(a) Suppose that for fixed s, $T_0(s,t)$ goes to zero like some finite inverse power of t for large $|t|$, so that the nth moment of $T_0(s,t)$ exists:

$$\int_0^\infty T_0(s, t=-q^2)q^{2n+1}dq<\infty\,. \tag{4}$$

Now an elementary calculation shows that

$$\left(\frac{d}{db^2}\right)^n H_0(b^2,s)=\left(\frac{-1}{2b}\right)^n \int_0^\infty q^{n+1}J_n(bq) \times T_0(s, t=-q^2)dq \tag{5}$$

or

$$\left(\frac{d}{db^2}\right)^n H_0(b^2,s)\,|_{b=0}=(-1)^n(2^{2n}n!)^{-1} \int_0^\infty q^{2n+1}T_0(s, t=-q^2)dq\,. \tag{6}$$

Hence the nth derivative of H_0 exists at $b=0$. But then the nth derivative of $H_0(b^2,s)/[1-I(s)H_0(b^2,s)]$ will also exist at $b=0$ [provided that $1-I(s)H_0(0,s)\neq 0$], which implies that the nth moment of $T(s,t)$ exists. The converse is easily seen to hold by expressing H_0 in terms of H and reversing the argument, provided that $1+I(s)H(0,s)\neq 0$. We conclude therefore that:

The nth moment of $T(s,t)$ (with respect to t) exists if and only if the nth moment of $T_0(s,t)$ exists, provided that $1-I(s)H_0(0,s)$ and $1+I(s)H(0,s)$ are different from zero.

(b) Suppose that for fixed s, $T_0(s,t)\sim\exp(-\alpha\sqrt{-t})$ for large $|t|$. This is the maximal rate of decrease for form factors consistent with the Jaffe bound.[5]

Since $J_0(bq)\sim(e^{ibq}-e^{-ibq})/(bq)^{1/2}$ for large q, it may be seen from (2a) that because $T_0(s, -q^2)\to_{q\to\infty}e^{-\alpha q}$, the Fourier-Bessel transform $H_0(b^2,s)$ is an analytic function of b in the strip $|\mathrm{Im}b|<\alpha$. Hence $H(b^2,s) =H_0(b^2,s)/[1-I(s)H_0(b^2,s)]$ is also analytic in the same strip.[6] We infer, therefore, from (2b) that $T(s, -q^2)$ should for large q fall off like $e^{-\beta q}$, with $\beta\geq\alpha$. By reversing the argument, we find $\alpha\geq\beta$, so that $\alpha=\beta$.

In the case considered by Abarbanel *et al.*, their input amplitude at large $|t|$ is dominated by the dipole term which goes like t^{-4}. It is therefore a special case of (a) with $n=2$. Abarbanel *et al.* had, in fact, reached similar conclusions for this case in their paper,[3] using somewhat different techniques. Their techniques can also be generalized to the case of arbitrary n.

[5] A. M. Jaffe, Phys. Rev. Letters **17**, 661 (1966).

[6] That $1-I(s)H_0(b^2,s)$ cannot vanish in the strip $|\mathrm{Im}b|<\alpha$ may be seen as follows. Suppose the contrary. Then $H(b^2,s)$ is infinite for some b in this strip and the integral

$$\int_0^\infty qdq\, J_0(bq)T(s, -q^2)$$

diverges. But we have

$$|J_0(bq)|=(1/\pi)\left|\int_0^\pi e^{ibq\cos\theta}d\theta\right|\leq(1/\pi)\int_0^\pi|e^{ibq\cos\theta}|d\theta$$
$$=(1/\pi)\int_0^\pi e^{-q\cos\theta\,\mathrm{Im}b}d\theta\leq(1/\pi)\int_0^\pi e^{|q\,\mathrm{Im}b|}d\theta=e^{q|\mathrm{Im}b|},$$

whereas $T\to_{q\to\infty}e^{-\alpha q}$ with $\alpha>\mathrm{Im}b$. Therefore the integral converges, which is a contradiction. Similarly, we have $1+I(s)H(b^2,s)\neq 0$ for $|\mathrm{Im}b|<\alpha$.

Reggeized $U(6)\otimes U(6)\otimes O(3)$ and Absorptive Correction Cuts for $K^+n\to K^0p$†

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The calculations of a previous paper on Reggeized $U(6)\otimes U(6)\otimes O(3)$ and absorptive corrections are extended to $K^+n\to K^0p$ at 5.5 GeV/c with the addition of no new parameters. Good agreement with experiment is obtained.

In a recent paper,[1] the reactions $\pi^-p\to\pi^0n$, $\pi^-p\to\eta^0n$, and $K^-p\to\bar{K}^0n$ were well explained using Reggeized $U(6)\otimes U(6)\otimes O(3)$ and absorptive correction cuts.

In I the reactions $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta^0n$ were used to determine the trajectories and residues of the ρ and A_2 poles, respectively. An absolute prediction was then made for the reaction $K^-p\to\bar{K}^0n$. When the predictions were compared with the data in the energy range $5.0\leqslant p_{\mathrm{lab}}\leqslant 12.3$ GeV/c, the agreement was found to be good.

High-energy data, namely, at 5.5 GeV/c,[2] are now

TABLE I. Absorption coefficients for K^+p elastic scattering.

p_{lab}	ν_1^{-1} (GeV^{-1})	C_1
2.3	0.38	1.00
3.0	0.36	0.91
5.5	0.31	0.68
8.0	0.28	0.55
12.0	0.27	0.51

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* Science Research Council Research Fellow.

[1] B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Phys. Rev. **187**, 1921 (1969), hereinafter referred to as I. All definitions, conventions, etc., used in the present paper are the same as in I, and for brevity will not be repeated.

[2] D. Cline, J. Penn, and D. Reeder, University of Wisconsin Report, 1969 (unpublished).

available for the charge-exchange reaction $K^+n \to K^0p$, and we have extended our calculations to this process. The results are shown in Fig. 1. The agreement with the experimental data at 5.5 GeV/c is good. In view of the fact that there is no resonance activity in the K^+n channel at 3.0 GeV/c,[3] the calculations were extended to this energy range. At 2.3 and 3.0 GeV/c,[4] the differential cross sections of the model have the correct t dependence, but the over-all normalization is too small by a factor of about 2. Possibly this is too low an energy for the asymptotic Regge formulation to apply. This difficulty also occurred in the application of previous Regge-pole models without absorptive cuts.[5,6] Regge models with ρ, ρ', and A_2 exchanges[5,7] were successful in explaining these four $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ charge-exchange reactions. However, the ρ' trajectory corresponds to no known particle.[8]

Above 5 GeV/c, the pole, the cut, and the pole-plus-cut amplitudes are largely real. This results from the destructive interference of the nearly exchange-degenerate ρ and A_2 poles. Consequently, the polarization is predicted to be almost zero.

No data exist on K^+n elastic differential cross sections at high energies. However, K^+n and K^+p total sections are almost the same,[9] so K^+p elastic scattering data were used to obtain the absorption parameters C_1 and $\nu_1{}^1$. Hopefully the parameters for K^+p and K^+n are not too different. These parameters are shown in Table I.

We wish to stress that this is a *no-parameter* fit to the $K^+n \to K^0p$ differential cross section.

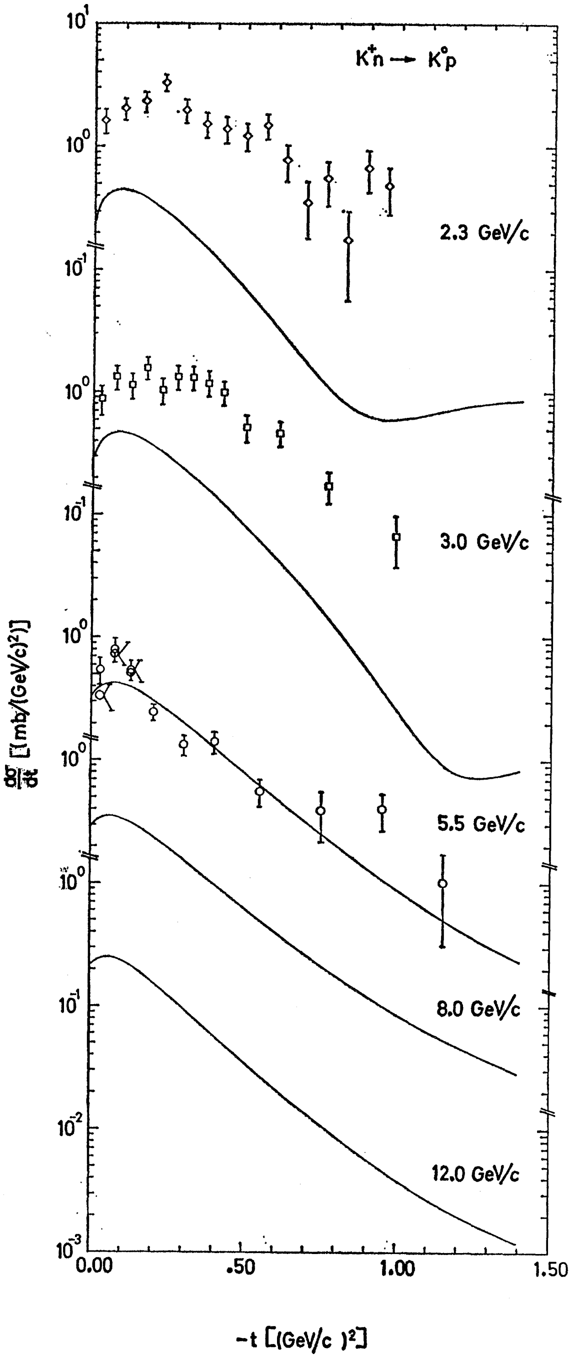

FIG. 1. Differential cross section for $K^+n \to K^0p$. Data from Refs. 2 and 4. Except for the lower three points in the forward direction, the data at 5.5 GeV/c contain the deuteron correction.

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