Implications of the shadow effect in theories of gravitation

C. C. Chiang*†
Center for Particle Theory, The University of Texas at Austin, Austin, Texas 78712

John J. Dykla‡§ and A. E. Hwang‖¶
Center for Relativity Theory, The University of Texas at Austin, Austin, Texas 78712
(Received 19 November 1973)

Following the idea of the theory of shadow states in quantum field theory, we construct a Newtonian theory and a Whitehead theory of gravitation with shadow gravitational potentials. As a heuristic approach, Einstein's theory of gravitation is modified by introducing the shadow effect. This is achieved by using the effective mass distribution which arises in the Newtonian theory of gravitation with shadow effect as the only nonvanishing component of the stress-energy-momentum tensor of a static point source. It is found that in the static spherically symmetric solution of Einstein's equations, with proper choice of the new dynamical parameter, there is no surface of infinite red shift and the formation of a black hole can be avoided, leaving a naked singularity at the origin.

I. INTRODUCTION

The theorems of Hawking and Penrose[1] and Geroch[2] establish that in Einstein's theory of general relativity spacetime singularities always arise in gravitational collapse. Hawking[3] points out that in spherically symmetric collapse the field exterior to the collapsed star is described by the Schwarzschild metric and has the following features:

(a) There is a spacetime singularity.

(b) The formation of a black hole is inevitable.

(c) The singularity is hidden within the event horizon (black hole) and is not visible to observers who are beyond the Schwarzschild radius.

He also shows that (a) and (b) are stable under slight perturbations away from spherical symmetry and argues that it is physically reasonable to assume that no naked singularities are formed even in the case of nonspherical collapse. If Hawking's conjecture is true, this might mean that the occurrence of singularities in gravitational collapse is relatively harmless in that these singularities cannot affect what happens outside the collapsed object.

It is interesting to note that the attitude towards singularities in general relativity is quite different from the attitude toward singularities in the theories of the other fundamental interactions. Whereas spacetime singularities in general relativity have led to new physics, namely the theory of black holes, the divergences present in theories of other fundamental interactions have generally been taken as indicators of a breakdown in the theories and have led to modifications aimed at eliminating these infinities. In the classical theory of the electron the infinite self-force due to the singular electromagnetic potential excited by the electron leads to the instability of the electron and is usually eliminated through renormalization. Unfortunately the renormalization procedure merely avoids and does not solve the problem. In quantum field theory divergent quantities are usually also eliminated with the help of renormalization. It is known, however, that apart from the ambiguities inherent in the renormalization procedure, unrenormalized quantities in particle physics are oftentimes as important as the renormalized quantities.

It is often suggested that infinities are due to the fact that we oversimplify the structure of elementary particles by regarding them as structureless point particles and that incorporating the structure of particles into the theory could very well eliminate the problem with infinities. One such attempt involves the introduction of the notion of fundamental length in the hope that incorporating a fundamental length into the formalism would lead to a natural cutoff for the divergent integrals in the calculations. Several authors[4] have indicated that a fundamental length arises naturally if gravitational effects are believed to contribute significantly in nongravitational interactions. For example, if an elementary particle is prohibited from being a black hole, the size of the particle must be greater than its Schwarzschild radius. Defining the Schwarzschild radius of the particle as its fundamental length then offers a unified explanation for the occurrence of singularities in all interaction theories. Indeed if all elementary particles have finite extent, then in principle, one would not expect infinities to arise in these theories. Unfortunately, there is as yet no workable theory in accord with such a picture available.

An alternate viewpoint is to adopt the attitude that the occurrence of infinities signals a need for a self-consistent, genuine finite field theory. From this perspective the divergence problem in quantum field theory is linked with the dynamical problem of interaction instead of particle structure. Considerable progress towards constructing a genuine finite field theory has been made in recent years by using an indefinite metric and the idea of shadow states.[5] The finite theory with an indefinite metric and shadow states was originally designed for quantum field theory and has successfully eliminated many of its divergences. When extended to classical electrodynamics it has been shown that with the introduction of a shadow electromagnetic potential the electron is stable in the point-particle limit.[5]

One can ask whether the introduction of the shadow effect into the theory of gravitation can solve any of the problems resulting from space-time singularities. In this paper we examine the implications of the shadow effect in the theory of gravitation and determine whether it has any bearing on black-hole physics. To introduce the notion of the shadow effect we briefly discuss in Sec. II the shadow effect in the classical theory of electrodynamics. In Sec. III the results of Sec. II are extended to the Newtonian theory of gravitation. In Sec. IV we examine the shadow effect in the Whitehead theory of gravitation. Finally, in Sec. V, using the effective-mass distribution which arises in the Newtonian theory of gravitation with shadow effect, we solve the general-relativistic field equations.

II. CLASSICAL ELECTRODYNAMICS

As mentioned in the Introduction there are sufficient reasons to search for a genuine finite quantum field theory instead of ignoring the divergence difficulties by some renormalization procedure. In the last few years it has become clearer that it is possible to formulate a genuine finite quantum field theory by making use of an indefinite metric and the notion of shadow states.[5] The main idea is to introduce some auxiliary field, quantized with the "wrong" sign for the commutation relations, such that the ill-defined divergent integrals in the conventional theory become well-defined finite integrals. Moreover, the Green's functions are chosen in such a way that the probability among the physical states, i.e., those states without any excitation of auxiliary quanta, is conserved. Thus to ensure probability conservation, the symmetric (principal value) Green's functions are used in addition to the causal ones used in the conventional theory.

It has been shown by one of the authors (C.C.C.) that the runaway modes which appear in the dipole approximation of the conventional theory of electrodynamics do not occur if a massive auxiliary photon field quantized with the "wrong" sign for the commutation relations is introduced.[6] A relativistic formulation of the S matrix for finite quantum electrodynamics has also been constructed by Chiang and Gleeson.[7] An important question here is whether the effect due to such an auxiliary photon field has a counterpart in the classical theory of the electron. Indeed it has been found that with the introduction of a massive auxiliary electromagnetic field with negative energy density the electron is stable in the point-particle limit.[8] This is simply because the auxiliary photon field provides an attractive force to stabilize the electron. The auxiliary electromagnetic field behaves in a similar manner (apart from the Green's function) as the ordinary electromagnetic field, but yields an attractive force between like charges.

Let us write down the Lagrangian for the system consisting of an electron, an electromagnetic field $A_\mu(x)$, and a shadow electromagnetic field $B_\mu(x)$:

$$\begin{aligned}\mathcal{L} = \tfrac{1}{2} m \dot{x}^\mu \dot{x}_\mu - \tfrac{1}{4}\int d^3x\, F_{\mu\nu}(x) F^{\mu\nu}(x) \\ + \tfrac{1}{4}\int d^3x\, G_{\mu\nu}(x) G^{\mu\nu}(x) \\ - \tfrac{1}{2}\int d^3x\, M^2 B^\mu(x) B_\mu(x) \\ + e\dot{x}^\mu[A_\mu(x) + B_\mu(x)]\ ,\end{aligned} \tag{2.1}$$

where the overdot denotes time differentiation and $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$, $G_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu$. Note that since A_μ and B_μ couple through the same coupling constant, only the linear combination $A_\mu + B_\mu$ enters into the interaction. The sign of the free-energy term of the auxiliary field B_μ is opposite to that of the usual one. The equations of motion satisfied by the electron and the fields can be obtained by the standard variational method. The fields satisfy the equations

$$\Box A_\mu = -4\pi j_\mu(x)\ , \tag{2.2}$$

$$(\Box + M^2) B_\mu = +4\pi j_\mu(x)\ . \tag{2.3}$$

Observe that there is a relative sign change between the A_μ and B_μ fields in the field equations. And it is precisely because of this sign change that the mutual effects of two charged particles which arise by virtue of their coupling to A_μ and B_μ are of opposite signs. We may say that while the charged particles interact with both the electromagnetic field $F_{\mu\nu}$ and the auxiliary electromagnetic field $G_{\mu\nu}$ in the same way, they act as sources for these two fields in opposite ways.

Since we will be interested mainly in the static case in the theory of gravitation, let us consider Eqs. (2.2) and (2.3) when the electron is at rest. In the static case, (2.2) and (2.3) can be written as

$$\nabla^2 \phi_A = -4\pi\delta(x) , \tag{2.4}$$

$$(\nabla^2 + M^2)\phi_B = 4\pi\delta(x) . \tag{2.5}$$

Here ϕ_A and ϕ_B are the time components of A_μ and B_μ, respectively. The solutions of (4) and (5) are

$$\phi_A = \frac{1}{r} , \tag{2.6}$$

$$\phi_B = -\frac{e^{-Mr}}{r} . \tag{2.7}$$

The total potential excited by an electron is then given by

$$\phi = \phi_A + \phi_B = \frac{1}{r} - \frac{e^{-Mr}}{r} . \tag{2.8}$$

Note that ϕ is finite at $r=0$.

In the conventional theory, the potential (2.6) is singular at $r=0$. In order to obtain a nonsingular potential, we may consider the electron as a charged particle with charge distribution

$$\rho(r) = +\frac{M^2}{4\pi}\frac{e^{-Mr}}{r} . \tag{2.9}$$

It is simple to verify that the total charge for this charge distribution is equal to 1; i.e.,

$$\int \rho(r)\, d^3r = 1 , \tag{2.10}$$

and that the corresponding electromagnetic potential is given by

$$\phi_A' = \frac{1}{r} - \frac{e^{-Mr}}{r} , \tag{2.11}$$

which is finite at the origin and is the same as the ϕ given in (2.8). Even though ϕ_A' and ϕ are described by the same expression their physical significance is quite different. With the charge distribution (2.9) the electron is not stable, but with the point-charge distribution, the usual field A_μ, and the auxiliary field B_μ, the electron is stable. Whereas the potential ϕ_A' is the sum of the potentials due to a distribution of charge given by Eq. (2.9) inside the electron, the potential ϕ is due to a point charge but with a different *law of force*.

III. NEWTONIAN THEORY OF GRAVITATION

Owing to the linear superposition character of the potentials, it is easy to extend the idea of the shadow effect, as discussed in Sec. II, to the Newtonian theory of gravitation. In Newtonian theory the gravitational potential ψ_N due to a mass distribution $\rho(r)$ satisfied the equation

$$\nabla^2 \psi_N = 4\pi\kappa\rho(r) , \tag{3.1}$$

where κ is the gravitational coupling constant. As in the static case in electromagnetic theory, for a point particle with mass m, the gravitational potential, solving (3.1), is

$$\psi_N = -\frac{\kappa m}{r} . \tag{3.2}$$

This potential is singular at $r=0$. Following the idea of the shadow effect let us introduce a shadow potential ψ_S which satisfies the equation

$$(\nabla^2 + M^2)\psi_S = -4\pi\kappa\rho(r) , \tag{3.3}$$

where M is a new parameter to be fixed. Solving (3.3) yields the following shadow potential for a point particle with mass m:

$$\psi_S = \frac{\kappa m e^{-Mr}}{r} . \tag{3.4}$$

The total potential is then given by

$$\psi = \psi_N + \psi_S = \frac{km}{r}(e^{-Mr} - 1) . \tag{3.5}$$

The shadow potential is a repulsive potential in contrast to the ordinary attractive potential. As in the case of electrostatic potential, the potential ψ is regular at $r=0$.

IV. WHITEHEAD'S THEORY OF GRAVITATION

The gravitational theory of Whitehead is an action-at-a-distance theory in which particles interact through retarded gravitational tensor potentials in the flat spacetime of special relativity theory.[9] The choice of the retarded effect, instead of the advanced effect or a mixture of the advanced and the retarded effects, is based on the common belief in causality. The gravitational tensor potentials are obtained by a direct generalization of the static Newtonian potential to a four-dimensional propagating potential. If we assume the existence of the shadow gravitational potential in the Newtonian theory as discussed in Sec. III, the corresponding modification to Whitehead's theory of gravitation to include the shadow effect can be done without any change in the basic assumptions of Whitehead's theory except that of the unique choice of the retardation effect, which has to be porperly modified in accordance with the idea of the shadow effect.

Let m_a denote the proper masses of a collection of interacting particles and x_a^μ their coordinates. The equations of motion for particles subject to two-body interactions can be obtained by the variational method from the action function

$$A=\tfrac{1}{2}\sum_a\int m_a\dot{x}_a{}^2\,ds_a+\tfrac{1}{2}\sum_{a,b}\iint G_{ab}\,ds_a\,ds_b\ , \tag{4.1}$$

where $ds_a{}^2$ are the line elements of the various particles in flat spactime, i.e.,

$$\begin{aligned} ds_a{}^2&=\eta_{\mu\nu}\,dx_a^\mu dx_a^\nu \\ &=(dx_a^4)^2-(dx_a^1)^2-(dx_a^2)^2-(dx_a^3)^2\ , \end{aligned} \tag{4.2}$$

and G_{ab} are invariant functions depending on the relative coordinates and velocities of the particles. To obtain the gravitational tensor potentials of Whitehead's gravitation theory, let us choose the following form for G_{ab}:

$$G_{ab}=-K_a m_a K_b m_b\,\dot{x}_a^\mu\dot{x}_a^\nu\,\delta((x_a-x_b)^2)\,\dot{x}_{b\mu}\,\dot{x}_{b\nu}\ , \tag{4.3}$$

the dot denoting differentiation with respect to the appropriate proper-time labels. With this choice we may write Eq. (4.1) in the form

$$A=\tfrac{1}{2}\sum_a m_a\int g_{a\mu\nu}\,\dot{x}_a^\mu\dot{x}_a^\nu\,ds_a\ , \tag{4.4}$$

where

$$g_{a\mu\nu}=\eta_{\mu\nu}-K_a\sum_b K_b m_b\int\delta((x_a-x_b)^2)\,\dot{x}_{b\mu}\dot{x}_{b\nu}\,ds_b\ . \tag{4.5}$$

The gravitational tensor potential of Whitehead's theory, $g^W_{a\mu\nu}$, can be obtained by solving the integral equation (4.5) with proper choice of the boundary conditions. To do this, let us untroduce the abbreviation

$$V_{ab\mu\nu}=K_b m_b\int\delta((x_a-x_b)^2)\,\dot{x}_{b\mu}\dot{x}_{b\nu}\,ds_b\ . \tag{4.6}$$

Since

$$\Box\delta(x^2)=-4\pi\delta(x)\ , \tag{4.7}$$

we see that

$$\Box V^{\mu\nu}_{ab}=4\pi K_b m_b\int\delta(x_a-x_b)\,\dot{x}_b^\mu\dot{x}_b^\nu\,ds_b\ . \tag{4.8}$$

The general solution of (4.8) is easily found to be

$$V^{\mu\nu}_{ab}=K_b m_b\left[\alpha\,\frac{(x_a^\mu-x_b^\mu)_R\,(x_a^\nu-x_b^\nu)_R}{w_R{}^3}+\beta\,\frac{(x_a^\mu-x_b^\mu)_A\,(x_a^\nu-x_b^\nu)_A}{w_A{}^3}\right] \tag{4.9}$$

with $\alpha+\beta=1$. Here the subscript R (A) means that x_b^μ is a retarded (advanced) point with respect to x_a^μ:

$$(x_a^\mu-x_b^\mu)(x_{a\mu}-x_{b\mu})=0 \tag{4.10}$$

and

$$x_a^4-x_b^4>0\quad(x_a^4-x_b^4<0) \tag{4.11}$$

and $w_R=(x_a^\mu-x_b^\mu)\,\eta_{a\mu}$ $[w_A=(x_a^\mu-x_b^\mu)\,\eta_{b\mu}]$, where η^μ is a unit tangent.

In Whitehead's theory only the retardation effect is considered, i.e., $\alpha=1$ and $\beta=0$, and thus the gravitational tensor potentials are

$$g_{a\mu\nu}=\eta_{\mu\nu}-K_a\sum_b K_b m_b\,\frac{(x_{a\mu}-x_{b\mu})_R\,(x_{a\nu}-x_{b\nu})_R}{w_R{}^3}\ .$$

For the special case in which the particles b are at rest relative to some Galilean reference system, we have

$$\begin{aligned} g_{a\kappa l}&=-\delta_{\kappa l}-K_a\sum_b\frac{K_b m_b}{r_b}\,\frac{(x_{a\kappa}-x_{b\kappa})}{r_b}\,\frac{(x_{al}-x_{bl})}{r_b}\ ,\\ g_{a\kappa 4}&=K_a\sum_b\frac{K_b m_b}{r_b}\,\frac{(x_{a\kappa}-x_{b\kappa})}{r_b}\ ,\\ g_{a44}&=1-K_a\sum_b\frac{K_b m_b}{r_b} \end{aligned} \tag{4.13}$$

For a two-body problem, namely particle a and particle b, Eqs. (4.13) can be expressed in terms of spherical coordinates, and the line element takes the familiar form

$$\begin{aligned} ds^2=&\left(1-\frac{K_aK_b m_b}{r_b}\right)dt^2+\frac{2K_aK_b m_b}{r_b}\,dr\,dt\\ &-\left(1+\frac{K_aK_b m_b}{r_b}\right)dr^2-r^2(d\theta^2+\sin^2\theta\,d\phi^2)\ . \end{aligned} \tag{4.14}$$

As mentioned previously, the choice (4.3) of the invariant function G_{ab} in Whitehead's theory is guided by correspondence with Newtonian theory. Now, as in the Newtonian theory, we may introduce the shadow potential to modify Whitehead's theory. In analogy with (4.1) let us write the modified action function as

$$A^m=\tfrac{1}{2}\sum_a m_a\dot{x}_a{}^2\,ds_a+\tfrac{1}{2}\sum_{a,b}\iint(G_{ab}+G^S_{ab})\,ds_a\,ds_b\ . \tag{4.15}$$

G_{ab} is defined in (4.3), and G^S_{ab} is the shadow-invariant function given by

$$G^S_{ab}=-K_a m_a K_b m_b\,\dot{x}_a^\mu\dot{x}_a^\nu\,\Delta(x_a-x_b)\,\dot{x}_{b\mu}\dot{x}_{b\nu}\ , \tag{4.16}$$

where (x_a-x_b) satisfies the equation

$$(\Box+M^2)\,\Delta(x_a-x_b)=4\pi\delta(x_a-x_b)\ . \tag{4.17}$$

Here M is a new parameter which can be regarded as a dynamical parameter similar to a coupling constant. As before, let us introduce the abbreviation

$$V^S_{ab\mu\nu} = -K_b m_b \int \Delta(x_a - x_b)\, \dot{x}_{b\mu} \dot{x}_{b\nu}\, ds_b \ . \qquad (4.18)$$

Then, with the aid of Eq. (4.17), we see that $V^S_{ab\mu\nu}$ satisfies the differentio-intergral equation

$$(\Box + M^2)\, V^S_{ab\mu\nu} = -4\pi K_b m_b \int \delta(x_a - x_b)\, \dot{x}_{b\mu} \dot{x}_{b\nu}\, ds_b \ . \qquad (4.19)$$

The general solution of (4.19) includes retarded and advanced effects with appropriate weights. According to the idea of the shadow effect, one should take the sum of half retarded and half advanced solutions as the solution for $V^S_{ab\mu\nu}$.

For the special case in which the particles b are at rest, we have

$$g_{a\kappa l} = -\delta_{\kappa l} - K_a \sum_b K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right) \times \frac{(x_{a\kappa} - x_{b\kappa})}{r_b} \frac{(x_{al} - x_{bl})}{r_b} ,$$

$$g_{a\kappa 4} = K_a \sum_b K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right) \frac{(x_{a\kappa} - x_{b\kappa})}{r_b} ,$$

$$g_{a44} = 1 - K_a \sum_b K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right). \qquad (4.20)$$

For a two-body problem, the line element in spherical coordinates is

$$ds^2 = \left[1 - K_a K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right)\right] dt^2 - 2K_a K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right) dr\, dt - \left[1 + K_a K_b m_b \left(\frac{1}{r_b} - \frac{e^{-Mr_b}}{r_b}\right)\right] dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2) \ . \qquad (4.21)$$

V. EINSTEIN'S THEORY OF GENERAL RELATIVITY

Although extension of the idea of the shadow effect to Whitehead's theory of gravitation is straightforward, incorporating this idea into Einstein's theory of general relativity is somewhat troublesome, due mainly to the nonlinear character of Einstein's field equation. However, so far as the static solution is concerned, we may use, as in the cases of the static theory of the electron and the Newtonian theory of gravitation discussed in Secs. II and III, an effective mass distribution for the only nonvanishing component of the stress-energy-momentum tensor. The form of the effective mass distribution is chosen in such a way that the linearized Einstein's equation with the effective-mass distribution should give us the effective Newtonian potential (3.5). We may therefore use the following effective mass distribution

$$\rho = \frac{1}{4\pi} m M^2 \frac{e^{-Mr}}{r} , \qquad (5.1)$$

where M is a constant, m is the Newtonian mass of the source, and r is the curvature radius. Whether M varies from source to source is an open question. Note, however, that M is constant for a given m. With this choice of the effective mass distribution we may proceed to find the spherically symmetric solution for the Einstein equations.

Let us write down the Einstein field equations in terms of the Ricci tensor

$$R_{\mu\nu} = -8\pi(T_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} T) , \qquad (5.2)$$

and in terms of Einstein's tensor

$$G_{\mu\nu} = -8\pi T_{\mu\nu}, \qquad (5.3)$$

where

$$T_{\mu\nu} = \begin{bmatrix} \rho g_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \qquad (5.4)$$

It is well known that for the static spherically symmetric case, the metric tensor $g_{\mu\nu}$ may be written in the following form:

$$g_{\mu\nu} = \begin{bmatrix} e^{\nu} & 0 & 0 & 0 \\ 0 & -e^{\lambda} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{bmatrix} . \qquad (5.5)$$

From (5.2), (5.4), and (5.5) we have

$$R_{00} = -8\pi\rho e^{\nu} (1 - \tfrac{1}{2} e^{\nu}) , \qquad (5.6a)$$

$$R_{11} = -8\pi\rho e^{\nu+\lambda} , \qquad (5.6b)$$

$$R_{22} = 0 , \qquad (5.6c)$$

$$R_{33} = 0 , \qquad (5.6d)$$

where the Ricci tensor components can be directly computed from the Christoffel symbols to yield

$$R_{00} = -\tfrac{1}{2} e^{\nu-\lambda}\left(\nu'' + \tfrac{1}{2}\lambda'\nu' - \tfrac{1}{2}\nu'^2 + \frac{2\nu'}{r}\right) \tag{5.7a}$$

$$R_{11} = \tfrac{1}{2}\left(\nu'' - \tfrac{1}{2}\lambda'\nu' + \tfrac{1}{2}\nu'^2 - \frac{2\lambda'}{r}\right), \tag{5.7b}$$

$$R_{22} = e^{-\lambda}[1 + \tfrac{1}{2} r(\nu' - \lambda')] - 1, \tag{5.7c}$$

$$R_{33} = R_{22}\sin^2\theta, \tag{5.7d}$$

and

$$R = -e^{-\lambda}\left(\nu'' - \tfrac{1}{2}\lambda'\nu' + \tfrac{1}{2}\nu'^2 + \frac{2}{r}(\nu' - \lambda') + \frac{2}{r^2}\right) + \frac{2}{r^2}. \tag{5.7e}$$

Then the Einstein tensor which is defined as

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R \tag{5.8}$$

has the following components:

$$G_{00} = e^{\nu-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{e^\nu}{r^2}, \tag{5.9a}$$

$$G_{11} = \frac{1}{r^2}(e^\lambda - 1) - \frac{\nu'}{r}, \tag{5.9b}$$

$$G_{22} = -\tfrac{1}{2} r^2 e^{-\lambda}\left(\nu'' + \tfrac{1}{2}\nu'^2 - \tfrac{1}{2}\lambda'\nu' + \frac{\nu' - \lambda'}{r}\right), \tag{5.9c}$$

$$G_{33} = G_{22}\sin^2\theta. \tag{5.9d}$$

Now we are in a position to find the solutions for the metric tensor $g_{\mu\nu}$. From (5.1), (5.4), and (5.9a) we have

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -2mM^2\frac{e^{-Mr}}{r}. \tag{5.10}$$

The radial component of the metric may be computed by multiplying (5.10) by r^2. We find that

$$e^{-\lambda} = 1 + 2Mme^{-Mr} - \frac{2m}{r}(1 - e^{-Mr}); \tag{5.11}$$

therefore

$$g_{11} = -\left[1 + 2Mme^{-Mr} - \frac{2m}{r}(1 - e^{-Mr})\right]^{-1}. \tag{5.12}$$

Here we see that in (5.11) or (5.12), in addition to the usual attractive part, $-2m/r$, there is a repulsive part, $2Mme^{-Mr} + 2me^{-Mr}/r$. In the post-Newtonian limit, i.e, for large r where the exponential terms become negligible, (5.12) leads to Schwarzschildlike solution

$$g_{11} \simeq -1/(1 - 2m/r). \tag{5.13}$$

For small r, g_{11} becomes

$$g_{11} \simeq -1/(1 - mM^2 r), \tag{5.14}$$

which is regular for $r \to 0$. Note that $e^{-\lambda}$ given in (5.11) might have zero value, depending on the value of M. It can easily be shown that we can avoid the occurrence of this singularity by requiring that

$$\tfrac{3}{5} M < \frac{1}{m}. \tag{5.15}$$

This gives us an upper bound on the strength or equivalently, a lower bound on the range $R = 1/M$ of the repulsive interaction.

To find the time component of the metric let us multiply (5.6b) by $e^{\nu-\lambda}$ and add it to (5.6a). We obtain

$$\tfrac{1}{2} e^{-\lambda}\left(\frac{2\nu'}{r} + \frac{2\lambda'}{r}\right) = 8\pi\rho. \tag{5.16}$$

Thus

$$\begin{aligned}\nu' &= -\lambda' + e^{-\lambda}\rho \\ &= -\lambda' + 2mM^2 e^{\lambda} e^{-Mr}.\end{aligned} \tag{5.17}$$

This equation can be solved by quadratures. While the exact form of the solution is not available, we may solve (5.17) by a numerical method. As shown in Fig. 1, it is found that $\nu \to 0$ as $r \to \infty$ and $\nu \to -\infty$ as $r \to 0$. Therefore $g_{00} \to 1$ as $r \to \infty$ and $g_{00} \to 0$ as $r \to \infty$. In contrast to the Schwarzschild solution this model has no surface of infinite red shift and avoids the formation of a black hole. An infinite red shift does occur at the origin, where there is a naked singularity and at which point the physics is not quite clear.

The active gravitational mass can be defined as $M \equiv 4\pi\int_0^\infty \rho r^2 dr$. A direct calculation shows that M agrees with the Newtonian mass m. Here we see that by identifying m with the Schwarzschild mass yields the same post-Newtonian limit g_{11} for both this and the Schwarzschild solutions. These features suggest that all the gravitational mass is associated with the effective density ρ and allows us to consider the possibility that the

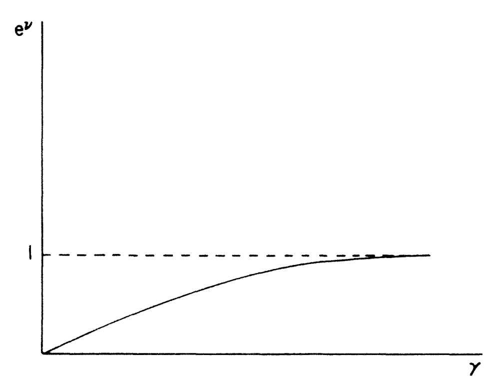

FIG. 1. General behavior of the solution for the metric tensor g_{00} (e^ν) [See Eq. (5.5)].

repulsive interaction which corresponds to equation (5.1) is a universal feature of gravity for any given mass.

In a separate paper[10] we have explored the consequences of applying this solution to macroscopic situations. It was found that our model is in agreement with the Schwarzschild solution for the classical tests of general relativity theory, namely the perihelion advance and the bending of light. Furthermore, it was also found that for various gravitationally bound systems condition (5.14) is always satisfied.

ACKNOWLEDGMENT

One of us (C.C.C.) would like to thank Professor E. C. G. Sudarshan for valuable discussions.

---

*Work supported in part by the U. S. Atomic Energy Commission under contract No. At(40-1)3992.

†Present address: Institute of Theoretical Physics, Fack, S-402 20 Göteborg 5, Sweden.

‡Work supported by NSF grant No. GP-3469X and by NSF grant No. GW-6716.

§Present address: Department of Physics, Loyola University of Chicago, Chicago, Illinois 60626.

‖University of Texas pre-doctoral fellow.

¶Present address: Bell Laboratories, Murray Hill, New Jersey 07974.

[1]S. W. Hawking and R. Penrose, Proc. R. Soc. A314, 529 (1969).

[2]R. Geroch, J. Math. Phys. 9, 450 (1968).

[3]S. W. Hawking, *Black Holes* (Gordon and Breach, New York, 1973).

[4]O. Klein, Kosmos 32, 33 (1954); Helv. Phys. Acta Suppl. 4, 58 (1956); and *Niels Bohr and the Development of Physics* (Pergamon, London, 1955), p. 96; S. Deser, Rev. Mod. Phys. 29, 417 (1957); C. Alden Mead, Phys. Rev. 135, B849 (1964).

[5]E. C. G. Sudarshan, Fields Quanta 2, 175 (1972); E. C. G. Sudarshan, Lecture at the Latin American Summer School, University of Mexico, 1972 (unpublished); C. A. Nelson and E. C. G. Sudarshan, Phys. Rev. D 6, 3658 (1972); E. C. G. Sudarshan and C. A. Nelson, Phys. Rev. D 6, 3678 (1972); A. M. Gleeson, R. J. Moore, H. Rechenberg, and E. C. G. Sudarshan, Phys. Rev. D 4, 2242 (1971); C. C. Chiang, Phys. Rev. D 7, 1725 (1973); J. L. Richard, Phys. Rev. D 7, 3617 (1973).

[6]C. C. Chiang, Phys. Rev. D 7, 3617 (1973).

[7]C. C. Chiang and A. M. Gleeson, Phys. Rev. D 10, 573 (1974).

[8]C. C. Chiang, Phys. Rev. D 8, 1025 (1973).

[9]A. N. Whitehead, *The Principle of Relativity* (Cambridge, Univ. Press, Cambridge, England, 1922). J. L. Synge, Proc. R. Soc. A211, 303 (1952); A. Schild, Proc. R. Soc. A235, 202 (1956).

[10]C. C. Chiang, John J. Dykla, and A. E. Hwang, Astrophys. J. 185, 87 (1973).