CP violation as a quantum effect*

Howard Georgi†
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138

A. Pais
Rockefeller University, New York, New York 10021
(Received 15 April 1974)

For gauge theories with perturbative spontaneous symmetry breakdown it may happen that the question of *CP* invariance is solely settled by the structure of the quantum corrections to the classical scalar potential. It is shown that this can occur if and only if there exist spinless bosons which are massless in the tree aproximation and which are non-Goldstone modes. This can be achieved in a natural way (in the technical sense) if an accidental symmetry is present. The general theorem is illustrated with a few formal examples, some of which yield *CP* invariance and some *CP* violation. Our theorem also implies that if the *leading* radiative corrections opt for *CP* invariance, then the theory is *CP*-invariant to all orders. Our examples further reveal the possibility that some gross features of the particle spectrum may also be solely determined by quantum effects.

I. INTRODUCTION

In this paper we comment on interesting suggestions made by Zee[1] and by Mohapatra.[2] They conjectured that in a theory which is *CP*-conserving in the tree approximation, the radiative corrections might induce a spontaneous breakdown of *CP* invariance ("conjecture A"). Zee also suggested[1] the possibility the the *CP* properties of the theory are not determined in the tree approximation so that radiative corrections are necessary to settle the question of *CP* invariance ("conjecture B"). It is the purpose of this paper to examine these conjectures in the context of gauge theories in which the spontaneous breakdown is perturbative. By this we mean that the symmetry breaking vanishes in the limit of small coupling constants. We shall show that conjecture B can then be realized but only if in the tree approximation there exist massless scalar fields *other* than Goldstone bosons.

Thus, in order to pursue further the conjectures, we must ask how spinless particles can occur which are naturally[3] massless to zeroth order. One way to realize this is for the theory to possess an accidental symmetry,[4] in which there are pseudo-Goldstone bosons which are massless in the tree approximation. Because of the accidental symmetry the physical content of the theory is not completely determined by the tree approximation. More specifically, in the theories we discuss, the radiative corrections solely determine whether the theory is *CP*-conserving or not. We will exhibit in Sec. III a class of models which illustrate these points and realize conjecture B. In these models, the radiative corrections also determine the gross features of particle mass spectra and we are led to realize a potential new source of natural[3] mass relations.

We will next prove in Sec. II our assertion for a general symmetry operation.

II. A GENERAL THEOREM

Consider a field theory involving a set of real spinless meson fields ϕ_j. If this theory is invariant under a symmetry in the tree approximation then $V(\phi)$, the tree approximation to the effective potential (which is just the classical potential) satisfies $V(\phi)=V(U\phi)$ where U is an orthogonal matrix describing the effect of the symmetry transformation on the scalar fields.[5] Suppose that the zeroth-order vacuum expectation value is consistent with the symmetry. That is, if the minimum of $V(\phi)$ occurs at $\phi=\lambda$, then we must have $U\lambda=\lambda$. Now radiative corrections will modify the effective potential by some function $\delta V(\phi)$. It is clear that this correction again satisfies $\delta V(U\phi) = \delta V(\phi)$. In general, this will induce a change $\delta\lambda$ in the vacuum expectation value of ϕ. We want to determine under what conditions $\delta\lambda$ also satisfies $U\delta\lambda=\delta\lambda$, since it is precisely these conditions which we wish to evade.

The derivative of the exact effective potential at the true vacuum expectation value must vanish, so we have

$$(V+\delta V)_j\,(\lambda+\delta\lambda)=0\,, \tag{1}$$

where we use the notation

$$\frac{\partial^n}{\partial\phi_{j_1}\cdots\partial\phi_{j_n}}\,F(\phi)=F_{j_1\cdots j_n}(\phi)\,.$$

Now assume that the change is perturbative in the sense that $\delta\lambda$ goes to zero as δV goes to zero.

Then we can expand Eq. (1) and write

$$0 = V_j(\lambda) + \sum_k V_{jk}(\lambda)\delta\lambda_k + \delta V_j(\lambda) + \cdots ,$$

where $\cdots$ are of higher order in small quantities. But $V_j(\lambda) = 0$ because $\phi = \lambda$ is a minimum of $V(\phi)$, so we have

$$\sum_k V_{jk}(\lambda)\delta\lambda_k + \delta V_j(\lambda) = 0 . \qquad (2)$$

Now suppose $F(U\phi) = F(\phi)$. Differentiating we find

$$F_{j_1 \cdots j_n}(\phi) = \sum_{k_1 \cdots k_n} U_{k_1 j_1} \cdots U_{k_n j_n} F_{k_1 \cdots k_n}(U\phi)$$

and in particular for $\phi = \lambda$

$$F_{j_1 \cdots j_n}(\lambda) = \sum_{k_1 \cdots k_n} U_{k_1 j_1} \cdots U_{k_n j_n} F_{k_1 \cdots k_n}(\lambda), \qquad (3)$$

because $U\lambda = \lambda$. Combining Eqs. (2) and (3) and using the orthogonality of U, we find

$$\sum_{k,l} V_{jk}(\lambda) U_{kl}\, \delta\lambda_l + \delta V_j(\lambda) = 0 . \qquad (4)$$

Subtracting Eq. (2) from Eq. (4), we derive

$$\sum_k V_{jk}(\lambda)[(U\delta\lambda)_k - \delta\lambda_k] = 0 . \qquad (5)$$

Now if there are no massless scalar-meson fields in the tree approximation, $V_{jk}(\lambda)$ is a nonsingular matrix and we can invert Eq. (5) to obtain $U\delta\lambda = \delta\lambda$. This result is still true if there are Goldstone bosons in the tree approximation. Goldstone bosons are associated with the spontaneous breakdown of a continuous symmetry, and by a symmetry transformation $\delta\lambda$ can be chosen orthogonal to the Goldstone-boson subspace. (See Appendix.) This completes the proof.

For the special case of a continuous symmetry, this theorem was proved by Georgi and Glashow.[6] In this case its interpretation is obvious. Consider the theory including radiative corrections. If the symmetry has been broken spontaneously there are Goldstone bosons. Now imagine turning off the radiative corrections. The spontaneous symmetry breaking in the form of the vacuum expectation value of a scalar field goes continuously to zero, because it is perturbative by assumption. But the Goldstone bosons remain until in the tree approximation we are left with an unbroken symmetry and massless scalar bosons which are no longer Goldstone bosons.

The case of a discrete symmetry is more subtle because its spontaneous breakdown does not imply the existence of Goldstone bosons. Nevertheless, the formal arguments given above apply to this case as well. The key is the assumption that the spontaneous symmetry breaking is perturbative. This assumption is justified in practice because we must calculate perturbatively in a loop approximation or some other such scheme, and if the symmetry breaking is nonperturbative there is no guarantee that it will not change drastically from one level of approximation to the next.[7]

This theorem is something of an embarrassment as far as both conjectures are concerned. Extraneous massless scalar-meson fields in the tree approximation are usually both unnatural[3] and unwelcome.[8] There is one well-known situation, however, where such fields appear naturally; that is when the model has an accidental symmetry.[4]

A field theory is said to have an accidental symmetry when the symmetry group $\mathcal{G}'$ of the renormalizable scalar-meson self-couplings is larger than the symmetry group $\mathcal{G}$ of the full Lagrangian. When spontaneous symmetry breakdown occurs in such a theory, the scalar fields which are massless in the tree approximation are the Goldstone bosons associated with breaking of $\mathcal{G}'$. Some of these are true Goldstone bosons associated with the breakdown of $\mathcal{G}$, but some can be pseudo-Goldstone bosons which will develop mass due to radiative corrections. One might think that conjecture A could be realized in such a model, but actually the situation is more complicated. The tree approximation only determines the vacuum expectation value of the scalar fields up to a $\mathcal{G}'$ transformation. But since the theory is not invariant under $\mathcal{G}'$, this means that the physical content of the theory is not determined classically. Only when radiative corrections are taken into account does the effective potential pick out the vacuum expectation value up to a $\mathcal{G}$ transformation and uniquely determine the physics. In a theory of this kind it is true in some sense that the spontaneous breakdown of CP invariance is due to the radiative corrections, but it is *not* true that the theory is CP-conserving in the tree approximation. In the tree approximation, the theory has simply not decided whether it is CP-conserving or not. As we shall see in the next section, it is possible to construct CP-violating theories of this type which are explicit examples of conjecture B. On the other hand, if we insist on achieving in a natural way the masslessness of spin-zero bosons in the tree approximation, then we believe that conjecture A cannot be realized perturbatively.

For our purposes we have so far only considered the rather familiar pseudo-Goldstone situation. We must ask, in addition, whether there are other ways in which mass-zero scalars can appear naturally in the tree approximation. No such alternatives are known to us.

We now describe a class of models which illustrate these ideas. We emphasize that these models have been contrived to make calculations of the relevant radiative corrections as simple as possible, and are not intended to describe the real world; but the points which they illustrate are equally applicable to more physical theories.

Before immersing ourselves in the specific examples, let us briefly outline the general strategy. Consider a gauge theory with a scalar field potential

$$V = V_{\rm cl}(\phi) + V_{\rm qu}(\phi), \tag{6}$$

where $V_{\rm cl}$ is the classical (tree) part and $V_{\rm qu}$ is the quantum (radiative correction) part. Now imagine that the vacuum expectation values of ϕ contain an angle θ which, however, cannot appear in $V_{\rm cl}(\phi)$, for accidental-symmetry reasons. In such a situation the extremal condition relative to θ becomes

$$\frac{\partial V_{\rm qu}(\phi)}{\partial \theta} = 0, \tag{7}$$

independently of the classical part. In the case at hand, θ will be the phase which will settle the CP properties of the theory. We must now examine the consequences of Eq. (7) and of $(\partial^2 V_{\rm qu}/\partial\theta^2)_{\rm extr} \geq 0$.

Fortunately no great labor is involved. We have only to consult the classic paper of Coleman and Weinberg.[9] They show how to calculate the one-loop corrections to the effective potential. In the pseudo-Goldstone case, the contribution of the scalar-boson loop is irrelevant because it still has the accidental symmetry. There remain contributions from the gauge-meson loop and from the fermion loop, given by

$$\frac{1}{64\pi^2}\left[3\,{\rm tr}M^4(\phi)\ln M^2(\phi)/M_0^2 - {\rm tr}\, m^4(\phi)\ln m^2(\phi)/M_0^2\right], \tag{8}$$

where $M^2(\phi)$ and $m^2(\phi)$ are, respectively, the vector-meson and fermion mass-square matrices as a function of the scalar-boson fields and M_0^2 is an arbitrary mass scale. Equation (8) will have in general a nontrivial dependence on θ. Again invoking accidental symmetry, Eq. (7) can now be written symbolically as

$$3\,{\rm tr}M^3\frac{\partial M}{\partial\theta}\ln M^2 - {\rm tr}\, m^3\frac{\partial M}{\partial\theta}\ln m^2 = 0\,. \tag{9}$$

This equation is the starting point for the next section.

III. ILLUSTRATIVE EXAMPLES

The models are gauge theories with a gauge group which is a direct product of N SU(2) factors. Denote the generators of the jth SU(2) by $\vec{T}_j$ which couple to gauge bosons $\vec{W}_j$. We also introduce N four-component scalar-meson fields, $\Sigma_{j\,j+1}$ for $j=1$ to $N-1$ and Σ_{N1}. We write each of these as a 2×2 matrix satisfying $\tau_2\Sigma^*\tau_2=\Sigma$, so that $\Sigma=\sigma+i\vec{\tau}\cdot\vec{\pi}$ where σ and $\vec{\pi}$ are real. Under the infinitesimal gauge transformation $1+i\sum_j\vec{\omega}_j\cdot\vec{T}_j$, these transform as follows:

$$\delta\Sigma_{jk} = i\vec{\omega}_j\cdot\frac{\vec{\tau}}{2}\Sigma_{jk} - i\vec{\omega}_k\cdot\Sigma_{jk}\frac{\vec{\tau}}{2}\;.$$

Thus each Σ transforms like a real 4-vector under the appropriate SU(2)×SU(2) subgroup. We demand invariance under the conventional charge conjugation operation, $W_j^{1,3}\to -W_j^{1,3}$, $W_j^2\to W_j^2$, and $\Sigma_{ij}\to\Sigma_{ij}^*$. For simplicity, we also demand invariance under the cyclic discrete symmetry:

$$\begin{aligned}&\vec{W}_1\to\vec{W}_2\to\cdots\to\vec{W}_N\to\vec{W}_1\,,\\ &\Sigma_{12}\to\Sigma_{23}\to\cdots\to\Sigma_{N-1\,N}\to\Sigma_{N1}\to\Sigma_{12}\,.\end{aligned} \tag{10}$$

This implies that all of the gauge couplings are equal.

Now suppose that the Σ's develop vacuum expectation values. We can use the gauge symmetry to put them into a very simple form. By a transformation in the second SU(2) group, we can choose $\langle\Sigma_{12}\rangle$ to be proportional to the unit matrix. Then by a transformation in the third SU(2) group, we can choose $\langle\Sigma_{23}\rangle$ to be proportional to the unit matrix. We can continue this process through $\langle\Sigma_{N-1\,N}\rangle$, but when we come to $\langle\Sigma_{N1}\rangle$, we cannot transform it under the first SU(2) group without disturbing $\langle\Sigma_{12}\rangle$. However, we can still transform it under the diagonal SU(2) generated by $\sum_j\vec{T}_j$, so we can diagonalize it. Thus we have finally

$$\langle\Sigma_{j\,j+1}\rangle\propto\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

for $j=1$ to $N-1$,

$$\langle\Sigma_{N1}\rangle\propto\begin{pmatrix}e^{i\theta}&0\\0&e^{-i\theta}\end{pmatrix}.$$

If all the vacuum expectation values are nonzero, this is the best we can do.

The physical content of the theory depends strongly on what value of θ is picked out. For $\theta=0$ or π, the theory is CP-conserving, there is an exact SU(2) gauge symmetry surviving after spontaneous symmetry breakdown, and there are three massless vector fields. For $\theta=\pi/2$ or $3\pi/2$ the theory is CP-conserving but only a single massless photon survives. For other values of θ, there is also a single massless photon but the theory is CP-violating (and C-violating since P

is always conserved in these simple models).

However, for $N>4$, the classical potential is independent of θ, because the first gauge-invariant coupling which depends on θ is $\mathrm{tr}(\Sigma_{12}\Sigma_{23}\cdots\Sigma_{N1})$ which is not renormalizable. The symmetry group of the renormalizable scalar-meson self-couplings is a direct product of $2N$ SU(2) factors, an independent SU(2)×SU(2) for each Σ. In other words, there is an accidental symmetry. When the Σ's develop vacuum expectation values, the accidental symmetry is spontaneously broken [down to $\mathrm{SU}(2)^N$] and there are $3N$ massless scalar bosons in tree the approximation. Only $3N-1$ (or $3N-3$ if $\theta=0$ or π) are true Goldstone bosons of the theory. The remaining one (or three) are pseudo-Goldstone bosons and will develop mass in the one-loop approximation.

The classical potential depends only on the magnitudes of the Σ fields ($|\Sigma|^2=\sigma^2+\vec{\pi}^2$), and cannot determine the value of θ since it is independent of θ. So to discover whether these theories are CP-conserving, we must include radiative corrections.

As a first example, let us consider the case where no other particles are present than the scalar-meson and gauge vector-meson system just described. Then the only radiative correction term to be considered is the M term in Eqs. (8) and (9). (Note that in this simple case it follows from our assumptions that the CP issue is independent of the magnitude of the gauge coupling constant.) This M term does have a nontrivial dependence on θ, and to this order, it is the only one. So the physical value of θ to leading order in the gauge coupling will be the θ which minimizes Eq. (8), with the magnitudes of the $\langle\Sigma\rangle$ fixed at their classical values.

To see what happens explicitly, assume that the parameters in the classical potential are such that all $\langle\Sigma\rangle$ have the same magnitude. This will happen for some region in the parameter space because of the cyclic symmetry, Eq. (10). Then we can easily diagonalize the vector-meson mass matrix. This matrix looks like

$$\tfrac{1}{2}\mu^2[(W_1^3-W_2^3)^2+(W_2^3-W_3^3)^2+\cdots+(W_N^3-W_1^3)^2]+\mu^2(|W_1^+-W_2^+|^2+|W_2^+-W_3^+|^2+\cdots+|e^{i\theta}W_N^+-e^{-i\theta}W_1^+|^2),$$

where $W^+=(W^1+iW^2)/\sqrt{2}$. Since only the shape of Eq. (8) as a function of θ is relevant, the overall mass scale is unimportant and we can set $\mu^2=1$. The neutral vector-meson masses are independent of θ and are therefore not relevant. The charged vector-meson mass squares are $2-2\cos[(2\theta+2m\pi)/N]$ where $m=0$ to $N-1$. The contribution of these charged vector-meson masses to the Coleman-Weinberg sum is proportional with positive coefficient to

$$-\sum_{k=1}^{\infty}\frac{\cos 2k\theta}{k(N^2k^2-1)(N^2k^2-4)},$$

for $N>4$. This function is obviously minimized at $\theta=0$ or π, so these theories pick out a CP-conserving solution.

Multiloop corrections will not change this result. This is clear because we can now regard the one-loop effective potential as a starting point and apply the theorem of Sec. II. Since there is no remaining accidental symmetry and therefore no pseudo-Goldstone bosons in the one-loop approximation, the theorem implies that there can be no perturbative breakdown of the CP invariance.

To get a CP-violating solution we must further complicate the model. We specialize to even $N\geq 6$ and include fermions in the theory. First, consider N doublets ψ_j transforming according to $\delta\psi_j=i\vec{\omega}_j\cdot\frac{1}{2}\vec{\tau}\psi_j$, with Yukawa couplings to the scalar mesons as follows:

$$\bar{\psi}_1\Sigma_{12}\psi_2+\bar{\psi}_2\Sigma_{23}\psi_3+\cdots+\bar{\psi}_N\Sigma_{N1}\psi_1+\text{H.c.} \tag{11}$$

This is invariant under the cyclic symmetry Eq. (10) and $\psi_1\to\psi_2\to\cdots\to\psi_N\to\psi_1$. Invariance under the conventional C operation implies that the Yukawa coupling constant is real. Now there is a θ-dependent contribution to the effective potential from the fermion loop, namely the m-type term in Eqs. (8) and (9). In this case, the eigenvalues of the fermion mass matrix are $2\cos[(\theta+2m\pi)/N]$, which give a contribution to Eq. (8) proportional to

$$\sum_{k=1}^{\infty}\frac{(-1)^{Nk/2}\cos k\theta}{k(N^2k^2-4)(N^2k^2-16)}. \tag{12}$$

This function has its minimum at either 0 or π (for $N=4l+2$ or $4l$, respectively) which again is CP-conserving. But we can also include fermions ϕ_j with Yukawa couplings

$$\bar{\phi}_1\Sigma_{12}\phi_2+\cdots+\bar{\phi}_{N-1}\Sigma_{N-1\,N}\phi_N-\bar{\phi}_N\Sigma_{N1}\phi_1+\text{H.c.} \tag{13}$$

invariant under Eq. (10) and $\phi_1\to\phi_2\to\cdots\to\phi_N\to\phi_1$. This gives a contribution to Eq. (8) of the form of Eq. (12) with $\theta\to\theta+\pi$. So if the Yukawa coupling constants for Eqs. (11) and (13) are equal, the contribution of the fermion loop to the effective potential is proportional to

$$\sum_{k\ \text{even}}\frac{\cos k\theta}{k(N^2k^2-4)(N^2k^2-16)},$$

which has a minimum at $\theta=\pi/2$ or $-\pi/2$. This is still CP-conserving, but if the Yukawa coupling constants for Eqs. (11) and (13) are slightly different, the minimum moves away from $\pi/2$ and $-\pi/2$. If, in addition, the Yukawa coupling constants are larger than the gauge coupling constant, then the contribution of the fermion loop will dominate that from the vector loop and the minimum of the full one-loop effective potential will be CP-violating.[10]

One reason that this class of models is so easy to work with is that at least one massless photon necessarily survives and one can divide the fields into charged and neutral sets. In more general (or more physical) theories, this does not always happen. It may be that the tree approximation does not determine whether an electromagnetic gauge invariance survives. The Coleman-Weinberg analysis can still be done but it is more complicated.

In any event, it is clear from our contrived examples that CP violation can be realized as a pure quantum effect, due to radiative corrections in a field theory. This mechanism should be kept in a mind as a possible byproduct of accidental symmetry, and as a possible quantum alternative to arrive at natural values for CP-violating phases, which so far have been discussed[3] via properties of the classical potential.

ACKNOWLEDGMENTS

It is a pleasure to thank Sidney Coleman and Judy Lieberman for useful discussions.

APPENDIX

We show that $\delta\lambda$ can be chosen orthogonal to the Goldstone-boson subspace.

If the Lagrangian is invariant under a continuous symmetry group G, let the generators of G on the scalar system be $T^a{}_{ij}$. Then the potential $V(\varphi)$ satisfies

$$\sum_{i,j} \varphi_i T^a{}_{ij} V_j(\varphi)=0.$$

The Goldstone-boson directions are

$$g^a{}_i = i\sum_j T^a{}_{ij}\lambda_j ,$$

satisfying

$$\sum_{i,j} V_{ij}(\lambda) g^a{}_j = 0,$$

so they are massless in the tree approximation.

Suppose $\delta\lambda$ has a component in a Goldstone-boson direction. Then we can write

$$\delta\lambda = \delta\lambda' + \sum_a \delta\alpha^a g^a ,$$

where $\delta\lambda'$ is orthogonal to the Goldstone-boson direction. But under an infinitesimal global transformation

$$\delta\varphi = i\sum_a \delta\alpha^a T^a \varphi ,$$

$$\delta\lambda_{\text{gauge}} = i\sum_a \delta\alpha^a T^a \lambda = \sum_a \delta\alpha^a g^a = \delta\lambda - \delta\lambda' .$$

So the component of $\delta\lambda$ in the Goldstone-boson direction can be obtained simply by making a gauge transformation, without any change in the potential. Then by making the inverse transformation, this part of $\delta\lambda$ can be transformed away, leaving $\delta\lambda'$ which is orthogonal to the Goldstone-boson direction.

---

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2232 and in part by the U. S. Air Force Office of Scientific Research under Contract No. F44620-70-C-0030 and the National Science Foundation under Grant No. GP30819X.

†Junior Fellow, Society of Fellows, Harvard University.

[1]A. Zee, Phys. Rev. D 9, 1772 (1974).

[2]R. N. Mohapatra, Phys. Rev. D 9, 3461 (1974).

[3]In this paper we use the terms natural and unnatural in the technical sense. See H. Georgi and A. Pais, Phys. Rev. D 10, 359 (1974).

[4]S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972).

[5]In general, there will also be some change in the space-time coordinates, $x_\mu \to x'_\mu$, but if $d^4x = d^4x'$, this part of the transformation is irrelevant to our argument, so we suppress all coordinate dependence.

[6]H. Georgi and S. L. Glashow, Phys. Rev. D 6, 2977 (1972).

[7]In some cases reliable calculations may be possible even if the symmetry breaking is nonperturbative.

[8]In the models of Refs. 1 and 2 neither conjecture can be realized perturbatively.

[9]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[10]If θ_0 is a minimum, then the same is true for $-\theta_0$ and one must choose either of these two by convention.