Since the A_2 couplings are rather crudely estimated, we have plotted the A_2 and the π contributions to $d\sigma/dt$ separately in Fig. 2 using $g_{A2NN}=1$ and $f_2=1$. The differential cross section in this model for different values of the A_2 couplings can be determined from Fig. 2.

From the differential cross sections shown in Fig. 1, we estimate total cross sections of about 5, 2, and 1 μb for incident photon energies of 4, 8, and 12 GeV, respectively. Total cross sections of about 16, 7, and 3 μb (for $\nu=4$, 8, and 12 GeV, respectively) would be indicated for $g_{A2NN}=1$ and $f_2=1$.

We would like to reemphasize our assumptions (Sec. I) and point out again that our estimates of g_{A2NN} and f_2 have been made with some reservations. When experimental cross sections are available it may be possible to say something further about the unknown couplings using a model such as the one presented here.

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Regge Model in High-Energy Lepton Processes*

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The application of Regge-model ideas to high-energy semileptonic reactions is taken up in some detail for the process $\nu+p\rightarrow\mu^-+p+\pi^+$ (and its electroproduction analogs), and, qualitatively, for more complicated processes of the sort $\nu+p\rightarrow\mu^-+p+X_1+X_2+\ldots$.

I. INTRODUCTION

ON the standard assumption of locality for the weak couplings of lepton pairs,[1] high-energy neutrino processes acquire a structure which is well suited to the testing of Regge-model notions.[2] In the simplest inelastic reaction, $l+N\rightarrow l+N+\pi$, rather detailed features of the theory become accessible for test in a striking way. This is particularly true for the neutrino reaction $\nu+N\rightarrow\mu+N+\pi$, which is the topic of Sec. II. Results for the corresponding electroproduction reactions (e or μ)$+N\rightarrow$ (e or μ)$+N+\pi$ are obtained in Sec. III. Certain qualitative features suggest themselves also for generalization to more complicated processes, again with definite observable consequences. These are taken up in Sec. IV.

It is a familiar implication of Regge-pole dominance[3] for strong two-body $\rightarrow$ two-body reactions that all helicity amplitudes for a given process share a common phase to leading order in energy, where this phase is determined by the signature factor of the dominant Regge trajectory. For the differential cross-section spectrum this entails the vanishing, to leading order in the energy, of correlations that are odd under reversal of all spin and momenta (e.g., correlations of the form $\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2$). Such quasi-$T$-violating effects first arise only in an energy order corresponding to interference between the leading and next ranking trajectories. It is this "phase" property of Regge theory that we shall especially focus on here. We suppose that it, along with other standard aspects on the theory, can be carried over to weak and electromagnetic analogs of two-body $\rightarrow$ two-body strong interactions, e.g., $\nu+N\rightarrow\mu+N+\pi$, $e+N\rightarrow e+N+\pi$. The experimental implications are especially rich for the neutrino-induced reactions. Here, because of the presence of parity-violating interactions, odd correlation terms formed solely out of momentum vectors (terms of the kind $\mathbf{k}_1\cdot\mathbf{k}_2\times\mathbf{k}_3$) are now permis-

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[1] General implications of locality are discussed by T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962); A Pais, Phys. Rev. Letters **9**, 117 (1962); A Pais and S. B. Treiman (unpublished). [In the last paper the factor σ on the left-hand-side of Eq. (18) should be replaced by $d^2\sigma/dsdt$.]

[2] Diffractive aspects of high-energy neutrino reactions have been discussed by C. A. Piketty and L. Stodolsky, in Proceedings of the Topical Conference on Weak Interactions, CERN, 1969 (unpublished).

[3] For definiteness we consider the Regge model to be a Regge-*pole* model. In what follows the, reader will find comments at appropriate places concerning the extent to which statements are actually independent of specific *pole* properties.

sible; and the question whether such correlations vanish in leading order in the energy provides a critical test of an important qualitative notion that derives from the Regge-pole model, but perhaps transcends its other more detailed features. It must immediately be stated that in taking over these ideas to weak semileptonic processes, we are explicitly assuming the validity of time-reversal invariance.

In Sec. II we take up in some detail the particular reaction[4] $\nu+p\rightarrow\mu^-+p+\pi^+$, a process specified by five independent variables (apart from spins, which are to be summed over). On standard assumptions of local V, A couplings of the leptons, the differential cross-section spectrum takes on an explicit and simple structure with respect to two of the variables. In its dependence on these variables the spectrum decomposes into nine distinctive terms, which serve to probe different aspects of the dynamics. The structure is displayed in full generality. In our parametrization, two distinct energy variables are dealt with. One of them, the laboratory energy ϵ of the incident neutrino, is a "trivial" variable in the sense described above. The other is the difference ν between the incoming and outgoing lepton energies. We are interested in the situation where ϵ and ν are both very large, and, for descriptive purposes, we count them as being of the same order. Estimates of the ν dependence of amplitudes are obtained on the basis of standard notions of the Regge-pole model. In leading approximation the reaction is presumed to be dominated by Pomeranchuk exchange. From G-parity considerations, it then follows that the leading amplitudes arise solely from the axial-vector current. Among the terms that survive in leading order there remains, so far as energy dependence alone is concerned, a certain characteristic odd correlation in the spectrum. However, its coefficient is supposed to vanish in leading order according to the idea that all helicity amplitudes arising from a single trajectory share a common phase. This leads to the nontrivial equation (18) given below. The correlation effect in question therefore constitutes a useful candidate for the testing of this qualitative feature of Regge-pole theory.

In lower order in the energy, this and other correlation effects can arise through interference from lower ranking trajectories of appropriate quantum numbers. In particular, the spectrum contains four distinctive correlation terms characteristic of V-A interference. These first arise in an energy order corresponding to interference between the Pomeranchuk trajectory and the leading odd-G-parity trajectory of appropriate quantum numbers. Here several remarkable circumstances are seen to obtain. First, if the vector current matrix element is dominated by a "natural-parity" trajectory [parity $=(-1)^J$], then the contribution to V-A interference has one more power of ν as compared to a dominance by an "unnatural-parity" trajectory [parity $=-(-1)^J$], for comparable trajectory function. We therefore consider the natural-parity case only. Secondly, we then find that the difference in phase of the signature factors associated with the interfering trajectories can be determined[5] from the relative strengths of the V-A correlation terms. Indeed, there are two different strength ratios which serve independently to determine this phase difference as a function of nucleon momentum transfer [see Eq. (22) below]. Although the experimental requirements here are very demanding, a unique opportunity is afforded for the testing of rather detailed features of the Regge-pole model.

In Sec. III we apply similar considerations to electroproduction of a single pion. It is noted there that the assumption of dominance by a trajectory (or trajectories) with natural parity leads to an expression for the differential cross section that contains only one unknown "form factor" that depends on only two of the five kinematic variables [see Eq. (28) below].

In Sec. IV we take up the application of Regge-like notions to multiparticle-production reactions induced by high-energy neutrinos, reactions of the sort $\nu+p\rightarrow\mu^-+p+X_1+X_2+\cdots+X_n$ $(n\geq 2)$. Even for the strong-interaction analog of multiparticle production, there is no longer a well-established set of ideas to be taken over, in particular, a lore which bears on the phases of amplitudes at high energy. For the weak semileptonic reaction it is of course true, so far as time-reversal invariance is valid, that all amplitudes must be relatively real if all final-state interactions can be ignored. Of course, these surely cannot be ignored. Suppose, however, that in the high-energy limit we restrict ourselves to the domain where the momentum transfer between the nucleons is held finite and the invariant mass of the system $\{X\}$ is also held finite, and consider the exchange of a given trajectory between the nucleon vertex and the vertex connecting the current to the system $\{X\}$. Under these conditions it seems reasonable to conjecture that the relative phase properties reflect final-state interactions *only* among the particles in $\{X\}$.

This qualitative view enables us to give several predictions that constitute checks for this uniperipheral model for multiparticle production, conjectured under the kinematical conditions just stated. These tests are described in Sec. IV. Qualitatively, their nature is to observe the existence of certain correlations which in general are nonzero, but which should vanish if averaged over certain kinematical variables, or if averaged over certain channels, or if averaged over both. All correlations in question are identically zero in electroproduction for unpolarized beams. Thus, they refer very largely to reactions induced by neutrinos, which once again appear as a vital tool to check on some aspects of

[4] For electroproduction and weak single-pion production at lower energies see S. L. Adler, Ann. Phys. (N. Y.) **50**, 189 (1968). This paper also contains detailed references to earlier contributions.

[5] For another example of a trajectory phase determination, see V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

the dynamics of hadrons. As shall be seen, the considerable experimental demands are somewhat alleviated by two circumstances: The consequences of these ideas can be verified with beams that need not be monochromatic, while, furthermore, considerable phase-space integrations can be made without washing out completely the effects in question.

The consequences of trajectory dominances in single- or multi-hadron production in neutrino reactions, discussed in this paper, are independent of the absolute values of the cross sections for the various channels. We can only hope that at least some channels will have a cross section sufficiently large to be amenable to analysis of this kind.

II. REACTION $\nu+p\rightarrow\mu^-+p+\pi^+$

A. Kinematics

Let q_1 and q_2 be, respectively, the four-momenta of incoming and outgoing leptons, p_1 and p_2 the respective momenta of incoming and outgoing nucleons, and k the momentum of the pion. It is convenient to introduce also the combinations

$$\begin{aligned} n&=q_1+q_2, \quad q=q_1-q_2,\\ P&=p_2+p_1, \quad \Delta=p_2-p_1=q-k. \end{aligned} \tag{1}$$

We shall systematically neglect the muon mass, so that $q_1^2=q_2^2=0$, whereas, with m and μ denoting, respectively, the nucleon and pion masses, $p_1^2=p_2^2=-m^2$, $k^2=-\mu^2$. Finally, let us denote by ϵ and ϵ' the respective laboratory energies of incoming and outgoing leptons, and let $\nu=\epsilon-\epsilon'$ to be the energy difference. Hence,

$$\epsilon=-q_1\cdot p_1/m, \quad \nu=-q\cdot p_1/m. \tag{2}$$

We take these as two of the five independent variables (apart from spins) that specify the reaction under discussion. Two further variables we take to be the invariant momentum transfer q^2 between the leptons, and Δ^2 between the nucleons. In terms of laboratory quantities, these are expressed by

$$\begin{aligned} q^2&=4\epsilon\epsilon'\sin^2(\tfrac{1}{2}\theta)\geq 0,\\ \Delta_2&\equiv -t=2mT_2\geq 0, \end{aligned} \tag{3}$$

where θ is the angle in the laboratory frame between the lepton three momentum vectors and T_2 is the kinetic energy of the outgoing nucleon. The fifth independent variable we take to be an azimuthal angle ϕ. In terms of laboratory quantities, it is defined explicitly by the equation

$$(\mathbf{q}\times\mathbf{n})\cdot(\mathbf{q}\times\mathbf{\Delta})=|\mathbf{q}\times\mathbf{n}|\cdot|\mathbf{q}\times\mathbf{\Delta}|\cos\phi. \tag{4}$$

We may note immediately that it is with respect to the variables ϵ and ϕ that the spectrum has a simple and explicit structure—if, as we suppose, the leptons couple locally. The other three variables enter into the hadronic matrix elements of the weak currents, and the behavior with respect to these quantities is a matter of detailed dynamics. However, for fixed values of the variables q^2 and Δ^2, the Regge model suggests the familiar asymptotic expansion appropriate to the domain where the variable ν is very large, and we shall be concerned with effects to leading orders in this variable. But let us first consider the most general structure of the spectrum with respect to dependence on the "trivial" variables ϵ and ϕ.

B. Cross-Section Structure

On the assumption that, effectively, the leptons couple locally to weak vector and axial-vector hadronic currents, we write the invariant amplitude in the form

$$\text{amp}=(G/\sqrt{2})\cos\theta_C\bar{u}(q_2)\gamma_\mu(1+\gamma_5)u(q_1)\times\langle p_2,k|V_\mu+A_\mu|p_1\rangle, \tag{5}$$

where G is the usual weak-coupling constant and θ_C the Cabibbo angle. The differential cross section, summed over final and averaged over initial spins, is given by the expression

$$d^4\sigma=\frac{G^2\cos^2\theta_C}{2(4\pi)^4}\frac{1}{\epsilon^2(q^2+\nu^2)^{1/2}}\times W(q^2,\Delta^2,\nu,\epsilon,\phi)dq^2d\Delta^2d\nu d\phi, \tag{6}$$

where

$$W=T_{\nu\mu}\tau_{\nu\mu} \tag{7}$$

and

$$\tau_{\nu\mu}=n_\mu n_\nu-q_\mu q_\nu+\delta_{\mu\nu}q^2+\epsilon_{\nu\mu\alpha\beta}n_\alpha q_\beta, \tag{8}$$

$$T_{\nu\mu}=\tfrac{1}{2}\sum_{\text{spins}}\langle p_2,k|V_\nu+A_\nu|p_1\rangle^*\langle p_2,k|V_\mu+A_\mu|p_1\rangle. \tag{9}$$

Let us write $T_{\nu\mu}=T_{\nu\mu}^{(+)}+T_{\nu\mu}^{(-)}$, where $T_{\nu\mu}^{(+)}$ refers to the sum of pure vector and pure axial-vector contributions, and $T_{\nu\mu}^{(-)}$ refers to V-A interference contributions. We define the pseudovector

$$V_\mu=\epsilon_{\mu\alpha\beta\gamma}P_\alpha\Delta_\beta q_\gamma. \tag{10}$$

It is clear that the most general structures for $T_{\nu\mu}^{(\pm)}$ are given by

$$\begin{aligned} T_{\mu\nu}^{(+)}&=A_1\delta_{\mu\nu}P_\mu P_\nu+A_3\Delta_\mu\Delta_\nu+\tfrac{1}{2}A_4(\Delta_\mu P_\nu+\Delta_\nu P_\mu)\\ &\quad+\tfrac{1}{2}iA_5(\Delta_\mu P_\nu-\Delta_\nu P_\mu),\\ T_{\mu\nu}^{(-)}&=\tfrac{1}{2}B_1(V_\mu P_\nu-V_\nu P_\mu)+\tfrac{1}{2}iB_2(V_\mu P_\nu+V_\nu P_\mu)\\ &\quad+\tfrac{1}{2}B_3(V_\mu\Delta_\nu-V_\nu\Delta_\mu)+\tfrac{1}{2}iB_4(V_\mu\Delta_\nu+V_\nu\Delta_\mu), \end{aligned} \tag{11}$$

where the A_i and B_i are real scalar functions of the variables q^2, Δ^2, and ν.

We are dealing here with a total of nine distinctive terms, five of them coming from $T_{\nu\mu}^{(+)}$, four from the V-A interference tensor $T_{\nu\mu}^{(-)}$. For the spectral function W, this leads to a corresponding decomposition into nine terms with distinctive and explicit dependence on the variables ϵ and ϕ. With

$$W=W^{(+)}+W^{(-)}, \tag{12}$$

we find

$$W^{(+)}=\alpha_1+\alpha_2\frac{\epsilon(\epsilon-\nu)}{\nu^2}+\alpha_3\frac{(2\epsilon-\nu)}{\nu}\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)^{1/2}\times\cos\phi+\alpha_4\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)\cos2\phi+\alpha_5\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)^{1/2}\sin\phi, \quad (13)$$

$$W^{(-)}=\beta_1\frac{2\epsilon-\nu}{\nu}+\beta_2\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)^{1/2}\cos\phi+\beta_3\frac{2\epsilon-\nu}{\nu}\times\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)^{1/2}\sin\phi+\beta_4\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}-\frac{q^2}{4\nu^2}\right)\sin2\phi, \quad (14)$$

where the α_i and β_i are linear combinations, respectively, of the A_i and B_i and depend only on the variables q^2, Δ^2, and ν.

The structure of the spectrum is at this point fully general and has interesting qualitative features in its own right. The odd correlation term proportional to $\sin2\phi$, for example, uniquely arises from *V-A* interference, irrespective of its dependence on ϵ and on the other variables ν, Δ^2, and q^2. The relative importance of *V-A* interference can thus be gauged from this term, with statistical economy, by integrating over ν, Δ^2, and q^2 and, for that matter, integrating appropriately also over the neutrino energy ϵ for the realistic circumstance of nonmonochromatic beams.

C. Regge-Pole Analysis

Let us now focus on the asymptotic properties of weak pion production at high energies, where we may appeal to the Regge-pole model for guidance. The kinematic domain in question corresponds to $\epsilon>\nu\gg m$ and $q^2\ll 2m\nu$, $\Delta^2\ll 2m\nu$. We are concerned here with leading effects in the energy-transfer parameter ν.

According to the standard classification, the dominant contributions arise from Pomeranchuk exchange. This is a trajectory with even *G* parity (and zero isospin), and it therefore contributes solely to the axial-vector matrix element. For the vector current, exchange of odd-*G*-parity trajectories (with $I=0$ or 1) is required and, for small Δ^2, we suppose that all such trajectories lie below the Pomeranchuk trajectory. The spectral function $W^{(+)}$ is therefore dominated to leading order in ν by Pomeranchuk exchange. Although it is of lower order in ν, we are also interested in the spectral function $W^{(-)}$. This is dominated by interference of the Pomeranchuk trajectory with the leading odd-*G*-parity trajectory having other appropriate quantum numbers to be specified below.

1. Axial-Vector Amplitude

The matrix element $\langle p_2k|A_\mu|p_1\rangle$ decomposes in a sum of eight distinct covariants, whose coefficients are scalar functions of the variables q^2, Δ^2, and ν. But two of the eight terms vanish on contraction with the lepton current $\bar{u}(q_2)\gamma_\mu(1+\gamma_5)u(q_1)$ when the muon mass is ignored. This much is general. However, one further term is eliminated on specialization to Pomeranchuk exchange—indeed, to exchange of any natural-parity trajectory. For the five surviving scalar amplitudes, the leading behavior in the energy variables ν is specified in the standard way by the Pomeranchuk trajectory function $\alpha_P(\Delta^2)$. With

$$\langle p_2,k|A_\mu|p_1\rangle=i\bar{u}(p_2)M_\mu{}^Au(p_1),$$

we find for large ν the structure

$$M_\mu{}^A=\left(\frac{\nu}{\nu_0}\right)^{\alpha_P(\Delta^2)}\left[a_1\Delta_\mu\gamma\cdot P+\left(\frac{\nu_0}{\nu}\right)\times[m^2a_2\gamma_\mu+a_3\Delta_\mu\gamma\cdot q+a_4P_\mu\gamma\cdot P]+\left(\frac{\nu_0}{\nu}\right)^2a_5P_\mu\gamma\cdot P\right], \quad (15)$$

where ν_0 is a scale factor and the a_i are complex scalar functions of q^2 and Δ^2.

It is a straightforward matter now to compute the Pomeranchuk contribution to the coefficients α_i in the spectral function $W^{(+)}$ of Eq. (13). To leading order in ν, we find that all the α_i display the same energy dependence; for fixed q^2 and Δ^2,

$$\text{all } \alpha_i\sim\nu^{2\alpha_P}, \quad (16)$$

with the further restriction, to this order, that

$$\alpha_4=2\alpha_1. \quad (17)$$

What is at issue, so far, is the natural-parity character of the leading (Pomeranchuk) trajectory and the energy-growth features of Regge theory. With only this much input, we see that for large values of the energy-transfer variable ν, $\nu\lesssim\epsilon$, all of the four independent terms [see Eq. (17)] in $W^{(+)}$ are formally of comparable size, including the odd correlation term proportional to $\sin\phi$. This odd correlation term, however, is of special interest here in connection with another aspect of Regge-pole theory: the feature that all of the helicity amplitudes [equivalently, all of the scalar amplitudes a_i in Eq. (15)] share a common phase in the high-energy limit. It may well be that this feature transcends the more specific details of the Regge-pole model that we have been employing so far and which we may wish to regard as only providing a guide to the relative sizes of the various correlation terms at high energies. In any event, on standard Regge-pole notions, the common phase of the a_i is determined by the Pomeranchuk signature factor $e^{-\frac{1}{2}\pi\alpha_P}/\sin\frac{1}{2}\pi\alpha_P$, and, as can be anticipated, the coefficients α_5 of the odd correlation term in $W^{(+)}$ vanish if the amplitudes a_i all have the same phase:

$$\alpha_5/\alpha_{i\neq5}\to 0, \quad \nu\to\infty\,. \tag{18}$$

2. *Vector Amplitude*

We turn now to the vector matrix element $\langle p_2,k|V_\mu|p_1\rangle$. This is of interest not for its contribution to $W^{(+)}$, which we are presuming to be dominated at high energies by the axial-vector current and Pomeranchuk exchange, but for its contribution to $W^{(-)}$ via interference with the axial-vector amplitude.

For the vector matrix element, we require trajectories with the quantum numbers G parity $=-1$, isospin $I=0$ or 1. It is also necessary, however, to consider separately the two spin-parity possibilities, i.e., natural or unnatural spin-parity. In interference with the (natural-parity) Pomeranchuk trajectory, it turns out that a natural-parity trajectory contributing to the vector current amplitude gives terms in $W^{(-)}$ which are larger by one whole power of ν than an unnatural-parity trajectory with comparable trajectory function. For the vector current matrix element we therefore focus on the leading trajectory with natural spin-parity. On present evidence, we can perhaps identify this with the ω-meson trajectory. At any rate, if only for notational purposes, we shall denote the trajectory function by $\alpha_\omega(\Delta^2)$.

The matrix element $\langle p_2,k|V_\mu|p_1\rangle$ decomposes, most generally, into a sum of six distinct covariants, whose coefficients are scalar functions of q^2, Δ^2, and ν. But on specialization to natural-parity exchange we find that only three terms survive. Again, we estimate their high-energy dependence on the variable ν according to the standard rules. Define

$$v_{\mu\nu}=\epsilon_{\mu\nu\alpha\beta}\Delta_\alpha q_\beta\,, \tag{19}$$

and let

$$\langle p_2,k|V_\mu|p_1\rangle=\bar{u}(p_2)M_\mu{}^V u(p_1)\,.$$

We find

$$M_\mu{}^V=\left(\frac{\nu}{\nu_0}\right)^{\alpha_\omega(\Delta^2)-1}\times\left[b_1 v_{\mu\nu}\gamma_\nu+b_2V_\mu\gamma\cdot P+\left(\frac{\nu_0}{\nu}\right)b_3V_\mu\gamma\cdot q\right], \tag{20}$$

where V_μ has been defined in Eq. (10) and where the b_i are complex scalar functions of q^2 and Δ^2,

The V-A interference function $W^{(-)}$ is now readily computed, and one finds that all of the β_i in Eq. (14) display the same high-energy behavior for fixed q^2 and Δ^2:

$$\text{all } \beta_i\sim\nu^{\alpha_P+\alpha_\omega}. \tag{21}$$

Moreover, if all of the amplitudes b_i in Eq. (20) have a common phase (determined by the signature factor of the ω trajectory) and if, as we have already assumed, all of the amplitudes a_i in Eq. (15) have a common phase (determined by the Pomeranchuk signature factor), then we find the remarkable result that

$$\beta_3/\beta_2=\tfrac{1}{2}(\beta_4/\beta_1)=\tan\{\tfrac{1}{2}\pi[\alpha_P(\Delta^2)=\alpha_\omega(\Delta^2)+1]\}, \quad \nu\to\infty\,; \tag{22}$$

i.e., the phase difference can be determined in two independent ways from ratios of the V-A correlation functions. Note that the factor $\alpha_P-\alpha_\omega+1$ refers specifically to the case that the trajectory dominating the vector amplitude has odd signature (as for the ω-meson trajectory). If the trajectory in question were to have even signature, one should drop the $+1$ term.

It is a critical test of Regge-pole theory that these two ratios must agree with each other and must furthermore depend only on the nucleon momentum transfer variable Δ^2. We are, of course, always restricting discussion to the Regge domain where ν is large, Δ^2 and q^2 small.

3. *Comments*

The key results for weak pion production are contained in Eqs. (16)–(18), (21), and (22). The specific form of the energy dependence implied by Eqs. (16) and (21) rests on the standard assumptions associated with exchange of Regge *poles* and would, for example, be altered in detail if the J-plane singularities involved Regge cuts. On the other hand, Eq. (17), which holds to order $1/\nu$, would not be affected to leading order by mild alterations in the energy-dependence estimates. However, this equation rests essentially on the natural-parity character of the leading (Pomeranchuk) J-plane singularity. As for Eq. (18), the only essential ingredient is the assumption, consistent with Regge-pole notions but perhaps of more general character, that the leading amplitudes all share a common phase at high energy. The relevance here of our more detailed appeal to Regge-pole analysis, embodied in Eq. (16), is that the coefficient α_5 might well be comparable in size to the remaining α_1 in the absence of this assumption on phases. Similarly, the left-hand equality of Eq. (22) rests essentially on the assumption that the leading axial-vector amplitudes all share a common phase and again that the leading vector amplitudes share a common phase; this, together with the assumption that the energy-dependence estimates provided by standard Regge-pole analysis are at least roughly right (to within corrections of order $1/\nu$). The right-hand equality of Eq. (22) goes farther, of course, in asserting that the ratios depend only on the variable Δ^2. This is more peculiarly a feature of Regge theory. At this level of detail, the phase difference $\alpha_P-\alpha_\omega$ can moreover be independently determined on the basis of trajectory information obtainable from the exponents in Eqs. (16) and (21), or from Regge-pole analysis of purely strong reactions.

The various issues discussed here will involve varying degrees of difficulty for their experimental testing, nothing being especially trivial. Some modest amelioration can, however, be noted. For none of the matters that we have focused on does the q^2 dependence of amplitudes come into question, although for other purposes this is an important issue. Here it is only necessary that we be in the asymptotic domain $\epsilon\gtrsim\nu\gg m$, $q^2\ll 2m\nu$, $\Delta^2\ll 2m\nu$. One can therefore integrate the spectrum in q^2 over some extensive fixed domain, with upper limit small compared to the minimum values of $2m\nu$ to be considered. If it should turn out that the spectrum falls rapidly with q^2, one could in fact integrate over all values of this variable.

Now in connection with Eqs. (17), (18), and the left-hand equality of Eq. (22), the dependence of correlation functions on the variable Δ^2 is also not in question. Thus here one can likewise integrate over some fixed domain in Δ^2, with upper limit small compared to $2m\nu_{\min}$. If, as we may anticipate, the correlation functions fall rapidly with Δ^2, it would in fact be safe to integrate over all values of Δ^2. We then consider the cross-section expression of Eq. (6) integrated over q^2 and Δ^2, and notice that $(q^2+\nu^2)^{1/2}\sim\nu$ for $\nu\gg m$, $q^2\ll 2m\nu$. The integrated expression is

$$d^2\sigma=\frac{G^2\cos^2\theta_C}{2(4\pi)^4}\frac{1}{\epsilon^2\nu}\tilde{W}(\epsilon,\nu,\phi)d\nu d\phi\,, \tag{23}$$

where $\tilde{W}$ has the decomposition of Eqs. (12)–(14), with α_i, $\beta_i\rightarrow\tilde{\alpha}_i(\nu)$, $\tilde{\beta}_i(\nu)$, and where Eqs. (17), (18), and the left-hand equality of Eq. (22) continue to hold for the tilde qualities. Indeed, these equalities are supposed to hold independently of the value of ν, provided it is sufficiently large, suggesting that a further integration is possible. However, as we see from Eqs. (13) and (14), to disentangle the $\tilde{\alpha}_3$ from $\tilde{\beta}_2$, $\tilde{\alpha}_5$ from $\tilde{\beta}_2$, and $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\tilde{\beta}_1$ from one another, it is necessary to study the cross section in its dependence on all three variables ν, ϵ, and ϕ. The experimental demands are therefore still considerable, but perhaps not hopeless.

The simplest situation, qualitatively, is presented by Eq. (18). Both $W^{(+)}$ and $W^{(-)}$ contain a correlation proportional to $\sin\phi$. The coefficients depend differently on the energies ϵ and ν, however, and for $\nu\approx\epsilon$, with ϵ very large, the $W^{(+)}$ correlations predominate. Here one can afford to integrate the spectrum over a range of values ν in the vicinity of ϵ, and even over a modest spread of neutrino energies in the case of nonmonochromatic beams. The question, for large energies, is whether a $\sin\phi$ correlation survives, with strength comparable to that of the other α-type correlations.

Most demanding experimentally is the right-hand equality of Eq. (22), which tests some rather detailed features of Regge-pole theory. Here it is necessary to retain the nucleon momentum transfer Δ^2 as a spectrum variable.

Obviously, the preceding discussion applies equally well to th e corresponding antineutrino reactions. In stead of the five α_i and four β_i of Eqs. (13) and (14), we now have a corresponding set $\bar{\alpha}_i$ and $\bar{\beta}_i$. The specialization to Pomeranchuk exchange as the leading mechanism to generate $W^{(+)}$ and $\bar{W}^{(+)}$ relates these two distributions in a simple way in the high-ν limit:

$$\bar{\alpha}_i=\eta_i\alpha_i;\quad \eta_i=1 \quad\text{if}\quad i\neq 5$$
$$=-1 \quad\text{if}\quad i=5. \tag{24}$$

Of course, all that is needed for this result is the fact that the Pomeranchuk is isospin pure. Furthermore, if the $W^{(-)}$ and $\bar{W}^{(-)}$ distributions come about through interference of the Pomeranchuk-dominated axial-vector amplitude with an isospin-pure-dominated vector amplitude, then

$$\bar{\beta}_i=\zeta\cdot\beta_i;\quad \zeta_i=\mp 1 \quad\text{for}\quad i=1,2$$
$$=\pm 1 \quad\text{for}\quad i=3,4, \tag{25}$$

where the upper (lower) sign holds for dominance by $I=0$ (1).

We finally note that the combination of Eqs. (24) and (18) yields

$$d^4\sigma(\nu p\rightarrow\mu^-p\pi^+)=d^4\sigma(\bar{\nu}p\rightarrow\mu^+p\pi^-),\quad \nu\rightarrow\infty\,. \tag{26}$$

III. PION ELECTROPRODUCTION

Closely related to the weak pion production reaction which we have been discussing are the pion electroproduction processes

$$\binom{e}{\mu}+p\rightarrow\binom{e}{\mu}+p+\pi^0,$$
$$\binom{e}{\mu}+p\rightarrow\binom{e}{\mu}+n+\pi^+,\text{etc.}$$

The general structure of the cross section is now (for $q^2\ll\nu^2$)

$$d^4\sigma=\frac{e^4}{(4\pi)^4}\frac{1}{q^4\epsilon^2(q^2+\nu^2)^{1/2}}\times\left[\gamma_1+\gamma_2\frac{\epsilon(\epsilon-\nu)}{\nu^2}+\gamma_3\frac{2\epsilon-\nu}{\nu}\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}\right)^{1/2}\cos\phi+\gamma_4\frac{\epsilon(\epsilon-\nu)}{\nu^2}\cos2\phi+\gamma_5\left(\frac{\epsilon(\epsilon-\nu)}{\nu^2}\right)^{1/2}\sin\phi\right]\times dq^2d\Delta^2d\nu d\phi\,, \tag{27}$$

where, again, $\gamma_i=\gamma_i(\Delta^2,q^2,\nu)$. Of course, for unpolarized beams, the coefficient γ_5 also vanishes identically. On the other hand, for longitudinally polarized muon beams, the coefficient γ_5 can make an appearance and, indeed, for full longitudinal polarization the kinematic analysis is exactly the same as for the weak-production case (of course with $\beta_i=0$).

In our discussion of weak pion production we did not write out the contribution to $W^{(+)}$ coming from the vector current, since we presumed $W^{(+)}$ to be dominated at large energies by the axial-vector contribution. Here, for the vector current we are concerned with trajectories with $G=+1$, $I=1$ (for the isoscalar part of the electromagnetic current), and $G=-1$, $I=0$ or 1 (for the isovector current). Known candidates for such leading trajectories (e.g., ω, ρ) have natural spin-parity character. If this is indeed the correct characterization, then the structure of the matrix element is as in Eq. (20). Without more detailed commitment to the identification of the leading trajectory, we shall denote the leading-trajectory function by $\alpha_V(\Delta^2)$. For $d^4\sigma$, we then find in the high-energy limit

$$d^4\sigma=\frac{e^4 f(q^2,\Delta^2)}{(4\pi)^4 q^4\epsilon^2(q^2+\nu^2)^{1/2}}\left(\frac{\nu}{\nu_0}\right)^{2\alpha_V(\Delta^2)} \times\left(1+\frac{4\epsilon(\epsilon-\nu)}{\nu^2}\sin^2\phi\right)dq^2d\Delta^2d\nu d\phi\,, \quad (28)$$

where $f(q^2,\Delta^2)$ is a scalar function expressible in terms of the b_i of Eq. (20). In the notation of Eq. (27), $\gamma_4=-\gamma_2=-2\gamma_1$, $\gamma_3=\gamma_5=0$. The absence of a $\cos\phi$ correlation ($\gamma_3=0$) in the high-energy limit should perhaps not be too difficult to test. For unpolarized lepton beams, the vanishing of γ_5 is, of course, an exact statement, but for longitudinally polarized (muon) beams, the (relative) vanishing of γ_5 in the high-energy limit is still automatic here, independent of phase assumptions. The structure of Eq. (28) rests essentially on the assumed natural-parity character of the leading trajectory. If, instead, the leading trajectory has unnatural spin-parity, we would find again the results of Eqs. (16) and (17), of course with $\alpha_P\to\alpha_V$, and the further assumption of common phases would again lead to

$$\gamma_5\to 0\,,\quad \nu\to\infty$$

in experiments with longitudinally polarized μ beams.

IV. HIGHER-MULTIPLICITY NEUTRINO PROCESSES

With increasing multiplicity, whether for strong, electromagnetic, or weak processes, multiparticle production reactions rapidly become too complex for fully detailed experimental analysis. (Detailed theoretical analysis of course fails at a much earlier level.) In the deep-inelastic region it is therefore natural to characterize happenings in cruder ways. For high-energy electroproduction and neutrino-induced processes, in particular, attention has fastened on the study of cross sections summed over all hadron channels open at a given invariant mass. At issue here is the behavior of the cross sections in their dependence on the lepton momentum-transfer and energy-transfer variables.

We discuss here a potentially useful next step into the details. Emphasis is placed on some speculative generalizations of the Regge notions that figured in Sec. III. Let us first consider, in the example of neutrino-induced processes, the production of a particular set of final-state hadrons:

$$\nu+p\to\mu^-\mp p+X_1+X_2+\cdots\,, \quad (29)$$

where, to enhance applicability of Regge-dominance notions, we insist on a nucleon in the final state and restrict ourselves to the domain of small momentum transfer Δ^2 between the nucleons.

In our earlier example of weak pion production, where the system $\{X\}$ consisted of a single particle, the reaction was specified, apart from spins, by the five variables ϵ, ϕ, ν, Δ^2, and q^2. With each additional particle in $\{X\}$, three new variables arise and matters soon get out of hand. At any rate, one of the new variables can be taken to be the invariant mass M of the system $\{X\}$, and the others can be chosen, in an obvious sense, as internal to the system $\{X\}$. Let us now consider the differential cross section for reaction (29) summed over all the internal variables, so that at given neutrino energy ϵ it is a function of ν, Δ^2, q^2, M^2, and ϕ. The structure is the same as given in Eqs. (6) and (12)–(14), except that the correlation functions α_i and β_i now depend also on M^2:

$$d^5\sigma=[G^2/\epsilon^2(q^2+\nu^2)^{1/2}]W(q^2,\Delta^2,\nu,M^2,\epsilon,\phi) \times dq^2d\Delta^2d\nu dM^2d\phi\,, \quad (30)$$

where the structure of W with respect to dependence on the variables ϵ and ϕ is given by Eqs. (12)–(14), with α_i and β_i now being functions of q^2, Δ^2, ν, and M^2. At this stage, the structure is general, resting only on the assumption that the leptons couple locally to vector and axial-vector hadron currents. It is clear that the cross section $d\sigma$ and the various correlation functions α_i and β_i should carry a label specifying the particular channel under consideration, but we shall allow this to remain implicit.

Once again there are useful qualitative features to be noticed at this general level. The angular correlation term $\sin 2\phi$, irrespective of the dependence of its coefficient on the other variables, arises uniquely from V-A interference. Thus, even upon integration over the other final-state variables, it can serve to gauge the importance of V-A interference as a function of neutrino energy for the particular channel under consideration.

A. Regge Matters

The formulation of Regge notions for multiparticle production reactions is very much in an unsettled state. The fashionable multiperipheral model for strong reactions focuses on the domain where all subenergies are large and all momentum transfers small, and one describes the amplitude here in terms of strings of exchanged trajectories. In the context of neutrino reactions, let us in contrast consider the domain where the energy ν is large, but the mass M of the system $\{X\}$ is small, as are also the nucleon and lepton momentum-transfer variables Δ^2 and q^2. This more closely resembles the standard situation where we can imagine a description in terms of Regge trajectories, exchanged singly, between the nucleon vertex and a vertex connecting the current to the system $\{X\}$. For Regge purposes, this is as if the system $\{X\}$ were regarded, at given mass M, as a superposition of single-particle states, distinguished by total angular momentum and other internal variables.

To fix ideas, let us first consider the example where $\{X\}$ is a two-pion system, and assume initially that M is below the threshold for inelastic π-π reactions. Suppose then that a partial-wave decomposition is made in angular momentum of the two-pion system, and consider, for a particular partial wave, the amplitudes corresponding to exchange of a given trajectory. If Regge ideas are to have any applicability, it seems natural to suppose that the amplitude has the standard structure corresponding to the spin and other quantum numbers of the two-pion state in question and corresponding also to the quantum numbers of the trajectory being exchanged, and, moreover, that the various helicity amplitudes to this state all share a common phase. Here, however, we expect this phase to be determined not only by the signature factor of the trajectory, but also by the scattering phase for the two-pion state in question. On this view, the variation of phase from one partial wave to another reflects final-state interactions solely among the pions, as if the outgoing nucleon were noninteracting in the final state.

Let us now see, on this example, what the experimental implications are. We imagine that the energy ν is sufficiently large so that only the leading trajectory need be considered. As it happens here, this means that the vector current amplitudes will predominate over the axial-vector current amplitudes so that all V-A interference effects in the spectrum must be absent in leading order. This is obvious enough. In addition, we are concerned with the question whether odd (i.e., quasi-time-reversal-violating) correlations can survive in leading order. In the presence of final-state interactions such correlations can indeed appear in the full spectrum, unintegrated over internal variables. (In the present example these internal variables are the two angles describing the orientation, in the dipion rest frame, of the relative momentum vector of the pions.) Upon integrating over internal variables, however, we are in effect summing over a complete set of final states connected by strong interactions, and odd correlations must disappear in leading order [Eqs. (18) and (13)].

To recapitulate: Let us focus on two classes of correlations in the unintegrated spectrum, namely, terms arising from V-A interference, denoted by γ_i^{V-A}, and terms $\gamma_i^{\rm odd}$ which are odd under reversal of all momenta. These are partly overlapping sets. Upon integration over internal variables they reduce, respectively, to the sets $(\beta_1,\beta_2,\beta_3,\beta_4)$ and $(\alpha_5,\beta_3,\beta_4)$ of Eqs. (13) and (14). Then, to leading order in ν:

Example 1. $\{X\}=2\pi$, M below inelastic threshold:

$$\gamma_i^{V,A}=0,\quad \gamma_i^{\rm odd}\ \text{generally} \neq 0,\quad \text{unintegrated spectrum,} \tag{31}$$

$$\beta_i=0,\quad \alpha_5=0,\quad \text{spectrum integrated over internal variables.} \tag{32}$$

By the same argument that led to Eq. (26), it follows from Eq. (32) that

$$d^5\sigma(\nu p\to p\mu^-\{\pi^+\pi^0\}_{\rm int}) = d^5\sigma(\bar\nu p\to p\mu^+\{\pi^-\pi^0\}_{\rm int}),\quad \nu\to\infty \tag{33}$$

where the subscript "int" means that the necessary integrations over the internal variables have been performed (for fixed M).

The conjectures illustrated above on a simple example may now be generalized to arbitrary multihadron systems $\{X\}$, whether above or below inelastic thresholds. Under the "Regge conditions" where M, Δ^2, and q^2 are all held finite as the energy transfer ν grows large, the amplitudes arising from exchange of a given trajectory all have a common phase factor determined by the trajectory signature, and the phases otherwise vary with internal variables of the system X in accordance with standard final-state theorems—as if, however, the outgoing nucleon were noninteracting. At large enough energies, where only the single leading trajectory need be considered, the phase arising from the signature factor is irrelevant in the squared amplitude, and all questions of phase in this limit depend solely on these final-state interaction effects. The dominant trajectory, we continue to suppose, is the Pomeranchuk ($G=+1$, $I=0$).

B. Implications

(a) An immediately obvious implication of the assumption that a single trajectory of definite G parity predominates at large energies is that all V-A interference effects vanish for systems $\{X\}$ composed solely of particles of definite G parity. Thus, for a system composed of an even number of pions, the Pomeranchuk contributes only to the vector current amplitudes; in the case of an odd number of pions, only to the axial-vector current amplitudes. Hence:

Example 2. $\{X\}=\{n\pi\}$. To leading order in ν:

$$\gamma_i^{V,A}=0,\quad \text{unintegrated spectrum.} \tag{34}$$

In contrast, for a system such as $K^+\bar{K}^0$, which represents a superposition of even- and odd-G-parity states, the Pomeranchuk can contribute to both the V and A amplitudes. However, if the spectrum is integrated over internal variables, all interference effects between the states of opposite G parity must vanish, as must also therefore all V-A correlations. Therefore:

Example 3. $\{X\}=\{K^+\bar{K}^0\}$. To leading order in ν:

$$\begin{aligned}\gamma_i^{V,A}&\neq 0, \quad \text{unintegrated spectrum;}\\ \beta_i&=0, \quad \text{spectrum integrated over internal variables.}\end{aligned} \tag{35}$$

Finally, the most general situation is illustrated in the example where we consider the three systems $\{K^+\bar{K}^0\pi^0\}$, $\{K^0\bar{K}^0\pi^+\}$, and $\{K^+K^-\pi^+\}$. For any one of the channels, V-A interference effects can survive in leading order even after integration over all internal variables. But on summing the differential cross section over all three channels one clearly eliminates interference between states of opposite G parity, and hence also all V-A interference correlations. Hence:

Example 4. $\{X\}=\{K\bar{K}\pi\}^{(A)}$, where (A) labels the (three) possible charge configurations. To leading order in ν, for given A:

$$\beta_i^{(A)}\neq 0 \tag{36}$$

$$\sum_{A=1}^{3}\beta_i^{(A)}=0 \quad \text{spectrum integrated over internal variables.} \tag{37}$$

In general, summation over all channels that differ only by the charge labels of the final-state particles in $\{X\}$ serves to eliminate V-A interference effects to leading order.

(b) For a given channel $\{X\}$, integration over all internal variables produces a reduced spectrum [Eqs. (12)–(14)] which contains the odd correlation coefficients α_5, β_3, and β_4. These last two arise from V-A interference and vanish in leading order (along with β_1 and β_2) when one sums over a related set of channels, as just described. At this level the odd correlation term α_5 may still survive.

However, from our hypothesis on the effective nonparticipation of the outgoing nucleon in final-state interactions, it now follows that α_5 must vanish in leading order when one sums over a complete set of channels $\{X\}$ connected by strong interactions. To leading order in ν:

$$\sum_{\text{all channels}}\beta_i=0 \tag{38}$$

$$\sum_{\text{all channels}}\alpha_5=0 \quad \text{spectrum integrated over internal variables,} \tag{39}$$

so that[6] [cf. Eqs. (26) and (33)]

$$d^5{}_{\text{tot}}\sigma(\nu p\to\mu^- p\{X\}) = d^5{}_{\text{tot}}\sigma(\bar{\nu}p\to\mu^+ p\{\bar{X}\}), \quad \nu\to\infty\,. \tag{40}$$

It is of course permissible, and experimentally most convenient, to sum over *all* systems $\{X\}$, but the result is supposed to hold separately for sums over zero-strangeness and unit strangeness systems. (In still greater refinement, for mass M below the $K\bar{K}$ threshold it should also hold separately for sums over systems with even and odd numbers of pions.) For statistical economy, it is also permissible to integrate in the variables Δ^2, q^2, M^2, and ν over a range restricted by $\Delta^2\ll 2m\nu$, $q^2\ll 2m\nu$, and ν "comparable" to the neutrino energy ϵ; and, in fact, strict neutrino-beam nonmonochromaticity is not crucial.

Of course, Eq. (39) is also of value in testing single-trajectory dominance in electroproduction and μ production with longitudinally polarized beams.

It must be reemphasized, in conclusion, that the qualitative ideas discussed in this subsection do not rest on the detailed structure of the Regge model, but rather, for (a), on the presumed dominance at high energies of exchange of definite G parity, and for (b) on the notion that the relative phases of amplitudes reflect final-state interactions in which the outgoing nucleon does not effectively participate—under the kinematic restrictions considered (q^2, Δ^2, and M^2 all held finite as the energy ν grows very large). Confirmation would be most persuasive if it were found that the odd correlation term α_5 survives at high energy for two or more individual channels, but vanishes, to leading order, on summation over channels.

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[6] The subscript "tot" means that all summations and integrations stipulated in Eqs. (38) and (39) have been performed.