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Model for elastic scattering at wide angle

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In a model for wide-angle elastic scattering each of the hadrons involved is pictured as being composed of a number of constituents. Each constituent of one hadron scatters at wide angle on at least one constituent of the other, in such a way that all the constituents remain near their mass shell. The resulting differential cross section is small because of the limited phase space available: It is required that all the constituents scatter through nearly the same angle, so that they can readily recombine to form the final-state hadrons. The differential cross sections calculated from the model have energy dependences that do not agree with those which would be obtained from simple dimensional counting, and indeed the mechanism of the model can dominate over other mechanisms that have been proposed for wide-angle scattering. The model is confronted with existing experimental data.

I. INTRODUCTION

There is a considerable quantity of accurate data for wide-angle proton-proton elastic scattering at moderate energies. For s between 15 and 60 GeV^2, and $|t| > 2.5$ GeV^2, an excellent fit[1] to these data is given by

$$\frac{d\sigma}{dt} \sim s^{-n} f(\theta), \tag{1.1}$$

with

$$n \approx 9.7. \tag{1.2}$$

Here θ is the center-of-mass scattering angle. The data for πp and Kp elastic scattering[2] appear also to be in agreement with the general form (1.1), though of course with different values of n and different functions f. However, these data are at rather lower energy, and are much less accurate, than in the pp case.

It is widely assumed that these simple features of the data indicate that some sort of asymptotic regime has set in and, further, that they are a manifestation of a constituent structure of the scattering hadrons. However, the precise mechanism by which such a constituent structure manifests itself is far from clear, and indeed different authors[3-9] have adopted rather different models.

It has been emphasized[1,10,11] that the asymptotic form (1.1) may well be overly simple. In particular, the data certainly allow the inclusion of a factor of a power of ln$s$, such as is obtained[12] in a covariant version of a particular constituent model.[3] Alternatively, (1.1) might be replaced by a rather more complicated form, such as is obtained in models[13,11] that emphasize the multiple exchange of some hypothetical elementary vector gluon, rather than a constituent structure of the hadrons.

The main purpose of this paper is to analyze another type of constituent model. In most of the previous types of model, only one constituent of each hadron plays an active role in the scattering, in the sense that it alone is exchanged,[3] or, alternatively, scatters directly[4,5,8] on a constituent of the other hadron, according to the precise version of the model. A consequence of this is that[12] the scattering depends on constituents that have a large component of momentum transverse to the momentum of their parent hadron. The differential cross section that results for the scattering is small because such constituents are found comparatively rarely; to calculate the differential cross section, one has to feed in assumptions about the distribution of large-transverse-momentum constituents within a hadron.

In the model that is described here, it is assumed that constituents with large transverse momentum are found so rarely within a hadron that any scattering mechanism that might depend on them is insignificant. Instead, each initial-state hadron is pictured as breaking up into a number of (virtual) constituents whose momenta are all more or less parallel to the momentum of their parents; each constituent of one hadron then scatters at wide angle on at least one constituent of the other hadron in such a way that after the scatterings the momenta of the constituents are so aligned that they can readily recombine to make up the final-state hadrons. The differential cross section for the over-all process is small, partly because the phase space available to the constituents after they have scattered is limited, if they are to recombine. (It may also be small because the unknown amplitudes for the wide-angle elastic scattering of the constituents may be small.)

In such a framework, the complexity of the interaction increases with the number of hadronic con-

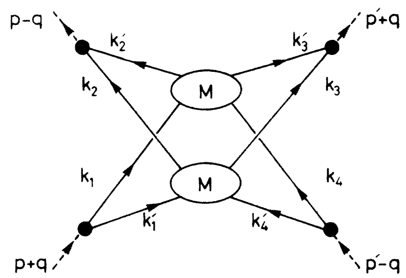

FIG. 1. Model used in Sec. II for $\pi\pi$ scattering, and in Sec. III for "core-core" scattering.

stituents. Thus, assuming that it is realistic to regard the pion as being composed of a single quark-antiquark pair, the simplest wide-angle elastic scattering would be $\pi\pi$ scattering. The corresponding amplitude is drawn in Fig. 1. At the four external vertices are coupling functions that restrict the momentum components of the quarks transverse to the momentum of their parent hadron. In a covariant formalism such as will be described in this paper, these coupling functions are functions of the squared four-momenta of the quarks concerned, and it turns out that the desired effect is obtained by supposing that the couplings go to zero sufficiently rapidly (like some inverse power) as either or both of the squared four-momenta become large. Thus, each of the wide-angle quark-quark scattering amplitudes M is evaluated with the quarks not too far off shell. Notice that, provided that the amplitudes M are assigned the appropriate crossing properties, the whole diagram of Fig. 1 will behave appropriately under crossing.

The model gives a result having the simple structure of (1.1). It turns out that the value of the constant n depends only on the behavior assumed for M, and not on the amount of damping provided by the vertex functions, provided only that the damping is sufficient. It is frequently assumed[4-8,14] that the differential cross section

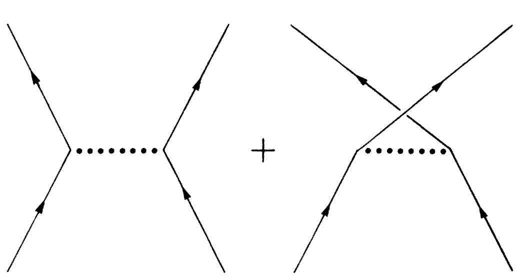

FIG. 2. Model for the quark-quark elastic amplitude at high energy and wide angle; for quarks near the mass shell, the coupling of the exchanged gluon is supposed to be pointlike.

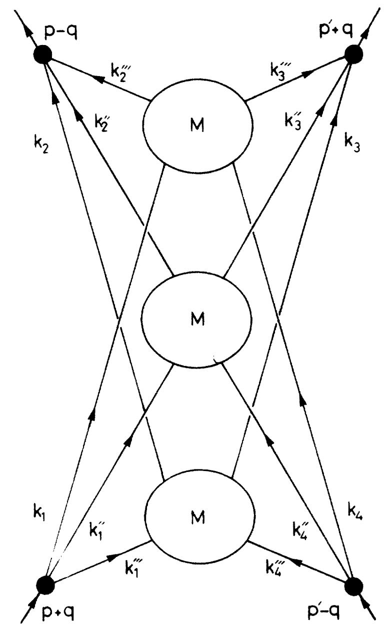

FIG. 3. Model for NN scattering.

$d\sigma/dt$ for quark-quark elastic scattering goes to a finite multiple of the kinematic factor s^{-2} at high energy and wide angle, and a simple structure for M that gives such a behavior will be arbitrarily assumed here. This is shown in Fig. 2 and corresponds to the simple exchange of an elementary scalar or pseudoscalar gluon, with pointlike coupling. It is also necessary to specify the spinor structure of the external vertex functions, and a simple γ_5 coupling of the pion to the quarks will be assumed, together with a form factor to provide the necessary damping.

One then obtains from Fig. 1 the result $n = 5$ for $\pi\pi$ scattering. The analysis is described in Sec. II. The main interest in this result is that it is in disagreement with the prediction based on simple dimensional counting,[6-8] which would give $n = 6$ here. The technical[15] reason for this is that the asymptotic behavior of Fig. 1 results from a "pinch contribution," in contrast with the simpler "end-point" contributions obtained from most other models.

The corresponding model for wide-angle nucleon-nucleon scattering, where each nucleon is regarded as a bound state of three quarks, is drawn in Fig. 3. This is analyzed in Sec. III and the results are discussed in the light of the existing data. In order to fit the model to the data, it is necessary to suppose that the mass μ on the internal quark propagators is small. In the final section of the paper, it is argued that this could perhaps explain the interesting experimental result[1,2] that, at

$\theta = 90°$ and incident momentum 10 GeV/c, the differential cross section for πp scattering is very much smaller than that for pp scattering (the ratio is a few percent). In a model where the differential cross section is simply related to the elastic form factor, the opposite would be expected, since at any large value of t the pion form factor is thought to be rather greater than the proton form factor.

II. $\pi\pi$ SCATTERING

To analyze Fig. 1, it is convenient to label the momenta as shown, so that

$$\begin{aligned} &p \cdot q = 0 = p' \cdot q\,, \\ &p^2 = p'^2 = -q^2 + m^2\,, \quad = \tau\,, \text{ say,} \end{aligned} \tag{2.1}$$

where m is the pion mass. Write

$$p \cdot p' = \lambda\tau\,. \tag{2.2}$$

Then in the high-energy wide-angle limit $\tau \to \infty$, λ fixed

$$\begin{aligned} &s \sim 2\tau(1+\lambda)\,, \\ &t \sim -4\tau\,, \\ &u \sim 2\tau(1-\lambda)\,. \end{aligned} \tag{2.3}$$

Write each of the internal momenta k as a linear combination

$$k = xp + yp' + zq + \kappa\,, \tag{2.4}$$

where κ is orthogonal to each of p, p', and q, and so is one-dimensional and spacelike. Then

$$k^2 = \tau[x^2 + y^2 - z^2 + 2\lambda xy] + m^2 z^2 - \kappa^2\,. \tag{2.5}$$

Consider the lower left-hand vertex. The coupling of the pion to the quarks will be taken to be

$$\gamma_5 g(k_1^{\,2}, k_1'^2)\,, \tag{2.6}$$

where the function g is supposed to have the property that when either (or both) of $k_1^{\,2}, k_1'^2$ is large, it goes to zero so rapidly that the dominant contribution to the integral obtained from Fig. 1 arises from values of $k_1^{\,2}, k_1'^2$ that remain bounded as $\tau \to \infty$. If one defines $\bar{y}_1$ and $\bar{z}_1$ by

$$\begin{aligned} &y_1 = \bar{y}_1/\sqrt{\sigma}\,, \quad z_1 = x_1 + \lambda y_1 + \bar{z}_1/2\tau\,, \\ &\sigma = \tau(\lambda^2 - 1)\,, \end{aligned} \tag{2.7}$$

this means that $\bar{y}_1$ and $\bar{z}_1$ remain bounded, with

$$\begin{aligned} &k_1^{\,2} \sim -x_1\bar{z}_1 + m^2 x_1^{\,2} - \bar{y}_1^{\,2} - \kappa_1^{\,2}\,, \\ &k_1'^2 \sim (1-x_1)\bar{z}_1 + m^2(1-x_1)^2 - \bar{y}_1^{\,2} - \kappa_1^{\,2}\,. \end{aligned} \tag{2.8}$$

Similarly, consideration of the other vertices leads to the introduction of further barred variables:

$$\begin{aligned} &y_2 = \bar{y}_2/\sqrt{\sigma}\,, \quad z_2 = -x_2 - \lambda y_2 - \bar{z}_2/2\tau\,, \\ &x_3 = \bar{x}_3/\sqrt{\sigma}\,, \quad z_3 = y_3 + \lambda x_3 + \bar{z}_3/2\tau\,, \\ &x_4 = \bar{x}_4/\sqrt{\sigma}\,, \quad z_4 = -y_4 - \lambda x_4 - \bar{z}_4/2\tau\,. \end{aligned} \tag{2.7$'$}$$

For Fig. 1 one needs the integration

$$\begin{aligned} \int d^4k_1 d^4k_2 d^4k_3 d^4k_4\, \delta^{(4)}(k_1+k_2+k_3+k_4-p-p') \sim \frac{1}{2^4\tau\sqrt{\sigma}} \int dx_1 \cdots d\bar{x}_4 d\bar{y}_1 \cdots dy_4 d\bar{z}_1 \cdots d\bar{z}_4 d\kappa_1 \cdots d\kappa_4\, \delta(\kappa_1 + \cdots + \kappa_4) \\ \times \delta(x_1 + x_2 - 1)\delta(y_3 + y_4 - 1)\delta(x_1 - x_2 + y_3 - y_4)\,. \end{aligned} \tag{2.9}$$

The δ functions make

$$\begin{aligned} &x_1 = y_4\,, \quad = \xi\,, \text{ say,} \\ &x_2 = y_3 = 1 - \xi\,. \end{aligned} \tag{2.10}$$

It turns out that $\bar{z}_1$ appears in the integrand only in the vertex function for the lower left-hand vertex, and in the denominators of the propagators of the quarks attached to that vertex, through its appearance in (2.8). According to usual ideas, $g(k_1^{\,2}, k_1'^2)$ is analytic in the $k_1^{\,2}, k_1'^2$ complex planes, with singularities just below the real axis; the propagators have similar properties. Thus, in order that the $\bar{z}_1$ integration be nonzero,

$$0 < \xi < 1\,; \tag{2.11}$$

otherwise, one can close the contour of integration in the $\bar{z}_1$ complex plane by an infinite semicircle in either the upper or the lower half plane. The condition (2.11) is needed also to make the other three $\bar{z}$ integrations nonzero.

If one inserts the structure of Fig. 2 for M into Fig. 1, a large number of terms result; quarks or antiquarks can scatter directly on either quarks or antiquarks; quarks can exchange-scatter with quarks, and antiquarks with antiquarks; or quarks can annihilate antiquarks. Each of the terms has the structure of one of the two diagrams in Fig. 4, or of a diagram obtained from one of these by crossing.

In Fig. 4(a), the exchanged particles require propagators

$$[(k_1 - k_2')^2(k_2 - k_1')^2]^{-1} \sim [16\tau^2\xi^2(1-\xi^2)]^{-1}\,, \tag{2.12}$$

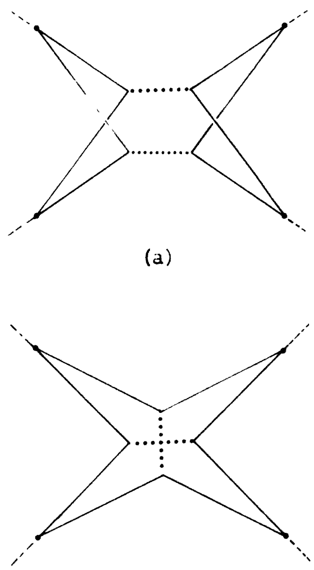

FIG. 4. Diagrams obtained on inserting the structure in Fig. 2 for M into Fig. 1. Crossed versions of the diagrams are also obtained.

and the traces around the fermion loops give $16\tau^2\mu^4(2\xi-1)^4$, where μ is the mass on the quark propagator. (In taking these traces, one uses the fact that $\int d\bar{y}\,\bar{y}f(\bar{y}^2)=0$.) Thus, the amplitude derived from Fig. 4(a) has the asymptotic form

$$\frac{C}{(stu)^{1/2}}, \tag{2.13}$$

where C is a constant:

$$C=\tfrac{1}{4}\mu^4\int_0^1\frac{d\xi_1 d\xi_2\,\delta(\xi_1+\xi_2-1)(\xi_1-\xi_2)^4}{\xi_1^{\,2}\xi_2^{\,2}} \times\int_{-\infty}^{\infty}d\kappa\,N^2(\xi,\kappa), \tag{2.14a}$$

where

$$N(\xi,\kappa)=\int d\bar{y}_1 d\bar{y}_2 d\bar{z}_1 d\bar{z}_2 d\kappa_1 d\kappa_2\,\delta(\kappa-\kappa_1-\kappa_2)G_1G_2, \tag{2.14b}$$

$$G_i=\frac{g(k_i^{\,2},k_i'^{2})}{(k_i^{\,2}-\mu^2)(k_i'^{2}-\mu^2)},\quad i=1,2 \tag{2.14c}$$

$$\begin{aligned}k_i^{\,2}&=-\xi_i\bar{z}_i+m^2\xi_i^{\,2}-\bar{y}_i^{\,2}-\kappa_i^{\,2},\\ k_i'^{2}&=(1-\xi_i)\bar{z}_i+m^2(1-\xi_i)^2-\bar{y}_i^{\,2}-\kappa_i^{\,2}.\end{aligned} \tag{2.14d}$$

Similarly, Fig. 4(b) gives $\frac{1}{2}C/(stu)^{1/2}$. Hence, since each of these results is crossing-symmetric, the differential cross section obtained from the many terms of Fig. 1 is

$$\frac{d\sigma}{dt}\sim\frac{\text{const}}{s^3tu}, \tag{2.15}$$

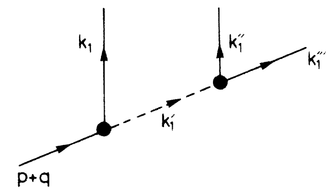

FIG. 5. Assumed structure for the coupling of the nucleon to three quarks. The broken line is the "core".

with the constant proportional to the eighth power of the mass on the quark propagator.

Notice the importance of the damping at large $k_i^{\,2}, k_i'^{2}$ provided by the vertex functions g. The precise nature of this damping does not affect the value obtained for n in (1.1), provided that the damping is sufficient. But if the damping were absent, so that the pions coupled to quarks in a pointlike fashion, a different value for n would result. This is because the integrations in (2.14a) would then diverge at $\xi=0$. The analysis would then have to be modified; to avoid the divergence at $\xi_1=0$, one would write

$$\xi_1=\bar{\xi}_1/\sqrt{\tau} \tag{2.16}$$

before taking the limit under the integral [the propagators (2.12) would then need to be evaluated more accurately]. The contribution from near $\xi_2=0$ would be dealt with similarly, and must be added on. The result is to change n from 5 to 4. Taking into account the four powers of τ here obtained in (2.15) from the traces around the fermion loops, this agrees with the result obtained for Fig. 4(a) by Halliday[16] in his calculation using α-space methods and ignoring spins.

III. NUCLEON-NUCLEON SCATTERING

For the analysis of Fig. 3, Eqs. (2.1)–(2.5) again apply, with m now the nucleon mass.

It is necessary to make assumptions about the structure of the external vertices, in particular about their spin structure. For simplicity, it will be supposed here that the nucleon first breaks up into a quark plus a zero-spin "core," and the core subsequently breaks up into two quarks (Fig. 5). Each of these breakups is described by a coupling function, and these are supposed to go to zero sufficiently rapidly when any of their variables $k_1^{\,2}, k_1'^{2}, k_1''^{2}, k_1'''^{2}$ becomes large. The spinor structure of Fig. 5 is taken to be the same as that which would be obtained if the core had the simplest point coupling. The core need not have a definite mass; its propagator can have a continuous spectrum. Indeed it need not have any real existence at all: Figure 5 can be interpreted just as describing the organization of the spin structure and the essential dependence of the vertex function on its scalar variables. Permutations of Fig. 5 are added, so as to obtain the appropriate symmetry of the vertex with respect to k_1, k_1'', k_1'''.

Inserting this vertex structure into Fig. 3, together with the structure of Fig. 2 for M, one obtains a large number of terms. Each quark of one nucleon may either scatter directly on a quark of the other nucleon or exchange-scatter with it. Each of the terms has the general structure of one of the two diagrams in Fig. 6. The "core-core" scattering amplitude T has a structure like Fig. 1, the external lines now being cores instead of pions.

The analysis of either diagram in Fig. 6 proceeds along the lines of that given for Fig. 1 in Sec. II, apart from the taking of the trace. In particular, the internal momenta are handled exactly as in Sec. II. Thus, the amplitude T has to be evaluated with Mandelstam variables

$$\begin{aligned} s' &= (k_1'+k_4')^2 \\ &\sim 2\tau(1+\lambda)(1-\xi)^2 = s(1-\xi)^2, \\ t' &= (k_1'-k_2)^2 \\ &\sim -4\tau(1-\xi)^2 = t(1-\xi)^2, \\ u' &= (k_1'-k_3)^2 \\ &\sim 2\tau(1-\lambda)(1-\xi)^2 = u(1-\xi)^2. \end{aligned} \tag{3.1}$$

Since T has the structure of Fig. 1, its asymptotic behavior may be evaluated in just the same way as was used for the $\pi\pi$ amplitude in Sec. II. There are two unimportant differences: The taking of the traces is a little different, because some of the quark propagators point in the reverse direction, and the external "masses" for T are not all equal. The result is [see (2.13)]

$$\frac{C(k_1'^2, k_2^2, k_3^2, k_4'^2)}{(s't'u')^{1/2}} = \frac{C}{(1-\xi)^3(stu)^{1/2}}, \tag{3.2}$$

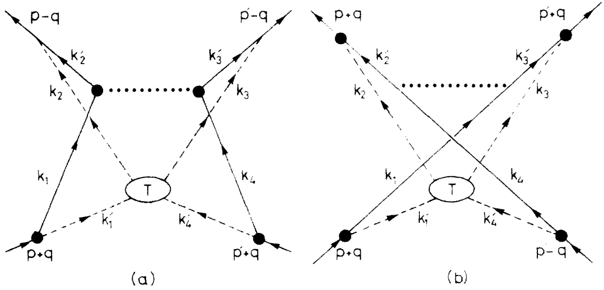

FIG. 6. Diagrams obtained on inserting the structures of Figs. 2 and 5 into Fig. 3. The amplitudes T represent "core-core" scattering.

where C is a function whose precise structure will not matter here, except that, as in (2.14a), it is proportional to the fourth power of the mass μ on the quark propagator.

The diagrams of Fig. 6 each obtain a factor

$$(2^4\tau\sqrt{\sigma})^{-1} = [4(stu)^{1/2}]^{-1}$$

from changing integration variables, as in (2.9). The two diagrams respectively require factors

$$\begin{aligned} &[(k_1-k_2')^2]^{-1} \sim -4\xi^2\tau, \\ &[(k_1-k_3')^2]^{-1} \sim -2\xi^2\tau(\lambda-1) \end{aligned} \tag{3.3}$$

for the exchanged propagators. Adding the diagrams together, squaring, and taking the appropriate traces to account for the spins of the external nucleons, one finds

$$\frac{d\sigma}{dt} \sim \frac{A}{s^4t^2u^2}, \tag{3.4}$$

where the constant A is proportional to μ^8. This corresponds to $n=8$ in (1.1), which again is in disagreement with the value $n=10$ obtained from dimensional arguments.[6-8]

The result (3.4) does not[1,10] agree well with existing data for wide-angle pp scattering. A possible remedy is to suppose that μ is very small, so that to compare with data at presently accessible energies one must replace (3.2) by the highest term in the asymptotic form of T whose coefficient is not proportional to a power of μ. (The power of μ came from taking internal traces, so now these traces are replaced by their $\mu\to 0$ limit.) Then, instead of (3.4),

$$\begin{aligned} \frac{d\sigma}{dt} &\sim \frac{1}{s^4t^2u^2}\left(\frac{A'}{t^2}+\frac{A'}{u^2}-\frac{B}{tu}\right)^2 \\ &\sim \frac{1}{(s\sin\theta)^{12}}[A'-\tfrac{1}{4}(2A'+B)\sin^2\theta]^2, \end{aligned} \tag{3.5}$$

where the relative magnitudes of the constants A', B depend on the isospin properties of the quark-quark scattering amplitude M. If the quarks do not undergo charge-exchange scattering at wide angle, $A'=B$; if they do, A' is a little different from B—just how different then also depends on the properties of the external wave functions in Fig. 3.

The bulk of the data on wide-angle pp scattering suggest[1] that n in (1.1) is near 10. However,[3,10] it is possible that, for $s \gtrsim 30\ \text{GeV}^2$, the value of n changes to 12. In Fig. 7 is plotted (3.5) with $A'=B$, that is,

$$\frac{d\sigma}{dt} \propto \frac{(1-\frac{3}{4}\sin^2\theta)^2}{(s\sin\theta)^{12}}, \tag{3.6}$$

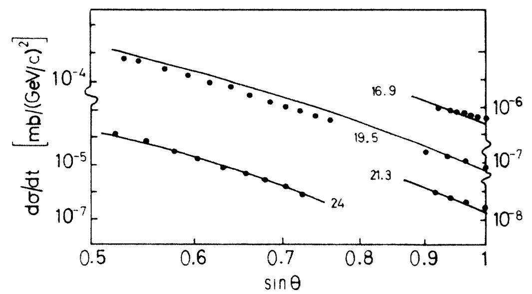

FIG. 7. Data for pp elastic scattering at various laboratory momenta, with the curves calculated from (3.6).

together with the higher-energy data. Not too much should be read into the good agreement, since it is over such a restricted range of energy. Also, of course, it is not clear why one should believe that μ is in fact small.

Apart from a flux factor s^{-2}, the result (3.4) is crossing-symmetric. Thus, the differential cross section for $p\bar{p}$ scattering would also be equal to (3.4). However, if (3.6) applies, one has

$$\frac{d\sigma/dt\,(p\bar{p})}{d\sigma/dt\,(pp)} \sim \left[\frac{(7-4\cos\theta+\cos^2\theta)(1+\cos\theta)^2}{4(4-3\sin^2\theta)}\right]^2 . \qquad (3.7)$$

That is, the ratio is 1 near the forward direction, about 3 at 90°, and 0 near the backward direction.

IV. DISCUSSION

The main result of this paper is that diagrams such as Fig. 1 and Fig. 3, in which the constituents of the scattering hadrons remain close to their mass shell, can give important contributions for wide-angle elastic scattering. In particular, the contributions are larger asymptotically than is expected from simple dimensional arguments[6-8] because[15] they correspond to pinch effects rather than end-point effects.

The simple discussion of Secs. II and III has been on the basis that the pion consists of just a quark-antiquark pair, and the nucleon of just three quarks. There is strong evidence from deep-inelastic electroproduction[17] that in fact the hadron structure involves also an infinite sea of quark-antiquark pairs. The interaction between the seas of two hadrons is supposed to correspond, in Regge language, to the exchange of a Pomeron between the hadrons. This suggests that one should consider Fig. 1 or Fig. 3 modified by the exchange of a Pomeron between any pair of external hadrons. If one makes the standard assumption of Reggeon calculus,[18] that the coupling of the Pomeron to a hadron goes to zero when the squared four-momentum of one of the hadron legs goes to infinity, one finds that such Pomeron exchange gives a negligibly small contribution. This is to be contrasted with the important effect[13,11] obtained from the exchange of an *elementary* particle of spin one, with pointlike coupling to the hadrons; here it will be assumed that there is no elementary hadron of spin one. Similar remarks apply to the exchange of a Pomeron, or of an elementary hadron of spin one, between a pair of internal lines or between an external and an internal line. So the conclusion is that the sea has no important effect.

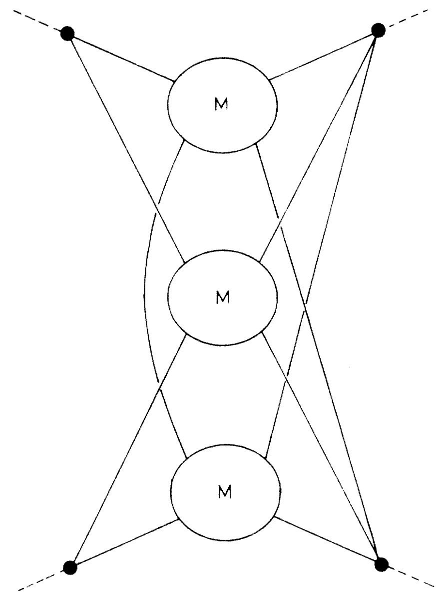

FIG. 8. Model for πN scattering.

It is of interest to discuss the elastic scattering of two hadrons with different numbers of constituents, particularly πN scattering. In order that each of the three constituents of the nucleon should have its momentum turned through a large angle (so that they can easily recombine after the scattering), one of the constituents of the pion must scatter twice (Fig. 8). This diagram gives $n=6$ in (1.1), and a factor μ^6, where μ is the mass on the internal quark propagators. With the assumption made in Fig. 2 for the amplitude M, that asymptotically it is represented by pure scalar, pseudoscalar, or indeed vector exchange, all the terms in $d\sigma/dt$ derived from Fig. 8 are proportional to μ^2 at least. This is because the trace associated with the left-hand fermion loop, which contains an odd number of fermion lines, is proportional to μ. If, as was suggested as a possibility in Sec. III, μ is small, the most important term in $d\sigma/dt$ at moderate energies is the one proportional to the lowest power of μ, that is μ^2; for this, $n=8$. It is conceivable that this is an explanation for the smallness of $d\sigma/dt$ in πN scattering, compared with NN scattering, which was remarked on in Sec. I.

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