*This work was supported in part by the National Science Foundation.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 4 15 AUGUST 1974

Current quarks and constituent quarks: Symmetry breaking and interaction*

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(Received 11 March 1974)

A study is made of the question of how different the correct transformation V from the current-quark to the constituent-quark basis is expected to be from the free mass-degenerate quark model discussed by Melosh. The effect of SU(3) mass breaking and of mutual interaction of the quarks is discussed in the context of a simple model. The algebraic properties of V are more complicated than that of V_{free}, the transformation constructed by Melosh; nevertheless, it still is tractable enough so an attack may be made on the problem of mass splitting in SU(6) multiplets.

There has been much discussion recently[1] of the relation between current quarks and constituent quarks. On the one hand, there is the SU(6) algebra of integrated weak and electromagnetic current densities[2] and related operators, which is denoted by $\text{SU}(6)_{W,\,\text{currents}}$; on the other hand, there is the SU(6) algebra of operators which form an approximate symmetry of the strong-interaction Hamiltonian,[3] which is denoted by $\text{SU}(6)_{W,\,\text{strong}}$. Assuming these two algebras to be connected by a unitary transformation V, there are several requirements that this V must satisfy; Melosh has described these requirements and furthermore, for the free-quark model with degenerate quark masses, he has constructed an operator V_{free} which satisfies the constraints. Although V_{free} most certainly does not have all the correct properties that V must have, by abstracting some of the algebraic structure of V_{free} which might reasonably be expected to carry over in a more realistic situation one may make predictions for pionic decays of meson and baryon resonances, recover many of the good results of the old $\text{SU}(6)_W$ scheme for the matrix elements of weak charges, and correct some of the poor results.

The basic problem which remains, however, is the determination of how different one should expect the correct transformation V to be from the explicitly constructed model transformation V_{free}. For example, the extremely simple property of

V_{free} that the axial charge Q_i^5 transforms as a sum of $(8, 1)+(1, 8)$ and $(3, \bar{3})+(\bar{3}, 3)$ representations of $SU(3)\times SU(3)_{\text{strong}}$ might not be generally true. It is surely astonishing that the series should terminate in but two terms.

In particular, we will address ourselves to the question of SU(3) breaking in the free-quark model, to see what effect this has on V; furthermore, we will study a simple model with interaction.

The transformation V is required to satisfy the following conditions (for notation, see Melosh, Ref. 1):

(i) $V^\dagger = V^{-1}$,

(ii) $[\mathcal{J}_1, VF_i^1 V^{-1}]=0$,
$[\mathcal{J}_1,[\mathcal{J}_1, VF_i^3 V^{-1}]]=VF_i^3 V^{-1}$,
$[\vec{\mathcal{J}}, VF_i V^{-1}]=0$,

(iii) V takes good operators into good operators,

(iv) $VF_i V^{-1}=F_i$ in the SU(3) limit,

(v) V has $C=+$, $R=+$,

(vi) $[J_3, V]=0$,

(vii) $[\Lambda_3, V]=0$.

How must V_0 be modified in order to be sure that the W_i, which are the $SU(3)_{\text{strong}}$ generators, do not change particle spin even if SU(3) is broken? That is, how do we satisfy the angular condition

$$[\vec{\mathcal{J}}, W_i]=0 , \tag{1}$$

where

$$W_i = VF_i V^{-1} ? \tag{2}$$

Clearly, Eq. (1) will be satisfied if

$$[V^{-1}\vec{\mathcal{J}}V, F_i]=0 , \tag{3}$$

which means that the transformed angular momentum operators, $V^{-1}\vec{\mathcal{J}}V$, must be invariant under $SU(3)_{\text{currents}}$. Since $\vec{\mathcal{J}}$ itself is no longer invariant, as for example

$$\mathcal{J}_1 = J_1\frac{P_0}{M}+\Lambda_2\frac{P_3}{M} , \tag{4}$$

we see that V must be chosen so as to eliminate all SU(3) breaking, which in the free-quark model is due to the inequality of the proton- and neutron-like quark masses, $m_{\mathcal{P}}$ and $m_{\mathfrak{N}}$, with that of the λ quark, m_λ. Denote the masses of the quarks by the usual 3×3 matrix $\underline{m}=\text{diag}(m_{\mathcal{P}}, m_{\mathfrak{N}}, m_\lambda)$. Thus, if $V^{-1}\vec{\mathcal{J}}V$ has any dependence on m then it will not commute with all the F_i. This is clear, because all operators are built out of expressions bilinear in quark fields; these bilinear forms must transform as either SU(3) singlets or the 8th component of an octet; if any of the octet component is used, the operator cannot transform as a singlet, no matter how many factors of bilinear form are used.

Therefore, we must conclude that $V^{-1}\vec{\mathcal{J}}V$ is independent of $\underline{m}$, although V and $\vec{\mathcal{J}}$ do depend on $\underline{m}$. This is true no matter how small the symmetry breaking is, so it is true in the symmetric limit. But this implies

$$V^{-1}\vec{\mathcal{J}}V = V_0^{-1}\vec{\mathcal{J}}_0 V_0 , \tag{5}$$

where the subscript zero means that the operator refers to the equal-mass free-quark model. Since neither side of the equation depends on $\underline{m}$, and since the substitution

$$(m, m, m)\to(m_{\mathcal{P}}, m_{\mathfrak{N}}, =m_{\mathcal{P}}, m_\lambda) \tag{6}$$

entails

$$\vec{\mathcal{J}}_0\to\vec{\mathcal{J}} , \tag{7}$$

then if the same replacement is made in V_0, the new V which results must also satisfy the condition. The other angular condition also holds: $[\mathcal{J}_1,[\mathcal{J}_1, W_i^3]]=W_i^3$. Of course, just as we had an ambiguity in Y_{free} of multiplying by a unitary operator U which commuted with all $\vec{\mathcal{J}}_{\text{free}}$ and with Y_{free}, we also have a similar ambiguity here of a possible U which commutes with all $\vec{\mathcal{J}}$ and with Y and which may depend on m.

Furthermore, recall that Melosh determined V_0 up to a multiplicative unitary transformation U, where

$$V_0 = UV_{\text{free}} . \tag{8}$$

V_{free} is explicitly given by Melosh and U commutes with $\vec{\mathcal{J}}$. But then, independent of U,

$$V_0^{-1}\vec{\mathcal{J}}_0 V_0 = V_{\text{free}}^{-1}\vec{\mathcal{J}}_0 V_{\text{free}}$$

must not have any $\underline{m}$ dependence. This is not true for V_{free}. Since V_{free} does not satisfy this condition, then we are led to the conclusion that V does not exist in the unequal-mass free-quark model, and thus it is doubtful that V exists [since in fact SU(3) is broken]. Of course, this would not invalidate a phenomenological role for V if all that was required was its approximately satisfying the angular conditions. (The possibility remains that the presence of interaction might allow the conditions to be satisfied.)

On the other hand, V as defined above is a solution to the angular conditions if only the strangeness-preserving W_i^α are considered. This situation parallels that in the equal-time formulation,[4] where the spacelike charges could not be transformed in such a way as to eliminate the SU(3) breaking due to $m_\lambda \neq m_{\mathcal{P}}=m_{\mathfrak{N}}$ and only an $SU(4)_W$ algebra could be defined under which the Hamiltonian was invariant.

Of course, one may choose to disregard Eq. (1) when the $SU(6)_{W,\text{strong}}$ multiplets are broken. This angular condition is not the only possibility since the W_i are not unique operators. The angular condition reflects our desire to maintain the association of states with definite momentum and spin together in irreducible representations of an SU(6) group even though there is no exact SU(6) symmetry. For the purposes of the following discussion, we will take V to be determined by the angular condition for nonstrange currents only.

Still within the unequal-mass free-quark model, we now consider another important quantity: the divergence of the axial-vector current, $\partial_\mu \mathfrak{F}_i^{\mu 5}$. The light-plane integrated operator satisfies

$$\left\langle A \left| \int d^4x\, \delta(x^+)\partial_\mu \mathfrak{F}_i^{\mu 5}(x) \right| B \right\rangle \equiv \langle A | v_i | B\rangle = (p_B^- - p_A^-)\langle A | F_i^3 | B\rangle \qquad (9)$$

for states A, B such that $p_A^+ = p_B^+$, $\vec{p}_{A\perp} = \vec{p}_{B\perp}$. Thus, the PCAC (partial conservation of axial-vector current) hypothesis implies that selection rules and relative amplitudes for π (and K) transitions are determined by the transformation properties of the generator F_i^3 under $SU(6)_{W,\,\mathrm{strong}} \times O(2)$; as Melosh noted, these correspond to the transformation properties of $V^{-1} F_i^3\, V$ under $SU(6)_{W,\,\mathrm{currents}} \times O(2)$. It is easy to see that

$$[F_i^3, P^-] = 0, \quad \text{for } i = 1, 2, 3, 8$$
$$\neq 0, \quad \text{otherwise}, \qquad (10)$$

$$[Y_{\mathrm{free}}, F_i^3] \neq 0, \quad \text{for all } i, \qquad (11)$$

$$V^{-1} P^- V = P^- . \qquad (12)$$

For the strangeness-preserving operators (that is, for those with $i = 1, 2, 3, 8, 0$)

$$W_i^3 = \frac{1}{\sqrt{2}} \int d^4x\, \delta(x^+) q_+^\dagger(x)\, \tfrac{1}{2}\lambda^i \times \left[\sigma^3 + 2\,\frac{(\partial_\perp{}^2\sigma^3 + i\kappa_{\mathrm{inv}}\beta\vec{\sigma}_\perp\cdot\vec{\partial}_\perp)}{\kappa_{\mathrm{inv}}{}^2 - \partial_\perp{}^2}\right] q_+(x)\,, \qquad (13)$$

where κ_{inv} is the operator defined by

$$\kappa_{\mathrm{inv}} = \frac{M}{P^+}|\partial_-| + m\,, \qquad (14)$$

$$\sqrt{2}\,|\partial_-| = m[1 - (1 + 2\partial_-{}^2/m^2)]^{1/2}\,, \qquad (15)$$

and the M/P^+ factor is understood to multiply integrated operators only. That is, one expands $|\partial_-|$ in a power series in $\partial_-{}^2$, integrates term by term, then multiplies each term by the appropriate power of M/P^+, and finally resumes the series to obtain W_i^3. Clearly, since M has an octet piece coming from the mass difference in the quark triplet, powers of M will give rise to terms in W_i^3 which transform as components of large SU(3) multiplets, i.e., those having exotic states contained in them. A similar result holds for W_i^3 with $i = 4, 5, 6, 7$. In terms of the transformation properties of the F_i^3 under $SU(3)_{\mathrm{strong}}$ this implies that F_i^3 has exotic transformation terms for all i, so that for example none of the F_i^3 transforms as a member of an $SU(3)_{\mathrm{strong}}$ octet. Similarly, examination of the operator v_i reveals that it also does not transform simply as an octet member under $SU(3)_{\mathrm{strong}}$.

From Eqs. (10) and (12) we see that

$$[W_i^3, P^-] = 0 \quad \text{for } i = 1, 2, 3, 8, \qquad (16)$$

which entails $m_\pi{}^2 = m_\rho{}^2$, $m_N{}^2 = m_\Delta{}^2$, etc. What one needs is some interaction which lifts these degeneracies but still leaves the old SU(6) results such as

$$m_K{}^2 - m_\pi{}^2 = m_{K^*}{}^2 - m_\rho{}^2 \qquad (17)$$

intact. In the presence of interaction, the Hamiltonian

$$P^- = i \int d^4x\, \delta(x^+)\bar{q}(x)\gamma^-\partial_- q(x) \qquad (18)$$

will no longer have the simple form

$$P^-_{\mathrm{free}} = i\sqrt{2}\int d^4x\, \delta(x^+) q_-^\dagger(x)\partial_- q_-(x) = \frac{i}{\sqrt{2}}\int d^4x\, \delta(x^+) \times \int d\xi\, \epsilon(x^- - \xi) q_+^\dagger(x_\perp, \xi)(-\partial_\perp{}^2 + m^2) q_+(x) \qquad (19)$$

when written in terms of the "good" fields q_+, $q_+^\dagger$ but will have additional pieces which depend on the quark source current j:

$$(\gamma\cdot\partial + im)q = j\ . \qquad (20)$$

One easily finds

$$P^- = P^-_{\mathrm{free}} + \frac{i}{\sqrt{2}}\int d^4x\, \delta(x^+)\int d\xi\, \epsilon(x^- - \xi)\,[q_+^\dagger(x_\perp, \xi)(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp - im)j(x) + j^\dagger(x_\perp, \xi)(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp + im)q_+(x) + j^\dagger(x_\perp, \xi)\gamma^-\gamma^+ j(x)]\,. \qquad (21)$$

The charge F_i^3 is not modified by the interaction since it is directly expressed in terms of q_+ fields alone; however, the axial-vector-current divergence will depend on interaction:

$$\partial_\mu \mathfrak{F}^{\mu 5} = i\,\bar{q}\{m, \lambda^i\}\gamma^5 q + \bar{j}\gamma^5\lambda^i q - \bar{q}\gamma^5\lambda^i j\ . \qquad (22)$$

The dependence on interaction is rather complicated, since the first term does depend on it implicitly through the expression of q in terms of q_+, q_- and of q_- in terms of q_+ and j; the remaining terms depend on interaction, both explicitly (j appears) and implicitly, as in the first term.

For a relatively simple example, we will study the effect of the interaction

$$\mathcal{L}_{\text{int}} = \tfrac{1}{2} g(\bar{q}q)^2 \equiv \tfrac{1}{2} g(u_0)^2 , \tag{23}$$

which, although nonrenormalizable, still provides some scattering of quarks so that pairs can be produced and so that momentum can be transferred between quarks in a hadron.[5] The quark source current is then

$$j = g\gamma^0 q\,\bar{q}q = g\gamma^0 q u_0 . \tag{24}$$

We have been unable to obtain closed expression for j in terms of $q_+^\dagger$, q_+ alone (or for q_- in terms of $q_+^\dagger$, q_+ alone). One may write j in terms of $q_+^\dagger$, q_+ and j; in this way, by an iterative procedure, it can be shown that P^- is a multilinear form in operators having the structure

$$\int d^4x\, \delta(x^+)\, d\xi\, d\xi'\, \epsilon(x^- - \xi)\, \epsilon(x^- - \xi') \times[q_+^\dagger(x)\, \mathcal{O}\, q(x_\perp, \xi) - q_+^\dagger(x_\perp, \xi)\, \mathcal{O}\, q_+ x)] \times[q_+^\dagger(x)\, \mathcal{O}'\, q(x_\perp, \xi') - q_+^\dagger(x_\perp, \xi')\, \mathcal{O}'\, q_+(x)], \tag{25}$$

where $\mathcal{O}$ and $\mathcal{O}'$ are operators depending on $\partial_\perp{}^2$, m^2, $\vec{\gamma}_\perp \cdot \vec{\partial}_\perp\, \partial/\partial x^-$, $\partial/\partial\xi$, and $\partial/\partial\xi'$. This implies that

$$[Y_{\text{free}}, P^-] \neq 0 , \tag{26}$$

even though all operators are integrated over lightlike planes. (For bilinear forms, this is usually sufficient to get commutativity.) Therefore, we cannot proceed as before by defining V to be obtained from V_{free} by replacing M_{free} by M, since M and V_{free} do not commute; there is an ambiguity as to where to place the M factor.

Although we cannot proceed further in this manner, we would nevertheless like to point out here some other recipes for constructing V which do not work either. First, $V \neq V_{\text{free}}$ since if we use M_{free} in constructing V then it will not be boost-invariant; this is easily seen by consideration of matrix elements of V between states with $M \neq M_{\text{free}}$. Second, one might attempt to satisfy $[\vec{J}, W_i] = 0$, where only matrix elements of rest states are taken. But then if there is mass splitting, matrix elements between unequal mass states are problematic since if one state is at rest then the other cannot be. Some approximation procedure may be devised, but then the question of mass breaking effects seems to be quite uncertain.

In any event, returning to Eq. (25), the interaction $(u_0)^2$ is not as simple as it appears at first glance. In terms of current quarks it leads to quark interaction terms of arbitrarily high order in the number of quarks; quark number is not conserved. These aspects are not surprising; what is striking, however, is that the interaction involves all angular momenta and, moreover, does not conserve L_z. This last result follows from the existence of $\vec{\gamma}_\perp \cdot \vec{\partial}_\perp$ dependence of the operators $\mathcal{O}$, $\mathcal{O}'$. For integrated bilocals, such terms could be integrated away by parts; for the expression of Eq. (25) this cannot be done. Since L_z is not conserved, the mass degeneracy of π and ρ will be lifted, presumably; this is not so obvious, since all we know is that $L_z(F)$, the L_z corresponding to the current-quark basis, is not conserved. We do not know if $L_z(W)$, the analogous quantity in the constituent-quark basis, is conserved, since to determine this we would need to know V, the transformation which takes us from one basis to the other.

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*This work was supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-1428.

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