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Light-cone analysis of the three-photon process*

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The three-photon process first treated by Brodsky, Gunion, and Jaffe in the parton model is analyzed in terms of light-cone singularities. It is found that certain physically reasonable assumptions suggested by the parton model lead to the dominance of the strongest singularities.

I. INTRODUCTION

Brodsky, Gunion, and Jaffe[1] have provided a parton-model analysis of the three-photon process shown in Fig. 1. They find that the model predicts (1) a scaling law and (2) a sum rule "which provides a definitive test of whether the constituents of the proton have fractional versus integral charge."

For the simpler process of electroproduction, there is a close relationship between the parton and light-cone treatments.[2] In this paper, we explore the possibility of a similar relationship between the two approaches for the three-photon process.

The three-photon process is controlled by the tensor[1]

$$V_{\mu\nu\lambda} \equiv \frac{4\pi^2 E_P}{M} \int d^4x \int d^4y\, e^{iq\cdot y} e^{ik\cdot x} \times \langle P | J_\nu(y) T^*[J_\lambda(0) J_\mu(x)] | P\rangle .$$

It is customary to begin discussions of electroproduction with a heuristic phase-variation argument which supposedly proves light-cone dominance. By itself, the phase-variation argument is not sufficient to give light-cone dominance. However, in conjunction with spectrum and causality restrictions, the correct result can be obtained. A more rigorous approach employs integral representations which incorporate spectrum and causality conditions automatically.[3] The success of either approach is based to a large extent upon the simplicity of the electroproduction process. (There are only two invariant variables.)

In the three-photon process four invariant variables are involved, and both of the approaches which have been used in electroproduction become vastly more complicated. The phase-variation argument is inconclusive because of the difficulties associated with the larger number of invariant variables and integrations. The crucial role played by spectrum and causality in electroproduction suggests that these requirements will be important in the three-photon process also. It would be desirable to use representations which include the broadest class of functions consistent with these conditions. Perturbation-theoretical integral representations are available[4] for the five-point function which appears in $V_{\mu\nu\lambda}$. Unfortunately, these representations are themselves rather complicated and difficult to work with.

The functions which appear in the following analysis are not the most general possible functions consistent with the general requirements of field theory. However, they are broad enough to include a wide range of singularity strengths (including rather smooth light-cone behavior) and to thereby suggest that the results obtained will be extendible to a more general class of functions.

We propose to analyze $V_{\mu\nu\lambda}$ in the following way:

1. We ask what the probable singularity structure of the three-current product is.
2. We ask whether the strongest of these singularities dominates the scaling region.
3. We ask whether a canonical singularity structure reproduces the parton-model result.

The results of this analysis will be that

1. if the ideas of light-cone operator-product expansions[5] are used to generate the structure of the singularities and of the bilocal operators in the three-current product, then
2. under the additional assumption that the forward matrix element of the bilocal operator satisfies certain physically reasonable conditions suggested by the parton model, the strongest singularities do dominate, and
3. if the singularities are canonical, the parton-model scaling law is reproduced.

The objection could be raised that the use of a light-cone expansion is not justified until it has been demonstrated that the process is light-cone dominated. We would argue that the light-cone expansion correctly reproduces the *singularities* of the three-current *operator* product. These singularities are always there. For some matrix elements in some kinematical regions, it will be found that the strongest singularities dominate the weaker ones and by implication also those nonsingular terms which have not been repre-

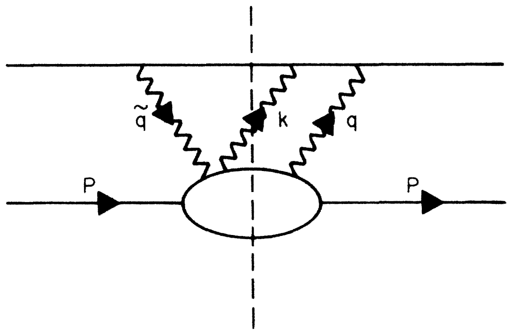

FIG. 1. An electron interacts via photons of momentum $\tilde{q}$, k, and q with a proton of momentum P. This is the diagram which controls the process $(e^- + p \to e^- + \gamma + X) - (e^+ + p \to e^+ + \gamma + X)$ studied in Ref. 1.

sented. For other processes, it will be found that the contributions from the nonleading singularities are as important as those from the leading singularities. In such a case, the irrelevance of the light-cone expansion is demonstrated *a posteriori*.

With this reasoning, we have a procedure for testing light-cone ideas on a process with a real photon. This approach may be useful in analyzing other processes such as those related to the nonforward two-current matrix element

$$\langle P | T[J_\mu(x) J_\nu(y)] | P' \rangle \ .$$

At this point, we should emphasize the importance of Regge behavior in these considerations. For electroproduction, it has been pointed out[6] that large q^2 is required in order to get to the light cone because the $-q^2/2q\cdot P = 0$ singularities due to leading Regge behavior must be avoided. This is connected with the question (which is a major concern of this paper) of whether the real photon in the three-photon process can spoil light-cone dominance. The analysis will deal with these problems, and the control of Regge behavior will emerge again as an important consideration.

II. THE STRUCTURE OF THE THREE-CURRENT PRODUCT

We begin by ignoring possible singular terms in the T^* product which destroy the covariance of the T product.[7] The T^* is then replaced by a T, and $V_{\mu\nu\lambda}$ becomes

$$V_{\mu\nu\lambda} = \frac{4\pi^2 E_P}{M} \int d^4y \int d^4x\, e^{iq\cdot y} e^{ik\cdot x} \theta(x^0) \times \langle P | [J_\nu(y), [J_\mu(x), J_\lambda(0)]] | P \rangle \ . \tag{1}$$

In arriving at Eq. (1), we have used the spectrum conditions with $k_0 > 0$ and $q_0 > 0$ to replace the T product by a retarded commutator and the resulting product with another commutator.

The Fritzsch-Gell-Mann light-cone algebra[8] makes a specific prediction for the singularity structure of the connected part of the double commutator:

$$[J_\nu(y), [J_\mu(x), J_\lambda(0)]] = \partial_{\rho'}[\epsilon(x^0)\delta(x^2)]\{\partial_\rho[\epsilon(y^0)\delta(y^2)] A^{\rho\rho'}_{\nu\mu\lambda}(y, x) + \partial_\rho[\epsilon(y^0 - x^0)\delta((y-x)^2)] B^{\rho\rho'}_{\nu\mu\lambda}(y, 0)\} \ . \tag{2}$$

A and B in Eq. (2) are various linear combinations of the bilocal generalizations of the octet currents.

Several features of Eq. (2) deserve comment:

1. Since the bilocal operators are assumed to be smooth at lightlike coordinate separations, Eq. (2) contains no term which has a product of three functions singular in x^2, y^2, and $(x-y)^2$, respectively. At least for the most singular part of the double commutator, such terms are present only in the disconnected part and do not contribute to our process.

2. Equation (2) was derived by a straightforward application of the formulas in Ref. 8 which, for this case, produced all the terms of interest. Unfortunately, this naive procedure is misleading. Consider a three-current product $J(x)J(y)J(z)$. According to the usual interpretation of the light-cone algebra, when all coordinate separations are nearly lightlike, this can be written (schematically)

$$\begin{aligned} J(x)J(y)J(z) &= J(x)C(y-z)F(y,z) \\ &= C(y-z)[C(x-y)F(x,z) \\ &\quad + C(x-z)F(x,y)] \ . \end{aligned} \tag{3}$$

The C's represent singular functions and the F's bilocal operators. However, by associating $J(x)$ and $J(y)$ first, we could also get

$$\begin{aligned} J(x)J(y)J(z) &= C(x-y)[C(x-z)F(y,z) \\ &\quad + C(y-z)F(x,z)] \ . \end{aligned} \tag{4}$$

Equations (3) and (4) do not appear to agree. The difficulty is that $J(x)$ can combine with nonsingular terms in the expansion of $J(y)J(z)$ to produce combinations such as

$$C(x-y)C(x-z)F(y,z) \tag{5}$$

which are not included in Eq. (3). An explicit calculation with currents constructed from free fields will demonstrate this point. A careful expansion of $J(x)J(y)J(z)$ will then contain all the terms we expect, including those of both Eqs. (3) and (4).[9]

The expression for the double commutator which we will use below will not contain all these terms. We anticipate that the terms which are neglected will contribute only to other physical processes. For example, the terms in Eq. (3) would correspond to graphs such as Figs. 2(a) and 2(b) which contribute to the three-photon process. The terms in Eq. (4) and represented in Fig. 2(c) and 2(d) do not contribute.

3. Finally we note that Eq. (2) contains free-field singularities. The ultimate origin of this aspect of the Fritzsch–Gell-Mann algebra is the observed scaling behavior of the SLAC electroproduction data. These experiments support the assumption of free-field singularities in the current correlation function.

We will generalize feature 3 of Eq. (2) by allowing other singularities besides those suggested by free-field theory. Since the tensor structure of the double commutator plays no essential role in the discussion of the dominance of the strongest singularities, we will simplify the notation by using scalar currents for a while. In fact, our first major assumption will be that the singularity structure of the double commutator

$$\theta(x^0)[J(y),[J(x),J(0)]]$$

is given by a sum of terms of the form suggested by Eq. (2):

$$C(x)[C'(y)F(y,x)+C''(y-x)F'(y,0)]. \qquad (6)$$

In Eq. (6), the F's are bilocal operators and the C's are singular functions which generalize the free-field singularities of Eq. (2).

At this point, we emphasize again that we are not assuming that Eq. (6) is a correct expression for the double commutator only for $x^2\cong 0$, $y^2\cong 0$,

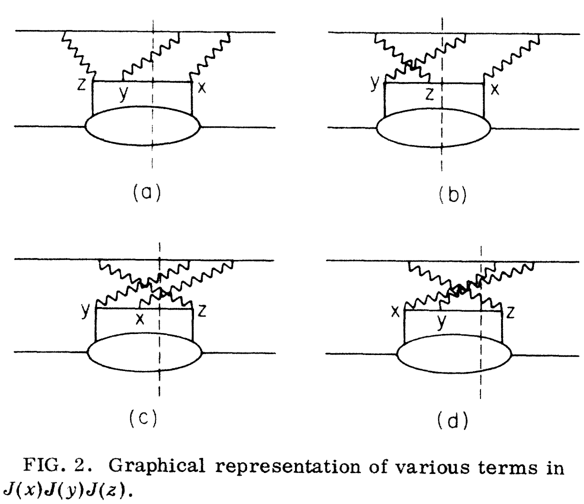

FIG. 2. Graphical representation of various terms in $J(x)J(y)J(z)$.

and $(x-y)^2\cong 0$. Rather, we are assuming that, regardless of the coordinate separations, the double commutator contains a series of terms like Eq. (6), and that these terms are the strongest singularities of the double commutator. In the next section, we ask whether or not the term of the form of Eq. (6) with the most singular C's dominates those with smoother C's. If the answer is yes, then this term will presumably also dominate the nonsingular parts of the double commutator, which have been completely ignored.

III. DOMINANCE OF THE STRONGEST SINGULARITIES

Substituting Eq. (6) into Eq. (1), we find that the contribution that a term of the form of Eq. (6) makes to V is

$$v\equiv\int d^4y\int d^4x\,e^{iq\cdot y}e^{ik\cdot x}C(x)[C'(y)G_1(y-x,P)+C''(y-x)G_2(y,P)]. \qquad (7)$$

In Eq. (7), we have introduced

$$G_1(y-x,P)\equiv\frac{4\pi^2E_P}{M}\langle P|F(y,x)|P\rangle=\frac{4\pi^2E_P}{M}\langle P|F(y-x,0)|P\rangle,$$

$$G_2(y,P)\equiv\frac{4\pi^2E_P}{M}\langle P|F'(y,0)|P\rangle.$$

By using the Fourier transforms

$$\tilde C(r)=\frac{1}{(2\pi)^4}\int d^4x\,e^{ir\cdot x}C(x),$$

$$\tilde G_1(l,P)=\frac{1}{(2\pi)^4}\int d^4x\,e^{-il\cdot x}G_1(x,P)$$

and similar expressions for $\tilde C'$, $\tilde C''$, and $\tilde G_2$, we can evaluate the x and y integrals in Eq. (7) to get

$$v=\int d^4l[\tilde C'(q+l)\tilde C(k-l)\tilde G_1(l,P)+\tilde C''(q+l)\tilde C(\bar q+l)\tilde G_2(l,P)], \qquad (8)$$

with $\bar q\equiv k+q$.

To proceed from here, we will make assumptions about the form of the C's and the general features of the $\tilde G$'s. In order to treat a broad range of singularity strengths, we will take the C's to be of the form

$$C'(y)=\frac{1}{(-y^2-i\epsilon y^0)^{d'}}-\text{c.c.},$$

$$C''(y)=\frac{1}{(-y^2-i\epsilon y^0)^{d''}}-\text{c.c.}, \qquad (9)$$

$$C(x)=\theta(x^0)\left[\frac{1}{(-x^2-i\epsilon x^0)^{d}}-\text{c.c.}\right],$$

with Fourier transforms[10]

$$\tilde{C}'(q+l) = i\pi^2 4^{2-d'} \frac{\Gamma(2-d')}{\Gamma(d')} \times \left\{ \frac{1}{[-(q+l)^2 - i\epsilon(q^0+l^0)]^{2-d'}} - \text{c.c.} \right\} ,$$

$$\tilde{C}''(q+l) = \tilde{C}'(q+l) \text{ with } d' \to d'' , \tag{10}$$

$$\tilde{C}(k-l) = i\pi^2 4^{2-d} \frac{\Gamma(2-d)}{\Gamma(d)} \times \frac{1}{[-(k-l)^2 - i\epsilon(k^0-l^0)]^{2-d}} ,$$

$$\tilde{C}(\tilde{q}+l) = \tilde{C}(k-l) \text{ with } k-l \to \tilde{q}+l .$$

Little can be said about v until some general restrictions are applied to the $\tilde{G}$'s. We will consult the parton model for guidance. The diagrams of interest correspond to the two terms in Eq. (8) and are shown in Fig. 3. The parton model assumes that the unamputated parton-proton scattering amplitude falls rapidly for l^2 much greater than a typical hadron scale such as the proton mass M. Together with the spectrum conditions

$$(l+q)^2 > 0, \quad l^0+q^0 > 0 ,$$
$$(P-l)^2 > 0, \quad P^0 - l^0 > 0 , \tag{11}$$

this limits the l integration so that, in the proton rest frame, none of the components of l gets bigger than the order of M/x. [x is the scaling variable $-q^2/(2q\cdot P)$, and $-q^2, q\cdot P \to \infty$ with x fixed is implied.] Although the conditions and the results are simple, the author has verified that a complicated calculation may result from an unfortunate choice of variables. We can recommend the parametrization introduced in Refs. 11 and 12 for use in the covariant parton model.

With all components of l limited to $\sim M/x$ in the proton rest frame, possible Regge behavior of the parton-proton scattering amplitude is tested only as $x \to 0$ (that is, in the photon-proton Regge region). In fact, since the Pomeron does not contribute to the odd-charge-conjugation parton-proton scattering, we can anticipate that the Regge behavior will be less troublesome than in electroproduction.

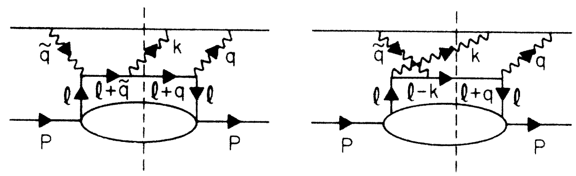

FIG. 3. Parton-model graphs.

We now abstract what we have learned from the parton model and make the following natural assumptions about the $\tilde{G}$'s:

A. The $\tilde{G}$'s fall rapidly for $l^2 \gtrsim M^2$.

B. Spectrum conditions, causality, and assumption A conspire in Eq. (8) to limit the l integrations so that no components of l exceed $\sim M/x$ in the proton rest frame.

In what follows, these assumptions will be seen to be crucial and, in fact, will lead to the dominance of the terms with the largest d' s (that is, the strongest light-cone singularities).

For our calculations, we work in the proton rest frame and orient the axes so that

$$q^\mu = (q^0, q^1, 0, 0), \quad q^1 > 0$$
$$k^\mu = (k^0, k^1, -k_2, 0), \quad -k_2 > 0 .$$

Then

$$(q+l)^2 \cong 2q\cdot P\left(\frac{l^0 - l^1}{M} - x\right) ,$$
$$(k-l)^2 \cong 2q\cdot P\left(\frac{k^2}{2q\cdot P} - \frac{k\cdot P}{q\cdot P}\,\frac{l^0-l^1}{M}\right) , \tag{12}$$
$$(\tilde{q}+l)^2 \cong 2q\cdot P\left(\frac{\tilde{q}^2}{2q\cdot P} + \frac{\tilde{q}\cdot P}{q\cdot P}\,\frac{l^0-l^1}{M}\right) .$$

The corrections are down by at least $(q\cdot P)^{1/2}$ relative to what is shown. Equations (12) are valid in scaling region $-q^2$, $q\cdot P$, $k\cdot P \to \infty$, x and $k\cdot P/q\cdot P$ fixed and of order unity. So far, we have not specialized to $k^2=0$ and have used only the condition $k^2/q\cdot P \lesssim 1$.

Equations (8), (10), and (12) combine to give

$$v = M\int_x^Y d\eta \left[\frac{-\pi^4 2^{2-d} 2^{2-d'} \Gamma(2-d)\Gamma(2-d')}{(q\cdot P)^{2-d}(q\cdot P)^{2-d'}\Gamma(d)\Gamma(d')} \left(\frac{1}{(x-\eta-i\epsilon)^{2-d'}} - \text{c.c.}\right) \left(\frac{1}{[(k\cdot P/q\cdot P)\eta - k^2/2q\cdot P - i\epsilon]^{2-d}}\right) H_1(\eta) \right.$$
$$+ \frac{-\pi^4 2^{2-d} 2^{2-d''} \Gamma(2-d)\Gamma(2-d'')}{(q\cdot P)^{2-d}(q\cdot P)^{2-d''}\Gamma(d)\Gamma(d'')} \left(\frac{1}{(x-\eta-i\epsilon)^{2-d''}} - \text{c.c.}\right)$$
$$\left. \times \left(\frac{1}{[-(\tilde{q}\cdot P/q\cdot P)\eta - \tilde{q}^2/2q\cdot P - i\epsilon]^{2-d}}\right) H_2(\eta)\right] . \tag{13}$$

In Eq. (13),

$$\eta = \frac{l^0 - l^1}{M} ,$$

$$H_i(\eta) \equiv \int d(l^0 + l^1) d^2 l_\perp \tilde{G}_i(M\eta, l^0 + l^1, \vec{1}_\perp, M) ,$$

and Y is a number of order x^{-1}. The lower bound of the η integration follows from the fact that

$$\frac{1}{(x - \eta - i\epsilon)^a} - \text{c.c.} = 0 \quad \text{for } \eta < x .$$

Several comments are in order.

(1) From Eq. (13), we can see that for $x \neq 0$ k^2 plays no special role, and we can take $k^2 = 0$.

(2) Problems could arise in the second term if $\tilde{q}^2 + 2\eta \tilde{q} \cdot P$ is near zero in the range of integration. Kinematical considerations reveal that this can happen only if k is at the part of its kinematical boundary where $k \propto q + xP$. We will assume that this region is avoided.

(3) Regge behavior (as discussed in connection with the parton model) may demand $\eta = 0$ singularities in the $H_i(\eta)$ and lead to a divergence in the η integral for $x = 0$.

Thus, if we avoid the point $x = 0$ and the boundary region $k \propto q + xP$ while satisfying

$$\begin{aligned} &-q^2 \to \infty , \\ &q \cdot P \to \infty , \\ &k \cdot P \to \infty , \qquad (14) \\ &x \sim 1 \text{ and fixed} , \\ &\frac{k \cdot P}{q \cdot P} \sim 1 \text{ and fixed} , \end{aligned}$$

the η integral in Eq. (13) will be finite. We can then conclude from the factors $(q \cdot P)^d (q \cdot P)^{d'}$ and $(q \cdot P)^d (q \cdot P)^{d''}$ which multiply scaling expressions in Eq. (13) that the terms with the largest d's (strongest light-cone singularities) will dominate. This is our main result. The reader should recall at this point that restrictions on the Fourier transforms of the bilocal operators (A and B above) were crucial.

It should be noted that large negative d's correspond to rather smooth light-cone behavior. Contributions from such terms are highly suppressed. This tends to support the conjecture that the nonsingular contributions to the double commutator which have not been considered will also be suppressed.

In the parton-model treatment[1] of the three-photon process, Brodsky, Gunion, and Jaffe required $q \cdot k$ to be large and negative.[13] We have found this restriction to be necessary. Diagrams of the type shown in Fig. 4 which were thought to be important for $q \cdot k > 0$ can be shown to vanish relative to those of Fig. 3 by employing the spectrum conditions and the assumption that amplitudes fall rapidly for virtual parton masses much larger than M. The argument parallels that leading to Eq. (12) above.

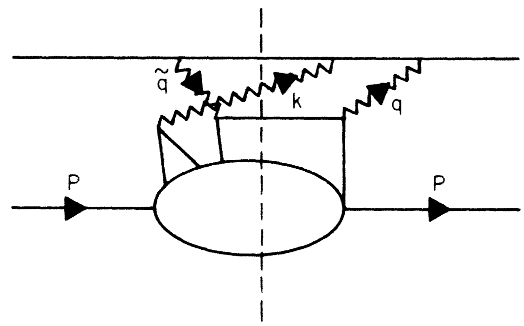

FIG. 4. Nonleading parton-model graph.

Finally, we comment on the contribution to the three-photon process which comes from the coupling of a vector meson, such as the ρ, to the real photon. An elementary ρ, with gauge-invariant coupling[14] to the photon, will not contribute because this coupling vanishes at $k^2 = 0$. The contribution from a composite ρ can be thought of as arising from complicated vertex corrections to the photon-parton vertex. However, we have seen that the parton model, with its version of assumption A, gives the result that the parton connecting the real and virtual photons is far off-shell in the scaling region considered. With one of the parton legs far off-shell, we can use the assumption that the parton-proton amplitude falls rapidly in the parton virtual mass to conclude that such vertex corrections must give nonleading contributions. We conclude then that assumption A of the light-cone treatment has indirectly eliminated the ρ contribution by associating it with weaker singularities which give nonleading contributions in the scaling region.

In the next section, we discuss the results which follow from assuming a canonical singularity structure for the three-current product.

IV. CANONICAL SINGULARITIES

In the last section, we found a set of assumptions which are sufficient to guarantee that the strongest singularities in the three-current product will give the dominant contribution to the matrix element. The electroproduction experiments suggest that the leading singularity of the two-current product is the same as that suggested by free-field theory.[8]

For the scalar case, which we discussed in Sec. III, free fields would suggest $d = d' = d'' = 1$. Equation (13) then becomes

$$v = \frac{+8\pi^4 M}{q\cdot P}\left[\frac{1}{-2xk\cdot P}H_1(x) + \frac{1}{\tilde{q}^2+2x\tilde{q}\cdot P}H_2(x)\right], \tag{15}$$

which is similar in structure to the parton-model result.[1]

For a brief discussion of the realistic case with vector currents, we can proceed by reasoning as follows:

(1) Section III has shown that the strongest singularities of $J_\nu(y)T[J_\lambda(0)J_\mu(x)]$ will dominate $V_{\mu\nu\lambda}$.

(2) The strongest singularities of current products are believed to be those given by free-field theory.[8]

(3) Fritzsch and Gell-Mann[8] have shown that this free-field singularity structure is effectively generated by using currents constructed from free parton (or in their case quark) fields. We can then follow Fritzsch and Gell-Mann (without assuming the specific quark charge assignments) and use free fields to generate the singularity and tensor structure of $J_\nu(y)T[J_\lambda(0)J_\mu(x)]$.

(4) This procedure will reproduce the parton-model result, since the parton model effectively uses parton fields and lowest-order perturbation theory to calculate $J_\nu(y)T[J_\lambda(0)J_\mu(x)]$.

V. CONCLUSION

We have analyzed the three-photon process in the scaling region given in Eq. (14). We conclude that the ideas of light-cone algebra when supplemented by assumptions A and B assure the dominance of the strongest light-cone singularities. The additional assumption of a free-field singularity structure will reproduce the parton-model result.

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$$\tilde{Q}^2 - Q^2 = -2k\cdot q \gg M^2.$$

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