shell amplitude, the integrations on s_1 and s_2 may be performed immediately, fixing each at m^2 and taking the residue. The result is

$$\bar{j}(k)=\frac{-\pi^2C}{(2\pi)^4}\int dt\int dt' J^{-1}(s,P\cdot k,p\cdot k,t,t') \times T^*(s,t)T(s,t'), \quad \text{(A11)}$$

with

$$s=P^2, \quad J=2^4\sqrt{\Delta(P,p,q,k)},$$

and

$$\Delta=\det\{(P^\mu,p^\mu,q^\mu,k^\mu)^T(P_\mu,p_\mu,q_\mu,k_\mu)\}.$$

This result, which is only a model with no *a priori* justification, looks like a distorted elastic unitarity integral, which carves out the contribution to the meson field of the quasi-elastic scattering process.

Many other similar models are possible within this framework. All may be tested by feeding into (A11), or into other models, a tractable parametrization of the off-shell elastic amplitude and treating the result as indicated in the previous sections. This approach would provide a completely covariant form for the investigation of uncorrelated, quasi-elastic meson production.

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Baryon Mass Spectra and Baryon Couplings

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General intermultiplet mass formulas between the decuplet and octet baryons are derived, based on the chiral $SU(3)\otimes SU(3)$ charge algebra and the asymptotic $SU(3)$ symmetry imposed only on the charge operator V_K which is the $SU(3)$ raising or lowering operator in the symmetry limit. In the absence of particle mixing, the formulas take on a simple form and are useful as a first guide in deducing baryon mass spectra. Inclusion of particle mixing is possible. The formulas imply that the equal squared-mass spacings of decuplet states $(\delta_a)^2$ (a specifies the quantum numbers of the a decuplet) are universal [i.e., $(\delta_a)^2=\text{const}$] and, furthermore, equal to the universal spacings of the Ξ and Σ members of any octet baryons. In the case of the usual $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, the formulas coincide with the well-known $SU(6)$ mass formula. Broken-$SU(3)$ sum rules (in the absence of mixing) for general transitions of the form (octet baryon) → (octet baryon)$+\pi$ (or $l+\bar{\nu}$) are also obtained. In particular, the strong decays, $\frac{5}{2}^-\to\frac{1}{2}^++\pi$ and $\frac{5}{2}^+\to\frac{1}{2}^++\pi$, are discussed in detail. The sum rules, in general, give rise to significant $SU(3)$-breaking effects. However, for the familiar axial-vector $\frac{1}{2}^+\to\frac{1}{2}^++l+\bar{\nu}$ transitions, our broken-$SU(3)$ sum rules assume the same forms as satisfied by the hypothetical exact $SU(3)$ couplings. This justifies the use of the original Cabibbo analysis [in broken-$SU(3)$ symmetry] in determining the value of the axial-vector Cabibbo angle.

I. INTRODUCTION

IT now seems that we have infinite varieties of baryon excited states, and even the recurrence of baryons with the same spin and parity has been confirmed.[1] For the $SU(3)$ mass splitting, the Gell-Mann–Okubo (GMO) mass formulas have been very successful. There, particle mixing, if it exists, plays an important role. In this paper, we are mainly concerned with the following two questions: (i) Is there any simple intermultiplet regularity among the complicated baryon mass spectra? (ii) How far is the use of exact $SU(3)$ symmetry for the baryon couplings justified in the real world? We approach these questions by using the well-known chiral $SU(3)\otimes SU(3)$ charge algebra[2] and the asymptotic $SU(3)$ symmetry[3] imposed only on the charge operator V_K which is the $SU(3)$ raising or lowering operator in the symmetry limit. The V_K may be written (for example, in a quark model) as

$$V_{K^0}=-i\int d^3x\, V_0^{K^0}(\mathbf{x})=\tfrac{1}{2}\int \bar{q}(\mathbf{x})\gamma_0(\lambda_6+i\lambda_7)q(\mathbf{x})d^3x.$$

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[1] See, for example, R. D. Tripp, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 179; and also H. Harari, *ibid.*, p. 195.

[2] M. Gell-Mann, Physics 1, 63 (1964).

[3] S. Matsuda and S. Oneda, Phys. Rev. **174**, 1992 (1968). For a review, see S. Matsuda and S. Oneda, Nucl. Phys. **B9**, 55 (1969). Our approach is not a saturation by low-lying states. However, our asymptotic symmetry allows us to truncate the sum over the complete set of intermediate states, though *only* in the appropriate infinite-momentum limit. Although we work in the infinite-momentum frame, the result is always manifestly covariant.

We also define other vector and axial-vector charges V_i and A_i ($i=\pi$, K, etc.) in an analogous way.

II. ASYMPTOTIC $SU(3)$ SYMMETRY

Our asymptotic symmetry assumes that even in the broken world the operator V_K behaves as an exact $SU(3)$ generator to a good approximation but *only* at the appropriate zero-momentum-transfer limit. In the presence of $SU(3)$ mass splitting, this limit can only be realized by taking an appropriate *infinite-momentum* limit. In this infinite limit, the V_K connects only the members of the same $SU(3)$ multiplet and the values of its matrix elements are known. If particle mixing is possible, then asymptotic symmetry will be applied after we introduce the mixing angle in this limit.[3] Together with the use of the usual current algebra, this asymptotic symmetry can deduce many broken-$SU(3)$ sum rules.[3] However, one now proceeds further and asks: What are the commutators involving the time derivative of the V_K? If we write the $SU(3)$-breaking Hamiltonian as $\epsilon H'$, $\dot{V}_K=\epsilon i[V_K,H']$. The commutators involving V_K will, in general, be complicated. However, the following commutators are particularly convenient and useful, since they are independent of the unknown coupling constant ϵ of octet $SU(3)$ breaking:

$$[V_{K^0},\dot{V}_{K^0}]=0 \quad\text{and}\quad [A_{K^0},\dot{V}_{K^0}]=0. \tag{1}$$

The latter is valid under a rather general class of models of $SU(3)$ breaking,[3] i.e.,

$$\epsilon H'=g\int\bar{q}(\mathbf{x})\lambda_8 q(\mathbf{x})d^3x+f\int d_{8ij}V_\mu{}^i(\mathbf{x})V_\mu{}^j(\mathbf{x})d^3x +\int hd_{8ij}A_\mu{}^i(\mathbf{x})A_\mu{}^j(\mathbf{x})d^3x.$$

Here g, f, and h are arbitrary constants, and d_{ijk} is the $SU(3)$ d symbol. Note that $[V_{K^0},V_{K^0}]=[A_{K^0},V_{K^0}]=0$ also holds. Therefore, these commutators may represent the rather abstract nature (independent of ϵ) of $SU(3)$ breaking.

III. DERIVATION OF INTERMULTIPLET MASS FORMULAS

We now show that, together with these commutators, our asymptotic symmetry leads to an interesting interrelation of masses, mixings, and coupling constants. Since our result is general, let us denote the individual baryon octets and decuplets by the symbols $B_a{}^8$ $(N_a,\Lambda_a,\Sigma_a,\Xi_a)$ and $B_a{}^{10}$ $(\Delta_a,Y_a,\Xi_a{}^*,\Omega_a)$, respectively. The subscript a specifies the quantum numbers such as J^P, etc. First we note that the commutator $[V_{K^0},\dot{V}_{K^0}]=0$ always leads to the *quadratic* GMO mass formulas.[3,4]

[4] For similar but not identical approaches, see, for example, S. Fubini and G. Furlan, Physics **1**, 229 (1965); G. Furlan, F. Lannoy, C. Rossetti, and C. Segrè, Nuovo Cimento **40**, 597 (1965); K. Nishijima and L. J. Swank, Phys. Rev. **146**, 1161 (1966).

Consider, for example, the matrix element

$$\langle n_a(\mathbf{q})|[V_{K^0},\dot{V}_{K^0}]|\Xi_a{}^0(\mathbf{q})\rangle=0$$

with $|\mathbf{q}|=\infty$. Then if there is *no mixing*, our asymptotic symmetry for the V_{K^0} (denoting the mass of Λ_a, for example, simply as Λ_a) gives

$$3(\Lambda_a{}^0)^2+(\Sigma_a{}^0)^2=2[(n_a)^2+(\Xi_a{}^0)^2] \quad (a\text{ is arbitrary}). \tag{2}$$

For the ground-state $\frac{1}{2}^+$ baryons (N,Λ,Σ,Ξ), this relation is well satisfied: 5.14 GeV^2 on the left, and 5.23 GeV^2 on the right. Barring the possibility of complex mixing, this indicates that the mixing between the usual $\frac{1}{2}^+$ octet and other higher-lying $\frac{1}{2}^+$ baryons is small. Exactly in the same way, the assumption $[V_{K^0},\dot{V}_{K^0}]=0$, taken between the appropriate decuplet states with infinite momentum, leads (again in the absence of mixing) to the GMO formulas

$$(\Omega_a{}^2)-(\Xi_a{}^*)^2=(\Xi_a{}^*)^2-(Y_a)^2 =(Y_a)^2-(\Delta_a)^2\equiv(\delta_a)^2 \quad (a\text{ is arbitrary}), \tag{3}$$

i.e., equal spacing in the squared-mass spectra. We always obtain quadratic mass formulas for baryons as well as for mesons.[3] For the ground-state $\frac{3}{2}^+$ decuplet (Δ,Y,Ξ^*,Ω), Eq. (3) experimentally reads (we write $M^2\pm\Gamma M$) $0.46\pm0.01=0.42\pm0.05=0.39\pm0.16$ in GeV^2. Therefore, compared with the case of ground-state $\frac{1}{2}^+$ baryons, there seems to be a little more room for mixing effects in the $SU(3)$ formula for the ground-state $\frac{3}{2}^+$ baryons. This could be rather significant. There is an experimental indication[1] of rather large $SU(3)$ breaking in the ground-state decuplet decays. Our sum rules without mixing[3,5] are in the right direction to explain experiment, but are not sufficient. However, with mixing, the explanation is certainly feasible. We also remark here that, as is shown below, the same quadratic GMO mass formulas for octets and decuplets can be derived (again in the absence of mixing) also from the commutator $[\dot{V}_{K^0},A_{K^0}]=0$, using the same asymptotic symmetry. This demonstrates the internal consistency of our asymptotic $SU(3)$ symmetry.

We now wish to demonstrate that, if we discard particle mixings, there exist simple intermultiplet regularities which hold in our approach on the same footing as the GMO mass formulas without mixing. If now we insert both the commutators, $[V_{K^0}$ (and $\dot{V}_{K^0}),A_{K^0}]=0$, between the $\Omega_a{}^-(\mathbf{q})$ state of $B_a{}^{10}$ and the $\Sigma_b{}^-(\mathbf{q})$ state of $B_b{}^8$ with $|\mathbf{q}|=\infty$, the asymptotic symmetry implies

$$\langle\Omega_a{}^-(\mathbf{q})|A_{K^0}|\Xi_b(\mathbf{q})\rangle\langle\Xi_b(\mathbf{q})|V_{K^0}|\Sigma_b{}^-(\mathbf{q})\rangle =\langle\Omega_a{}^-(\mathbf{q})|V_{K^0}|\Xi_a{}^*(\mathbf{q})\rangle\langle\Xi_a{}^*(\mathbf{q})|A_{K^0}|\Sigma_b{}^-(\mathbf{q})\rangle,$$

$$\langle\Omega_a{}^-(\mathbf{q})|A_{K^0}|\Xi_b(\mathbf{q})\rangle\langle\Xi_b(\mathbf{q})|\dot{V}_{K^0}|\Sigma_b{}^-(\mathbf{q})\rangle =\langle\Omega_a{}^-(\mathbf{q})|\dot{V}_{K^0}|\Xi_a{}^*(\mathbf{q})\rangle\langle\Xi_a{}^*(\mathbf{q})|A_{K^0}|\Sigma_b{}^-(\mathbf{q})\rangle.$$

[5] For full details of decuplet → octet+π transitions, see G. Fourez, Nucl. Phys. (to be published).

These two equations are compatible only if

$$E(\Xi_b)-E(\Sigma_b)=E(\Omega_a)-E(\Xi_a{}^*)$$

with $|\mathbf{q}|=\infty$, where $E(\Xi_b)$, for example, denotes $[|\mathbf{q}|^2+(\Xi_b)^2]^{1/2}$. Then we obtain an intermultiplet mass formula between the a decuplet and the b octet, $(\Omega_a)^2-(\Xi_a)^2=(\Xi_b)^2-(\Sigma_b)^2$. This, together with Eq. (3), gives $(\Xi_b)^2-(\Sigma_b)^2=(\delta_a)^2$. Since a and b are arbitrary, we thus derive hybrid-type intermultiplet mass formulas,

$$(\Xi_b)^2-(\Sigma_b)^2=(\delta_a)^2=\text{const}\quad (a \text{ and } b \text{ are arbitrary}). \quad (4)$$

Thus, neglecting the mixing, we predict that the equal squared-mass spacings of decuplet states $(\delta_a)^2$ are universal $[(\delta_a)^2=\text{const}]$ and, furthermore, are equal to the universal spacing of the Ξ and Σ members of any octet baryons. Thus we feel that the scale of baryon mass spacings is established.[6] Our intermultiplet mass formula, Eq. (4), suggest that the squared mass of the baryon with spin and parity J^P, hypercharge Y, and isotopic spin I can be expressed as follows:

$$M^2(J^P,Y,I)=\alpha(J^P)+\beta Y+\gamma\{I(I+1)-\tfrac{1}{4}Y^2\}.$$

Here α is a constant independent of I and Y. β and γ are the universal constants which take the values $\beta\simeq 0.42$ GeV^2 and $\gamma\simeq 0.086$ GeV^2, respectively. Quadratic mass formulas are, in fact, very natural. One naturally expects that the linear mass splitting should become small for higher-lying $SU(3)$ multiplets. Our quadratic mass formulas, Eq. (4), are consistent with this expectation.

We note, however, that these intermultiplet mass formulas will be modified more significantly than the GMO formulas if particle mixing takes place. We have shown that the GMO formulas are derived from the commutators involving two V_K's, i.e., $[V_{K^0},\dot{V}_{K^0}]=0$. If mixing arises, the formulas will be modified to involve $\sin^2\theta$ and $\cos^2\theta$, where θ is the mixing angle. However, our intermultiplet formulas are based on the $[A_{K^0},\dot{V}_{K^0}]=0$ which involves only one V_K. Thus the modifications of these formulas depend linearly on $\sin\theta$ and $\cos\theta$. Therefore, even a rather small mixing angle, which hardly affects the GMO formulas, may modify the intermultiplet formulas [Eq. (4)] appreciably. Nevertheless, we believe that the simple formulas [Eq. (4)] obtained without mixing are useful as a first guide in determining the baryon mass spectra, as were the GMO formulas without mixing. Once mixing is involved, Eqs. (4) become sum rules which involve not only the masses and mixing angles but also coupling constants. These sum rules will not be very useful until more experimental information becomes available. At present, no higher-lying baryons are unambiguously established. Nevertheless, we consider the octets $\frac{5}{2}^-$ and $\frac{5}{2}^+$ of baryons, assuming that their present spin-parity assignments are correct. We use the following experimental mass values and notation: $\frac{5}{2}^-$ [$N'(1688)$, $\Lambda'(1815)$, $\Sigma'(1765)$, $\Xi'(1930)$],[1,7] $\frac{5}{2}^+$ [$N''(1688)$, $\Lambda''(1815)$, $\Sigma''(1940)$, $\Xi''(2030)$].[1,8] For Eq. (4) we then obtain $(\Xi)^2-(\Sigma)^2=0.31$, $(\Xi')^2-(\Sigma')^2=0.60\pm0.22$,[9] $(\Xi'')^2-(\Sigma'')^2=0.36\pm0.22$ in GeV^2, whereas the average of the quadratic mass spacing of the ground-state $\frac{3}{2}^+$ decuplet is $(\delta)^2=0.42\pm0.16$ in GeV^2. The result looks rather reasonable considering the fact that the first-order mixing effect has been neglected.

IV. SUM RULES FOR TRANSITION $B_a{}^8\to B_b{}^8+P$ (OR $l+\bar{\nu}$)

We now discuss the strong and weak axial-vector semileptonic decays between octet baryons, $B_a{}^8\to B_b{}^8+P$ (or $l+\bar{\nu}$). In exact $SU(3)$, these couplings are usually parametrized by the D and F couplings with a unique D/F ratio. We show below that this parametrization cannot be expected in general to work well in broken symmetry, with the important exception of the $B_a{}^8\to B_a{}^8+l+\bar{\nu}$ decay with $a=\frac{1}{2}^+$. We consider, for example, the commutator $A_{K^+}=-[V_{K^0},A_{\pi^+}]$ and insert it between the appropriate $\langle B_b{}^8(\mathbf{q})|$ and $|B_a{}^8(\mathbf{q})\rangle$ states with $|\mathbf{q}|=\infty$. With our asymptotic symmetry we obtain six linearly independent sum rules which are given, for example, by

$$\begin{aligned}
&\langle p_b|A_{K^+}|\Lambda_a\rangle=\langle\Sigma_b{}^+|A_{\pi^+}|\Lambda_a\rangle-(\surd\tfrac{3}{2})\langle p_b|A_{\pi^+}|n_a\rangle,\\
&\langle n_b|A_{K^+}|\Sigma_a{}^-\rangle=-(\surd\tfrac{1}{2})\langle\Sigma_b{}^0|A_{\pi^+}|\Sigma_a{}^-\rangle\\
&\qquad\qquad+(\surd\tfrac{3}{2})\langle\Lambda_b|A_{\pi^+}|\Sigma_a{}^-\rangle,\\
&\langle p_b|A_{K^+}|\Sigma_a{}^0\rangle=\langle\Sigma_b{}^+|A_{\pi^+}|\Sigma_a{}^0\rangle\\
&\qquad\qquad+(\surd\tfrac{1}{2})\langle p_b|A_{\pi^+}|n_a\rangle, \quad (5)\\
&\langle\Lambda_b|A_{K^+}|\Xi_a{}^-\rangle=-(\surd\tfrac{3}{2})\langle\Xi_b{}^0|A_{\pi^+}|\Xi_a{}^-\rangle\\
&\qquad\qquad+\langle\Lambda_b|A_{\pi^+}|\Sigma_a{}^-\rangle,\\
&\langle\Sigma_b{}^+|A_{K^+}|\Xi_a{}^0\rangle=-(\surd\tfrac{1}{2})\langle\Sigma_b{}^+|A_{\pi^+}|\Sigma_a{}^0\rangle\\
&\qquad\qquad+(\surd\tfrac{3}{2})\langle\Sigma_b{}^+|A_{\pi^+}|\Lambda_a\rangle,\\
&\langle\Sigma_b{}^0|A_{K^+}|\Xi_a{}^-\rangle=(\surd\tfrac{1}{2})\langle\Xi_b{}^0|A_{\pi^+}|\Xi_a{}^-\rangle\\
&\qquad\qquad+\langle\Sigma_b{}^0|A_{\pi^+}|\Sigma_a{}^-\rangle.
\end{aligned}$$

We also consider $\langle n_b(\mathbf{q})|[\dot{V}_{K^0},A_{K^0}]|\Xi_a(\mathbf{q})\rangle=0$ with $|\mathbf{q}|=\infty$, which gives

$$\begin{aligned}
&(\surd\tfrac{1}{2})(n_b{}^2-\Sigma_b{}^2)\langle\Sigma_b|A_{K^0}|\Xi_a{}^0\rangle\\
&\qquad-(\surd\tfrac{3}{2})(n_b{}^2-\Lambda_b{}^2)\langle\Lambda_b|A_{K^0}|\Xi_a{}^0\rangle\\
&=-(\surd\tfrac{1}{2})(\Sigma_a{}^2-\Xi_a{}^2)\langle n_b|A_{K^0}|\Sigma_a{}^0\rangle\\
&\qquad+(\surd\tfrac{3}{2})(\Lambda_a{}^2-\Xi_a{}^2)\langle n_b|A_{K^0}|\Lambda_a{}^0\rangle. \quad (6)
\end{aligned}$$

Equation (5) reduces to the exact $SU(3)$ sum rules in the symmetry limit, whereas in broken $SU(3)$ Eq. (6)

[6] Similar intermultiplet sum rules for the boson cases are $K^2-\pi^2=K^{*2}-\rho^2=K^{*2}(1420)-A_2{}^2=K_A{}^2-A_1{}^2=\cdots$, $m_\rho=m_\omega$, etc: S. Matsuda and S. Oneda, Phys. Rev. **179**, 1301 (1969).

[7] We take $\Xi'=1930\pm20$ MeV from J. Alitti *et al.*, Phys. Rev. Letters **21**, 1119 (1968). See also N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

[8] We take $\Sigma''=1940\pm20$ MeV and $\Xi''=2030\pm10$ MeV from V. E. Barnes *et al.*, Phys. Rev. Letters **22**, 479 (1969).

[9] Here the discrepancy seems rather large. We note that $\Sigma'<\Lambda'$ in the present assignment.

provides, in general, an additional constraint.[10] We first consider the case $a=b=\frac{1}{2}^+$ and note that, for example, $\lim_{|\mathbf{q}|\to\infty}\langle p(\mathbf{q})|A_{K^+}|\Lambda(\mathbf{q})\rangle = g_{p\Lambda}$. We have chosen the spin states of $\langle p(\mathbf{q})|$ and $\langle\Lambda(\mathbf{q})|$ as $(1,0,0,|\mathbf{q}|/(E_p+p))$ and $(1,0,0,|\mathbf{q}|/(E_\Lambda+\Lambda))$, respectively. $g_{p\Lambda}$ denotes the axial-vector semileptonic coupling at zero momentum transfer for the decay $\Lambda^0\to p+l+\bar\nu$. Then Eqs. (5) give broken-$SU(3)$ sum rules[3] such as $g_{p\Lambda}=g_{\Sigma^+\Lambda}-(\sqrt{\frac{3}{2}})g_{pn}$, etc. We emphasize the remarkable fact that physical masses do not appear in these sum rules and they assume the same forms as satisfied by the hypothetical exact $SU(3)$ couplings. However, this is by no means true for other cases such as $\frac{3}{2}^+\to\frac{1}{2}^++l+\bar\nu$. It implies that the usual way of determining the axial-vector Cabibbo angle θ_A by using exact $SU(3)$ sum rules is justified (rather accidentally) in our asymptotic symmetry to the same degree of accuracy as that of the vector angle θ_V. There could be small correction due to the neglect of mixing between the usual $\frac{1}{2}^+$ octet and other higher-lying $\frac{1}{2}^+$ baryons. However, as mentioned before, we expect that this effect will not be very large, in view of the success of the GMO mass formula for the usual $\frac{1}{2}^+$ octet. Thus the present experimental indication $\theta_A\simeq\theta_V$ (by using these sum rules)[11] may indeed imply that we have only one Cabbibo angle. We also remark that Eq. (6) does not lead to new information in this case. Instead, Eqs. (5) and (6) with $a=b$ always lead to the GMO mass formula, Eq. (2)—i.e., our sum rules are always compatible with GMO mass splitting. We now use pion PCAC (partially conserved axial-vector current) for the A_π in Eq. (5). Since here we discuss only the pion decays, it is more convenient to consider

$$\langle n_b(\mathbf{q})|[V_{K^0},A_{\pi^-}]|\Sigma_a^+(\mathbf{q})\rangle=0$$

and

$$\langle\Sigma_b^-(\mathbf{q})|[V_{K^0},A_{\pi^-}]|\Xi_a^0(\mathbf{q})\rangle=0$$

with $|\mathbf{q}|=\infty$ instead of Eqs. (5). For $a=b=\frac{1}{2}^+$, noting, for example, that

$$\lim_{|\mathbf{q}|\to\infty}\langle\Sigma^+(\mathbf{q})|A_{\pi^+}|\Lambda(\mathbf{q})\rangle=F_\pi(\Sigma+\Lambda)^{-1}g_{\Sigma^+\Lambda\pi^-},$$

we have obtained[3,10]

$$g_{np\pi^-}+(\sqrt{\tfrac{1}{2}})\left(\frac{p}{\Sigma}\right)g_{\Sigma^+\Sigma^0\pi^-}-(\sqrt{\tfrac{3}{2}})\left(\frac{2p}{\Sigma+\Lambda}\right)g_{\Sigma^-\Lambda\pi^+}=0,\quad(7)$$

$$g_{\Xi^-\Xi^0\pi^+}-(\sqrt{\tfrac{1}{2}})\left(\frac{\Xi}{\Sigma}\right)g_{\Sigma^0\Sigma^+\pi^-}-(\sqrt{\tfrac{3}{2}})\left(\frac{2\Xi}{\Sigma+\Lambda}\right)g_{\Lambda\Sigma^+\pi^-}=0.\quad(8)$$

Here all the pion couplings are defined with a pion off the mass shell ($q_\pi^2=-m_\pi^2=0$). The appearance of physical masses in Eqs. (7) and (8) no longer allows us to use an exact $SU(3)$ parametrization. Again Eq. (6) does not provide additional information for the case $a=b$, but it is always satisfied if the GMO formula, Eq. (2), is valid.

V. STRONG $\frac{5}{2}^-\to\frac{1}{2}^++\pi$ AND $\frac{5}{2}^+\to\frac{1}{2}^++\pi$ DECAYS

We now consider the cases $a\neq b$, for example, $\frac{5}{2}^-\to\frac{1}{2}^++\pi$ and $\frac{5}{2}^+\to\frac{1}{2}^++\pi$. The method can easily be extended to other cases. We merely need to note,[12] for example, that

$$\lim_{|\mathbf{q}|\to\infty}\langle\Sigma'^+(\mathbf{q})|A_{\pi^+}|\Lambda^0(\mathbf{q})\rangle=\tfrac{1}{8}F_\pi(\Sigma'+\Lambda)^2\Sigma'^{-2}(\Sigma'-\Lambda)g_{\Sigma'^+\Lambda\pi^-}\quad\text{for}\quad\Sigma'\equiv\Sigma(\tfrac{5}{2}^-)$$

and

$$\lim_{|\mathbf{q}|\to\infty}\langle\Sigma''^+(\mathbf{q})|A_{\pi^+}|\Lambda^0(\mathbf{q})\rangle=\tfrac{1}{8}F_\pi(\Sigma''-\Lambda)^2\Sigma''^{-2}(\Sigma''+\Lambda)g_{\Sigma''\Lambda\pi^-}\quad\text{for}\quad\Sigma''\equiv\Sigma(\tfrac{5}{2}^+).$$

Then the sum rules corresponding to Eqs. (7) and (8) are given by

$$\frac{(n'+p)^2(n'-p)}{n'^2}g_{n'p\pi^-}+(\sqrt{\tfrac{1}{2}})\frac{(\Sigma'+\Sigma)^2(\Sigma'-\Sigma)}{\Sigma'^2}g_{\Sigma'^0\Sigma^+\pi^-}-(\sqrt{\tfrac{3}{2}})\frac{(\Sigma'+\Lambda)^2(\Sigma'-\Lambda)}{\Sigma'^2}g_{\Sigma'^-\Lambda\pi^+}=0,\quad(9)$$

$$\frac{(\Xi'+\Xi)^2(\Xi'-\Xi)}{\Xi'^2}g_{\Xi'^-\Xi^0\pi^+}-(\sqrt{\tfrac{1}{2}})\frac{(\Sigma'+\Sigma)^2(\Sigma'-\Sigma)}{\Sigma'^2}g_{\Sigma'^0\Sigma^+\pi^-}-(\sqrt{\tfrac{3}{2}})\frac{(\Lambda'+\Sigma)^2(\Lambda'-\Sigma)}{\Lambda'^2}g_{\Lambda'\Sigma^+\pi^-}=0,\quad(10)$$

$$\frac{(n''-p)^2(n''+p)}{n''^2}g_{n''p\pi^-}+(\sqrt{\tfrac{1}{2}})\frac{(\Sigma''-\Sigma)^2(\Sigma''+\Sigma)}{\Sigma''^2}g_{\Sigma''^+\Sigma^0\pi^-}-(\sqrt{\tfrac{3}{2}})\frac{(\Sigma''-\Lambda)^2(\Sigma''+\Lambda)}{\Sigma''^2}g_{\Sigma''^-\Lambda\pi^+}=0,\quad(11)$$

$$\frac{(\Xi''-\Xi)^2(\Xi''+\Xi)}{\Xi''^2}g_{\Xi''^-\Xi^0\pi^+}-(\sqrt{\tfrac{1}{2}})\frac{(\Sigma''-\Sigma)^2(\Sigma''+\Sigma)}{\Sigma''^2}g_{\Sigma''^0\Sigma^+\pi^-}-(\sqrt{\tfrac{3}{2}})\frac{(\Lambda''-\Sigma)^2(\Lambda''+\Sigma)}{\Lambda''^2}g_{\Lambda''\Sigma^+\pi^-}=0.\quad(12)$$

[10] The accuracy of Eqs. (5) and (6) can be different from that of Eq. (4). If mixing is large for decuplets but small for octets, Eqs. (4) will be affected while Eqs. (5) and (6) will not.

[11] S. P. Desai, S. Matsuda, and S. Oneda, Phys. Rev. **178**, 2129 (1969).

[12] For the technique of evaluation, see Ref. 5.

Also from Eqs. (5) we can derive the relations

$$\langle\Sigma'^-(\mathbf{q})|A_{\pi^-}|\Lambda^0(\mathbf{q})\rangle=\langle\Lambda'(\mathbf{q})|A_{\pi^+}|\Sigma^-(\mathbf{q})\rangle$$

and

$$\langle\Sigma''^-(\mathbf{q})|A_{\pi^-}|\Lambda^0(\mathbf{q})\rangle=\langle\Lambda''(\mathbf{q})|A_{\pi^+}|\Sigma^-(\mathbf{q})\rangle$$

with $|\mathbf{q}|=\infty$, which lead to

$$\frac{(\Sigma'+\Lambda)^2(\Sigma'-\Lambda)}{\Sigma'^2}g_{\Sigma'\Lambda\pi}=\frac{(\Lambda'+\Sigma)^2(\Lambda'-\Sigma)}{\Lambda'^2}g_{\Lambda'\Sigma\pi}\,, \quad (13)$$

$$\frac{(\Sigma''-\Lambda)^2(\Sigma''+\Lambda)}{\Sigma''^2}g_{\Sigma''\Lambda\pi}=\frac{(\Lambda''-\Sigma)^2(\Lambda''+\Sigma)}{\Lambda''^2}g_{\Lambda''\Sigma\pi}\,. \quad (14)$$

Thus physical masses always appear in these sum rules. To obtain some feeling for numerical results, we write, for example, Eqs. (9), (11), (13), and (14) by using current experimental masses. They are

$$g_{n'p\pi}+(0.88)(\surd\tfrac{1}{2})g_{\Sigma'\Sigma\pi}-(0.95)(\surd\tfrac{3}{2})g_{\Sigma'\Lambda\pi}=0\,,$$

$$g_{n''p\pi}+(0.85)(\surd\tfrac{1}{2})g_{\Sigma''\Sigma\pi}-(0.83)(\surd\tfrac{3}{2})g_{\Lambda''\Sigma\pi}=0\,,$$

$$g_{\Lambda'\Sigma\pi}=(0.98)g_{\Sigma'\Lambda\pi}\,,$$

and

$$g_{\Lambda''\Sigma\pi}=(1.22)g_{\Sigma''\Lambda\pi}\,,$$

respectively. The decimal numbers in parentheses will be unity in the symmetry limit. The $SU(3)$ breaking indicated is of a reasonable order of magnitude and is more conspicuous for the $\frac{5}{2}^+\to\frac{1}{2}^++\pi$ decays since the sum rules involve the squares of the mass differences. Thus, the usual parametrization in terms of unique D/F ratio is not expected to work very well.

Equation (6) now provides another constraint. In order to use PCAC, we write Eq. (6) by expressing A_K in terms of A_π by using, for example, Eqs. (5):

$$[(\Xi_b^2-n_b^2)-(\Xi_a^2-n_a^2)]\langle\Sigma_b^+|A_{\pi^+}|\Sigma_a^0\rangle$$
$$=\tfrac{1}{2}\sqrt{3}[(\Sigma_b^2-\Lambda_b^2)-(\Sigma_a^2-\Lambda_a^2)]\langle\Sigma_b^+|A_{\pi^+}|\Lambda_a\rangle\,. \quad (15)$$

Equation (15), together with Eqs. (9)–(14), enables us to express all the couplings in terms of one independent coupling [in exact $SU(3)$ one does this by fixing the D/F ratio]. However, Eq. (15) involves expressions like $(\Xi_b^2-n_b^2)-(\Xi_a^2-n_a^2)$, so that it is very sensitive to the present experimental errors. Therefore, at present, the two-parameter expressions (9)–(14) will be more useful in comparisons with experiment.[1,7,8] The above computations can be extended to other baryons in a straightforward way.

Note added in proof. If we apply the same approach and kaon PCAC as well as pion PCAC to the $\frac{1}{2}^-\to\frac{1}{2}^++P$ transition, we see a more drastic effect of $SU(3)$ breaking. Namely, if the $Y_0^*(1405)$ is a $\frac{1}{2}^-$ $SU(3)$ singlet, our approach predicts for the ratio of the $Y\to pK$ and $Y\to\Sigma\pi$ couplings, $R\equiv(g_{YpK}/g_{Y\Sigma\pi})\simeq(Y-p)(Y-\Sigma)^{-1}(F_K/F_\pi)^{-1}\simeq-2.2$, with $F_K\simeq F_\pi$. In exact $SU(3)$, $R=1$. This result is also obtained recently by Gell-Mann, Oakes, and Renner [M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968)] by using a reasonable approximation in broken-$SU(3)$ symmetry. As a matter of fact, one can show that this approximation can be derived from our asymptotic $SU(3)$ symmetry. In the forthcoming paper we discuss the broken-$SU(3)$ sum rules for the general transition $B'\to(\frac{1}{2})^\pm+P$, where B' is a nonet or octet baryon with arbitrary spin and parity.

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