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Measuring the Gravitational Interaction of Elementary Particles*

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An approach to the measurement of static gravitational couplings is discussed. A plausible mathematical treatment of the nongravitational interactions of particles in the presence of a small, constant, classical tensor field without the assumption of universal coupling to the field is outlined. Some nonstandard features, principally the breakdown of strict Lorentz covariance, are pointed out. Calculations based on this formalism, or, alternatively, on simple energy-momentum conservation, indicate that some particle processes which are forbidden if the coupling to the tensor field is universal are allowed if it is not. The processes considered in some detail are spontaneous emission of a photon by a free particle, spontaneous photoproduction of a particle-antiparticle pair, and spontaneous neutrino decay. These occur under certain conditions involving the energies and the coupling constants to the tensor field. The tensor field is identified with the local gravitational field and rates are estimated. Certain existing data on the absence of such reactions give evidence that the gravitational coupling of the muon in particular is equal to that of the photon to within parts per ten thousand, and that all particles with mc^2 less than a few GeV, excepting possibly the neutrinos and the graviton itself, have gravitational coupling which is at most the normal amount—to varying degrees of accuracy. The latter result is subject to an assumption that the algebraic sign of the local gravitational field is what one expects on intuitive grounds.

I. INTRODUCTION

ONE expects elementary particles to participate in a fourth interaction in addition to the strong, electromagnetic, and weak interactions. The theories of this "gravitational" interaction which have been put forth[1] share one basic feature: All particles are to interact with equal strength with a gravitational field. (A possible exception would be the graviton itself.[2]) I will call this principle "universal gravitation," after Isaac Newton.

In spite of the fact that universal gravitation is widely believed, the actual evidence for it—or indeed for any general statement about the gravitational interaction—is meager: The classical experiments which are usually associated with general relativity, the Eötvös-type experiments, and a few scattered free-fall measurements[3] tell us about the gravitational interaction of the constituents of ordinary matter and of light rays, namely, the electron, the proton, the neutron, and the photon.[4] In addition, the Mercury perihelion effect is generally considered to be evidence for the gravitational interaction of the graviton itself, although it may be possible to avoid this conclusion.[2] Also, there exists very convincing evidence that the K^0 and the $\bar{K}^0$ have the *same* gravitational interaction, not necessarily the normal one.[5] Finally, there is a single semiquantitative argument by Schiff which concerns other particles.[6] However, the number of species of known elementary particles is enormous.[7] Thus, even though no deviations

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[1] Most such theories of course construe gravitation to be a property of "matter" rather than of elementary particles. A partial list is as follows: Newton's original theory; Einstein's geometrical theory (general relativity); the operationally equivalent tensor field theory [see W. Thirring, Ann. Phys. (N. Y.) **16**, 96 (1961), containing references to earlier work]; scalar and/or scalar-tensor theories [see C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961), containing references to earlier work]; linear tensor theory [see S. Deser and B. E. Laurent, Ann. Phys. (N. Y.) **50**, 76 (1968)]. For a particle-physics approach using group theory, see S. Weinberg, Phys. Rev. **138**, B988 (1965).

[2] S. Deser and B. E. Laurent (Ref. 1).

[3] For individual atoms, see I. Estermann, O. C. Simpson, and O. Stern, Phys. Rev. **71**, 238 (1947); for neutrons, A. W. McReynolds, Phys. Rev. **83**, 172 (1951); for electrons, F. C. Witteborn and W. M. Fairbank, Phys. Rev. Letters **19**, 1049 (1967).

[4] Further, one only learns about the gravitational interaction at zero momentum transfer to the field, as the gradient of a laboratory classical field is always extremely small. This is true also in the case of all conclusions derived from the present work.

[5] M. L. Good, Phys. Rev. **121**, 311 (1961).

[6] L. I. Schiff, Proc. Natl. Acad. Sci. U. S. **45**, 69 (1959).

[7] For a listing of presently known particles and their properties, see N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).

from universal gravitation have been observed to date, more experiments seem desirable, to say the least.[8]

Of course, it has been extremely difficult to devise experiments on the gravitational interaction because the experimental effects are in general so small as to be unobservable. For example, one may consider measuring the free fall of the longest-lived of the unstable particles, the muon, at the earth's surface. If we assume universal gravitation, then a typical horizontally traveling muon turns out to fall of order 10^{-9} cm during its lifetime. Other particles with even shorter lifetimes of course fall even less. It is hard to see how one can measure such small distances in practice. Other, more subtle schemes which one can devise seem no more promising.[9]

The claim of this paper is that the experimental problem of measuring gravitational couplings of elementary particles is essentially solved. The method is to search for certain elementary-particle reactions at high energies. It turns out that some reactions are forbidden if universal gravitation is valid, but are allowed otherwise. One purpose of this work is to give a general discussion of the method. Another is to point out that, indeed, some specific experiments have already been done (unknowingly), and to discuss the results.[10]

The basic idea can be simply put, and is as follows:

Let there be an elementary-particle reaction $A \to B$, where A and B represent the initial and final collections of particles, respectively. We assume that energy and momentum are conserved. If the invariant mass of the initial state is less than the sum of the masses of the final-state particles, then energy cannot be conserved and the reaction is kinematically forbidden.

(If the initial invariant mass is equal to the sum of the masses in the final state, then energy is conserved, but momentum is not conserved with finite momenta for the final-state particles. Thus the final-state phase space is zero and the reaction is still forbidden.)

Now suppose that a classical gravitational field ϕ is present.[11] (We take it to be a scalar under Lorentz transformations for this argument.) According to universal gravitation this adds a (negative) potential energy $-E\phi$ for each particle, where E is the energy of the particle.[12] Thus we multiply the initial- and final-state energies by the same factor $(1-\phi)$. This does not alter the energy inequality and the reaction is still forbidden.

Now, however, suppose that universal gravitation fails. If, say, one or more of the *initial* particles has an anomalously *small* gravitational interaction, so that its potential energy is $-EC\phi$ with $C<1$, then less negative (or, equivalently, more positive) energy is added to the initial state than to the final state. If, also, the quantity $EC\phi$ is large enough, then the energies balance and the reaction is no longer kinematically forbidden. If, on the other hand, one or more of the *final* particles has an anomalously *large* interaction, then the same conclusion follows.

If either of these two possibilities actually occurs and if there is also a dynamical mechanism which would cause the reaction $A \to B$ to proceed if only energy and momentum could be conserved, then the reaction does indeed proceed.

Thus we can in principle measure the gravitational interaction by looking for the occurrrence, in a given field ϕ, of reactions which are normally forbidden only by energy-momentum conservation. If we observe the reaction in question, then we know either that one or more of the initial particles is "too light"[13] to be described by universal gravitation, or that one or more of the final particles is "too heavy," or both. Further, we ought to be able to tell just which particles are

[8] The following historical note seems apt. For some years it was believed that the basic "bare" coupling constants responsible for muon and nucleon β decay, respectively, are precisely the same. Indeed, this belief persisted even though there *was* evidence of a small discrepancy. At present, of course, it is reasonably well verified that these constants differ by a factor $\cos\theta$, where θ is the Cabibbo angle. N. Cabibbo, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Calif., 1967).

[9] For example, a charged particle falling with an *anomalous* gravitational interaction turns out to produce a transverse electromagnetic field at large distances. Roughly speaking, this is because the particle and the normally gravitating photons comprising its self-electromagnetic field fall at different rates. If we have many particles, say, a short burst of them from an accelerator, then the net field is coherent at sufficiently low frequencies and can be detected by a radio receiving antenna. The relative effect of stray electromagnetic fields decreases with the energy of the particle. However, even at an energy of 100 GeV, the earth's magnetic field turns out to cause an effect of order 10^8 times the effect in question. In addition, there are severe problems of apparatus alignment because one must isolate the effect from the normal longitudinal induction field.

One may also consider a "double Stern-Gerlach" technique as suggested by D. Greenberger (private communication): One divides a horizontal beam of electrically neutral objects in half; allows one beam to traverse some distance at a greater height than the other, thereby causing a relative phase difference in the de Broglie waves; recombines the beams; and measures the phase difference by observing the rotation of the spin vector. It is clear that one must know the difference in optical path lengths to well within a wavelength. This may well be feasible for ordinary molecules; however, it is not feasible for neutral combinations of elementary particles in general: In the case of muonium, for example, the largest wavelength one can get in practice is of order 10^{-10} cm, and it is hard to see how one can measure the difference in optical paths to anywhere near that figure.

[10] The present work is a generalization of two earlier papers: E. F. Beall, Phys. Rev. Letters **21**, 1364 (1968); **21**, 1507(E) (1968); University of Maryland Technical Report No. 927, 1968 (unpublished). The notations and procedures used in these two papers differ considerably from those used in the present work.

[11] The word "field" rather than the word "potential" is used throughout this work in accordance with contemporary usage. To avoid confusion, the following is noted: What is ordinarily called, for example, "the earth's gravitational field" is actually the gradient of the "gravitational field" discussed here. The Riemann tensor, also sometimes called a "field," is obtained by a certain second-order differential operation upon the "gravitational field" discussed here.

[12] We neglect nonlinearities in this argument.

[13] The phrases "too light" and "too heavy" are frequently used in this work to denote anomalously small and anomalously large gravitational coupling, respectively. Thus the mnemonic refers to gravitation and not to inertia.

anomalous and by how much by investigating such things as which portions of the final-state phase space are enhanced. On the other hand, if we do not observe the reaction under favorable conditions, we can set limits on the amount of anomalous gravitational interaction.[14]

There is one difficulty with this method: We do not really know the value of the gravitational field. We could compute it from something like Einstein's equations if the large-scale structure of the universe were sufficiently understood and also if a prescription were somehow given for assigning a gauge (or "coordinate condition"). However, this is not the case in reality and all we can do is set a plausible limit on the field. Thus, if we were to actually observe one of the reactions under study, we would only be able to say that there is an anomaly, not measure it quantitatively.

Nonetheless, if we do not observe a reaction under favorable conditions, we can still set limits on the anomalous gravitational interaction of the particles involved. It is in this sense that the experimental problem is "essentially" solved, as stated earlier: The experimental problem is not completely solved only if there is an anomaly, in which case we cannot measure it quantitatively.

A modest amount of formalism based upon the above general considerations and upon relativistic quantum mechanics is developed in Sec. II—to the point where one can actually calculate reaction kinematics and rates with some degree of confidence. Then calculations are presented for the following basic types of process:

(1) The electromagnetic process $P \rightarrow P+\gamma$, where P is any particle with appreciable electromagnetic coupling. This process has a classical analog: the Čerenkov effect. We know that the photon is described very closely by universal gravitation.[15] Therefore, we expect from the earlier discussion that this process will proceed if the initial particle P is "too light." (The final-state particle P is, of course, also too light. However, it has less energy than does the initial particle and a little thought shows that the argument still applies.) One expects the reaction rates to be characterized by electromagnetic times, multiplied by some function of the gravitational potential. Indeed, the rates for particles in the TeV energy range turn out to be so large that, in a given case, a particle with anomalously small gravitational coupling would lose all of its excess energy above a certain threshold in a time which is short in comparison with lab times. Thus, in such a case one only need investigate the stability of the energy of the particle. If the particle is observed to retain its energy, then it is not necessary to actually search for radiation from the particle in order to set a lower limit on its gravitational interaction.

On the basis of existing data, we will see that the muon, in particular, is not in fact too light by any appreciable amount.

(2) The related process $\gamma \rightarrow P+\bar{P}$. This process is allowed for P or $\bar{P}$ or both too heavy. The rates are again extremely large, and one may set upper limits on the gravitational interaction of all particles to which the photon couples by observing the very existence of a sufficiently high-energy photon over appreciable lab times.

Existing data show that, with the possible exceptions of the neutrinos and the gravitons, no known particle with mass mc^2 of order GeV or less is in fact too heavy by any appreciable amount.

(3) The weak process $\nu \rightarrow e+\mu+\nu'$, where ν and ν' denote generic neutrinos. There are actually four distinct reactions. They are described by the same matrix element as is ordinary μ decay. One expects the process to occur if the initial neutrino is too light. The rates are quite sensitive to the energy excess above a certain threshold and can be extremely large or extremely small, depending on details. Existing data do not allow any conclusions in this case.

Brief attention is also given to processes involving gravitons.

I make four general remarks before proceeding to the main business of this work.

(i) The strict validity of energy-momentum conservation is assumed throughout this work, and all conclusions are based on this assumption. This assumption is made because it is known to apply to a vast number of particle phenomena with no known violations. Indeed, it is frequently used in particle-physics experiments essentially as engineering, in the same way as it is used here.[16]

However, it has sometimes been argued that a breakdown of universal gravitation necessarily implies violation of energy conservation. Some discussion is therefore

[14] It should be clear that the particle processes discussed in the present work have nothing to do with particle processes in *variable* gravitational fields (see Ref. 11) which have often been discussed. [For example, there is the old question of whether or not radiation by a normally gravitating charged particle occurs as a result of acceleration in a gravitational field. B. S. de Witt and R. W. Brehme, Ann. Phys. (N. Y.) **9**, 220 (1960); T. Fulton and F. Rohrlich, *ibid.* **9**, 499 (1960).] Such processes can occur under certain conditions even if universal gravitation is valid by means of actual energy and momentum *transfer between* the gravitational field and the particles. In the present case the field is constant to good approximation, thus its Fourier transform is zero, and therefore there can be no energy-momentum exchange to or from the field itself.

[15] The gravitational coupling of the photon is known to within about 1% from the terrestrial red-shift measurements. R. V. Pound and J. L. Snider, Phys. Rev. **140**, B788 (1965). In terms of the coupling constant C_γ introduced later in the present work, one has $C_\gamma=0.999$ with a statistical uncertainty of ± 0.0076 and a possible systematic error of 0.01.

[16] For example, the existence of a highly unstable particle is often inferred by observation of its decay products in a bubble chamber and by then using energy-momentum conservation to "measure" its mass, energy, and direction.

in order. A generalized version of this argument and a criticism of it as follows.[17]

Suppose a collection of particles A with energy E converts into a collection of particles B, with the same energy E, which couple to gravitation *less* than does the collection A. We may raise B in a gravitational field to some point by exerting a certain amount of work. Now let B reconvert to A with energy E at the new point. Then lower A to the original point, thereby getting back a certain amount of work. This work is more than what was exerted in the raising operation because we raised something which weighs less than what we lowered. Thus we extract net work from a closed cycle, energy is not conserved, we can make perpetual motion machines, etc.

(If A weighs more than B, we just interchange A and B in the above argument. The same result is obtained.)

Notice that the argument assumes the energy of A created at the upper point to be the same as that of A destroyed at the lower point, namely, E. However, A has a different gravitational potential energy at the two points. Thus it is assumed that the potential energy is not involved in the mechanism which causes A to convert to B and *vice versa*. That is, it is assumed that the energy which is transferred between A and B is the total energy minus the gravitational potential energy. The criticism is, then, that this is purely an assumption; it need not be fulfilled in reality.

The present work assumes, for the reason stated earlier, that the total energy is what is transferred in $A \leftrightarrow B$. It is then easily shown that no net work is obtained from the cycle in the above argument, to first order in the gravitational field and, again, assuming it to be a scalar.

(ii) I discuss the Schiff argument[6] briefly here. In response to a suggestion that antimatter is repelled by gravitation, Schiff argues that "virtual" positrons are present in an atomic nucleus by virtue of vacuum polarization, and therefore may be "weighed" using the Eötvös experiment: Virtual positrons are present in differing amounts in different nuclei; thus the gravitational/inertial mass ratios of different nuclei would be different if the positron's gravitational coupling were anomalous. Schiff makes quantum-mechanical calculations of the expected effects and obtains finite results under the assumption that the kinetic energy of the positron is normal but that its rest mass is repelled by gravitation. The effects are of order 10^{-7} as compared with an Eötvös-experiment sensitivity of $\sim 10^{-8}$. (Further, the latter has decreased to $\sim 10^{-11}$ since Schiff's article.[18]) Schiff also notes that one can make stronger, although necessarily qualitative, statements about particles which are produced virtually by the strong nuclear interactions, i.e., the hadrons.

I think that the Schiff argument gives good evidence for universal gravitation for all presently known particles except the muon, the neutrinos, and the graviton.[19] I say this because other self-energy effects are invariably of the order of magnitude one expects. Nonetheless, self-energies in general are not theoretically understood at present. In particular, it is possible that the effects discussed by Schiff are renormalized away in each nucleus in the following manner.

In the context of ordinary first-order renormalization theory with no gravitational field, one of course has a self-energy term calculated by some theoretical prescription involving intermediate states and a "counter term" in the free Hamiltonian. These terms may or may not be divergent. It is well known that only the sum of the two terms, i.e., the physical self-energy or inertial mass, is strictly meaningful. In the present case it is easily seen that there is also in general a term calculated by some prescription which is proportional to the gravitational field and which involves the gravitational mass of particles present "virtually," and a "counter term" in the field-dependent part of the free Hamiltonian. The sum of these two terms gives the gravitational mass of the nucleus (multiplied by the field), and again it is only this sum which is strictly meaningful. There is no way to compute the finite part of either of the two counter terms (field-dependent and field-independent) without imposing some further assumption. (One such assumption would be the imposition of universal gravitation itself. This would say, in effect, that the four terms must be such that the sum of the two field-dependent terms is equal to the sum of the two field-independent terms. Other possibilities arise if one has a believable theory of the internal structure of the nucleus.) In particular, we do not know that these two counter terms are the same, which is what Schiff apparently assumes when he says that a certain field-dependent divergent integral "cannot be removed by renormalization."[6]

Thus it is quite possible that the physical sum of the two field-dependent terms is equal to the physical sum of the two field-independent terms, thereby giving the nucleus normal gravitation, independent of the field-dependent term which is computed using the gravitational mass of "virtual" particles. In other words, nature may be such that an anomaly in the gravitational mass of a "virtual" particle is automatically canceled by a corresponding anomaly in a counter term in the

[17] For versions of this argument see, for example, P. Morrison, Am. J. Phys. **26**, 358 (1958); H. Frauenfelder, *The Mössbauer Effect* (W. A. Benjamin, Inc., New York, 1962), p. 62. For a slightly different criticism of the argument, see P. Thieberger, Nuovo Cimento **35**, 688 (1965).

[18] P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N. Y.) **26**, 442 (1964).

[19] The largest effect caused by muons is the muonic vacuum polarization (virtual photoproduction of μ^+-μ^- pairs). It gives roughly the same order of magnitude as does the present Eötvös experiment sensitivity (Ref. 18), and therefore any conclusion would be marginal. "Virtual" neutrinos are produced only by second-order weak interactions, perhaps giving an effect of order 10^{-14}. The gravitational self-energy of a nucleus is of course hopelessly small.

free Hamiltonian of the parent nucleus. We cannot exclude this possibility because we do not know how to compute such counter terms.[20]

I frankly think that the occurrence of a situation such as that just discussed in every nucleus is far-fetched and that the Schiff argument is good evidence, as stated earlier. Nonetheless, it is the only evidence we have for the gravitational coupling of the great majority of elementary particles (aside from the evidence adduced in the present work). Accordingly, since it is not at all conclusive, as argued above, it still leaves universal gravitation on a shaky experimental basis.

(iii) Weinberg has argued on very general grounds that Lorentz covariance of the S matrix for particle processes requires universal gravitation.[21] The argument is made by examination of the S matrix for production of soft gravitons, i.e., classical linearized gravitational waves, in accompaniment to elementary-particle reactions. The formal development presented later in the present work seems to bear out Weinberg's conclusion from a different direction: The S matrix for particle processes in a *static* gravitational field violates Lorentz covariance unless universal gravitation is imposed. In the present case, the noncovariance appears in a natural way as being associated with the Lorentz frame in which the sources of the gravitational potential are at rest. It is not at all clear that there is anything objectionable about it in the present context. Discussion of this point is deferred until the situation is actually encountered in Sec. II.

(iv) Finally, for the sake of completeness, I give two possible reasons why one might want universal gravitaion to be violated.

There is, first, the well-known $\mu - e$ puzzle: Their strong, electromagnetic, and weak interactions are the same to within experimental accuracy; yet their masses differ by $\sim$100 MeV. One could imagine that they interact differently with gravitation. Alternatively, perhaps their respective neutrinos interact differently with gravitation. It is of course hard to explain 100 MeV by means of a gravitational self-energy. However, self-energies in general are not really understood and, further, the gravitational interaction is highly divergent, since it is proportional to the square of the momentum.

The present work presents evidence that the gravitational interaction of the μ and the e are indeed the same, and the question of the neutrinos is left open.

Secondly, there is the problem of gravitational collapse. As is well known, there arises a real, physical singularity. If we take the philosophy that such a singularity is intolerable, then we must find a way out. Suppose that there is a particle which does not couple to gravitation and which is produced copiously in hot stars. As the star compresses, it becomes heated and produces such particles, they escape, and energy is carried away. If energy were carried away fast enough, it would seem that the star could drop below critical mass before the singularity occurs. Whether or not this actually happens would of course depend upon details, but it seems intuitively possible.[22]

II. PARTICLES IN CLASSICAL GRAVITATIONAL FIELD

In this section I outline very briefly an elementary mathematical description of the propagation and interaction of particles in a classical gravitational field—without the assumption of universal gravitation.[23] The procedure is essentially to (a) assume the existence of a "background" field which is a second-rank tensor under Lorentz transformations, (b) write down the most general description of particles in the presence of this field which is consistent with the usual invariance requirements, and (c) quantize particle fields in the usual canonical fashion.

We make two assumptions throughout this work which greatly simplify the mathematics: (1) The gravitational field in dimensionless units is much less than unity, so that we may work to first order; (2) the gravitational field is constant, meaning constant over the deBroglie wavelengths of particles, and, indeed, over the neighborhood of the earth.[24] The characteristic dimension of the gravitational field is at least the radius of the local cluster of galaxies, which is $\sim 10^{15}$ times the radius of the earth, so that this is clearly a good approximation. The first approximation may or may not be valid, depending on the large-scale structure of the universe and the boundary conditions imposed upon the field equation governing the gravitational field.

The notation throughout this paper is as follows: $\hbar = c = 1$; Greek indices run from 0 to 3 and Latin indices from 1 to 3; the summation convention is implied for both types; the Minkowski tensor $\eta^{\mu\nu}$ has signature $(-+++)$; indices are raised and lowered using the Minkowski tensor; Dirac matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$; and a comma denotes differentiation.

The gravitational field itself is taken to be a dimensionless second-rank tensor, called $h^{\mu\nu}$. The sign convention is such that $h^{\mu\nu}$ is positive for such static solu-

[20] In a relativistic theory there are actually three self-energy terms in general: a field-independent term, a term proportional to the tensor gravitational field, and a term proportional to its trace. In general the field-dependent and field-independent terms diverge to different degrees. Thus it is clear that the corresponding counter terms are not in any sense "the same."

[21] S. Weinberg, Phys. Letters **9**, 357 (1964). See also S. Weinberg (Ref. 1).

[22] A similar suggestion has been made by S. Deser and B. E. Laurent (Ref. 1).

[23] It should be clear that the purpose of the present work is not to give a complete, sophisticated theory of particles in a classical gravitational field. Such discussions exist; however, they invariably assume universal gravitation in the form of general relativity or the universal tensor field approach. See, for example, R. Utiyama, Phys. Rev. **125**, 1727 (1962), and references therein.

[24] In the present work we assume that all particle processes of interest take place at or near the earth's surface. This means that we do not consider astrophysical data. This assumption can be relaxed at the cost of increasing the number of parameters (coupling constants) in the formalism.

tions of the gravitational field equation as the Schwarzschild solution with a sufficiently small source.

A. Classical Free Particles

In the absence of a gravitational field, a classical particle is of course described by a scalar characteristic of the particle, called the mass (m), and by a vector whose value depends upon the state of the particle, called the momentum (p_μ), such that

$$p_\mu p_\nu \eta^{\mu\nu} + m^2 = 0. \tag{1}$$

In the presence of the field $h^{\mu\nu}$ we assume that the particle is still characterized by p_μ and m, and we write the most general modification of Eq. (1) which is bilinear in p_μ and m and which satisfies Lorentz covariance. It is

$$p_\mu p_\nu (\eta^{\mu\nu} - H^{\mu\nu}) + (A + m^2)K = 0, \tag{2}$$

where the tensor $H^{\mu\nu}$ and the scalars A and K are arbitrary functions of $h^{\mu\nu}$. In general, we expect these quantities to be different for different types of particle.

Equation (2) simplifies considerably within the framework of the approximations stated earlier: The quantities A and K are constant all over the earth where we measure particle masses.[24] Then we see from Eq. (2) that they merely correct the mass by an unobservable amount.[25] Therefore we may as well set $A = 0$, $K = 1$. Further, since $h^{\mu\nu}$ is small, we may expand $H^{\mu\nu}$ in a power series. The zeroth-order term must be proportional to $\eta^{\mu\nu}$ and also merely supplies a mass correction. This is also true of that part of the first-order term which is proportional to the trace of $h^{\mu\nu}$. We keep the rest of the first-order term only and are left with

$$p_\mu p_\nu (\eta^{\mu\nu} - C h^{\mu\nu}) + m^2 = 0, \tag{3}$$

where C is some constant characteristic of the particle. Equation (3) is then the most general energy-momentum-mass relation which has observable consequences within the present framework of approximations.

We may identify the constant C by momentarily assuming general relativity. The generally covariant generalization of Eq. (1) is $p_\mu p_\nu g^{\mu\nu} + m^2 = 0$, where $g^{\mu\nu}$ is the contravariant metric tensor and where we must consider p_μ and m to be a vector and a scalar, respectively, under general coordinate transformations. Reinterpreting gravitation as a field, we may write $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$. (Consideration of the linearized theory with static sources shows that the minus sign is correct.) Comparing with Eq. (3), we see that $C = 1$ corresponds to universal gravitation. Further, $h^{\mu\nu}$ turns out to be proportional to Newton's gravitational constant G in lowest order. Thus we may interpret $CG^{1/2}$ to be the gravitational "coupling constant" of the particle in a more conventional system of units.

One may identify the concept of "gravitational mass" by appealing to the nonrelativistic approximation to (3): Let the total energy minus the rest-mass energy, $p_0 - m$, be such that $p_0 - m \ll m$. Then if $h^{\mu\nu}$ is sufficiently small, one obtains from Eq. (3) the result

$$p_0 - m \simeq p_i p^i / 2m - \tfrac{1}{2} m C h^{00} - p_i C h^{0i}. \tag{4}$$

The first term is familiar. The gradient of the last term is a "velocity-dependent force." In the limit $p_i \to 0$, the second term evidently gives the "gravitational potential energy." Thus it is easily seen that the "gravitational mass" is

$$m_G = Cm. \tag{5}$$

Clearly, a particle which is "too light" has $C < 1$, one which is "too heavy" has $C > 1$, a nongravitating particle has $C = 0$, and a particle which is repelled rather than attracted by gravitation has $C < 0$.

Notice that Eq. (3) in general allows spacelike momenta. In particular, it is easily shown that this occurs for $C > 0$ if a Lorentz frame exists in which $h^{\mu\nu}$ is diagonal, positive, and sufficiently large. Thus this happens even if universal gravitation is valid. It can be shown that the resulting causal anomalies are unobservable to first order in $h^{\mu\nu}$ provided that space and time intervals are measured using instruments constructed of matter which has the same value of C. Thus this problem disappears if universal gravitation is valid. Nonetheless, real causal anomalies can occur if the particle and the measuring instruments possess different values of C. We will return to this point later in connection with the quantum-mechanical formalism.

We take Eq. (3) with C characteristic of a given particle to be the basic energy-momentum-mass relation for the particle, replacing Eq. (1). Now suppose we have N_I initial particles and N_F final particles undergoing a reaction. We would then wish to solve the set of equations

$$p_{\mu I} p_{\nu I} (\eta^{\mu\nu} - C_I h^{\mu\nu}) + m_I^2 = 0 \quad (I = 1, \ldots, N_I),$$

$$p_{\mu F} p_{\nu F} (\eta^{\mu\nu} - C_F h^{\mu\nu}) + m_I^2 = 0 \quad (F = 1, \ldots, N_F),$$

$$\sum_I^{N_I} p_{\mu I} = \sum_F^{N_F} p_{\mu F},$$

where the last expression reflects the assumption of energy-momentum conservation. The reaction is allowed if the set of equations has a solution. This solution gives the relevant energy threshold and other kinematic constraints. As in the case of normal kinematics, solutions with some negative energies occur in the solution unless they are eliminated by assumption. In the present case one can require that all energies are positive in some specified Lorentz frame, e.g., the "laboratory frame." This condition is not maintained

[25] If any scalar field associated with gravitation were to exist, as in the Brans-Dicke theory (see Ref. 1), then it would also merely supply an unobservable mass correction within our framework of approximations. Thus the conclusions of the present work are independent of whether or not such a field exists.

in general in other Lorentz frames because spacelike momenta are allowed in general, as noted above.

The kinematics which one obtains by solving sets of equations such as the above for various processes is quite similar to the kinematics involving faster-than-light particles which has been discussed in detail by Feinberg.[26] The principal difference is that direction dependence can exist in the present case. I do not give further discussion here because the kinematics in a specific case is automatically obtained from an S-matrix treatment, as will be seen. However, it is important to note that whether or not a reaction is allowed is independent of any quantum-mechanical considerations.

Finally, it is necessary to note the following: As was mentioned in the Introduction and as will be seen later, the dynamics of particle interactions violates Lorentz covariance in general. Thus there may be no good reason to require Lorentz covariance in the kinematics of free particles either. If we suppose that the kinematics depends upon some specific Lorentz frame, then one can show that Eq. (3) would be replaced by

$$p_\mu p_\nu(\eta^{\mu\nu}-Ch^{\mu\nu}-Dhn^\mu n^\nu)+m^2=0\,, \qquad (6)$$

where D is a constant, $h=h_\mu{}^\mu$ and n^μ is a unit timelike vector with components (1,0,0,0) in the specified frame. The analog of the nonrelativistic Eq. (4) would be

$$p_0-m\simeq p_ip^i/2m-m(\tfrac{1}{2}Ch^{00}+\tfrac{1}{2}Dhn^0n^0) -p_i(Ch^{0i}+Dhn^0n^i)\,. \qquad (7)$$

"Gravitational mass" becomes a somewhat tenuous concept. For example, if we define it to be the coefficient of $-\frac{1}{2}h^{00}$, then it would be instead of (5),

$$m_G=(C-Dn^0n^0)m\,, \qquad (8)$$

and would thus be frame-dependent. Further, it would not completely describe the gravitational properties of the particle in the limit $p_i\to 0$, since an additional residual term $-\frac{1}{2}mD(h^{11}+h^{22}+h^{33})n^0n^0$ remains in (7) in this limit.

Although there may be no reason to exclude the possibility of noncovariance in the kinematics as well as in the dynamics, neither does there seem to be any reason to include it. In this paper, I assume that the kinematics is covariant, so that Eq. (3), or Eq. (6) with $D=0$, is valid.[27]

B. Quantum Mechanics, Second Quantization, S Matrix

A free scalar particle in the absence of gravitation is described by the Klein-Gordon equation in the wave function $\phi(x)$:

$$\Box\phi-m^2\phi\equiv\phi_{,\mu\nu}\eta^{\mu\nu}-m^2\phi=0\,.$$

The required generalization follows easily from the same considerations as those leading to Eq. (3). It is

$$\phi_{,\mu\nu}(\eta^{\mu\nu}-Ch^{\mu\nu})-m^2\phi=0\,, \qquad (9)$$

subject to the same approximations. Equation (9) has plane-wave solutions of the form $e^{(\pm ip_\mu x^\mu)}$ provided that p_μ satisfies Eq. (3). Evidently sharp wave packets which satisfy (9) correspond to the classical particles just discussed.

It can be shown that the momentum eigenfunctions of (9) corresponding to real energies do not form a complete set for arbitrary $h^{\mu\nu}$. However, there is no difficulty if $h^{\mu\nu}$ is small in comparison with unity.

In the case of a spin-$\frac{1}{2}$ particle, the principal results are as follows: The generalized Dirac equation is

$$\Gamma^\mu\psi_{,\mu}+m\psi=0\,, \qquad (10)$$

where the Γ matrices satisfy

$$\Gamma^\mu\Gamma^\nu+\Gamma^\nu\Gamma^\mu=2(\eta^{\mu\nu}-Ch^{\mu\nu})\,. \qquad (11)$$

Operation on (10) with the operator $\Gamma^\mu\partial_\mu-m$ just gives back the generalized Klein-Gordon equation (9). Thus plane-wave solutions of the form $u(\mathbf{p})\exp(ip_\mu x^\mu)$, where $u(\mathbf{p})$ is some four-component object, exist if p_μ satisfies Eq. (3).

One possible representation of the Γ matrices in terms of ordinary γ matrices is $\Gamma^\mu=b_\lambda{}^\mu\gamma^\lambda$, where to first order $b_\lambda{}^\mu$ is given by

$$b_\lambda{}^\mu\simeq\delta_\lambda{}^\mu-\tfrac{1}{2}Ch_\lambda{}^\mu\,. \qquad (12)$$

The detailed properties of the spinor functions are not needed here. However, one result is of interest: It can be shown that the spinors may be chosen in such a way that the generalizations of the usual "projection operators" used in reducing spinor products to traces are, to first order,

$$\sum_{\text{spins}}\bar{u}_a{}^{(\pm)}(\pm\mathbf{p})u_b{}^{(\pm)}(\pm\mathbf{p}) \propto[i\gamma^\mu(p_\mu-\tfrac{1}{2}Ch_\mu{}^\lambda p_\lambda)\mp m]_{ba}\,, \qquad (13)$$

where a and b denote spinor indices. The proportionality constant depends on one's normalization convention. This differs from the usual result by the factor proportional to $h_\mu{}^\lambda$.

There are no "spin effects" manifested in Eq. (10). This is a consequence of the neglect of the derivative of $h^{\mu\nu}$.

We now consider second quantization. We confine ourselves to the case of the scalar field. One might think that it is easiest to first quantize the field with $h^{\mu\nu}$ zero and then apply some sort of perturbation theory. It turns out that this is the hard way, and furthermore it obscures the physics. Instead, we quantize directly the scalar field in the presence of $h^{\mu\nu}$, that is, the scalar field which satisfies Eq. (9).

A Lagrangian density from which one can derive

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[26] G. Feinberg, Phys. Rev. **159**, 1089 (1967).

[27] More generally, processes which are ordinarily forbidden would still occur if some values of C were not unity even if some values of D were nonzero. However, the situation would obviously be more complicated, and the quantitative (or semiquantitative) conclusions of this work would have to be modified in such a case.

Eq. (9) using the Euler-Lagrange procedure is

$$\mathcal{L}=-\tfrac{1}{2}[\phi_{,\mu}\phi_{,\nu}(\eta^{\mu\nu}-Ch^{\mu\nu})+m^2\phi^2]. \quad (14)$$

The canonical momentum density obtained from Eq. (14) is

$$\pi=\partial\mathcal{L}/\partial\phi_{,0}=-(\phi_{,0}\eta^{00}-Ch^{0\mu}\phi_{,\mu}). \quad (15)$$

We will interpret $\phi(x)$ and $\pi(x)$ to be operators satisfying canonical equal-time commutation rules in the usual fashion.

At this point we assume that a Lorentz frame exists in which all $h^{0i}=0$. (As noted later, it is not at all clear that this assumption is valid.) We quantize in precisely this frame. The commutation rules are, as usual, taken to be

$$[\phi(\mathbf{x},x^0),\phi(\mathbf{y},x^0)]=[\pi(\mathbf{x},x^0),\pi(\mathbf{y},x^0)]=0\,,\quad [\phi(\mathbf{x},x^0),\pi(\mathbf{y},x^0)]=\delta(\mathbf{x}-\mathbf{y})\,. \quad (16)$$

We introduce the usual creation and destruction operators

$$[a(\mathbf{p}),a(\mathbf{q})]=[a^\dagger(\mathbf{p}),a^\dagger(\mathbf{q})]=0\,,\quad [a(\mathbf{p}),a^\dagger(\mathbf{q})]=\delta_{\mathbf{pq}}. \quad (17)$$

We wish a plane-wave expansion of the operator $\phi(\mathbf{x},x^0)$ in terms of the a and $a^\dagger$ operators above such that the field equation (9) and the commutation rules (16) are satisfied in the Lorentz frame with $h^{0i}=0$. The solution is easily obtained and is

$$\phi(x)=\frac{i}{\sqrt{V}}\sum_{\mathbf{p}}\frac{e^{ip_\mu x^\mu}a(\mathbf{p})+e^{-ip_\mu x^\mu}a^\dagger(\mathbf{p})}{[-2(\eta^{00}-Ch^{00})p_0]^{1/2}}\,, \quad (18)$$

where $\mathbf{p}$ and p_0 satisfy the energy-momentum-mass relation (3) with the positive root for p_0 and where V is the normalization volume. Notice that this is quite similar to the usual expansion and, indeed, reduces to it as $h^{\mu\nu}\to 0$.

Although it is not necessary for the applications considered later, it is instructive to attempt the construction of an invariant Δ function satisfying the covariant commutation rule $[\phi(x),\phi(y)]=-i\Delta(x-y)$. In the special Lorentz frame one may derive a differential equation in $\Delta(x)$ using the field equation (9) and initial conditions using the commutation rules (16). The solution of the Cauchy problem then turns out to be

$$\Delta(x)=\Delta^+(x)+\Delta^-(x)\,, \quad (19)$$

where

$$\Delta^\pm(x)=\pm\frac{i}{(2\pi)^3}\int d^4p\; e^{ip_\mu x^\mu}\times\delta[p_\mu p_\nu(\eta^{\mu\nu}-Ch^{\mu\nu})+m^2]\theta(\pm p_0)\,. \quad (20)$$

This expression is applicable at least in the Lorentz frame where $h^{0i}=0$.

Now, the δ function in (20) in general allows spacelike momenta, so that the step function $\theta(\pm p_0)$, and therefore expression (20) itself is not covariant in general. In cases involving intermediate states, the S matrix contains the usual propagation function

$$\Delta_C(x)=\theta(x)\Delta^+(x)-\theta(-x)\Delta^-(x)\,, \quad (21)$$

which is also not Lorentz-covariant since it involves $\Delta\pm(x)$. Therefore the S matrix, at least for processes with intermediate states, is not Lorentz-covariant in general.

One could in principle quantize in a different Lorentz frame, but the result cannot be covariant since it would have to apply to the $h^{0i}=0$ frame as well, thereby reducing to (20). Thus we cannot obtain a covariant theory by the canonical procedure unless $Ch^{\mu\nu}$ is such that the δ function in (20) excludes spacelike momenta. We will return to this point later.

A formally covariant version of (20) may be written with the introduction of a unit timelike vector n^μ with components (1,0,0,0) in the special Lorentz frame. It is

$$\Delta^\pm(x)=\pm\frac{i}{(2\pi)^3}\int d^4p\; e^{ip_\mu x^\mu}\times\delta[p_\mu p_\nu(\eta^{\mu\nu}-Ch^{\mu\nu})+m^2]\theta(\pm n^\mu p_\mu)\,. \quad (22)$$

In constructing the S matrix for some given process we assume that S contains the usual space-time integral over field operators, positioned in such a way as to destroy and create the initial and final particles, respectively. We work in the special frame with all $h^{0i}=0$. In obtaining reaction probabilities, extraction of the result of the field operators (18) acting upon the state vectors yields a factor for each particle which is, for the Jth particle,

$$1/[-2(\eta^{00}-C_Jh^{00})p_{0J}{}^V].$$

With the exception of this result, the reduction is exactly the same as usual.

For later applications we require the differential probability per unit time that an initial state consisting of a single particle I will convert to a final state consisting of several particles F. The result is

$$d\Gamma=\frac{(2\pi)^4}{-2(\eta^{00}-C_Ih^{00})p_{0I}}\left(\prod_F\frac{1}{(2\pi)^3}\frac{d^3p_F}{-2(\eta^{00}-C_Fh^{00})p_{0F}}\right)\times\delta(p_I-\sum_F p_F)|M|^2, \quad (23)$$

where the various C values are obtained from Eq. (3) applied to the various particles, and where the positive roots are to be taken for the energies. The quantity M is some matrix element left over in the S matrix after the kinematic factors are removed.

The *formally* covariant version of (23) is

$$d\Gamma=\frac{(2\pi)^4}{-2(\eta^{0\mu}-C_Ih^{0\mu})p_{\mu I}}\left(\prod_F\frac{d^4p_F}{(2\pi)^3}\theta(n^\mu p_{\mu F})\delta[p_{\mu F}p_{\nu F}\times(\eta^{\mu\nu}-C_Fh^{\mu\nu})+m_F{}^2]\right)\delta(p_I-\sum_F p_F)|M|^2, \quad (24)$$

where we have again introduced the vector n^μ with components (1,0,0,0) in the special Lorentz frame where $h^{0i}=0$. Again, notice that one cannot reduce the step function to $\theta(p_{0F})$ unless the corresponding δ function excludes spacelike momenta. Thus decay probabilities are not in general Lorentz-covariant, independent of whether or not intermediate-state propagators occur in the matrix element M.

Equation (24), together with the relation obeyed by the initial particle,

$$p_{\mu I}p_{\nu I}(\eta^{\mu\nu}-C_I h^{\mu\nu})+m_I^2=0\,, \tag{25}$$

and with knowledge of M, are all that is needed to compute measurable quantities, in principle. Kinematic constraints turn out to result from the θ and δ functions as usual. Notice that the entire formalism reduces to the usual formalism[28] in the limit $h^{\mu\nu}\rightarrow 0$.

One can also develop equations analogous to Eq. (24) in cases where there are two or more initial particles, but these are not needed for the calculations in Sec. III and therefore are not given here.

At this point several remarks are in order.

(1) Negative energies and acausality in general seem to be inherent in the propagation of free particles, as noted earlier, and Lorentz covariance fails in the S-matrix formalism governing interactions, as just noted. One naturally wonders if these features can be made to go away.

Indeed, one can fix up the formalism, provided that $h^{\mu\nu}$ has certain properties, in the following manner. We note that surely there is no acausality, etc., if universal gravitation is valid. In that case what one does is to make a general coordinate transformation into a coordinate system in which $h^{\mu\nu}$ disappears, or, equivalently, in which all particles have zero effective values of C.[29] Then we just have the usual nongravitational S-matrix formalism, and there are no problems. If all values of C are not the same, one can choose the particle which has the largest value of C and make a general transformation such that that particle is in free fall. It turns out that all other particles are then subject to a repulsive "inertial" field, or, equivalently, have effective values of $C\ll 0$. If $h^{\mu\nu}$ in the original coordinate system is positive and diagonal (in some Lorentz frame), then it can easily be shown that spacelike momenta are forbidden and that Lorentz covariance is thereby assured.

The above prescription would suffice if we were assured that $h^{\mu\nu}$ has the requisite properties in the gauge chosen by nature. As it turns out, however, we are not so assured in general. Indeed, if the source of the gravitational field is dynamic, so that we have a gravitational wave, it is well known that in the linear approximation $h^{\mu\nu}$ is not both diagonal and positive in all components. Thus the above prescription cannot be considered a general theoretical panacea.

The alternative and more natural approach is simply to live with the situation, since it is not at all clear that it is objectionable. First, causal anomalies associated with faster-than-light travel have been discussed and seem to be tolerable.[26] Second, negative-energy quanta and violation of Lorentz covariance do occur in the case of the Čerenkov effect. That negative energies occur is easily seen by considering the rest frame of the initial particle. The particle recoils from rest and therefore must absorb energy when it emits the quantum. As for covariance, there clearly is a preferred frame, namely, the one in which the medium is at rest. A complete theory of this situation has been given.[30]

In the present case we have every right to expect gravitational physics to depend upon motion with respect to the source. As it turns out, the preferred Lorentz frame introduced earlier, in which all $h^{0i}=0$, is indeed the frame in which a nonrotating source is at rest (in a certain gauge, as noted later). The vector n^μ just turns out to be the four-velocity of the source. Thus we merely have a situation in which the interaction of a collection of particles depends upon motion with respect to the source of a background field.

A more detailed analysis of the situation would of course be desirable but will not be given here.

(2) If Lorentz covariance is lost, then one has a right to suspect the loss of invariance under the translation subgroup of the Poincaré group as well: One expects gravitational physics to depend upon the position with respect to the source as well as the motion. Thus energy-momentum conservation, which is generated by this subgroup in the usual picture, is called into question. However, one expects any such effects to be negligible if the source is large enough in extent. This latter assumption is of course part of the basic framework of approximations used here.

(3) It has been assumed in the development of Eq. (24) that a Lorentz frame with $h^{0i}=0$ exists. This is true for the gravitational sources considered in this work in a certain gauge which is discussed later. However, gauge invariance is violated if universal gravitation is violated, and the gauge chosen by nature may be such that no such Lorentz frame exists with the given sources. We still assume that (24) is valid in lowest order for the purpose of estimating rates in Sec. III in the hope that it is not far off. (It turns out that the vector n^μ disappears in lowest order.) Clearly, however, the purely theoretical problem of describing anomalously gravitating particles is not solved until one understands what to do about this point.

My perspective toward the development of Eq. (24) and toward the difficulties referred to in the above remarks is as follows. First, there is no question but

[28] See, for example, G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1964).

[29] General coordinate transformations are discussed in the Appendix.

[30] J. M. Jauch and K. M. Watson, Phys. Rev. **74**, 950 (1948); **74**, 1485 (1948); **75**, 1249 (1949).

that particles are in fact created and destroyed in accordance with the above basic formalism at least in the limit $h^{\mu\nu}\to 0$, because this is just the ordinary formalism which works very well.[31] Second, there must be a first-order formalism if universal gravitation is violated—which possibility we must assume if we are to obtain evidence that it does not happen. Third, Eq. (24) above was developed in the most natural fashion imaginable (according to canonical quantization, etc.). It is thus simplest to assume that it is correct and that the difficulties are to be lived with. Lastly, the classical development (the discussion in Sec. II A) is all that is needed to find out whether or not a given reaction is allowed. Since we do not know $h^{\mu\nu}$ quantitatively anyway, Eq. (24) is really only useful to estimate rates. It is hard to see how it could be very far off the mark for this purpose.

(4) It is clear from Eq. (24) that one must know the matrix element M for a given case in order to make calculations. It is known to zeroth order in $h^{\mu\nu}$ in most cases directly from experiment—up to a phase. However, we do not know it to first order without making further assumptions. A first-order term in the matrix element in some cases turns out to modify the rates by factors of order unity.[32]

One approach to the problem would to be assume some theoretical interaction Hamiltonian in each case and to use Feynman-Dyson perturbation theory. The alternative, which is used in the present work, is simply to construct the matrix element from general principles in the same way that Eqs. (3) and (9) were constructed. We will usually assume, for no really good reason, that the frame-dependent vector n^μ does not occur, and that the same combination of field operators occurs in the first-order term as in the zeroth-order term. In this way one obtains a term proportional to $h^{\mu\nu}$ with an unknown coefficient which corresponds, roughly speaking, to the gravitational coupling constant of the "interaction energy."[33]

Thus, for the purposes of this work, the lack of fully developed theories of interacting particles in a gravitational field is simply translated into an unknown constant for each case.

This completes the discussion of the preceding formal development. Now I show in a simple (and artificial) case that one indeed obtains the results expected from the discussion in the Introduction.

[31] This is well known in the case of electrodynamics. In the case of weak interactions, with the use of the semiphenomenological $V-A$ Hamiltonian (with such additions as the Cabibbo theory, form factors, etc.) to compute the matrix element, Eq. (24) in the limit $h^{\mu\nu}\to 0$ explains at least the leptonic and semileptonic decays very well. In the case of strong interactions, in the absence of "resonances" final-state differential distributions usually conform well with the predictions of "phase space." See G. Källén (Ref. 28).

[32] The statement made in the first paper of Ref. 10 to the effect that the "phase space" itself is of first order in $h^{\mu\nu}$ is incorrect.

[33] There is also a term proportional to $\eta^{\mu\nu}h$, but it only serves to correct the interaction coupling constant (e.g., the charge) within the present framework of approximations.

Let a zero-mass particle with $C=-1$ (so that it is "repelled") convert into two particles, each with mass m, with $C=0$ (so that they are "weightless"). All three particles are "too light" but the initial particle is more so than are the final particles; thus we expect the reaction to be allowed. In the normal case with $h^{\mu\nu}=0$ the reaction is forbidden.

Let the momenta of the initial and two final particles be k, p, and q, respectively. We assume that the matrix element is constant and take the constant to be unity. We take $h^{\mu\nu}$ to be diagonal with all components positive in some specified Lorentz frame. Equations (24) and (25) become, respectively,

$$\Gamma=\frac{1}{(2\pi)^2}\frac{1}{-2(\eta^{0\mu}+h^{0\mu})k_\mu}\int d^4p\,d^4q\;\theta(p_0)\theta(q_0)$$
$$\times\delta(p_\mu p_\nu\eta^{\mu\nu}+m^2)\delta(q_\mu q_\nu\eta^{\mu\nu}+m^2)\delta(k-p-q)$$

and

$$-k_\mu k_\nu\eta^{\mu\nu}=k_\mu k_\nu h^{\mu\nu}.$$

The integral in the first equation depends on $h^{\mu\nu}$ only through the vector k_μ. It is a standard Lorentz-invariant integral and has the value

$$\tfrac{1}{2}\pi[1-4m^2/(-k_\mu k_\nu\eta^{\mu\nu})]^{1/2}\theta(k_0)\theta(-k_\mu k_\nu\eta^{\mu\nu})$$
$$\times\theta[(-k_\mu k_\nu\eta^{\mu\nu})^{1/2}-2m].$$

From the second θ function, k_μ must be timelike. The quantity $(-k_\mu k_\nu\eta^{\mu\nu})^{1/2}$ in the third θ function plays the role of an effective mass. Its square is indeed positive by virtue of the second equation above, the right-hand side of which is positive because $h^{\mu\nu}$ is diagonal with all components positive in some frame. This "mass" must be greater than $2m$ according to the third θ function. Thus the threshold in the special frame is given by

$$k_0^2h^{00}+|k|^2h^{11}\geq 4m^2,$$

where the 1 axis is the direction of propagation. To a good approximation $k_0\simeq|\mathbf{k}|$, and we get the threshold condition

$$k_0\geq 2m(h^{00}+h^{11})^{-1/2}$$

in the frame where $h^{\mu\nu}$ is diagonal.

Dropping a term of higher order in $h^{\mu\nu}$, the rate is

$$\Gamma\simeq\frac{1}{16\pi k_0}\left(\frac{1-4m^2}{k_\mu k_\nu h^{\mu\nu}}\right)^{1/2}\theta(k_0)\theta(k_\mu k_\nu h^{\mu\nu})$$
$$\times\theta[(k_\mu k_\nu h^{\mu\nu})^{1/2}-2m)].$$

Notice that $1/\Gamma$ does indeed transform like the zeroth component of a vector so that the correct time-dilation factor is ensured.

It is easily seen that the initial particle has a "rest frame" in which the three momentum is zero, in spite of its zero-mass value. One can work out the detailed kinematics of the final particles in this frame in the usual way: Each particle has energy $\frac{1}{2}(k_\mu k_\nu h^{\mu\nu})^{1/2}$, the momenta are equal and opposite, and the direction of one of them is arbitrary. One may then transform into the lab frame with a velocity parameter which is easily shown to be

$$\beta \simeq 1-\tfrac{1}{2}(h^{00}+2h^{01}+h^{11})$$

to first order with $h^{\mu\nu}$ evaluated in the lab frame (not necessarily the special frame). The 1 axis again refers to the propagation direction.

Clearly the reaction occurs, as expected, and the working out of further details would be straightforward.

III. SPECIFIC CASES

It is reasonably straightforward to apply the formalism in Sec. II to the processes involving photons and neutrinos mentioned in the Introduction. The computational techniques differ in only two ways from standard ones: First, any θ function of the form $\theta(p_\mu+h_\mu{}^\lambda q_\lambda)$ is expanded in a power series in $h_\mu{}^\lambda$. The zeroth order term is kept, while the first-order term, proportional to a δ function, and all higher orders are dropped. (In this way it turns out that all noncovariance goes away in practice, to first order.) Second, it is convenient to make general coordinate transformations into systems in which certain particles are in free fall. This is discussed in the Appendix. Other than that, there are no significant differences from ordinary particle-physics computational techniques, and most of the details are omitted in the following.

It is emphasized again that all thresholds and kinematic constraints quoted below depend only on the energy-momentum-mass relations and on energy-momentum conservation. The expressions for the rates themselves depend upon the validity of Eq. (24).

A. Estimation of Field

It is convenient first to discuss the numerical value of $h^{\mu\nu}$.

In spite of one's intuition, it does not seem to be the case that the field due to the entire universe can even be estimated qualitatively from something like the Robertson-Walker model with a knowledge of the Hubble constant and the density of the universe. This is independent of the question of whether or not the universe is closed. Indeed, the algebraic sign itself is not determined.[34]

The procedure I follow here is to compute the field due to the largest local subunit of the universe and then argue that this gives a limit on the magnitude, if not the sign, of $h^{\mu\nu}$. If the sign is wrong, certain conclusions are altered.

One obtains a differential equation in $h^{\mu\nu}$ either from the linear approximation to Einstein's equations or from the tensor field approach to gravitation. With the present conventions it is

$$\Box h^{\mu\nu}+\eta^{\mu\lambda}\eta^{\nu\sigma}h_{,\lambda\sigma}-\eta^{\nu\lambda}h^{\mu\alpha}{}_{,\alpha\lambda}-\eta^{\mu\lambda}h^{\nu\alpha}{}_{,\alpha\lambda}=16\pi G(T^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}T),$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the source and $T=T_\mu{}^\mu$. The equation is invariant under the gauge transformation

$$h^{\mu\nu}\to h^{\mu\nu}+\eta^{\nu\lambda}\xi^\mu{}_{,\lambda}+\eta^{\mu\lambda}\xi^\nu{}_{,\lambda}. \quad (26)$$

We choose the gauge

$$(h^{\mu\nu}-\tfrac{1}{2}h\eta^{\mu\nu})_{,\mu}=0, \quad (27)$$

where $h=h_\mu{}^\mu$. One then finds that the above field equation becomes a standard wave equation

$$\Box h^{\mu\nu}=16\pi G(T^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}T). \quad (28)$$

We assume that $T^{\mu\nu}$ refers to a nonrotating, homogeneous, uncompressed, spherical body with mass M and radius R. With the boundary condition that $h^{\mu\nu}\to 0$ at ∞, the interior solution of (28) is

$$h^{\mu\nu}=(2GM/R)(2n^\mu n^\nu+\eta^{\mu\nu}), \quad r<R \quad (29)$$

where n^μ has components (1,0,0,0) in the rest frame of the body. (n^μ is, in fact, the four-velocity of the body.) Thus $h^{\mu\nu}$ in the gauge (27) is indeed positive and diagonal in the rest frame of the body. It is also isotropic, and it is constant everywhere within the body.[35]

We take the body in question to be the local cluster of galaxies. Using the experimental parameters,[36] one obtains

$$2GM/R=8\times10^{-7}\simeq10^{-6}. \quad (30)$$

Note that (30) is small, so that the linear approximation is probably justified.

The solution (29) is subject to the gauge ambiguity (26). Further, we do not really know the boundary condition at infinity: If $h^{\mu\nu}\to$ const at infinity, then the constant is to be added to the above solution (assuming that the constant is small). Whatever is the gauge chosen by nature, we expect it to depend upon large distances characteristic of the source so that any gauge

[34] We may write the Robertson-Walker metric as $ds^2=-dt^2+R^2(t)(d\xi^2+d\eta^2+d\zeta^2)[1+\frac{1}{4}k\rho^2]^{-2}$, where ξ, η, and ζ are dimensionless markers, $\rho^2=\xi^2+\eta^2+\zeta^2$, and $k=\pm1$ or 0. The density of the universe and the nebular red shift can be used in principle to measure $R(t_0)$ and k by a well-known procedure. This is all that is interesting if general relativity is valid. However, if it is not then we must relate ξ, η, and ζ to some "absolute" coordinates measured by "weightless" matter in order to express $g^{\mu\nu}$ as $\eta^{\mu\nu}-h^{\mu\nu}$. For example, it may be that $\xi=x/r_0$, etc., where x, y, and z are the absolute coordinates and r_0 is some characteristic length. In that case there is no way to measure r_0 using normal astronomical matter (which gravitates normally). Therefore we cannot even estimate $h^{\mu\nu}$ by such means because the metric tensor contains a completely unknown parameter. Some other, no doubt unusual approach to the problem is needed before one can say anything. I would like to thank E. Toton for a discussion of this point.

[35] The gauge function which carried Eq. (29) into the linear approximation to the normal Schwarzschild interior solution is $\eta^{\mu\lambda}\xi^\nu{}_{,\lambda}+\eta^{\nu\lambda}\xi^\mu{}_{,\lambda}=(2GM/R)[-\frac{1}{2}(1+s^2)n^\mu n^\nu+s^\mu s^\nu-\eta^{\mu\nu}]$, where s^μ is a spacelike vector with components $s^i=x^i/R$, $s^0=0$ in the rest frame of the body, with x^i the coordinates of the field point measured from the center of the body.

[36] The radius R is estimated from the volume and the latter is given by C. W. Allen, *Astrophysical Quantities* (The Athlone Press, University of London, London, England, 1963). The mass M is calculated neglecting any mass other than the masses of the galaxies themselves, and assuming 1.2×10^{11} solar masses per galaxy.

function added to the solution is also effectively constant in the neighborhood of the earth. Thus we expect (29) and (30) to be correct to within an additive constant tensor. If we assume this constant to be nonnegative in all components, then we have a lower limit on the field,

$$h^{\mu\nu} \gtrsim 10^{-6}(2n^\mu n^\nu + \eta^{\mu\nu}). \tag{31}$$

The magnitude of $h^{\mu\nu}$ satisfies the limit (31) even if the unknown additive constant tensor is negative in all components, unless the constant is fortuitously of just the right magnitude to cancel (or nearly cancel) the numerical value on the right-hand side of (31). This latter possibility is extremely unlikely, since the constant depends at least in part on the large-scale structure of the universe. Further, a cancellation would imply that the interior of the local cluster of galaxies occupies a privileged position in the universe; the same cancellation would not occur in some other cluster with different values of M and R.[37] We assume that no such cancellation occurs.

If $h^{\mu\nu}$ were to be negative rather than positive, the conclusions of this work would be essentially reversed, e.g., $P \to P+\gamma$ would be allowed if P were too heavy rather than too light.[38]

Case 1: $P \to P+\gamma$

We may call this process "elastic decay," after Feinberg.[26] It turns out that it only proceeds if the energy of P is above a certain threshold, as in the Čerenkov effect. It is of course necessary that the particle P have some kind of electromagnetic coupling. In the following the rate is estimated to first order in $h^{\mu\nu}$ and in the fine-structure constant.

In this case, Eq. (24) is best transformed into a general coordinate system where the particle P is in free fall. (See the Appendix.) After dropping terms explicitly proportional to $h^{\mu\nu}$, one obtains

$$\Gamma \simeq \frac{1}{(2\pi)^2}\frac{1}{2p_0}\int d^4q d^4k\, \theta(q_0)\theta(k_0)\delta(q_\mu q_\nu \eta^{\mu\nu}+m^2) \times \delta\{k_\mu k_\nu[\eta^{\mu\nu}-(1-C)h^{\mu\nu}]\}\delta(p-q-k)\,|M|^2,$$

with

$$p_\mu p_\nu \eta^{\mu\nu}+m^2 \simeq 0.$$

Here, p, q, and k refer to P initially, P finally, and the photon, respectively, and m and C are the mass and C value, respectively, of P.

If P is a *point* charge, it is well known that the matrix element to zeroth order in $h^{\mu\nu}$ is, up to a sign,

$$M = e(p_\mu + q_\mu)\eta^{\mu\nu}\epsilon_\nu,$$

where e is the charge[39] and where ϵ_ν is the photon polarization vector. In a Lagrangian theory this is the matrix element of the operator $j_\mu \eta^{\mu\nu} A_\nu$. We assume that the appropriate momentum space projections of j_μ and A_ν still occur to first order in $h^{\mu\nu}$, in which case the appropriate generalization is to merely add a term in $h^{\mu\nu}$ to $\eta^{\mu\nu}$.[33] Further, in the case of an unpolarized spin-$\frac{1}{2}$ particle, one picks up another factor in the matrix element proportional to $h^{\mu\nu}$ due to the properties of the projection operators, Eq. (13). In summary, for either spin 0 or spin $\frac{1}{2}$, we have for a point-charge particle

$$M = e(p_\mu + q_\mu)(\eta^{\mu\nu} - Kh^{\mu\nu})\epsilon_\nu, \tag{32}$$

where K is some constant. (We assume here and in the following that the particle is unpolarized.)

If we do *not* have a point charge, then (32) becomes, neglecting magnetic and higher electromagnetic moments,

$$M = eF(Q^2)(p_\mu + q_\mu)(\eta^{\mu\nu} - Kh^{\mu\nu})\epsilon_\nu, \tag{33}$$

where Q^2 is the invariant momentum transfer,

$$Q^2 = (p_\mu - q_\mu)\eta^{\mu\nu}(p_\nu - q_\nu),$$

and where $F(Q^2)$ is the form factor. It could also be a function of invariants formed using $h^{\mu\nu}$ and, since one has intermediate states, using the frame-dependent vector n^μ.

From consideration of the properties of photons in a constant static field, analogous to Eqs. (9) and (10), it can be shown that $k_\mu \eta^{\mu\nu}\epsilon_\nu$ is zero as usual in the electromagnetic gauge we are using.

In a given Lorentz frame we define a unit vector $\boldsymbol{\kappa}$ in the direction of the photon momentum k and an anisotropic "inverse index of refraction" $n^{-1}(\boldsymbol{\kappa})$ such that $k_0 = |\mathbf{k}|n^{-1}(\boldsymbol{\kappa})$. The quantity n^{-1} is obtained from the root of the second δ function in the above rate formula. It is

$$n^{-1} = \frac{(1-C)(-h^{0i}\kappa_i) + \{(1-C)^2(h^{0i}\kappa_i)^2 + [1+(1-C)h^{00}][1-(1-C)h^{ij}\kappa_i\kappa_j]\}^{1/2}}{1+(1-C)h^{00}}$$
$$\simeq 1 - \tfrac{1}{2}(1-C)[h^{00} + 2h^{0i}\kappa_i + h^{ij}\kappa_i\kappa_j]. \tag{34}$$

Without specific assumptions concerning $h^{\mu\nu}$, the rate formula reduces to

$$\Gamma \simeq \frac{1}{(2\pi)^2 4p_0}\int k_0 dk_0 d\Omega_k \theta(k_0)\theta(p_0-k_0)|M|^2 \times \delta[k_\mu k_\nu(1-C)h^{\mu\nu} - 2p_\mu \eta^{\mu\nu}k_\nu],$$

and one obtains a kinematic condition on the photon-emission angle and energy:

$$\cos\theta = \frac{n^{-1}}{\beta}\left(1 + \frac{k_0}{2p_0}\frac{[1-(n^{-1})^2]}{(n^{-1})^2}\right),$$

where β is the initial velocity of P.

[37] A similar argument has been given by M. L. Good (Ref. 5).

[38] It is of course possible in principle for some components of $h^{\mu\nu}$ to be positive and others negative. We assume that this does not happen.

[39] The units are such that $e^2 = 4\pi\alpha$, where α is the fine-structure constant.

The quantity n^{-1} is a function of angle, and it is not necessarily the case that it is easiest to emit in the forward direction. Nevertheless, the threshold for emission in the forward direction itself, which we label the 1 axis, is given by

$$\beta \geq n^{-1}{}_{(\kappa=\kappa_1)} \simeq 1-\tfrac{1}{2}(1-C)(h^{00}+2h^{01}+h^{11}). \quad (35)$$

Thus, since $\beta \leq 1$, emission in the forward direction, at least, does not occur for positive $h^{\mu\nu}$ unless $C<1$, as expected.

For a point charge, the rest of the problem is easily solved analytically if we assume that $h^{\mu\nu}$ is diagonal and isotropic in some Lorentz frame, as in Eq. (29). We compute in this frame. It is the lab frame if we use Eq. (29) with $n^\mu=(1,0,0,0)$ and if we neglect our motion with respect to the local cluster, which we do. In this case, Eq. (35) with $h^{01}=0$ gives the true threshold; the energy distribution is

$$d\Gamma \simeq \alpha\beta \sin^2\theta dk_0, \quad (36)$$

with maximum energy p_0; and the total rate is

$$\Gamma \simeq \alpha p_0[(1-n^{-1})-(1-\beta)]^2/(1-n^{-1}), \quad (37)$$

where higher orders in both $1-n^{-1}$ and $1-\beta$ have been neglected. The quantity n^{-1} is given by Eq. (35) with $h^{01}=0$ and $\cos\theta$ is determined above from k_0. The azimuthal distribution is of course uniform. Equation (37) applies to the point-charge case only.

The above results are independent of the constant K in Eq. (33). The radiation is polarized in the (pk) plane, as in the Čerenkov effect. Both of these circumstances are a consequence of the assumption that $h^{\mu\nu}$ is isotropic.

We make numerical estimates in the lab frame assuming the limiting value $\sim 10^{-6}$ from Eq. (31) for $h^{00}=h^{11}=h^{22}=h^{33}$. The velocity threshold is

$$1-\beta \lesssim 10^{-6}(1-C),$$

which with $(1-C)$ of order unity corresponds to energies in the 100-GeV range for particles with mass in the 100-MeV range. Now consider as an example the rate, Eq. (37), neglecting $1-\beta$ and taking p_0 to be 1 TeV. One then obtains a mean radiation length

$$1/\Gamma \sim 3\times 10^{-9}\ \text{cm}/(1-C).$$

If $C<1$, the particle of course radiates energy until its velocity drops to the threshold value. The above result clearly shows that this happens in a distance which is much smaller than what can be resolved with normal measuring instruments. Therefore, for experimental purposes the *actual observation of the particle above threshold* shows that the reaction does not occur. In that case we immediately obtain a lower limit on the value of C for the particle.

It is easily seen that the above consideration is not modified if the particle is not a point charge. Consider a crude model in which the form factor in the matrix element, Eq. (33), is a step function which cuts off at some reasonable momentum transfer, say, $Q^2\sim 1$ GeV^2. It is easily seen from the kinematics that

$$Q^2=k_0^2[1-(n^{-1})^2]/(n^{-1})^2.$$

This gives $Q^2\sim 2(1-C)10^{-6}k_0^2$ with the present estimate of $h^{\mu\nu}$. We cut off the integration of the differential formula, Eq. (36), at the value of k_0 corresponding to $Q^2=1$ GeV^2. This value is $k_0\sim 10^3/(2(1-C))^{1/2}$ GeV. With $p_0\sim 10^3$ GeV as just discussed this cutoff is of the same order as the limit imposed by conservation of energy. Therefore the rate, Eq. (37), is modified by a factor of order unity, the radiation length is still far too small to be resolved normally, and we again need only *observe* the particle above threshold to set a limit on C.

In the case of a neutral hadronic particle, we have a form factor which is zero at *small* Q^2 but which acquires appreciable structure at $Q^2\sim(0.1\ \text{GeV})^2$ and above. An appreciable amount of the radiation occurs in this range, and the same conclusion follows for this case as well: We only need observe the particle above threshold to set a limit on C.

All presently known particles except the neutrinos and the graviton (and the photon) are either charged or are hadrons. The above qualitative argument is also inapplicable to those particles whose decay length is too short to be resolved anyway (π^0, η, N^*, etc.). It is applicable to all others.

It is easily seen from Eq. (35) that the higher is the velocity at which we observe the particle, the smaller is the limit we can set on $1-C$.

The existing experimental situation is, in part, as follows:

Except in the case of the electron, the threshold energies are in the 100-GeV range, assuming the lower limit (31) for $h^{\mu\nu}$. This is above the range of existing particle accelerators and therefore, for the present, one must turn to cosmic rays to see what may be said.

In practice the identification of charged particles above $\sim$100 GeV using techniques such as magnetic spectroscopy is extremely difficult, if not impossible, when the source is weak and not collimated, as is the case with cosmic rays. Among the charged component one can distinguish hadrons, electrons, and muons from each other because of their qualitatively different interactions in matter, and that is about all one can say about an individual particle.

In the case of hadrons one could attempt some sort of self-consistent analysis of existing hadronic shower data: If the π or the K, say, had essentially zero decay length because of a value of C less than unity, then the development of a shower started by a proton might be expected to have qualitatively different features. A study along these lines appears worthwhile but it will not be pursued here.[40]

In the case of the muon, we may actually set a limit using the well-known underground experiment of

[40] I would like to thank G. B. Yodh for a discussion of the cosmic-ray situation.

Bergeson *et al.*[41] The maximum muon energy observed in this experiment is ~ 10 TeV, which is well above threshold.

The limit for the muon is sensitive enough so that we must really take into account the possibility that the photon itself is slightly anomalous.[15] Consideration of the formulas earlier in this section shows that one must replace the quantity $1-C$, where C refers to the particle P, by $C_\gamma - C$, where C_γ refers to the photon. (Thus the elastic decay process really measures the *difference* between the gravitational coupling of the particle and that of the photon.)

With the above proviso, one obtains a limit[42]

$$C_\gamma - C_\mu = (1-C_\mu) - (1-C_\gamma) \lesssim 5\times 10^{-5}. \quad (38)$$

This assumes the existence of muons at 10 TeV and the limit on $h^{\mu\nu}$, Eq. (31).

Evidently we have good evidence that the gravitational coupling of the μ is *at least* that of the photon, provided that the boundary condition on $h^{\mu\nu}$ at infinity does not turn out to change its sign. This latter possibility is discussed later in connection with the process $\gamma \to P + \bar{P}$.

A study of existing electromagnetic and hadronic shower data should allow one to put very precise lower limits on C for electrons and protons (the latter because the proton must be the primary object which starts most of the hadronic showers).[43] Another interesting possibility would be to obtain a limit for the electron from the Eötvös experiment and to use air-shower data together with the $\gamma \to P + \bar{P}$ effect discussed next to set a precise limit on C_γ itself. However, these questions are not considered here.

Case 2: $\gamma \to P + \bar{P}$

We will call this process "spontaneous pair production." It clearly is related to the preceding process. It should proceed if either P or $\bar{P}$ or both is too heavy. However, we assume here that the particle and antiparticle have the same gravitation. We again calculate to first order in $h^{\mu\nu}$ and in the fine-structure constant.

The decay rate may be written in a general coordinate system where P and $\bar{P}$ are in free fall:

$$\Gamma \simeq \frac{1}{(2\pi)^2}\frac{1}{2k_0}\int d^4p\,d^4q\;\theta(p_0)\theta(q_0)\delta(p_\mu p_\nu \eta^{\mu\nu} + m^2) \times \delta(q_\mu q_\nu \eta^{\mu\nu} + m^2)\delta(k-p-q)|M|^2, \quad (39)$$

where the photon momentum k_μ satisfies

$$-k_\mu k_\nu \eta^{\mu\nu} \simeq (C-1)k_\mu k_\nu h^{\mu\nu}, \quad (40)$$

and where C refers to P and $\bar{P}$. The problem is very similar to the example discussed at the end of Sec. II and one can easily show that the "rest-frame" kinematics is the same if $C>1$ and if $h^{\mu\nu}$ has reasonable properties. However, the following formulas apply to the Lorentz frame in which $h^{\mu\nu}$ is given, presumably the lab frame.

The matrix element for a point charge is the same as Eq. (32) except that the relative sign of p_μ and q_μ is interchanged. We also assume for simplicity that the unknown constant K is zero. Then the point-charge solution with the incident photon traveling along the 1 axis for *arbitrary* $h^{\mu\nu}$ is as follows: The threshold is

$$k_0 \geq k_{0\min} = 2mn^{-1}[(n^{-1})^2 - 1]^{-1/2}; \quad (41)$$

the emission angle and energy of either final particle satisfy

$$\cos\theta = \frac{n^{-1}}{\beta}\left(1 - \frac{k_0}{2p_0}\frac{(n^{-1})^2-1}{(n^{-1})^2}\right),$$

where β is the velocity of the particle; the energy distribution is

$$d\Gamma \simeq \tfrac{1}{2}\alpha(\beta^2 p_0^2/k_0^2)dp_0 \sin^2\theta; \quad (42)$$

the energy limits are

$$p_{0\min}{}^{\max} = \tfrac{1}{2}k_0\left\{1 \pm \frac{1}{n^{-1}}\left[1 - \left(\frac{k_{0\min}}{k_0}\right)^2\right]^{1/2}\right\};$$

and the total rate is

$$\Gamma \simeq \tfrac{1}{6}k_0\alpha(1 - k_{0\min}^2/k_0^2)^{3/2}(n^{-1}-1). \quad (43)$$

In the above formulas, n^{-1} is to be evaluated using Eq. (34) with $\boldsymbol{\kappa} = (1,0,0)$, so that

$$n^{-1} - 1 \simeq \tfrac{1}{2}(C-1)(h^{00} + 2h^{01} + h^{11}). \quad (44)$$

We see from Eqs. (41) and (44) that the reaction proceeds for positive $h^{\mu\nu}$ if $C>1$, as expected.

The threshold for a final pair with mass in the 100-MeV range is in the 100-GeV range and the rates are extremely large, just as in the electromagnetic elastic decay case. If we neglect $k_{0\min}/k_0$ in Eq. (43) and take k_0 to be 1 TeV, say, then Eqs. (43) and (44) with the numerical limit Eq. (31) for $h^{\mu\nu}$ give a photon decay length

$$1/\Gamma \lesssim 1.6\times 10^{-8}\ \text{cm}/(C-1).$$

As in the elastic decay case, this circumstance is easily seen to not be altered appreciably if the particle is a hadron, charged or neutral, with appreciable charge structure for $Q^2 \sim (0.1\ \text{GeV})^2$ and above. Thus for appreciable values of $C-1$ the photon would be so unstable that it would not be observed to travel any appreciable lab distance. Therefore, if we observe a *single* photon (in principle) at some energy, we can

[41] H. E. Bergeson, J. W. Keuffel, M. O. Larson, E. R. Martin, and G. W. Mason, Phys. Rev. Letters **19**, 1487 (1967). It should be noted that higher muon energies than those quoted in this work have been quoted elsewhere; however, I have not given the other data the attention sufficient to feel justified in using it here.

[42] The appropriate number quoted in the first paper of Ref. 10 is in error.

[43] I would like to thank C. W. Misner for suggesting these possibilities.

conclude with confidence that spontaneous pair production does not occur at that energy or lower. This allows us to set a limit on $C-1$ for *any* particle which couples to a photon and whose mass is low enough so that the threshold energy, Eq. (41), is exceeded.

Individual photons with energies up to 10 TeV, at least, have in fact been observed.[44] Quite probably one could find examples at higher energies by carefully examining air-shower data; however, I shall use the 10-TeV figure. This indeed allows one to set good limits for most of the known particles with reasonably small mass. As in the electromagnetic elastic decay process, one really sets limits on $C-C_\gamma$ rather than $C-1$. The limits using Eq. (31) are given in Table I for several particles. Not all the known particles are listed, but it is easy to add other entries since the limits given are simply proportional to the square of the mass. A few of the lighter of atomic nuclei are listed for completeness.

The only known particles to which the present effect does not apply even in principle are the neutrinos and the graviton. Thus, with inspection of Table I, we have good evidence that the gravitational coupling of all particles with mass a few GeV or less is *at most* that of the photon, excepting possibly the neutrinos and the graviton. This again assumes that we have the correct algebraic sign for $h^{\mu\nu}$. If we have the wrong sign it is easily seen from Eq. (44) that the conclusion is reversed: In that case there is good evidence that the gravitational coupling of all particles with mass less than a few GeV is *at least* that of the photon, excepting possibly the neutrinos and the graviton.

We know that the muon, at least, neither undergoes elastic decay nor is spontaneously photoproduced. Thus we have evidence that the gravitational coupling of the μ is not far from normal, independent of the sign of $h^{\mu\nu}$. Quantitatively, the absolute value $|C_\mu-C_\gamma|$ is no larger than the largest of the two limits, (38) and the value from Table I. The largest number is the latter. Thus, independent of the sign of $h^{\mu\nu}$ we have

$$\frac{C_\mu-C_\gamma}{C_\gamma}=\frac{G_\mu{}^{1/2}-G_\gamma{}^{1/2}}{G_\gamma{}^{1/2}}=0\pm\sim 2\times10^{-4}, \qquad (45)$$

where distinct Newton constants have been introduced to cast the result in a more conventional system of units.

Equation (45) is the final result of this work concerning the muon. The quoted "error" assumes that the astrophysical quantities used to compute Eq. (30) are correct, although one should probably not trust them too closely.

[44] See the review article by Y. Fujimoto and S. Hayakawa, *Handbuch der Physik* (Springer-Verlag, Berlin, 1967), Vol. 46/2, p. 115. The photons are observed in "emulsion chambers"; the photon is produced, traverses some distance, and produces a shower all in the same apparatus. There is no question but that the particles are photons.

TABLE I. Upper limits on $C-C_\gamma$ from the formula $k_{0\min}=2m/[(C-C_\gamma)(h^{00}+h^{11})]^{1/2}$, assuming $k_{0\min}=10$ TeV, $h^{00}=h^{11}=10^{-6}$.

Particle	m (GeV/c^2)	$(C-C_\gamma)_{\max}$
e	5.1×10^{-4}	5.2×10^{-9}
μ	0.106	2.2×10^{-4}
π^+	0.140	3.9×10^{-4}
K^0	0.498	5.0×10^{-3}
ρ	0.77	0.012
p	0.94	0.018
Λ	1.12	0.025
$N_{3/2}{}^*(1238)$	1.24	0.031
Ξ^-	1.32	0.035
$Y_1{}^*(1385)$	1.39	0.039
Ω^-	1.67	0.056
d (H^2)	1.88	0.071
α (He^4)	3.73	0.28
C^{12}	11.2	2.5

Case 3: $\nu\rightarrow e+\mu+\nu'$

This actually refers to four distinct reactions:

$$\nu_\mu(\bar\nu_\mu)\rightarrow e^\pm+\mu^\mp+\nu_e(\bar\nu_e)$$

and

$$\nu_e(\bar\nu_e)\rightarrow e^\mp+\mu^\pm+\nu_\mu(\bar\nu_\mu).$$

They are described by the same matrix element as is ordinary μ decay. The appropriate process is expected to proceed if the respective initial neutrino is too light.

It turns out that no conclusion can be made on the basis of existing data in this case. Further, the outlook for further experiments is not promising. Nonetheless, I give a short discussion in the hope that some clever idea or another (or perhaps a better estimate of $h^{\mu\nu}$) will eventually make an appropriate experiment feasible.

It is assumed for simplicity that the interaction is described by the usual four-fermion $V-A$ interaction in first order with no form factor effects, that the interaction energy is weightless in the general coordinate system in which the problem is reduced, and that the final-state neutrino, the muon, and the electron have normal gravitation. We also neglect the electron mass in comparison with the muon mass, and we neglect radiative corrections. We compute as usual to first order in $h^{\mu\nu}$.

The rate formula is best written in a general coordinate system in which all but the initial neutrino are in free fall. It is

$$\Gamma\simeq\frac{1}{(2\pi)^5}\frac{1}{2k_0}\int d^4p\,\theta(p_0)\delta(p_\mu p_\nu\eta^{\mu\nu}+m^2)d^4qd^4q'\theta(q_0)\times\theta(q_0')\delta(q_\mu q_\nu\eta^{\mu\nu})\theta(q_\mu'q_\nu'\eta^{\mu\nu})\times\delta(k-p-q-q')|M|^2, \qquad (46)$$

with

$$-k_\mu k_\nu\eta^{\mu\nu}\simeq(1-C)k_\mu k_\nu h^{\mu\nu}, \qquad (47)$$

where k, q', q, and p refer to the initial neutrino, final neutrino, electron, and muon, respectively, C refers to the *initial* neutrino, and m is the mass of the muon.

The matrix element is assumed to be

$$M=(g/\sqrt{2})\eta^{\lambda\sigma}\bar{u}_e\gamma_\lambda(1+\gamma_5)u_{\nu_e}\bar{u}_{\nu_\mu}\gamma_\sigma(1+\gamma_5)u_\mu,$$

where g is the Fermi weak-interaction constant. Positive- and negative-energy spinor functions of the appropriate three-momenta are to be used in the standard way in accordance with which of the four reactions is under consideration. We assume that no polarizations are observed. Then, as usual, one squares the above matrix element, and averages over initial and sums over final spin states. This leads to an effective $|M|^2$ which is, to first order in $h^{\mu\nu}$,

$$|M|_{\text{eff}}^2 \simeq 64g^2[k_\mu-\tfrac{1}{2}(1-C)h_\mu{}^\alpha k_\alpha]\eta^{\mu\nu}q_\nu q_\lambda'\eta^{\lambda\sigma}p_\sigma$$

if the initial neutrino is ν_μ or $\bar{\nu}_\mu$, or

$$|M|_{\text{eff}}^2 \simeq 64g^2[k_\mu-\tfrac{1}{2}(1-C)h_\mu{}^\alpha k_\alpha]\eta^{\mu\nu}p_\nu q_\lambda\eta^{\lambda\sigma}q_\sigma'$$

if the initial neutrino is ν_e or $\bar{\nu}_e$.

In the same way as in the simple case discussed at the end of Sec. II, it is easily seen that the initial neutrino has a Lorentz-invariant effective mass which, from Eq. (47), is

$$\mathfrak{M}=[(1-C)k_\mu k_\nu h^{\mu\nu}]^{1/2}, \qquad (48)$$

and a "rest frame" in which the effective mass must be greater than the sum of the final state masses in order for the decay to be allowed. This in turn leads to a lab threshold

$$k_0 \gtrsim m/[(1-C)(h^{00}+2h^{01}+h^{11})]^{1/2}, \qquad (49)$$

with the neutrino traveling along the 1 axis.

Equation (46) is easily reduced in the "rest frame." The detailed energy and angular distributions are somewhat complicated and only the total rate is given here. It turns out to be independent of the type of initial neutrino. We define a quantity

$$\rho=m/\mathfrak{M} \qquad (50)$$

and write the result to lowest order in $h^{\mu\nu}$:

$$\Gamma\simeq\left(\frac{g^2}{192\pi^3}m^5\right)\tfrac{3}{2}f(\rho) \qquad (51)$$

$$f(\rho)=(1/\rho^5)\{(1-\rho^4)[(1+\rho^2)^2-10\rho^2]-24\rho^4\ln\rho\}. \qquad (52)$$

[The coefficient $\frac{3}{2}$ in (51) would be unity if we had neglected the term in $h^{\mu\nu}$ in the effective matrix element.] There are two useful limiting expressions for $f(\rho)$:

$$f(\rho)\to 1/\rho^5 \quad \text{as } \rho\to 0, \qquad (53)$$

corresponding to a neutrino energy far above threshold, and

$$f(\bar{\nu})\to(192/15)(1-1/\rho)^5 \quad \text{as } \rho\to 1, \qquad (54)$$

corresponding to a neutrino energy only slightly above threshold.

The γ factor which transforms (51) into the lab frame is easily seen to be, to lowest order,

$$\gamma\simeq[(1-C)(h^{00}+2h^{01}+h^{11})]^{-1/2}.$$

We also make use of the fact that the expression in large parentheses in (51) is just the $V-A$ theoretical expression for the μ-decay rate, neglecting radiative corrections. We take it to be the actual μ-decay rate. Then the lab mean life of the neutrino to lowest order in $h^{\mu\nu}$ is

$$\tau\simeq\frac{2}{3}\frac{\tau_\mu}{[(1-C)(h^{00}+2h^{01}+h^{11})]^{1/2}} \times\frac{1}{f\{m/k_0[(1-C)(h^{00}+2h^{01}+h^{11})]^{1/2}\}}, \qquad (55)$$

where τ_μ is the muon mean life at rest, $\sim 2.2\times 10^{-6}$ sec $=6.6\times 10^4$ cm, and where $f(\rho)$ is defined by (52)–(54).

The experimental situation is as follows:

We again assume the limit (31) for $h^{\mu\nu}$ and the energy threshold from (49) becomes

$$k_0\gtrsim 75/(1-C)^{1/2}\ \text{GeV}.$$

Thus we again must consider cosmic rays.

Since neutrinos are so difficult to detect, the most promising approach would be to actually search for the decay in regions where neutrinos are known to exist. It would have the appearance of two charged particles being created in vacuum, or perhaps one charged particle if the opening angle were too small to be resolved. Alternatively, the lifetime (55) may be so small that the decay occurs for practical purposes right at the source of the neutrino, e.g., from a μ decay. In that case one would observe a charged particle becoming three charged particles. If the opening angle were not resolvable, there would nevertheless be a sudden increase in ionization in a medium. We examine (55) in the high-energy limit $\rho\to 0$. Using (53), this gives

$$\tau\to\tfrac{2}{3}\tau_\mu(m^5/k_0^5)[(1-C)(h^{00}+2h^{01}+h^{11})]^{-3}.$$

[This shows that, unlike the electromagnetic processes previously considered, the neutrino lifetime (55) is extremely sensitive both to $h^{\mu\nu}$ and to the neutrino energy.] Let us imagine that we have a neutrino of energy 750 GeV. Assuming (31), this gives a decay length of $\sim 300/(1-C)^3$ cm, which is in a reasonable range for observation. Of course $h^{\mu\nu}$ could be much larger, making τ much smaller, but we can always observe the decay at the source of the neutrino as discussed above. However, one easily sees that there are essentially zero neutrinos in *existence* at such energies from, say, μ decay because munons have too long a lab decay length at such energies. Numerically, using the known cosmic-ray muon flux near the earth,[41] one estimates $\sim 6\times 10^{-17}$ neutrino productions per cm^2 sr sec in a detector of length 1 meter above $\sim$750 GeV. This of course is hopeless.

At lower energies than $\sim$750 GeV the neutrino production improves rapidly. However, the neutrino decay length rapidly becomes so long as to not be correlatable with the source. For example, using the same figure for $h^{\mu\nu}$ and with $C=0$, one has at 150 GeV ($\rho=0.5$), $\tau\sim 7\times 10^6$ cm [using the exact expression (52) for $f(\rho)$]. In this case, we could simply look for isolated events. All of the μ decays in the atmosphere from appropriate directions contribute to the neutrino flux at the earth's surface. However, a rough numerical estimate shows that even with a huge apparatus one would be lucky to detect one event per year.

Thus it is clear that the experimental situation is discouraging.

If there should arise some way of convincing oneself that $h^{\mu\nu}$ is in fact much larger than what is assumed in this work, then the experimental situation would improve rapidly as one could consider neutrinos at much lower energies.

One further remark is in order: If we believe that the "direct" elastic scattering interaction between e and ν_e and/or between μ and ν_μ exists, then the processes $\nu\rightarrow e+\bar{e}+\nu'$ and/or $\nu\rightarrow\mu+\bar{\mu}+\nu'$ are allowed if the initial neutrino is too light. One could calculate the rates in much the same way as above. The experimental situation would still be discouraging for the ν_μ case but not necessarily for the ν_e case. There is at present no laboratory evidence that the matrix element exists, and for this reason I have not pursued the problem.

Case 4: Gravitons

The most promising method of testing the gravitational coupling of the graviton itself is still probably to observe anomalies in the orbits of the planets (although to be sure that is not very promising[45]). Nonetheless this would be a test involving "virtual" gravitons in much the same way as the Eötvös experiment involves "virtual" photons. Since we now have evidence that dynamic gravitational fields exist,[46] it would be interesting to see if any of the techniques discussed in the present work can be applied.

Processes such as $g\rightarrow\gamma+\gamma$, $\gamma\rightarrow g+\gamma$, $\nu\rightarrow g+\nu$, and $g\rightarrow\nu+\bar{\nu}$, where g denotes the graviton, are allowed with respective appropriate C values. However, one does not have to calculate anything to see that a lot has to be overcome before one can discuss real experiments. From simple dimensional analysis it is easily shown that the rate for any of these processes is of the form

$$\Gamma\sim GE^3 f(h^{\mu\nu}),$$

where E is a characteristic energy and f is some dimensionless function of $h^{\mu\nu}$. At an energy corresponding to 1660 cps,[46] the quantity $1/(GE^3)$ is of order 10^{85} cm. At a characteristic energy of 1 GeV it is of order 10^{24} cm. If we assume the function f to be proportional to $h^{\mu\nu}$ in first order, the decay lengths are multiplied by $\sim 10^6$. The size of the universe itself is of order 10^{28} cm. Therefore it is clear that there is not much hope along these lines.

IV. CONCLUSION

A greatly abbreviated phenomenological description of the behavior of elementary particles in a constant, small classical gravitational field has been given. While far from rigorous, this description follows standard lines which are known to apply elsewhere, and therefore possesses plausibility. If only classical energy-momentum conservation is valid, certain processes which are normally forbidden are allowed if universal gravitation is violated. If the validity of the straightforward quantum-mechanical development is also assumed, then it is easy to derive reaction rates. In the case of electromagnetic processes in particular, the rates turn out to be such that any modifications of the formalism caused by some hypothetical, more correct approach are not likely to make any difference for practical experimental purposes.

A class of experiments with which one can (in principle) measure the gravitational coupling of elementary particles at zero momentum transfer has thus been brought to light. (In practice, even though there exists no prescription for calculating the gravitational field precisely, one can at least set limits on the difference between the actual gravitational coupling constant of a particle and that specified by universal gravitation.) At the simplest level (the only one considered in any detail here), one only need look for the presence or absence of certain reactions at the requisite energies.

One sees that, indeed, some relevant experimental data already exist. From these data, a concrete, quite small limit on the anomalous gravitational interaction of the muon has been given [Eq. (45)], and limits which are upper limits if the algebraic sign of the gravitational field is what one expects intuitively have been given for all particles with mass a few GeV or less which have appreciable electromagnetic coupling (Table I).

Some future lines of effort immediately suggest themselves: (a) One could attempt an improved theoretical estimate of the gravitational field by some means. Experimental data in a much more reasonable energy range become usable if the magnitude of the gravitational field can be shown to be considerably larger than the limit assumed here. (b) It would be useful to examine existing hadronic air-shower data, contrasting those features in the shower development that are observed with those that would be expected if one or more of the common hadrons (p, π, and K, principally) had an anomalously short decay length. In this way one could possibly learn something about the gravitational coupling of these particles. (c) A study of electromag-

[45] R. Sexl (private communication); S. Deser and B. E. Laurent (Ref. 1).

[46] J. Weber, Phys. Rev. Letters 22, 1320 (1969).

netic air-shower data should allow one to set extremely precise limits on the gravitational coupling of the electron. (d) More thought could be given to the question of the neutrinos.

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APPENDIX

The following method of making general coordinate transformations is used to simplify rate calculations based upon Eq. (24). (Actually, the transformations are not completely general. One only needs general linear transformations in consequence of the assumption that the gravitational field is constant.)

Suppose we have a set of N particles. The energy-momentum-mass relations, Eq. (3), are

$$p_{\mu J}p_{\mu J}(\eta^{\mu\nu}-C_J h^{\mu\nu})+m_J{}^2=0 \quad (J=1,\ldots,N). \quad \text{(A1)}$$

We define a set of vectors $\bar{p}_{\lambda J}$ by

$$p_{\mu J}=a_\mu{}^\lambda \bar{p}_{\lambda J}\,, \quad \text{(A2)}$$

where $a_\mu{}^\lambda$ is some set of numbers. Then (A1) becomes

$$\bar{p}_{\lambda J}\bar{p}_{\sigma J}(\eta^{\mu\nu}-C_J h^{\mu\nu})a_\mu{}^\lambda a_\nu{}^\sigma+m_J{}^2=0 \quad (J=1,\ldots,N). \quad \text{(A3)}$$

We choose $a_\mu{}^\lambda$ such that, for a particular particle X,

$$(\eta^{\mu\nu}-C_X h^{\mu\nu})a_\mu{}^\lambda a_\nu{}^\sigma=\eta^{\lambda\sigma}. \quad \text{(A4)}$$

Setting $J=X$ in (A3) then gives a new energy-momentum-mass relation for particle X:

$$\bar{p}_{\mu X}\bar{p}_{\nu X}\eta^{\mu\nu}+m_X{}^2=0. \quad \text{(A5)}$$

A solution of (A4) which defines $a_\mu{}^\lambda$ to within a Lorentz transformation is the matrix

$$a_\mu{}^\lambda=(\delta_\mu{}^\lambda-C_X h_\mu{}^\lambda)^{-1/2},$$

which is defined by its power-series expansion

$$a_\mu{}^\lambda=\delta_\mu{}^\lambda+\tfrac{1}{2}C_X h_\mu{}^\lambda+\tfrac{3}{8}C_X{}^2 h_\mu{}^\alpha h_\alpha{}^\lambda+\cdots.$$

We substitute this expression into (A3), drop the higher-order terms, and obtain

$$\bar{p}_{\mu J}\bar{p}_{\nu J}[\eta^{\mu\nu}-(C_J-C_X)h^{\mu\nu}]+m_J{}^2\simeq 0 \quad (J=1,\ldots,N). \quad \text{(A6)}$$

Equation (A6) defines a set of energy-momentum-mass relations which to first order in $h^{\mu\nu}$ is equivalent to the original set. One can use whichever set is convenient in a specific case.

We may, if we wish, say that $\bar{p}_{\mu J}$ and $p_{\mu J}$ denote the same vector expressed in different general coordinate systems. Equation (A5), or Eq. (A6) with $J=X$, shows clearly that particle X is not subject to a gravitational field in the barred coordinate system. That is to say, particle X is in free fall in this system. Also, the magnitude of the effective gravitational field (or, equivalently, of the appropriate coupling constant) acting upon any other particle is reduced in accordance with Eq. (A6).

Matrix elements which occur in calculations based upon Eq. (24) transform as follows. If A_μ and B_ν are any two vectors, then

$$A_\mu(\eta^{\mu\nu}+Kh^{\mu\nu})B_\nu\simeq\bar{A}_\mu[\eta^{\mu\nu}+(C_X+K)h^{\mu\nu}]\bar{B}_\nu \quad \text{(A7)}$$

to first order.

The Jacobian factors involved in the transformation of differential elements d^4p and δ functions may be approximated by unity, and calculations based upon Eq. (24) are still valid to lowest nonvanishing order. The same is true of transformed θ functions, as noted in the text.

The "bar" notation is not used in the text; the general coordinate system in use in each case is stated in the context.