obtained by replacing e^2 with $\sum_\alpha Q_\alpha{}^2 e^2$.
[13] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Wiley, New York, 1959); K. Hepp, Commun. Math. Phys. 2, 301 (1966).
[14] W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory*, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), Vol. 1, p. 395.
[15] The t^S operations are defined in Ref. 14.

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Couplings of the ρ' meson in the quark-pair-creation model

William B. Kaufmann and Richard J. Jacob
Arizona State University, Tempe, Arizona 85281
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We show that the quark-pair-creation model can account for the strong suppression of the 2π mode relative to the 4π mode in the decay of the ρ' meson.

The elusive $\rho'(1600)$ meson has been the subject of several recent articles.[1] One curious feature of the meson is the nearly total absence of the decay mode into two pseudoscalar mesons. This has led to the suggestion that the $\rho'(1600)$ may be a kinematical effect[2] in the same spirit as the Deck effect. Although this is a viable alternative, it was our thought to see if conventional quark models for the $\rho'(1600)$ also might be able to account for the suppressed decay mode.[3] In this note we present a calculation of the decay of the ρ' meson assuming that its wave function corresponds to either a radially excited or an orbitally excited ρ meson. The decay dynamics is provided by a recent formulation of the quark-pair-creation (QPC) model.[4] Since this model contains an over-all arbitrary constant corresponding to the vacuum-$q\bar{q}$ coupling, we are able to predict only the ratio of the $\rho'\pi\pi$ and $\rho\pi\pi$ couplings. We find that *nodes* in the ρ' wave function (for either radial or orbital excitations) suppress the ratio, provided that the ρ' mass is near the observed one.

Two further points should be noted. First, symmetry principles alone [such as SU(6)] do not relate ρ and ρ' couplings. Second, couplings have recently been deduced from experimental data,[5] albeit in an approximate way. Thus we are able to put the QPC model to a new and nontrivial test.

In what follows, we first summarize the relevant quark-model wave functions. Then, the QPC dynamics is reviewed and the results are presented.

From standard harmonic-oscillator wave functions $\psi_{nl}(q)Y_{lm}(\hat{q})$ and SU(6) quark spinors[6] we can construct wave functions for the mesons that we need. We use the notation $n\,^{2S+1}L_j$ to denote radial, orbital, and spin quantum numbers. The π and ρ mesons correspond to $1\,^1S_0$ and $1\,^3S_1$ states, respectively. There are two likely models for the ρ' meson within the simple quark model. A radial excitation of the ρ meson, i.e., the $2\,^3S_1$ state, (Model I) is simple and attractive. A second and more popular model (II) is the orbitally excited ρ meson, with configuration $1\,^3D_1$. Actually the true wave function could be a mixture of these configurations, the mixing coming about through the coupling to $q\bar{q}q\bar{q}$ states. Previous calculations using a coupled-channel Schrödinger model indicate that the coupling may be quite large.[7] Finally, we need a model for the σ meson, which is believed to be one of the main decay products of the ρ' meson decay. To obtain J^P of 0^+, we need a spin-triplet state with one unit of orbital angular momentum. We have chosen the SU(6) state to be the same as the ω meson; however, an SU(3) singlet form is also possible.

Denoting the ρ, π, and ω meson spinors[6] by ρ_λ, π, and ω_λ, and the harmonic-oscillator state $|nlm\rangle$ by $\psi_{nl}Y_{lm}$, the explicit wave functions are

$$\psi_\pi = \pi\psi_{10}Y_0\,, \tag{1}$$

$$\psi_\rho\lambda = \rho_\lambda\psi_{10}Y_0\,, \tag{2}$$

$$\psi_{\rho}^{\lambda} = \begin{cases} \rho_\lambda\psi_{20}Y_0\,, & \text{model I} \quad (3)\\ \displaystyle\sum_{\nu,\mu} C^{121}_{\nu\mu\lambda}\,\rho_\nu\,Y_{2\mu}\psi_{12}\,, & \text{model II} \quad (4)\end{cases}$$

$$\psi_\sigma = \sum_{\nu,\mu} C^{110}_{\nu\mu 0}\,\omega_\nu\,Y_{1\mu}\psi_{11}\,, \tag{5}$$

where the C's are the usual Clebsch-Gordan coefficients.

According to the QPC model, a meson decays into two others through the creation of a vacuum pair of quarks (see Fig. 1). In order that the quantum numbers of the pair be that of the vacuum, the $q\bar{q}$ system must be in a $^3P_0$ state. The $\times$ in the figure indicates the phenomenological interaction which creates the pair. The strength of the coupling and the range of the oscillator wave function are the only parameters in the theory. The first parameter does not appear if we look at ratios of coupling strengths; the second can be determined through independent considerations[8] such as the slope of the ρ Regge trajectory.

The decay amplitude (Fig. 1) corresponds to the convolution of the product of the meson wave functions and the vacuum pair wave function. In the case of spinless particles for the final-state mesons, the explicit form is

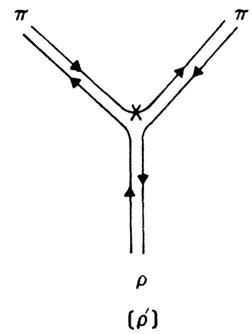

FIG. 1. The quark-pair-creation model. The $\times$ indicates a phenomenological interaction which produces the quark-antiquark pair from the vacuum.

$$\langle BC|T|A^J_M\rangle = \delta_{M0}\gamma\sum_m C^{110}_{m-m0}C^{LSJ}_{m-m0}\langle\Phi_B\Phi_C|\Phi^m_A V^{-m}\rangle\int\psi_B(-\vec{k})\psi_C(\vec{k})\psi_A^{-m}(\vec{k}_B+\vec{k})\mathcal{Y}^m_1(\vec{k}_B-\vec{k})d^3k\,, \tag{6}$$

where V^{-m} is the spin triplet and SU(3) singlet state corresponding to the vacuum pair. The Φ's are SU(6) spinors and $\mathcal{Y}^m_l$ is a solid harmonic. The matrix element is compared with the phenomenological Lagrangian, e.g.,

$$\mathcal{L}_{\rm int} = -f_{\rho\pi\pi}\vec{\pi}\times\partial_\mu\vec{\pi}\cdot\vec{\rho}_\mu\,,$$

to evaluate the coupling $f_{\rho\pi\pi}$. Here we have assumed that the axis of quantization is the same as the direction $\vec{k}_B$. A similar form can be written if the final-state particles have spin. The reader is referred to the original paper (Ref. 4) for further details.

In Ref. 4 the matrix element [our Eq. (6)] is evaluated for the case of the $f_{\rho\pi\pi}$ coupling. The integral is most easily computed in cylindrical coordinates, and the SU(6) matrix elements are evaluated from the explicit wave functions given in the book by Feld.[6] The resulting coupling constant is

$$f_{\rho\pi\pi} = c\,m_\rho^{3/2}R_\rho^{3/2}\exp[-k_\pi^2R_\rho^2/12]\,, \tag{7}$$

where c is a constant proportional to γ. Similar calculations using the two models for the ρ' meson (3) and (4) lead (after some algebra) to

$$\begin{aligned}\alpha_{\rm I} &\equiv (f_{\rho'\pi\pi}/f_{\rho\pi\pi})_{\rm I}\\ &= (m_{\rho'}/m_\rho)^{3/2}(\tfrac{3}{2})^{1/2}(\tfrac{5}{9})[1-\tfrac{2}{15}(k'_\pi R)^2]\\ &\quad\times\exp[-(k'^2_\pi - k_\pi^2)R^2/12]\end{aligned} \tag{8}$$

for model I and

$$\begin{aligned}\alpha_{\rm II} &\equiv (f_{\rho'\pi\pi}/f_{\rho\pi\pi})_{\rm II}\\ &= (\tfrac{4}{5})^{1/2}\alpha_{\rm I}\end{aligned}$$

for model II. The momentum of a pion in the decay ρ $(\rho')\to 2\pi$ is denoted by k_π (k'_π). In the spirit of the simple quark model we have set all oscillator radius parameters equal.[9] It is interesting that both models give the same vertex functions with only a very small difference in the coupling strength. Thus even if there is considerable configuration mixing between orbitally and radially excited states, there is a strong suppression of the coupling strength if $k'^2_\pi \simeq 15/2R^2$.

The value of the oscillator radius parameter R has been determined by the Orsay group by several independent methods, the preferred value being 8 GeV^{-2}.[8] Figure 2 gives the ratio α_1 as a function

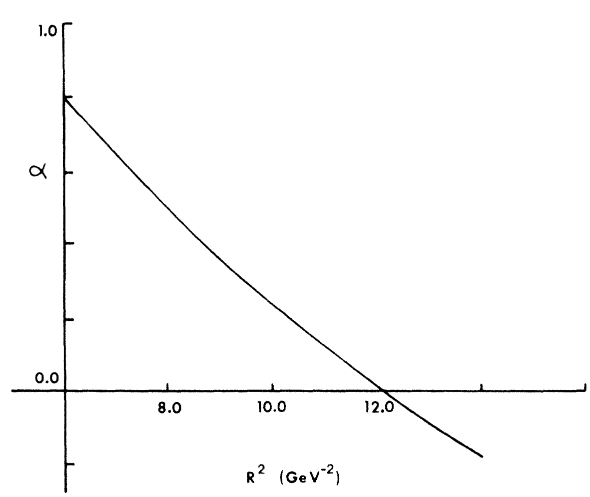

FIG. 2. Coupling ratio $\alpha_1 = (f_{\rho'\pi\pi}/f_{\rho\pi\pi})$ vs oscillator radius parameter R. The zero is ultimately due to the node in the ρ' wave function.

of R^2. The zero at about $R^2 \simeq 12$ GeV^{-2} arises from the polynomial multiplying the Gaussian. The Gaussian factor varies only slightly over the range of R^2 shown. The most recent estimate[5] of α is 0.2 which obtains for $R^2 \simeq 10$ GeV^{-2} in reasonable agreement with the earlier determination. This value of α is subject to considerable uncertainty, however.[4]

The ρ' meson decays predominantly through the 4π mode, with only about 1% into the 2π mode. In our interpretation, this is due to the nearness of the zero in the polynomial factor in the expression for α [Eq. (8)]. The $\rho' \to 2\rho$ decay is found to have the same polynomial factor as in Eq. (8) with k'_π being replaced by k'_ρ, but the decays $\rho' \to 2\rho \to 4\pi$ would not experience significant suppression since the polynomial and Gaussian factors are nearly one due to the smaller value of k'^2_ρ. This decay has, however, relatively little phase space unless the ρ' mass is somewhat larger than 1.6 GeV. (See Cason *et al.*, Ref. 1, who find a mass of 1.71 GeV and claim the 2ρ mode dominates.)

We have also investigated the decay mode $\rho' \to \rho\sigma \to 4\pi$ in the two models.[10] This mode has more phase space and has the advantage of having zero orbital angular momentum in the $\rho\sigma$ system. The polynomial-exponential factor which appears in the coupling function for Model I is

$$[1 - \tfrac{1}{3}(k_\sigma R)^2 + \tfrac{2}{27}(k_\sigma R)^4] \exp[-(kR)^2/12],$$

which has no real roots. It has a minimum at $(kR)^2 \simeq 2.6$. Assuming $m_\sigma = 0.6$ GeV, $m_\rho = 0.77$ GeV, $m_{\rho'} = 1.6$ GeV, and $R^2 = 10$ GeV^{-1}, the factor has a value 0.563. This is to be compared with only 0.103 for the polynomial-exponential factor

$$[1 - \tfrac{2}{15}(k_\pi R)^2] \exp[-(k_\pi R)^2/12]$$

which occurs for the 2π mode, and with 0.901 which occurs in the analogous factor for the 2ρ mode.

In summary, the ordinary harmonic-oscillator quark model augmented by the quark-pair-creation dynamics gives a simple explanation for a strongly suppressed two-pion decay mode of the ρ' meson while allowing the 4π mode to proceed unimpeded.

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[1] A. Bramon and M. Greco, Nuovo Cimento Lett. 1, 739 (1971), *ibid.* 3, 693 (1972); G. Barbarino, M. Grilli, E. Iarocci, P. Spillantini, V. Valente, R. Visentin, F. Ceradini, M. Conversi, L. Paoluzi, R. Santonico, M. Nigro, L. Trasatti, and G. T. Zorn, *ibid.* 3, 689 (1972); H. H. Bingham, W. B. Fretter, W. J. Podolsky, M. S. Rabin, A. H. Rosenfeld, G. Smadja, G. P. Yost, J. Ballam, G. P. Chadwick, Y. Eisenberg, E. Kogan, K. C. Moffeit, P. Seyboth, I. O. Skillicorn, H. Spitzer, and G. Wolf, Phys. Lett. 41B, 635 (1972); N. M. Cason, N. N. Biswas, V. P. Kenney, W. B. Madden, O. R. Sander, and W. D. Shephard, Phys. Rev. D 7, 1971 (1973).

[2] T. Ferbel and P. Slattery, Phys. Rev. D 9, 824 (1974).

[3] The 2π mode suppression has also been accounted for by a gauge model [K. Fujikawa and P. J. O'Donnell, Phys. Rev. D 8, 3994 (1973)]. We would like to thank R. Buchl for bringing this work to our attention.

[4] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 8, 2223 (1973).

[5] S. Matsuda, C. Huang, and S. Oneda, Phys. Rev. D 8, 4133 (1973). This article contains further references.

[6] B. T. Feld, *Models of Elementary Particles* (Blaisdell, Waltham, Mass., 1969), p. 327.

[7] A. Ahmadzadeh and W. Kaufmann, Phys. Rev. D 3, 1923 (1971).

[8] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Orsay Report No. LPTHE 72/6 (unpublished).

[9] E. Leader and B. Nicolescu [Phys. Rev. D 7, 836 (1973)] have suggested that the slope of the ρ' trajectory is about one third of that of the ρ. We have used the *same* radius parameter and hence the same slope, however, in the spirit of the simple quark model. It is possible (see Ref. 7) to obtain a smaller slope for the ρ' meson than for the ρ meson dynamically within a coupled-channel quark model.

[10] We would like to thank Robert B. Clark for pointing out to us the possible importance of this decay mode.