Electromagnetic Decays of Baryon Resonances in the Symmetric Quark Model*

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We study the radiative decays of the $D_{13}(1515)$, $S_{11}(1525)$, $F_{15}(1690)$, and $Y_0^*(1520)$ resonances within the framework of the symmetric quark model with harmonic-oscillator wave functions. We point out that if it is assumed that the quarks have large anomalous magnetic moments, then it becomes necessary to consider the so-called "spin-orbit" term in the nonrelativistic electromagnetic interaction Hamiltonian. Including the effect of this term, we show that it is possible to understand the vanishing contribution of the $D_{13}(1515)$ resonance to backward or forward pion photoproduction in terms of a cancellation between electric and magnetic multipoles. Requiring such a cancellation gives a value of the oscillator range parameter, $\alpha=105$ MeV. Using this value and assuming that the quark scale magnetic moment μ is equal to the proton moment, we calculate the multipole amplitudes involved in the decays of interest, and compare the results for the $D_{15}(1515)$ and $S_{11}(1525)$ resonances with the recent phenomenological multipole analysis by Chau, Dombey, and Moorhouse of pion photoproduction in the second resonance region. Agreement is reasonably good. For the $Y_0^*(1520)$ we obtain a radiative width $\Gamma_\gamma=0.051$ MeV to be compared with a recent experimental result $\Gamma_\gamma=0.15\pm0.03$ MeV, and an angular distribution compatible with experiment. We comment on the contribution of the $F_{15}(1690)$ resonance to pion photoproduction.

I. INTRODUCTION

THE quark model has achieved some measure of success in describing the electromagnetic decay (or excitation) of baryon resonances[1]; qualitatively, in the existence of selection rules discovered by Becchi and Morpurgo,[2] and by Moorhouse,[3] and quantitatively, in the calculation[2] of the magnetic dipole amplitude for photoexcitation of the $P_{33}(1236)$ resonance in fair agreement with the value deduced by Dalitz and Sutherland[4] from the data on pion photoproduction. In this paper we extend the quantitative analysis to a study of the radiative decays of the $D_{13}(1515)$, $S_{11}(1525)$, $F_{15}(1690)$ and $Y_0^*(1520)$ resonances, motivated chiefly by the information deriving from the recent phenomenological analysis by Chau, Dombey, and Moorhouse[5] (CDM) of pion photoproduction in the second resonance region.

We take as our starting point the symmetric quark model, introduced by Greenberg,[6] in which the baryons are assumed to be shell-model bound states of three spin-$\frac{1}{2}$ quarks which obey para-Fermi statistics of order three. Dalitz[7] and Faiman and Hendry[8] have shown, using harmonic-oscillator functions for the spatial parts of the three-quark wave functions, that the sequence of orbital excitations arising in such a scheme is capable of accommodating the known baryon resonances up to a mass of about 2 GeV. Section II is devoted to a brief review of the allocation of the states of interest within the framework of such a shell model.

In Sec. III we discuss the information available on radiative decays from the analysis of photoproduction and, in the case of $Y_0^*(1520)$, from the direct observation of the radiative decay by Mast *et al.*[9]

Section IV contains comment on the electromagnetic properties of quarks, and indicates the need to consider the so-called "spin-orbit" term in the coupling of the quarks to the radiation field. We give the formalism for calculation of multipole amplitudes, including the contribution of the spin-orbit term. The need to choose a particular value for the quark mass (or gyromagnetic ratio) is obviated by the use of the Heisenberg equivalence for the momentum operator.

In Sec. V we give the results of computing the multipole amplitudes involved in the radiative decays of the $D_{13}(1515)$, $S_{11}(1525)$, $F_{15}(1690)$, and $Y_0^*(1520)$ resonances, in terms of the oscillator range parameter α and the quark scale magnetic moment μ, which we take to be equal to the proton moment of 2.79 μ_N. We determine the range parameter α by requiring that the $D_{13}(1515)$ resonance should give no contribution to forward or backward pion photoproduction, in accord with observation, and obtain numerical values for the various amplitudes of interest. Recent work of a similar nature by other authors[10,11] is discussed.

We conclude, in Sec. VI, that the model is in reasonably good agreement with available data on electromagnetic decays, with one possible exception, and allows an understanding of the apparent absence from forward or backward pion photoproduction of the $D_{13}(1515)$ resonance.

* Based in part on a thesis submitted to the University of Sussex in 1969, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[1] R. H. Dalitz, in *Les Houches Lectures, 1965* (Gordon and Breach, Science Publishers, Inc., New York, 1965).

[2] C. Becchi and G. Morpurgo, Phys. Letters **17**, 352 (1965); Phys. Rev. **140**, B687 (1965).

[3] R. G. Moorhouse, Phys. Rev. Letters **16**, 771 (1966).

[4] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

[5] Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. **163**, 1632 (1967).

[6] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[7] R. H. Dalitz, in Proceedings of the Topical Conference on πN Scattering, Irvine, California, 1967 (unpublished).

[8] D. Faiman and A. W. Hendry, Phys. Rev. **173**, 1720 (1968).

[9] T. S. Mast, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, L. K. Gershwin, F. T. Solmitz, R. D. Tripp, and B. R. Webber, Phys. Rev. Letters **21**, 1715 (1968).

[10] D. Faiman and A. W. Hendry, Phys. Rev. **180**, 1572 (1969).

[11] L. A. Copley, G. Karl, and E. Obryk, Phys. Letters **29B**, 117 (1969); Nucl. Phys. (to be published).

II. SHELL MODEL

The shell model which we employ has been discussed at length in Ref. 8, where the eigenfunctions of the shell-model Hamiltonian

$$H_{\rm sm}=\sum_{i=1}^{3}\frac{\mathbf{p}_i^2}{2M}+\tfrac{1}{2}M\omega^2\sum_{i=1}^{3}\mathbf{r}_i^2 \quad (1)$$

are listed. These eigenfunctions are in one-to-one correspondence with the eigenfunctions of the Hamiltonian for three quarks interacting via harmonic-oscillator forces, provided that spurious states[12] are eliminated by restricting the c.m. wave function in the shell model to one particular state. In fact we utilize an observation made by Elliott and Skyrme,[12] and investigated in some detail by Gartenhaus and Schwartz,[13] who noted that multipole amplitudes may be calculated using shell-model wave functions directly provided that the wave functions are restricted to the nonspurious set where the c.m. system is in a $1s$ state. The only correction for c.m. motion then needed is multiplication by an energy-dependent normalization factor.

The ground state of the three-quark system is the totally symmetric $(1s)^3$ shell giving the usual[14] **56**, $L^p=0^+$ supermultiplet containing the nucleon and $P_{33}(1236)$.

The first nonspurious excited state is the mixed-symmetric $(1s)^2(1p)$ configuration, giving the supermultiplet **70**, $L=1^-$, to which the negative-parity resonances $D_{13}(1515)$, $S_{11}(1525)$, $S_{31}(1630)$, $D_{33}(1670)$, $D_{15}(1675)$, $S_{11}(1715)$, and $D_{13}(1730)$ are assigned together with the unitary singlet states $Y_0^*(1405)$ and $Y_0^*(1520)$. Faiman and Hendry[8] note that the physical S_{11} and D_{13} states are, in principle, mixtures of the quark states with total quark spin $\frac{1}{2}$ and $\frac{3}{2}$, and accordingly they define mixing angles θ_s and θ_d for which they obtain the values $\theta_s\approx35°$ or 90° and $\theta_d\approx35°$ or 127°, from an analysis of πN decay widths. The physical Y_0^* states can also be mixtures of the unitary singlet and unitary octet quark states. For the $Y_0^*(1520)$ Tripp *et al.*[15] define a mixing angle θ_y for which they obtain a value of about 16°. We take into account these mixing effects in our subsequent calculations.

The second excited state necessitates consideration of the configurations $(1s)^2(2s)$, $(1s)^2(1d)$, and $(1s)(1p)^2$. The nonspurious supermultiplets are **56**, $L=0^+$, **70**, $L=0^+$, **56**, $L=2^+$, **70**, $L=2^+$, and **20**, $L=1^+$, which are capable of accommodating the known resonances $P_{11}(1460)$, $P_{33}(1690)$, $F_{15}(1690)$, $P_{11}(1785)$, $P_{13}(1855)$, $F_{35}(1880)$, $P_{31}(1905)$, $F_{37}(1940)$, and $F_{17}(1983)$. Only the F_{37} and F_{17} can be allocated uniquely, however, the others being mixtures of configurations, at least in principle. In particular, three F_{15} quark-model states are available to accommodate $F_{15}(1690)$, although analysis of the πN widths suggests[8] that this resonance is predominantly a **56**, $L=2^+$ state.

[12] J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) A232, 561 (1955); Nuovo Cimento **4**, 164 (1956).

[13] S. Gartenhaus and C. Schwartz, Phys. Rev. **108**, 482 (1957).

[14] See, e.g., Ref. 7. L^p denotes the total orbital angular momentum and parity of the three-quark system, while the spin/unitary spin part of the wave function is specified by the $SU(6)$ representation **56**, **70**, or **20**.

[15] R. D. Tripp, D. W. G. Leith, A. Minten, R. Armenteros, M. Ferro-Luzzi, R. Levi Setti, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Nucl. Phys. **B3**, 10 (1967).

TABLE I. Connection between the CGLN amplitudes and the conventional multipole classification.

πN partial wave	CGLN amplitude	Multipolarity
S_{11}	E_{0+}	$E1$
D_{13}	E_{2-}	$E1$
	M_{2-}	$M2$
F_{15}	E_{3-}	$E2$
	M_{3-}	$M3$

III. DATA

The main source of our information on the electromagnetic decays of baryon resonances is the CDM multipole analysis of pion photoproduction from protons. The analysis spans the c.m. energy range 1375–1575 MeV which contains three πN resonances: $D_{13}(1515)$, $S_{11}(1525)$, and $P_{11}(1460)$. The status of the $P_{11}(1460)$ is unclear; an analysis of photoproduction by Schmidt and Engles[16] finds little or no evidence for a contribution from this resonance, in distinction to the results of CDM, while the problem of mixing of states in the second excited level of the shell model means that an unambiguous assignment of the $P_{11}(1460)$ to a particular configuration is not really possible. In view of these difficulties, we concentrate on the negative-parity states $D_{13}(1515)$ and $S_{11}(1525)$.

Conventionally, discussion of pion photoproduction is in terms of amplitudes introduced by Chew, Goldberger, Low, and Nambu[17] (CGLN); $E_{l\pm}$ ($M_{l\pm}$) denotes an electric (magnetic) transition into a πN state of orbital angular momentum l and total angular momentum $J=l\pm\frac{1}{2}$. In Table I we relate these amplitudes to the ordinary multipole classification of electromagnetic transitions. CDM parametrize the resonating amplitudes E_{2-}, M_{2-}, and E_{0+} leading to the D_{13} and S_{11} partial waves by

$$(qk)^{1/2}E_{2-}=c_{r\gamma}c_{r\pi}e^{i\delta_r}\sin\delta_r+A+iB\,, \quad (2)$$

and so on, with q, k the magnitudes of the 3-momenta of pion and photon, respectively. The strong interaction parameter $c_{r\pi}=(\Gamma_\pi/\Gamma)^{1/2}$ and the phase shift δ_r are determined from a Breit-Wigner fit to the elastic πN phase-shift analysis of Baryere *et al.*[18] for each partial wave. Then $c_{r\gamma}=(\Gamma_\gamma/3\Gamma)^{1/2}$; A and B are parameters determined from the fit to photoproduction data. CDM

[16] W. Schmidt and J. Engels, Phys. Rev. **169**, 1296 (1968).

[17] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[18] P. Bareyre, C. Bricman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965).

TABLE II. Values obtained by CDM for the parameter $c_{r\gamma}=(\Gamma_\gamma/3\Gamma)^{1/2}$, in units of 10^{-2}.

πN partial wave	CGLN amplitude	$c_{r\gamma}$ Solution (1)	Solution (2)	Solution (3)
S_{11}	E_{0+}	1.409	0.947	0.62[a]
D_{13}	E_{2-}	2.415	2.389	...
	M_{2-}	0.817	0.787	...

[a] Solution of K-matrix fit to η photoproduction.

give two solutions to the fit, while a K-matrix fit to η photoproduction yields a third value of $c_{r\gamma}(E_{0+})$. Table II lists the results of interest to us. The relative sign of E_{2-} and M_{2-} is determined from the fit since the photoproduction data includes angular distribution and polarization measurements. Both solutions are consistent with $c_{r\gamma}(E_{2-})/c_{r\gamma}(M_{2-})=3.0$; CDM note that this supports the simplest explanation for the experimental observation that there is little evidence of the dominant D_{13} resonance in forward or backward photoproduction, namely, that the $J_z=\frac{1}{2}$ helicity amplitude $=0$, implying that $E_{2-}=3M_{2-}$.

The solution for $c_{r\gamma}(E_{0+})$ is unstable; CDM attribute this instability to the inadequacy of the Breit-Wigner parametrization of the S_{11} partial wave in the neighborhood of the η threshold which lies in the second-resonance region at 1488 MeV.

Logan and Uchiyama-Campbell[19] have also analyzed η photoproduction and give two possible solutions for the partial width for radiative decay of $S_{11}(1525)$: $\Gamma_\gamma=0.34$ or 0.13 MeV corresponding[20] to $c_{r\gamma}(E_{0+})=3.70\times10^{-2}$ or 2.32×10^{-2}.

Dombey[21] has suggested that the best estimate of this quantity is to be obtained from a direct comparison of η production by pions and by photons in the region of the S_{11} resonance, and he obtains the result $\Gamma_\gamma/\Gamma_\pi=3.2\times10^{-3}$ which, combined with $\Gamma_\pi/\Gamma=0.34$, gives $c_{r\gamma}(E_{0+})=1.90\times10^{-2}$.

Mast *et al.*[9] have observed the decay $Y_0^*(1520)\to\Lambda+\gamma$ and give $\Gamma_\gamma=0.15\pm0.03$ MeV together with an angular distribution consistent with a pure $E1$ decay.

There is no analysis comparable with that of CDM for pion photoproduction in the third-resonance region. However, the striking feature is the absence of $F_{15}(1690)$ from forward or backward photoproduction; Beder[22] has suggested that the $J_z=\frac{1}{2}$ helicity amplitude vanishes, implying that $E_{3-}=2M_{3-}$.

IV. CALCULATIONS

We make the usual assumption that the quark magnetic moments are simply proportional to their charges and may thus be described by a single parameter μ, the so-called scale moment. Becchi and Morpurgo[2] have shown that μ is equal to the proton magnetic moment μ_p. If we assume that the quarks are very massive, it is apparent that they must have large anomalous moments since

$$\mu=(1+\kappa)e/2M=2.79e/2M_p. \quad (3)$$

Thus the quark gyromagnetic ratio g is given by

$$g=(1+\kappa)=2.79M_q/M_p. \quad (4)$$

We ignore the possibility of enhancement of the quark Dirac moment as proposed by Bogoliubov *et al.*[23]

The nonrelativistic coupling of a spin-$\frac{1}{2}$ particle of charge e, mass M, and Pauli moment $e\kappa/2M$ to the radiation field is given by the Foldy-Wouthuysen Hamiltonian[24]

$$\mathcal{H}=-\frac{e}{M}\mathbf{A}\cdot\mathbf{p}-\frac{e}{4M^2}(1+2\kappa)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p}-\frac{e}{2M}(1+\kappa)\boldsymbol{\sigma}\cdot\mathbf{H} \quad (5)$$

to order M^{-3}, which we may write in terms of the total moment μ as

$$\mathcal{H}=-\frac{e}{M}\mathbf{A}\cdot\mathbf{p}-\frac{1}{2M}\left(2\mu-\frac{e}{2M}\right)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p}-\mu\boldsymbol{\sigma}\cdot\mathbf{H}. \quad (6)$$

The second term is called the spin-orbit coupling term, since when $\mathbf{E}$ is a fixed central field rather than the radiation field, it is responsible for spin-orbit splitting. Its importance in other contexts has been noted by Barton and Dombey.[25] If we take Eq. (6) to describe the coupling of a quark to the radiation field, then we stress that although the spin-orbit term is formally of order M^{-2}, the large value of the quark moment does not allow us to discard it *a priori*.

We take the coupling to the radiation field of three quarks in the shell-model potential to be a sum of terms like Eq. (6), with complete neglect of exchange effects:

$$\mathcal{H}=\sum_{j=1}^{3}\left[-\frac{e_j}{M_q}\mathbf{A}\cdot\mathbf{p}_j-\frac{1}{2M_q}\left(2\mu_j-\frac{e_j}{2M_q}\right)\times\boldsymbol{\sigma}_j\cdot\mathbf{E}\times\mathbf{p}_j-\mu_j\boldsymbol{\sigma}_j\cdot\mathbf{H}\right]. \quad (7)$$

In fact such an assumption leads to paradoxical results, noticed by Barton and Dombey[25] and by Barton.[26] Brodsky and Primack[27] and Osborn[28] have pointed out

[19] R. K. Logan and F. Uchiyama-Campbell, Phys. Rev. **153**, 1634 (1967).

[20] We use the most recent values for total widths, taken from the Rosenfeld Tables, Rev. Mod. Phys. **41**, 109 (1969).

[21] N. Dombey (private communication); Phys. Rev. **174**, 2127 (1968).

[22] D. S. Beder, Nuovo Cimento **33**, 146 (1964).

[23] N. N. Bogoliubov, B. Struminski, and A. Tavkhelidze, Dubna Report Nos. JINR D1968, D2015, and D2141, 1965 (unpublished); see also H. J. Lipkin and A. Tavkhelidze, Phys. Letters **17**, 331 (1965).

[24] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950); H. Neuer and P. Urban, Acta Phys. Austriaca **15**, 380 (1962).

[25] G. Barton and N. Dombey, Phys. Rev. **162**, 1520 (1967).

[26] G. Barton, University of Sussex Report, 1967 (unpublished).

[27] S. J. Brodsky and J. Primack, Phys. Rev. **174**, 2071 (1968).

[28] H. Osborn, Phys. Rev. **176**, 1523 (1968).

that a subtle modification to the Thomas part of the coefficient of the spin-orbit term is required. However, we make use of Eq. (4), which tells us that the quark Dirac moment is very small compared with the anomalous moment and hence with the total moment, to write Eq. (7) as

$$\mathcal{H}=\sum_{j=1}^{3}\left[-\frac{e_j}{M_q}\mathbf{A}\cdot\mathbf{p}_j-\frac{\mu_j}{M_q}\boldsymbol{\sigma}_j\cdot\mathbf{E}\times\mathbf{p}_j-\mu_j\boldsymbol{\sigma}_j\cdot\mathbf{H}\right]. \quad (8)$$

The standard expressions[29] for electric and magnetic multipole amplitudes are

$$\mathcal{E}_{lm}=\frac{(2l+1)!!}{k^{l+1}(l+1)}\int d\tau\,\mathbf{j}\cdot(\boldsymbol{\nabla}\times\mathbf{L})[j_l(kr)Y_{lm}^*(\Omega)] \quad (9)$$

and

$$\mathfrak{M}_{lm}=-\frac{i(2l+1)!!}{k^l(l+1)}\int d\tau\,\mathbf{j}\cdot\mathbf{L}[j_l(kr)Y_{lm}^*(\Omega)], \quad (10)$$

where $\mathbf{L}=-i\mathbf{r}\times\boldsymbol{\nabla}$.

The contribution of the first and third terms in Eq. (8) to the current $\mathbf{j}$ are the familiar expressions[30]

$$\mathbf{j}_j^{(1)}=(\psi_b^*e_j\boldsymbol{\nabla}_j\psi_a-\psi_a e_j\boldsymbol{\nabla}_j\psi_b^*)/2iM_q \quad (11)$$

and

$$\mathbf{j}_j^{(3)}=\boldsymbol{\nabla}_j\times(\psi_b^*\mu_j\boldsymbol{\sigma}_j\psi_a), \quad (12)$$

which satisfy the continuity equation

$$\boldsymbol{\nabla}_j\cdot(\mathbf{j}_j^{(1)}+\mathbf{j}_j^{(3)})=ik\rho_j, \quad (13)$$

where $k=E_a-E_b$, the energy difference between initial and final states, and

$$\rho_j=\psi_b^*e_j\psi_a. \quad (14)$$

The additional term in $\mathbf{j}$ arising from the spin-orbit part of Eq. (8) is (see the Appendix)

$$\mathbf{j}_j^{(2)}=k\psi_b^*\mu_j(\boldsymbol{\sigma}_j\times\boldsymbol{\nabla}_j)\psi_a/M_q. \quad (15)$$

It is then possible, with the help of Eq. (13), the identity

$$(\boldsymbol{\nabla}\times\mathbf{L})[j_l(kr)Y_{lm}^*(\Omega)]\equiv i\boldsymbol{\nabla}\left[\frac{\partial}{\partial r}(rj_l(kr))Y_{lm}^*(\Omega)\right]+ik^2\mathbf{r}j_l(kr)Y_{lm}^*(\Omega), \quad (16)$$

and the Heisenberg equivalence

$$\mathbf{p}=iM[H,\mathbf{r}], \quad (17)$$

to cast Eqs. (9) and (10) in the following form:

$$\mathcal{E}_{lm}=\frac{(2l+1)!!}{k^l(l+1)}\sum_{j=1}^{3}\left\{\int d\tau\frac{\partial}{\partial r_j}(r_jj_l(kr_j))Y_{lm}^*(\Omega_j)\psi_b^*e_j\psi_a+\tfrac{1}{2}k^2\int d\tau\, j_l(kr_j)Y_{lm}^*(\Omega_j)\psi_b^*e_j\mathbf{r}_j^2\psi_a+ik\int d\tau\, j_l(kr_j)Y_{lm}^*(\Omega_j)\boldsymbol{\nabla}_j\cdot[\psi_b^*\mu_j(\boldsymbol{\sigma}_j\times\mathbf{r}_j)\psi_a]-ik\int d\tau\frac{\partial}{\partial r_j}(r_jj_l(kr_j))Y_{lm}^*(\Omega_j)\boldsymbol{\nabla}_j\cdot[\psi_b^*\mu_j(\boldsymbol{\sigma}_j\times\mathbf{r}_j)\psi_a]\right\} \quad (18)$$

and

$$\mathfrak{M}_{lm}=-\frac{(2l+1)!!}{k^l(l+1)}\sum_{j=1}^{3}\left\{\int d\tau\, j_l(kr_j)Y_{lm}^*(\Omega_j)\boldsymbol{\nabla}_j\cdot[\psi_b^*e_j\mathbf{L}_j\psi_a-\psi_a e_j\mathbf{L}_j\psi_b^*]/2M_q+\int d\tau\frac{\partial}{\partial r_j}(r_jj_l(kr_j))Y_{lm}^*(\Omega_j)\boldsymbol{\nabla}_j\cdot[\psi_b^*\mu_j\boldsymbol{\sigma}_j\psi_a]-k^2\int d\tau\, j_l(kr_j)Y_{lm}^*(\Omega_j)\psi_b^*\mu_j\mathbf{r}_j\cdot\boldsymbol{\sigma}_j\psi_a-k^2\int d\tau\, j_l(kr_j)Y_{lm}^*(\Omega_j)\boldsymbol{\nabla}_j\cdot[\psi_b^*\mu_j(\boldsymbol{\sigma}_j\times\mathbf{r}_j)\times\mathbf{r}_j\psi_a]\right\}. \quad (19)$$

The last term in each expression is the additional contribution resulting from the spin-orbit coupling.

The multipole operators for the transition from the state $\psi_a(J,J_z)$ to the state $\psi_b(J',J_z')$ (emission) are adjoints of tensor operators and hence we can define reduced amplitudes through the Wigner-Eckart theorem

$$\begin{aligned}\mathcal{E}_{lm}&=\langle J,J_z|J',l;\,J_z',m\rangle\langle J\|T_l^{(E)}\|J'\rangle,\\ \mathfrak{M}_{lm}&=\langle J,J_z|J',l;\,J_z',m\rangle\langle J\|T_l^{(M)}\|J'\rangle,\end{aligned} \quad (20)$$

adopting the phase convention of Brink and Satchler.[31]

The radiative width of the state ψ_a for a given multipole is then

$$\Gamma_\gamma=\frac{2(l+1)k^{2l+1}}{l[(2l+1)!!]^2}|\langle J\|T_l^{(E\text{ or }M)}\|J'\rangle|^2. \quad (21)$$

[29] See, e.g., F. M. Renard, Nucl. Phys. **B2**, 537 (1967); and K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956). Note that our units are such that $\hbar=c=1$, $e^2/4\pi=1/137$, and that l now denotes the total angular momentum of the photon.

[30] L. D. Landau and E. M. Lifschitz, *Quantum Mechanics* (Pergamon Press, Oxford, 1965), 2nd ed., p. 436.

[31] D. M. Brink and G. R. Satchler, *Angular Momentum* (Oxford University Press, New York, 1962).

TABLE III. Reduced amplitudes for the decays studied.

Decay	Reduced amplitudes
$D_{13}^{+}\rightarrow p\gamma$	$\langle\frac{3}{2}\Vert T_1^{(E)}\Vert\frac{1}{2}\rangle=\frac{1}{6(6\pi)^{1/2}}\left[e\left(\frac{1}{\alpha}+\frac{3k^2}{8\alpha^3}-\frac{k^4}{16\alpha^5}\right)+\mu\left(\frac{k}{2\alpha}-\frac{k^3}{4\alpha^3}\right)\right]\exp\left(-\frac{k^2}{6\alpha^2}\right)\cos\theta_d$
	$\langle\frac{3}{2}\Vert T_2^{(M)}\Vert\frac{1}{2}\rangle=-\frac{5\mu}{6(6\pi)^{1/2}}\left(\frac{1}{\alpha}+\frac{k^2}{2\alpha^3}\right)\exp\left(-\frac{k^2}{6\alpha^2}\right)\cos\theta_d$
$S_{11}^{+}\rightarrow p\gamma$	$\langle\frac{1}{2}\Vert T_1^{(E)}\Vert\frac{1}{2}\rangle=\frac{1}{6(6\pi)^{1/2}}\left[e\left(\frac{1}{\alpha}+\frac{3k^2}{8\alpha^3}-\frac{k^4}{16\alpha^5}\right)-\mu\left(\frac{k}{\alpha}-\frac{k^3}{2\alpha^3}\right)\right]\exp\left(-\frac{k^2}{6\alpha^2}\right)\cos\theta_s$
$Y_0^{*}\rightarrow\Lambda\gamma$	As for $D_{13}^{+}\rightarrow p\gamma$ except that $\cos\theta_d$ is replaced by $\frac{1}{2}(\cos\theta_y+\sin\theta_y)$
$F_{15}^{+}\rightarrow p\gamma$	$\langle\frac{5}{2}\Vert T_2^{(E)}\Vert\frac{1}{2}\rangle=\frac{1}{27}\left(\frac{5}{2\pi}\right)^{1/2}\left[e\left(\frac{3}{2\alpha^2}+\frac{3k^2}{\alpha^4}-\frac{k^4}{8\alpha^6}\right)+\mu\left(\frac{2k}{\alpha^2}-\frac{k^3}{2\alpha^4}\right)\right]\exp\left(-\frac{k^2}{6\alpha^2}\right)$
	$\langle\frac{5}{2}\Vert T_3^{(M)}\Vert\frac{1}{2}\rangle=-\frac{7}{36}\left(\frac{5}{2\pi}\right)^{1/2}\mu\left(\frac{1}{8\alpha^2}-\frac{3k^2}{2\alpha^4}\right)\exp\left(-\frac{k^2}{6\alpha^2}\right)$

It is then straightforward to compute $c_{r\gamma}=(\Gamma_\gamma/3\Gamma)^{1/2}$.

To determine the relative sign of the contribution of multipoles to a given transition it is necessary to be precise about phase conventions for the reduced matrix elements. Rose and Brink[32] have examined in detail the problem of obtaining phase-consistent probability amplitudes and angular distribution formulas in terms of phase-defined reduced matrix elements. We refer the reader to their paper for details and merely note that in our notation and units the probability amplitude for emission of a photon along $\mathbf{k}$ with circular polarization $q(=\pm1)$ is

$$A^q(J_z,J_z',\mathbf{k})=-\left(\frac{k}{2\pi}\right)^{1/2}\sum_{lm\sigma}(-iq)^\sigma\frac{(ik)^l}{(2l-1)!!}\times\left[\frac{(l+1)}{2l(2l+1)}\right]^{1/2}(-1)^{2l}\langle J,J_z|J',l;J_z',m\rangle\times\langle J\Vert T_l^{(\sigma)}\Vert J'\rangle\mathfrak{D}_{mq}^{l}(R)\,,\quad(22)$$

where $\sigma=0$ for electric radiation so that the superscript (0) means (E) and $\sigma=1$ for magnetic radiation. The rotation R takes the z axis to the direction of $\mathbf{k}$. When the direction of $\mathbf{k}$ is taken as quantization axis, Eq. (22) simplifies to

$$A^m(J_z,J_z')=-\left(\frac{k}{2\pi}\right)^{1/2}\sum_{lm\sigma}\frac{(-im)^\sigma(ik)^l}{(2l-1)!!}\left[\frac{(l+1)}{2l(2l+1)}\right]^{1/2}\times\langle J,J_z|J',l;J_z',m\rangle\langle J\Vert T_l^{(\sigma)}\Vert J'\rangle\,,\quad(23)$$

with $m=\pm1$ only.

TABLE IV. Calculated values of radiative amplitudes and widths, assuming $\alpha=105$ MeV, to be compared with the data of Sec. III.

Process	Amplitude or width	Mixing angle	Calculated value
$D_{13}^{+}\rightarrow p\gamma$	$c_{r\gamma}(E_{2-})$	$\theta_d=35°$	2.27×10^{-2}
		$\theta_d=127°$	1.68×10^{-2}
	$c_{r\gamma}(M_{2-})$	$\theta_d=35°$	0.76×10^{-2}
		$\theta_d=127°$	0.56×10^{-2}
$S_{11}^{+}\rightarrow p\gamma$	$c_{r\gamma}(E_{0+})$	$\theta_s=35°$	1.42×10^{-2}
$Y_0^{*}\rightarrow\Lambda\gamma$	Γ_γ	$\theta_y=16°$	0.051 MeV

[32] H. J. Rose and D. M. Brink, Rev. Mod. Phys. **39**, 306 (1967).

V. RESULTS

In Table III we display the reduced amplitudes involved in the radiative decays of the $D_{13}(1515)$, $S_{11}(1525)$, $Y_0^*(1520)$, and $F_{15}(1590)$ resonances. The effects of the mixing discussed in Sec. II have been included and it should be noted that there is no contribution to the decays of $D_{13}(1515)$ and $S_{11}(1525)$ from the states with total quark spin $S=\frac{3}{2}$ by virtue of a selection rule discovered by Moorhouse.[3] The rule depends on the vanishing matrix elements of the magnetic-moment operator, sandwiched between appropriate $SU(6)$ wave functions, and hence remains valid in the presence of the spin-orbit term. As a consequence of our use of the Heisenberg equivalence [Eq. (17)] the reduced amplitudes depends only on α and μ, the quark scale moment, and not on the quark mass or gyromagnetic ratio.

Using Eq. (21) we have computed the magnitudes of the various amplitudes of interest for a range of values of α. To determine whether or not the model reproduces correctly the relative sign of electric and magnetic contributions to the decay (or production) of the $D_{13}(1515)$, we utilize the experimental result[33] that there is little evidence of any contribution to forward

[33] S. D. Ecklund and R. L. Walker, Phys. Rev. **159**, 1195 (1967); see also Ref. 19.

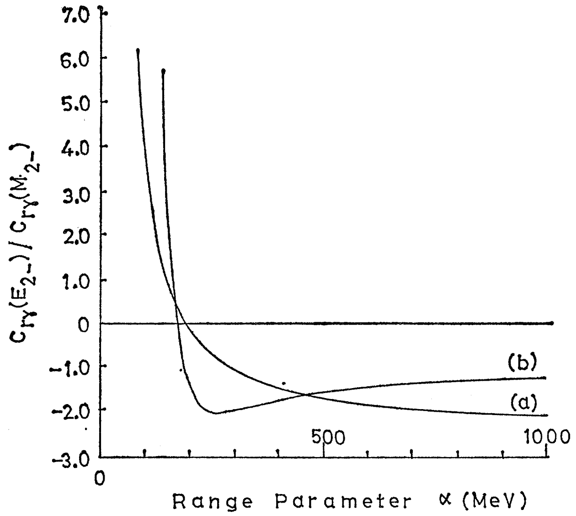

FIG. 1. Plot of $c_{r\gamma}(E_{2-})/c_{r\gamma}(M_{2-})$ versus range parameter α (a) including and (b) excluding the spin-orbit contribution.

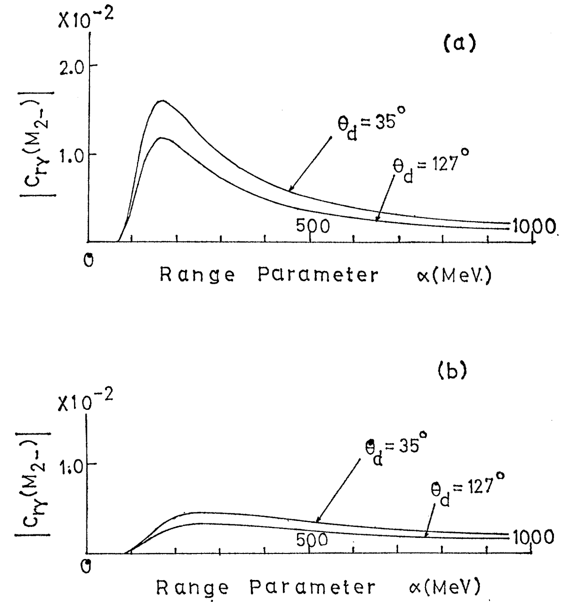

FIG. 3. Plot of $|c_{r\gamma}(M_{2-})|$ versus range parameter α (a) including and (b) excluding the spin-orbit contribution, for the two values of the mixing angle $\theta_d = 35°$ and $127°$.

or backward photoproduction from this resonance. For the process $\pi+N \rightarrow \gamma+N$ it is clear that in the forward direction conservation of angular momentum restricts the z component of angular momentum of the resonance to the values $J_z = \pm\frac{1}{2}$. Thus $m = \pm 1$ and $J_{z'} = \pm\frac{1}{2}$ since m can only be $+1$ or -1 in the forward direction and Eq. (23) tells us that the reduced amplitudes $\langle\frac{3}{2}\|T_1^{(E)}\|\frac{1}{2}\rangle$ and $\langle\frac{3}{2}\|T_2^{(M)}\|\frac{1}{2}\rangle$ must have the same sign for destructive interference. Hence the sign of the ratio $c_{r\gamma}(E_{2-})/c_{r\gamma}(M_{2-})$ is the relative sign of the reduced amplitudes. Armed with this knowledge we plot, in Fig. 1, the ratio of $c_{r\gamma}(E_{2-})/c_{r\gamma}(M_{2-})$ as a function of α; we see that for $\alpha = 105$ MeV it is possible to obtain the value $+3.0$, and hence to understand the vanishing

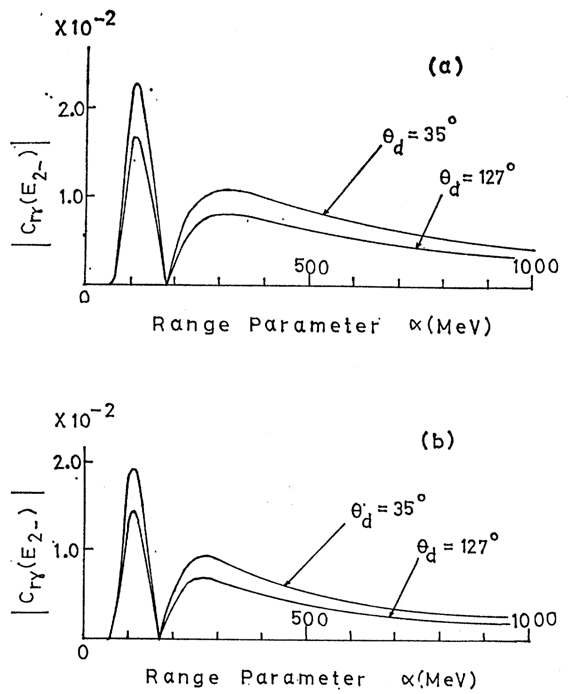

FIG. 2. Plot of $|c_{r\gamma}(E_{2-})|$ versus range parameter α (a) including and (b) excluding the spin-orbit contribution, for the two values of the mixing angle $\theta_d = 35°$ and $127°$.

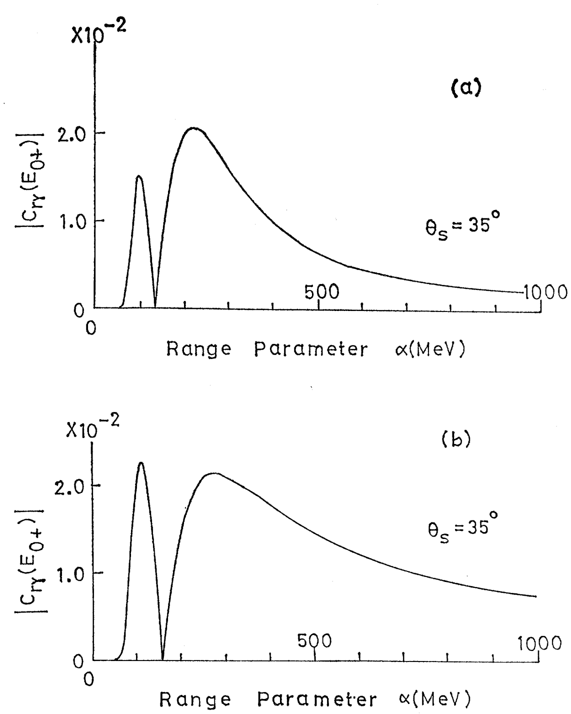

FIG. 4. Plot of $|c_{r\gamma}(E_{0+})|$ versus range parameter α (a) including and (b) excluding the spin-orbit contribution, with the mixing angle $\theta_s = 35°$.

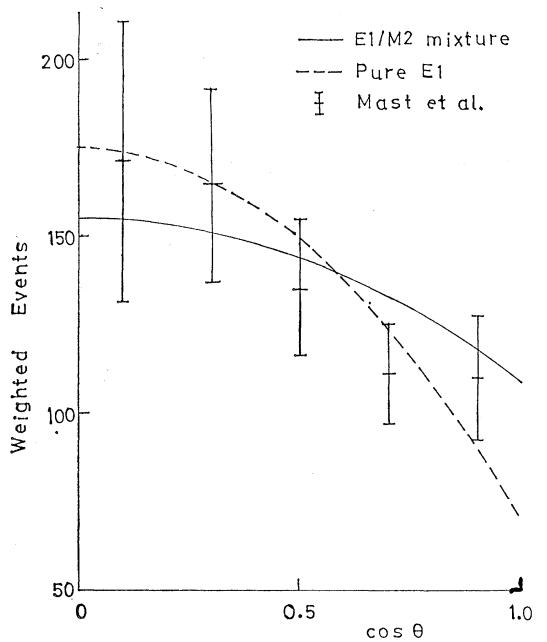

FIG. 5. Angular distribution of γ rays in the c.m. frame for the reaction $K^-p \to Y_0^*(1520) \to \Lambda\gamma$. The solid and broken curves are normalized to the total number of events.

contribution of the $D_{13}(1515)$ resonance in terms of a cancelation between electric and magnetic multipoles.

In Figs. 2–4 we plot the magnitudes of $c_{r\gamma}(E_{2-})$, $c_{r\gamma}(M_{2-})$, and $c_{r\gamma}(E_{0+})$, respectively, as functions of α, indicating the role played by the spin-orbit term. Table IV summarizes the numerical values obtained for the various amplitudes by setting $\alpha = 105$ MeV and comparison with the data of Sec. III indicates that the agreement appears to be good, with the possible exception of the $Y_0^*(1520)$ width, which is a factor of 3 too small. We cannot argue firmly in favor of particular values of the mixing angles θ_d and θ_s because our results are subject to some uncertainty, through the value of k used, to the extent that the "exact" mass of each resonance is not known. We estimate that a change of 50 MeV in the mass value assumed will produce a change of $\approx 20\%$ in the calculated values of $c_{r\gamma}$. The value of the total width, Γ, used in computing $c_{r\gamma}$ is also subject to uncertainty,[21] particularly for the $S_{11}(1525)$. The case of no mixing cannot be excluded, and in fact gives just as reasonable a fit; indeed the value of $c_{r\gamma}(E_{0+})$ is increased to 1.96×10^{-2}, in good agreement with Dombey's estimate. It can be seen from Figs. 1–4 that neglect of the spin-orbit contribution significantly worsens the agreement with available data. It is still possible to arrange the cancellation, with $\alpha = 149$ MeV, but, for instance, we obtain

$$c_{r\gamma}(E_{0+}) = 0.42 \times 10^{-2}.$$

In Fig. 5 we plot the angular distribution of γ rays to be expected in the c.m. frame of the reaction $K^-p \to Y_0^*(1520) \to \Lambda\gamma$, using the formalism of Rose and Brink.[32] For the sake of comparison, we have normalized the distribution to the total number of events seen by Mast *et al.*,[9] and it can be seen that the data are not incompatible with an $E1/M2$ mixture predicted by the model.

Finally, we have computed the ratio $c_{r\gamma}(E_{3-})/c_{r\gamma}(M_{3-})$ on the assumption that the $F_{15}(1690)$ is a pure **56**, $L = 2^+$ state, with the results shown in Fig. 6. With $\alpha = 105$ MeV, we obtain $c_{r\gamma}(E_{3-})/c_{r\gamma}(M_{3-}) = +1.8$. Exact cancellation of the $F_{15}(1690)$ contribution to backward pion photoproduction can be obtained by choosing the slightly smaller value $\alpha \approx 98$ MeV, but we maintain our previous criterion for the choice of α in view of the fact (mentioned in Sec. III) that not only is the experimentally observed contribution of the $D_{13}(1515)$ resonance to forward or backward pion photoproduction small but both solutions of the CDM analysis give $c_{r\gamma}(E_{2-})/c_{r\gamma}(M_{2-}) = 3.0$. Our ignorance of the amount of configuration mixing in the case of $F_{15}(1690)$ prevents us from making a useful statement on the radiative width of that state.

Work of a similar nature to that reported here has been performed independently by Faiman and Hendry,[10] who have calculated electromagnetic widths of several πN resonances, and by Copley, Karl, and Obryk,[11] who have calculated the contributions to pion photoproduction of πN resonances below 2 GeV as well as the electromagnetic widths of $Y_0^*(1520)$ and $Y_0^*(1405)$. Both groups of authors are forced to take a particular value of the gyromagnetic ratio g or, equivalently, of the quark mass,[34] because they make no use of the Heisen-

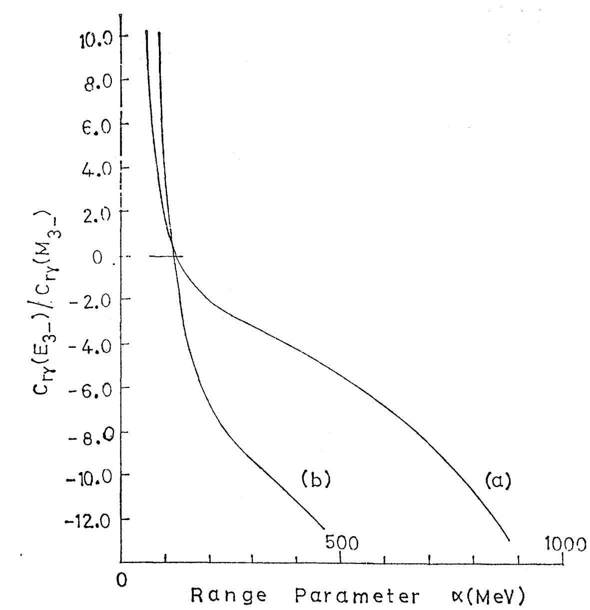

FIG. 6. Plot of $c_{r\gamma}(E_{3-})/c_{r\gamma}(M_{3-})$ versus range parameter g (a) including and (b) excluding the spin-orbit contribution.

[34] It can be seen from Eq. (4) that choosing a particular value of g determines the effective quark mass which appears in the interaction Hamiltonian.

berg equivalence. In amplification of this remark, we note that by using the Heisenberg relation we are eliminating the (unknown) quark mass in favor of the experimentally known quantity k; in a world where our model is exact, the two are of course related, with $k\approx\omega$, the energy interval between oscillator states ($\omega=\alpha^2/M$). The only explicit quark-mass dependence left in our amplitudes arises from the Thomas part of the spin-orbit term, but, as noted in Sec. IV, this may be neglected if we assume that $g\gg 1$. Faiman and Hendry do not treat α as a free parameter but take $\alpha^2=0.10$ GeV^2 from previous work on πN decay widths using the same model. Treating the gyromagnetic ratio as a free parameter, they estimate that $g\approx 1$ will give reasonable agreement with available data on electromagnetic widths. They do not include the spin-orbit term in the interaction Hamiltonian, but presumably there is *a posteriori* justification for this in the small value of g used. Copley, Karl, and Obryk take $g=1$ to be a "reasonable" value and obtain $\alpha^2=0.17$ GeV^2 by requiring zero contribution from $F_{15}(1690)$ to backward photoproduction. They also find that where data are available, agreement with experiment is reasonable. Further, they give a new selection rule for pion photoproduction off neutrons. The interesting feature of this rule is that it is violated by the spin-orbit term in the interaction Hamiltonian (8); hence, when more precise data on photoproduction from neutrons become available, it should be possible to distinguish between a model with $g\approx 1$ and the present model with large g and a large spin-orbit interaction.

The value we obtain for the range parameter corresponds to $\alpha^2=0.011$ GeV^2, considerably smaller than either of the values given above, and presumably reflects the large anomalous moment we have assumed for the quarks. Once the nature of the electromagnetic interaction has been elucidated, it will presumably be possible to distinguish between the various values of α^2 by utilizing electron-scattering data, as has been suggested by Thornber.[35]

VI. CONCLUSIONS

We have studied the electromagnetic decays of those baryon resonances for which some empirical information is available, within the framework of the symmetric quark model with harmonic-oscillator wave functions. Assuming that the quark gyromagnetic ratio is large, we have indicated the importance of the spin-orbit coupling term in the nonrelativistic electromagnetic interaction Hamiltonian and, including this term, have shown that it is possible to understand the vanishing contribution of the $D_{13}(1515)$ resonance to backward (or forward) pion photoproduction in terms of a cancellation between electric and magnetic multipoles. Requiring such a cancellation yields a value of the oscillator range parameter, $\alpha=105$ MeV, which gives numerical values for the various radiative amplitudes and widths considered. Agreement with available data is generally good, with the possible exception of the $Y_0^*(1520)$ radiative width. The model also predicts a substantial cancellation between electric and magnetic multipoles in the contribution of the $F_{15}(1690)$ resonance to backward pion photoproduction; the cancellation can be made complete by slight adjustment of the range parameter α at the expense of complete cancellation for the $D_{13}(1515)$.

[35] N. S. Thornber, Phys. Rev. 169, 1096 (1968).

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APPENDIX

We give a prescription for finding the contribution of the spin-orbit term to the multipole amplitudes $\mathcal{E}_{lm}$ and $\mathfrak{M}_{lm}$. The spin-orbit term in the Hamiltonian Eq. (6) is

$$\mathcal{H}_{so}=-(\mu/M)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p}. \tag{A1}$$

Now using a semiclassical picture, $\mathbf{E}=-\partial\mathbf{A}/\partial t=-ik\mathbf{A}$ (emission) and hence we can write Eq. (A1) as

$$\mathcal{H}_{so}=(i\mu k/M)\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{p}. \tag{A2}$$

But

$$\int\psi_b^*\mathcal{H}_{so}\psi_a d\tau=-\int\mathbf{j}^{(2)}\cdot\mathbf{A}d\tau$$

$$=\frac{i\mu k}{M}\int\psi_b^*(\boldsymbol{\sigma}\cdot\mathbf{A}\times\mathbf{p})\psi_a d\tau,$$

$$=\frac{-i\mu k}{M}\int\mathbf{A}\cdot\psi_b^*(\boldsymbol{\sigma}\times\mathbf{p})\psi_a d\tau,$$

and hence

$$\mathbf{j}^{(2)}=(i\mu k/M)\psi_b^*(\boldsymbol{\sigma}\times\mathbf{p})\psi_a \tag{A3}$$

is the contribution of the spin-orbit term to the current $\mathbf{j}$ appearing in Eqs. (9) and (10). Insertion of Eq. (A3) in Eqs. (9) and (10) then gives the contribution to the multipole amplitudes $\mathcal{E}_{lm}$ and $\mathfrak{M}_{lm}$.