec{\rho}_\mu^b$, for the special case $m = m'$, (3.29b) takes the form

$$\mathcal{L}_{b,VV\gamma} = eA_\mu[(\rho_\nu^{a,1}\overleftrightarrow{\partial}_\mu\rho_\nu^{a,2}) + (\rho_\nu^{b,1}\overleftrightarrow{\partial}_\mu\rho_\nu^{b,2})] + eF_{\mu\lambda}[2\rho_\mu^{a,1}\rho_\nu^{a,2} + (2-2\alpha)\rho_\mu^{b,1}\rho_\nu^{b,2}]. \quad (3.30b)$$

According to (3.15) and (3.19b), one has in this case $m_a = m(1+2\alpha)^{-1/2}$, $m_b = m$; hence for $\alpha > 0$, $m_a < m_b$, $\mu(\rho^a) = 2$, $\mu(\rho^b) = 2 - 2\alpha < 2$, i.e., the lighter vector meson has magnetic moment 2 and the heavier vector meson carries a magnetic moment which is smaller than 2. On the other hand, for $\alpha < 0$, $m_a > m_b$, $\mu(\rho^a) = 2$, $\mu(\rho^b) = 2 - 2\alpha > 2$, i.e., in this case the lighter vector meson carries a magnetic moment which is *greater* than 2, in contrast with the conjecture of Shtokhamer and Singer.[4] Our study of the simplified situation $m = m'$ in (3.3) and (3.3b) already showed that the lightest charged vector meson can carry any magnetic moment, depending on the mixing scheme and on the mixing parameter α. For the general case $m \neq m'$, one has also nondiagonal magnetic terms, i.e., terms of the form $F_{\mu\nu}\rho_\mu^{a,1}\rho_\nu^{b,2}$, in (3.30) and (3.30b). The A_μ terms always diagonalize to the unit matrix, as can be easily seen and as is obviously required.

The inclusion of other fields does not pose serious problems and follows closely the treatment in Ref. 2. Let ψ represent all other matter fields and let $\mathcal{L}_m(\psi, \partial_\mu\psi)$ be the isospin-invariant matter Lagrangian. Then the total strong Lagrangian is given by $\mathcal{L}_\rho + \mathcal{L}_m(\psi, D_\mu\psi)$, where $\mathcal{L}_\rho$ is our previous (3.3) or (3.3b), and where $D_\mu\psi$ is related to the matrix representation $-i\vec{T}$ of the isospin generators on ψ by

$$D_\mu\psi = \partial_\mu\psi + \vec{T}\cdot(G\vec{\rho}_\mu + G'\vec{\rho}_\mu')\psi, \quad (3.31)$$

with

$$G + G' = g. \quad (3.32)$$

The em interaction with the additional matter fields is now obtained by using (3.20) in $\mathcal{L}_m$. It is easy to show, following the reasoning of Ref. 2 and of this section, that gauge invariance and the relation (3.2) hold unchanged.

IV. DISCUSSION

The introduction of *mutually interacting* vector mesons has been shown to be compatible with the vector-dominance idea. In fact the latter allows for more than one form of interaction between the vector mesons. In Sec. III only the simplest case with two vector mesons was analyzed. However, this analysis can, in principle, be extended—without serious modifications—to any number of vector mesons.

We have also seen that the interaction scheme between the vector mesons strongly influences their magnetic couplings to the em field. These can in principle be studied in photoproduction experiments.[9] Specifically, one looks at $\gamma N \to \rho^\pm N$ or $\gamma N \to \rho^\pm \Delta$. There are strong indications[10] that vector-meson exchanges play a significant role in these processes. This provides a good opportunity to study the vertices $\rho\gamma\rho$, $\rho\gamma\rho'$. In the simplest cases, $\alpha = 0$ or $m = m'$, only the $\rho\gamma\rho$ vertex appears since there are no mixing terms in (3.30) and (3.30b). Generally, however, the possible relevance of $\rho', \rho'', \ldots$ exchanges has to be kept in mind.

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