Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in **Physical Review Letters** *and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in* **The Physical Review** *or* **Physical Review Letters**; *(2) addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters**, *in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section should be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

Multiplicity distribution and meson interaction*

T. F. Hoang
Argonne National Laboratory, Argonne, Illinois 60439
(Received 13 December 1973; revised manuscript received 19 March 1974)

The multiplicity distributions from high-energy pp collisions are analyzed using a Poisson-type distribution which has two input parameters expressed in terms of the measures of the average multiplicity and the width parameter. A discussion is presented on its statistical and physical properties, as well as the mechanism of meson interaction in analogy with the photon statistics due to Scully and Lamb.

One of the salient features of the multiparticle production from high-energy hadron collisions is that the multiplicity distribution appears to be of the Poisson type. This is observed by plotting $\log(n!\sigma_n)$ against n, σ_n being the cross section for the multiplicity n.[1] It has been noticed that the plot is close to a straight line; the latter is to be expected in the case of a Poisson distribution.

In this paper we present results of a further analysis of multiplicity distributions using the following probability distribution for n particles:

$$p_n = N\frac{\alpha^n}{(n+\beta)!}, \quad (1)$$

where α and β are two input parameters to be determined in the following, and N is the normalization constant. We note that this distribution reduces to Poisson's law in the case $\beta = 0$.

We are led to consider this distribution because of its specific properties, statistical as well as physical. First, we note that the parameters α and β can easily be expressed in terms of the measures of the first and second moments of the experimental distribution (cf. infra). This enables us to analyze the data without any free parameterization. Such a procedure is statistically more meaningful than the usual least-squares method. In this regard, we note that with our method the conditions imposed by the moments on the parameters are identically satisfied *a priori*. Next, from the physics point of view, we note that this kind of Poisson-type distribution has been derived by Scully and Lamb for the photon statistics in their quantum theory of a maser.[2] Thus the physics underlying such a distribution is well known. We may interpret its departure from Poisson's law, which, as is well known, holds only for particles in a coherent state.[3]

We proceed to outline some properties of the probability function under consideration. From (1) we obtain for the average multiplicity

$$\langle n\rangle = \sum_{n=0}^{\infty} np_n = (\alpha-\beta)+\epsilon, \quad (2)$$

where $\epsilon = N\beta/\beta!$. Note that for $n=0$ we have $p_0 = N/\beta!$, which represents the percentage of zero-multiplicity events. Therefore we may express ϵ in terms of experimental cross sections as follows:

$$\epsilon = \beta\sigma_0/\sigma_{\text{inel}}, \quad (3)$$

where

$$\sigma_{\text{inel}} = \sum_{n=0}^{\infty}\sigma_n$$

is the total inelastic cross section. Since $\sigma_0/\sigma_{\text{inel}} < 1$, we note that $\epsilon < \beta$.

As for the second moment, we find

$$\langle n^2\rangle = \alpha + (\alpha-\beta)^2 + (\alpha-\beta)\epsilon. \quad (4)$$

From (2) and (4) we deduce the width parameter $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$ as follows:

$$f_2 = \beta - \epsilon(\alpha-\beta+1) - \epsilon^2. \quad (5)$$

Solving for α and β from (4) and (5), we obtain

$$\alpha=(1+\epsilon)\langle n\rangle+f_2\,,\quad \beta=f_2+(1+\langle n\rangle)\epsilon\,, \qquad (6)$$

where ϵ is given by (3).

In order to simplify the computation, we note that if we drop ϵ in the above equations we have for the first approximation $\alpha\simeq\langle n\rangle+f_2$ and $\beta\simeq f_2$. As f_2 is rather small according to the experimental data, we write

$$\epsilon\simeq f_2\sigma_0/\sigma_{\text{inel}}\,. \qquad (7)$$

In this way, we simplify considerably the computations of α and β. The validity of the approximation thus made will be justified *a posteriori* by the consistency tests which will be discussed in detail afterwards.

We now proceed to analyze the multiplicity distributions from high-energy *pp* collisions. We shall use the currently available data ranging from 303 to 29 GeV/*c*,[4-8] and limit ourselves to the negative prongs only. The characteristics of these experiments are listed in Table I. For each set of these data we compute ϵ using (7), then α and β by (6). The fits are performed by requiring that the total inelastic cross section of the fitted distribution be the same as that of the experimental value σ_{inel} as listed in Table I. This amounts to multiplying (1) by σ_{inel}/N, N being the normalization factor. The fitted curves are shown in Fig. 1. The values of ϵ, α, β, σ_{inel}/N, and χ^2/point are presented in Table I.

A comparison of the fitted curves with the histograms and an inspection of the values of χ^2/point in Table I indicate that the fits we have obtained without free parameters are indeed very satisfactory.

As for self-consistency tests, we have evaluated the average multiplicity $\langle n\rangle$ and the width parameter f_2 from the fitted distributions; we should find the same values as the input ones computed from the experimental data. For a further check, we have also computed the third moment μ_3. The values thus obtained are listed in Table I. The plots in Fig. 2 show the comparison of these computed values with those from the experimental data. The fact that all points lie very close to the bisector verifies that the probability function (1) here considered is indeed adequate and that our method of analysis with no free parameterization as well as our approximation as regards the computation of ϵ by (7) are both valid.

It is interesting to note that the scale factors (see Table I) are practically constant for P_{in} above 50 GeV/*c*; this behavior may be regarded as the scaling property of the high-energy inelastic interaction.

It should be mentioned that the correction terms involving ϵ, although small in general, are essential for a faithful reproduction of moments higher than the first order. We have investigated this point by refitting the data with $\epsilon=0$ and found that although the values of $\langle n\rangle$ are not affected much, both f_2 and μ_3 deviate significantly, as much as 50%, from the expected values of the experimental data.

Finally, we mention that if we treat α and β as free parameters and try to fit the multiplicity distributions with (1), we find that the estimates of α and β thus obtained are consistent, within statistical errors, with the values computed from expressions (6).

We now turn to the physics content as revealed by the multiplicity distributions which we have analyzed using the probability function (1). For this purpose, we use the analogy with the photon distribution given by the theory of Scully and

TABLE I. Experimental data and results of analysis. ϵ is a correction term for α and β; see Eq. (9). α and β are input parameters for fits; see Eqs. (7) and (8).

P_{in} (GeV/*c*)	303	205	102	69	50	29
			Experimental			
σ_{incl} (mb)	31.80 ± 0.79	32.70 ± 1.20	32.79 ± 1.50	31.32 ± 0.31	31.12 ± 0.93	27.82 ± 0.71
$\langle n\rangle$	3.43 ± 0.06	2.83 ± 0.08	2.19 ± 0.07	1.96 ± 0.02	1.68 ± 0.04	1.27 ± 0.03
f_2	1.37 ± 0.59	0.94 ± 0.21	0.33 ± 0.10	0.12 ± 0.06	−0.04 ± 0.12	−0.21 ± 0.09
μ_3	96.7 ± 1.7	58.8 ± 1.7	29.6 ± 1.0	21.7 ± 0.2	14.3 ± 0.4	6.8 ± 0.2
			Characteristics of fits			
ϵ	0.075	0.111	0.046	0.019	−0.008	−0.062
α	5.04	4.17	2.62	2.12	1.63	1.67
β	1.64	1.43	0.48	0.18	−0.06	−0.31
Scale factor	10.5	8.0	8.5	7.2	5.3	4.4
χ^2/point	1.9	1.3	0.5	2.3	1.8	1.2
$\langle n\rangle$	3.49	2.87	2.20	1.96	1.67	1.22
f_2	1.14	0.84	0.24	0.09	−0.04	−0.17
μ_3	96.7	60.0	29.2	21.7	14.4	6.5

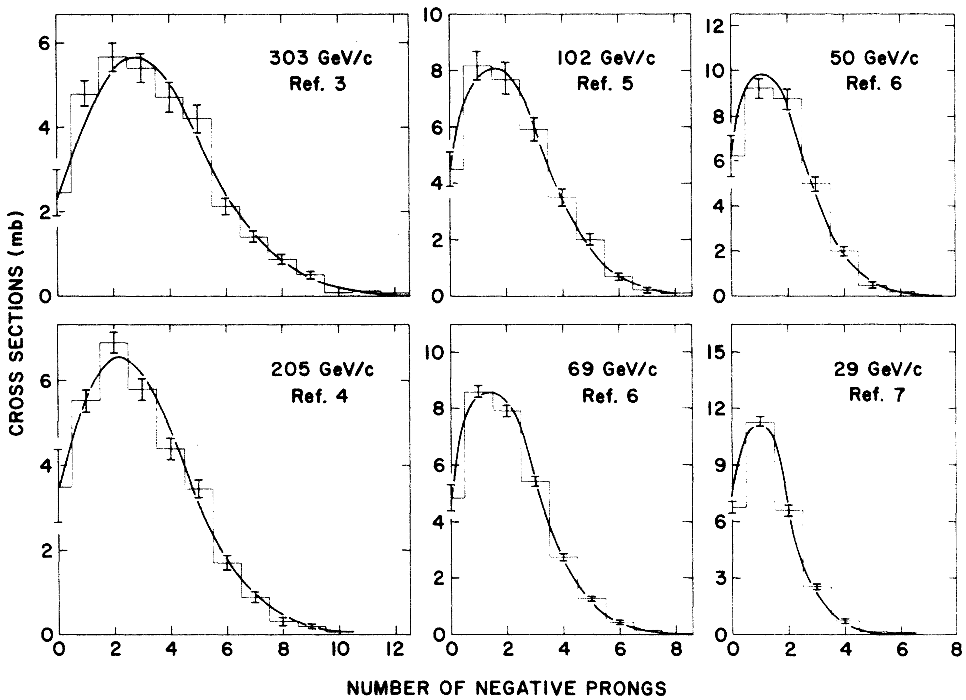

FIG. 1. Comparison of fits with experimental multiplicity distributions from pp collisions. The parameters and the characteristics of the fits are listed in Table I.

Lamb.[2] According to their result, the probability for observing n particles is

$$p_n \sim \frac{(A^2/BC)^n}{(n+A/B)!}\ , \qquad (8)$$

which is formally identical with (1). We refer to Ref. 2 for the derivation of (8), and recall that A, B, C are three coefficients characterizing the transitions between neighboring states of n and $n \pm 1$ particles. They are as follows: A and C are related to the emission and absorption of a single particle, whereas B corresponds to a double transition with an emission followed by an absorption of one particle; this transition plays the role of a damping.

The important point is that the distribution (8), although originally derived on the basis of the quantum electrodynamics, is indeed very general and holds also for other kinds of interactions proceeding through the similar transitions specified by three coefficients A, B, and C. This is because, as pointed out by Scully and Lamb,[2] this distribution can be obtained in a straightforward way by applying the detailed balance to the transitions between two neighboring states described above.

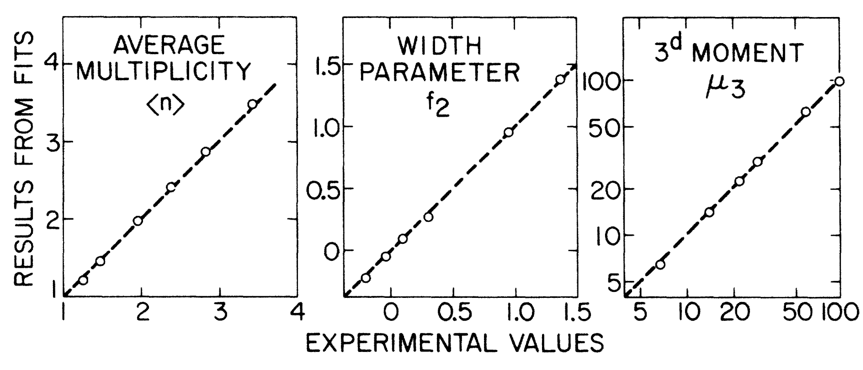

FIG. 2. Consistency tests for $\langle n\rangle$, f_2, and μ_3. The values are listed in Table I.

Thus, if we assume that the mechanism of meson interaction proceeds through such transitions between neighboring states of n and $n\pm 1$ mesons as is described by the theory of Scully and Lamb,[2] we obtain, *mutatis mutandis*, for the meson multiplicity distribution the same expression as for the photon distribution (8). We may express the parameters α and β, characteristics of our meson multiplicity distribution (1), in terms of the coefficients A, C, and B of the theory of Scully and Lamb.[2] For this purpose, we compare (1) with (8) and find

$$\alpha = \frac{A^2}{BC}, \quad \beta = \frac{A}{B} . \tag{9}$$

Now we can interpret the physical meaning of the distribution (1). Recalling that $f_2 \simeq \beta$, we note that the multiplicity distribution would be reduced to Poisson's law, if $B \to \infty$; this means no damping. Clearly, β must be positive-definite. In this regard we note that in Table I there are some negative values for β; however, this should not be taken literally, since all the β's would be positive if instead of negative secondaries only, we considered both positive and negative prongs. We refer to a previous paper for a discussion on this point.[9]

Consider now the average multiplicity $\langle n\rangle \simeq \alpha - \beta$; we find

$$\langle n\rangle = \left(\frac{A}{B}\right)\left(\frac{A-C}{C}\right) . \tag{10}$$

First, we note that A must be greater than C; this indicates that in the second-order process the creation dominates the annihilation of a meson. Next, we note that the same damping factor A/B appears also in the expression for $\langle n\rangle$. This may remind us of the bremsstrahlung process of meson production.[10] However, we have to bear in mind that the damping mechanism in the theory of Scully and Lamb[2] is of a different nature; it is due to the nonlinear gain. Note that the condition mentioned before for observing a Poisson distribution, namely $B=\infty$, leads to a self-contradictory result, because then $\langle n\rangle = 0$. This rules out the Poisson distribution, which describes idealistic processes without random fluctuations.

To sum up, guided by the analogy with the photon statistics, we tentatively interpret the deviation of multiplicity distributions from Poisson's law as due to a damping characterized by the factor A/B, its effect being to translate the number of particles n by an amount $\beta = B/A \simeq f_2$ and $\langle n\rangle$ to $\langle n\rangle + f_2$. It would be interesting to investigate whether the analogy we have invoked also has applications for other problems of multiparticle production.

Finally, we note that although the distribution (1) has the specific properties discussed above, it does not satisfy the semi-inclusive scaling postulated by Koba, Nielsen, and Olesen,[11] and that it cannot be decomposed into two parts to fit within the framework of a two-component model.[12]

Acknowledgment is due to Professor E. C. G. Sudarshan and Dr. G. Thomas for comments and discussions.

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]See, e.g., T. F. Hoang, Phys. Rev. D 7, 2799 (1973); W. R. Frazer, R. D. Peccei, S. S. Pinsky, and C.-I Tan, *ibid.* 7, 2647 (1973). The use of a Poisson distribution to analyze the multiplicity distributions was first made by C. P. Wang [Nuovo Cimento 64A, 546 (1969); Phys. Rev. 180, 1463 (1969)]. For its consistency test, we refer to T. F. Hoang, Nuovo Cimento 2A, 467 (1971).

[2]M. Scully and W. E. Lamb, Jr., Phys. Rev. Lett. 16, 853 (1966); Phys. Rev. 159, 208 (1967).

[3]See, e.g., E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill, New York, 1962), Chap. 2, and J. R. Klauder and E. C. G. Sudarshan, *Fundamentals of Quantum Optics* (Benjamin, New York, 1968), Chap. 7.

[4]F. T. Dao, D. Gordon, J. Lach, E. Malamud, T. Meyer, R. Poster, and W. Slater, Phys. Rev. Lett. 29, 1627 (1972).

[5]G. Charlton, Y. Cho, M. Derrick, R. Engelmann, T. Fields, L. Hyman, K. Jaeger, U. Mehtani, B. Musgrave, Y. Oren, D. Rhines, P. Schreiner, H. Yuta, L. Voyvodic, R. Walker, J. Whitmore, H. B. Crawley, Z. Ming Ma, and R. G. Glasser, Phys. Rev. Lett. 29, 515 (1972).

[6]J. W. Chapman, N. Green, B. P. Roe, A. A. Seidl, D. Sinclair, J. C. Vander Velde, C. M. Bromberg, D. Cohen, T. Ferbel, P. Slattery, S. Stone, and B. Werner, Phys. Rev. Lett. 29, 1686 (1972).

[7]France–Soviet Union Collaboration (V. V. Ammosov *et al.*), Phys. Lett. 24B, 519 (1972).

[8]D. B. Smith, R. J. Sprafka, and J. A. Anderson, Phys. Rev. Lett. 23, 1964 (1969).

[9]Hoang, Ref. 1.

[10]R. P. Feynman, in *High Energy Collisions*, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969) pp. 249 ff.; Phys. Rev. Lett. 23, 1415 (1969).

[11]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 3176 (1972).

[12]Assuming $\alpha = \langle n\rangle + f_2$ and $\beta = f_2$, we may expand (1) as follows:

$$P_n = N\frac{\langle n\rangle^n}{n!}\left\{1 + f_2\left[\frac{n}{\langle n\rangle} - \psi(n+1)\right]\right\},$$

where $\psi(x) = (d/dx)\ln\Gamma(x)$ denotes the digamma function. Clearly, the first part represents a pure Poisson distribution. However, the second part in brackets has no physical meaning because it has no definite sign unless $\langle n\rangle$ is less than 0.48.

PHYSICAL REVIEW D VOLUME 10, NUMBER 3 1 AUGUST 1974

Scalar contribution to the e^+e^- annihilation in asymptotically free theories*

J. S. Kang

Brandeis University, Department of Physics, Waltham, Massachusetts 02154

(Received 25 March 1974)

The neutral scalar mesons are incorporated into the asymptotically free theories to see if this can explain the experimental approach of the e^+e^- total cross section to its possible scaling limit. The corresponding fourth-order vacuum polarizations are calculated. The scalar contributions are found to be unable to explain the experiments as the limiting behavior in the semiasymptotic region.

Recently much attention has been given to the asymptotically free theories which exhibit Bjorken scaling in the deep Euclidean region.[1,2] In these theories the effective couplings vanish asymptotically and scaling is attained up to logarithmic corrections, which are explicitly calculable from the perturbation expansion. Of particular interest is the e^+e^- annihilation cross section into hadrons, where the leading term is scale-invariant and the correction depends only on the group structure of the theory.[3] Unfortunately, the calculation indicates an approach to the scaling limit from above, while experimental cross sections are still rising.[4]

The purpose of this paper is to see if this difficulty can be overcome by the incorporation of scalar mesons into the theory. Let us consider a non-Abelian gauge theory of the strong interactions which involves a number of fermion multiplets and scalar multiplets, limited in number to preserve asymptotic freedom. Assuming the strong gauge group to be electrically neutral, we require that it commutes with the gauge group of the weak and electromagnetic interactions. Hence, the scalar mesons are neutral scalar gluons. Since we have scalar mesons in the theory we are led to the study of renormalization-group equations with three coupling constants: the strong gauge coupling g, the scalar self-coupling λ, and the Yukawa coupling G. The equations for the effective coupling constants $\bar\lambda$, $\bar g$, and $\bar G$ are[5]

$$\frac{d\bar\lambda}{dt} = a_1\bar\lambda^2 + a_2\bar\lambda\bar g^2 + a_3\bar g^4 + a_4\bar G^4 + a_5\bar\lambda\bar G^2 + \text{higher-order terms}, \qquad (1a)$$

$$\frac{d\bar g^2}{dt} = -b\bar g^4 + \text{higher-order terms}, \qquad (1b)$$

$$\frac{d\bar G^2}{dt} = c_1\bar G^4 + c_2\bar G^2\bar g^2 + c_3\bar G^2\bar\lambda^2 + \text{higher-order terms}. \qquad (1c)$$

Higher-order terms can be obtained from higher-order perturbation calculations. However, we do not need them because the origin in the three-dimensional coupling-constant space is an ultraviolet-stable fixed point and they are not dominant in the asymptotic limit. The various coefficients a_i, b, c_i can be calculated from the relevant Feynman diagrams. For the purpose of our discussion, however, we only need to know a_1, b, c_1, and c_2, which are

$$a_1 = \frac{1}{16\pi^2}(N_s + 8),$$

$$b = \frac{1}{16\pi^2}\left[\tfrac{1}{3}(22\tau - 8\tau_2 - \theta_2)\right]. \qquad (2)$$

$$c_1 = \frac{1}{16\pi^2}(\gamma_1 + 2\gamma_2 + 2\gamma_3),$$

$$c_2 = -\frac{1}{16\pi^2}6\tau_1,$$