the ϵ isoscalar meson[20] to the isospin-even amplitudes; however, since the a^+ and b^+ results of Raman which neglected the contributions of the ϵ meson were in excellent agreement with experiment, the contribution is probably negligible and was omitted in our calculation. As can be seen from Table I, the results are in agreement with experiment except perhaps for the a_{1-}^- scattering length. The difficulty which Schnitzer had with b_{0+}^- has been overcome, while a_{1-}^- is about the same as Schnitzer's calculation but is an improvement over Raman's value. We note that the σ term mentioned in Sec. I would not affect our calculation if we adopt the usual symmetric form[3] for it. It would then contribute only to the isospin-even amplitudes.

[20] B. Dutta-Roy, I. Lapidus, and M. Tausner, Phys. Rev. **177**, 2529 (1969).

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Nucleon-Nucleon T-Violating Force from Electromagnetic Interaction

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A T-violating nucleon-nucleon potential is calculated with a model in which the basic violation is in the electromagnetic interaction. The conditions of Hermiticity and current conservation are applied to choose a T-violating matrix element of the nucleon electromagnetic current. Since current conservation requires that the T-violating term vanish on the mass shell, the longest-range force comes from diagrams corresponding to the exchange of one photon and one pion. In leading nonrelativistic order, there are three spin-dependent T-violating forces of range roughly equal to the Compton wavelength of the pion and of "strength" about 0.01% of the one-pion-exchange potential.

I. INTRODUCTION

In 1964 Christenson *et al.*[1] found evidence for the 2π CP-violating decay of the K_2^0 meson. If the TCP theorem is valid, as is generally assumed, this experiment implies that T must also be violated. Various suggestions have been made that the violation is in the weak interaction,[2] in a "superweak" interaction,[3] or in the electromagnetic interaction.[4–6] Extensive experiments, especially with K mesons, have not been able so far to distinguish which of these theories is correct. Indeed, experiments involving searches for transverse polarization, polarized γ correlations, polarization-asymmetry inequality, static electric dipole moments, and violation of reciprocity in scattering have so far produced no direct evidence for T violation.

One of the more elegant theories of T violation is that of Lee and co-workers,[4–6] who propose that the violation is in the electromagnetic interaction. One basis for this suggestion is the fact that the ratio of the rate of the T-violating K decay to the normal decay is of the order of $\alpha/2\pi$. In Lee's theory, each fundamental interaction is separately invariant under its own T, C, and P operators. The product operator TCP is the same for every interaction. Violations can arise when the operator of one interaction is defined differently from that of another interaction. Thus, T (and C) violation arises because of the mismatch of the C operators for the strong (C_{strong}) and electromagnetic (C_γ) interactions, just as P is violated because of the mismatch of the parity operators of the weak (P_{weak}) and electromagnetic (P_γ) interactions. The mismatch of the operators could arise because of the existence of hypothetical "a" particles[5] which satisfy

$$C_\gamma a^+ C_\gamma^{-1} = a^-,$$

but

$$C_{\text{strong}} a^+ C^{-1}_{\text{strong}} = a^+,$$

or from the minimal electromagnetic interaction of a system of vector mesons.[6]

This electromagnetic violation of T will manifest itself in nuclear forces by time-reversal-violating (TRV)

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turley, Phys. Rev. Letters **13**, 138 (1964).

[2] R. G. Sachs, Phys. Rev. Letters **13**, 286 (1964); see also L. B. Okun, Usp. Fiz. Nauk **89**, 603 (1966) [English transl.: Soviet Phys.—Usp. **9**, 574 (1967)], for a list of references on theories of CP violation.

[3] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

[4] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[5] T. D. Lee, Phys. Rev. **140**, B959 (1965).

[6] T. D. Lee, Phys. Rev. **140**, B967 (1965).

contributions to the electromagnetic current of the nucleon. If both the initial and final nucleons are on the mass shell, the matrix element of the current reduces to the form[4] ($\hbar=c=1$; the Dirac, relativistic conventions, and gauge are those used by Bjorken and Drell[7])

$$\langle p'|J^\mu|p\rangle=\bar{u}(p')[F_1(q^2)\gamma^\mu+iF_2(q^2)\sigma^{\mu\nu}q_\nu +iF_3(q^2)q^\mu]u(p)\,, \quad (1)$$

where $q^\mu=p'^\mu-p^\mu$. Each form factor F is understood to contain an isoscalar and isovector part:

$$F_j(q^2)=F_j^{(s)}(q^2)+F_j^{(v)}(q^2)\tau^{(3)}\,. \quad (2)$$

The first two terms of Eq. (1) are the well-known charge and anomalous magnetic moment matrix elements, and the third term violates T, since $T\mathbf{q}T^{-1}=+\mathbf{q}$ but $T\mathbf{J}T^{-1}=-\mathbf{J}$. However, the condition of current conservation requires $q_\mu J^\mu=0$ or $F_3(q^2)=0$. Therefore, there is no long-range TRV force from such obvious graphs as one-photon exchange. This point has been noted by Bernstein *et al.*[4] If the photon connects to states off the mass shell, both the F_3 term and others that violate T may appear. Then the lowest-order TRV nucleon-nucleon force will arise from graphs like photon-meson exchange (Fig. 1), with a TRV vertex (shown by $\otimes$ in Fig. 1), and the force will be longest-range if the mediating particle represented by the dashed line is a pion.

It is the longest-range TRV force that is likely to have the strongest effect in nuclear experiments, since[8] the mean separation of nucleons in the nucleus is $\sim$2 F, which is even larger than the range of the one-pion-exchange potential (OPEP). Furthermore, hard-core effects tend to keep the nucleons from approaching more closely than $\sim$0.5 F. Therefore, in this paper we derive the one-pion–one-photon–exchange TRV nucleon-nucleon force based on the model of a chargeless C-even current[4,5] for use in analyzing nuclear tests of T invariance. In the following paper, we apply it to direct-reaction reciprocity tests.

The derivation is performed by calculating the matrix elements corresponding to the Feynman graphs of Fig. 1. Although perturbation theory is not valid for nuclear forces, it may be valid for the long-range (low-momentum-transfer) part of the forces. Thus we cannot at present calculate the pion-nucleon interaction and do not even know whether it can be described in a local Lagrangian theory. However, it is known that the nucleon-nucleon force beyond about $\mu^{-1}=1.4$ F (μ is the mass of pion) is described fairly well by the pseudoscalar coupling[7]

$$\mathcal{H}_{\rm int}=ig\bar{\psi}\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi\,,$$

where $\boldsymbol{\tau}$ is the isospin operator of the nucleons and $\boldsymbol{\phi}$ is the π-meson field operator. This coupling has been used to calculate one- and two-pion exchange forces[9] and is employed here. The resulting TRV force should be approximately correct beyond the range μ^{-1}.

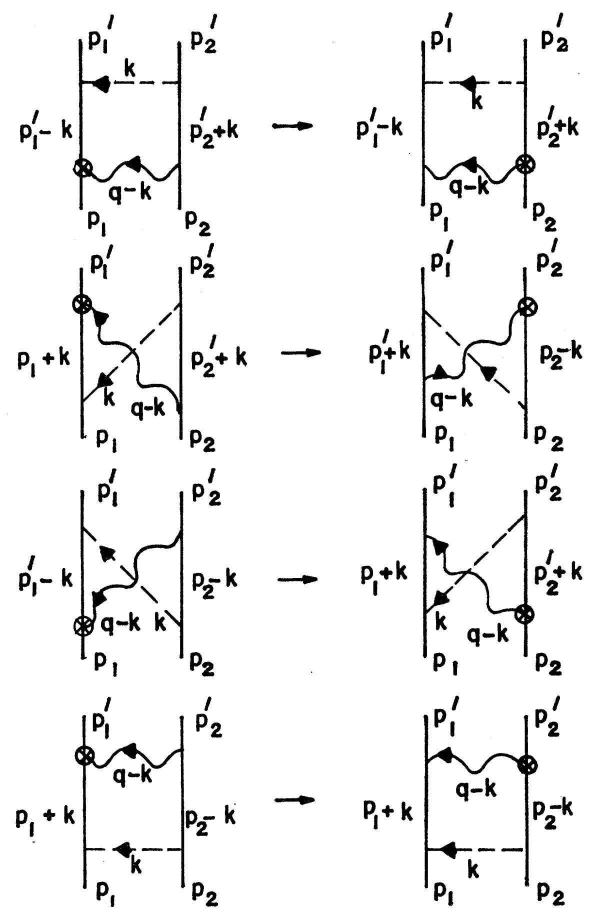

FIG. 1. Feynman diagrams for T-violating contributions to the π-γ exchange potential. The $\otimes$ indicates vertices at which a T-odd electromagnetic interaction occurs.

The magnitude of the TRV nucleon-nucleon potential can be estimated as follows: The off-the-mass-shell character of the force introduces a factor $\sim(\mu/m)^2 \cong 1/50$ (m is the mass of nucleon). There are factors of $g^2(\mu/2m)^2$ from the strong vertices, α for the electromagnetic vertices, and $q/m\cong\mu/m$ (q is the momentum transfer) for the TRV form factor. Then

$$V_{\rm TRV}\sim g^2(\mu/2m)^2\alpha(\mu/m)^3\,.$$

The TRV force will have the range $\sim\mu^{-1}$, and so may be compared with OPEP:

$$V_{\rm OPEP}\sim g^2(\mu/2m)^2\,.$$

Thus $V_{\rm TRV}$ is expected to be 0.002% of $V_{\rm OPEP}$. (This estimate is discussed further in Sec. IV.)

[7] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Co., New York, 1964).
[8] D. H. Wilkinson, Comments Nucl. Particle Phys. 2, 48 (1968).
[9] For two-pion exchange potentials (TPEP) see, for example, Progr. Theoret. Phys. (Kyoto) Suppl. 3 (1956); E. M. Henley and M. A. Ruderman, Phys. Rev. 92, 1036 (1953).

In Sec. II, we examine the TRV photon-nucleon vertex more closely and choose a model current satisfying the conditions of Hermiticity and Ward's identity. Section III describes the calculation of the nucleon-nucleon TRV force. A discussion follows in Sec. IV.

Lipshutz[10] has analyzed the electromagnetic current of the nucleon and chosen a TRV term to calculate T violation in Compton scattering. Our procedure is similar to his, but is applied to nucleon-nucleon scattering. We will specify explicitly the conditions to be satisfied by the electromagnetic current, choose a TRV term, compute the S matrix for Feynman diagrams like Fig. 1, and reduce the S matrix to a nonrelativistic potential.

II. CHOICE OF MODEL CURRENT

In order to choose a reasonable model of T violation in the nucleon electromagnetic current, we begin by studying the properties of the current. The general off-the-mass-shell nucleon electromagnetic current must satisfy the known conditions of covariance (and parity), current conservation (Ward's identity), and Hermiticity. If only the conditions of covariance (and parity) are applied—that is, if all possible four-vectors are constructed—then there is a total of 12 terms[10,11] for the current matrix element $\langle p'|J^\mu|p\rangle$. (To get terms with the improper transformation under parity, a γ^5 must be inserted.) The 12 form factors are functions of the dynamical scalars. We take the squares of the initial and final momenta and the momentum transfer as a possible set:

$$F_j(p'^2,p^2,q^2)\,.$$

If only one of the initial or final nucleons is off the mass shell ($p'^2\neq m^2$ *or* $p^2\neq m^2$), the 12 terms reduce to six, and if both the initial and final nucleons are on the mass shell ($p'^2=p^2=m^2$), only the three terms of Eq. (1) remain. (The third term does not vanish until Ward's identity is applied.) It is straightforward to apply the conditions of Hermiticity and time reversal to the 12 terms, but Ward's identity gives a complicated relationship among most of the 12 terms.[10] To proceed it is necessary to choose certain terms to be zero so that Ward's identity simplifies. We will take the three terms of Eq. (1) (including their off-the-mass-shell dependence) to be nonzero:

$$\langle p'|J^\mu|p\rangle=\bar{u}(p')[F_1(p'^2,p^2,q^2)\gamma^\mu+iF_2(p'^2,p^2,q^2)\sigma^{\mu\nu}q_\nu+iF_3(p'^2,p^2,q^2)q^\mu]u(p)\,. \quad (3)$$

We first study the properties of the current under Hermiticity and time reversal. The condition of Hermiticity is $J^\dagger=J$ or

$$\langle p'|J^\mu|p\rangle=\langle p|J^\mu|p'\rangle^*\,,$$

and requires

$$F_j^*(p'^2,p^2,q^2)=F_j(p^2,p'^2,q^2)\,,\quad j=1,\,2,\,3\,. \quad (4)$$

The condition of time-reversal invariance on the current is

$$\langle p'|\mathbf{J}|p\rangle=-\langle p_T|\mathbf{J}|p_T'\rangle\,,$$
$$\langle p'|J^0|p\rangle=+\langle p_T|J^0|p_T'\rangle\,,$$

where $|p_T\rangle\equiv T|p\rangle$. The properties of the current matrices under time reversal are usually studied by using TP,[10] since we already know the current has the correct property under P; alternatively, CP can be used, since it is equivalent to T if TCP holds, as is usually assumed. In any case, time-reversal invariance requires

$$F_j(p'^2,p^2,q^2)=F_j(p^2,p'^2,q^2)\quad \text{for}\quad j=1,\,2\,,$$
$$F_3(p'^2,p^2,q^2)=-F_3(p^2,p'^2,q^2)\,. \quad (5)$$

(For a more complete derivation of the conditions of Hermiticity and time-reversal invariance on the form factors see, for example, Refs. 10 or 12.) The relations (4) and (5) are summarized in Table I, where we list the conditions which must be satisfied by the form factors if J^μ is to satisfy Hermiticity and *violate* T. A violation of T occurs if any one of the three form factors obeys the conditions given in Table I.

The conditions of Hermiticity and time-reversal invariance, Eqs. (4) and (5), imply that the form factors must be either symmetric or antisymmetric in the momentum variables p'^2 and p^2. Since essentially nothing is known about the shape of the off-the-mass-shell form factors, it is simplest to assume that the dependence factors

$$F_j(p'^2,p^2,q^2)=F_j(q^2)G_j(p'^2,p^2)\,. \quad (6)$$

Without loss of generality we may take $G_j(m^2,m^2)=1$. The functions $F_1(q^2)$ and $F_2(q^2)$ are known partially from experimental measurements; for the TRV term, this dependence may be calculated from some theory such as Lee's phenomenological Lagrangian for the "a" particles.[5] We have taken all the F_j's constant for this work, since we do not expect the one-pion–one-photon–exchange potential to depend critically on the factors $F_j(q^2)$, at least beyond $r\cong\mu^{-1}$, where it has a chance of being valid.

TABLE I. Conditions forced on the form factors of Eq. (1) to satisfy Hermiticity and violate T invariance. All form factors vanish on the mass shell.

	F_1	F_2	F_3
Symmetry under $p'^2\leftrightarrow p^2$	odd	odd	even
Real (R) or imaginary (I)	I	I	R

[10] N. R. Lipshutz, Phys. Rev. **158**, 1491 (1967).
[11] A. M. Bincer, Phys. Rev. **118**, 855 (1960).
[12] A. H. Huffman, thesis, University of Washington, 1968 (unpublished).

Our model current must contain both T-normal and TRV terms, since a detection of T violation requires an interference of normal and violating terms. For the T-normal terms we take the F_1 and F_2 terms of Eq. (3), which are known to contribute. The conditions of Eqs. (4) and (5) then require that the functions $G_1(p'^2,p^2)$ and $G_2(p'^2,p^2)$ be real and symmetric. Since only the lowest-order term in the expansion of off-shell momentum dependence, p'^2-p^2, is likely to have any validity, we take

$$G_1(p'^2,p^2)=G_2(p'^2,p^2)=1.$$

The next step is to apply the restrictions of current conservation or Ward's identity,

$$q_\mu J^\mu=0.$$

For an "improper" vertex, Ward's identity can be expressed[13] as

$$q_\mu \bar{u}(p+q)\Gamma^\mu(p+q,\, p)=\bar{u}(p+q)\boldsymbol{q}. \qquad (7)$$

An "improper" vertex is distinguished from a "proper" vertex in that it has self-energy corrections on the external legs. The second term of Eq. (3) does not contribute to Ward's identity, Eq. (7), but for the first term, we must add a counterterm so that Ward's identity will be satisfied.

Possible counterterms are

$$[1-F_1(q^2)]\boldsymbol{q}q^\mu/q^2$$

or

$$[1-F_1(q^2)]\boldsymbol{q}P^\mu/P\cdot q,$$

where $P=p+p'$. The second possibility is ruled out, since it has a pole on the mass shell. Such a pole is unphysical because graphs like one-photon exchange would be infinite. (Higher-order terms of G_1 can cancel the pole, as we will see below.) The first possibility has been used in calculations[13] and agrees with Ward's identity.[11] Thus, a satisfactory T-normal current is

$$J^\mu=F_1(q^2)\gamma^\mu+iF_2(q^2)\sigma^{\mu\nu}q_\nu+[1-F_1(q^2)]\boldsymbol{q}q^\mu/q^2.$$

We can now choose TRV terms from the conditions of Table I. The three candidates are

$$(1)\ \ F_1'(q^2)G_1'(p'^2,p^2)\gamma^\mu,$$
$$(2)\ \ iF_2'(q^2)G_2'(p'^2,p^2)\sigma^{\mu\nu}q_\nu,$$
$$(3)\ \ iF_3'(q^2)G_3'(p'^2,p^2)q^\mu.$$

The primes on these form factors serve to remined the reader that they are *different* functions from the T-normal form factors. To violate T, Table I says that G_1' and G_2' must be imaginary and *antisymmetric* in p'^2 and p^2, and G_3' must be real and symmetric. The simplest choices for (1) and (2) are

$$G_j'(p'^2,p^2)=i(p'^2-p^2),\quad j=1,\,2.$$

[13] L. K. Morrison, Ann. Phys. (N. Y.) **50**, 6 (1968); thesis, University of Washington, 1967 (unpublished).

Term (2) automatically leaves Ward's identity satisfied in Eq. (7), but term (1) needs a counterterm. The only possibility is $-\boldsymbol{q}P^\mu$,

$$(1)\ \ iF_1'(q^2)(\gamma^\mu P\cdot q-\boldsymbol{q}P^\mu).$$

Term (3) is more complicated. It first must have a counterterm for Ward's identity. The only possibility,

$$-iF_3'(q^2)G_3'(p'^2,p^2)q^2P^\mu/P\cdot q,$$

has a pole on the mass shell which is unphysical, as discussed above; however, a proper choice for G_3' can be made to cancel the pole. Since G_3' must be symmetric and real for T violation, the simplest possibility is

$$G_3'=(p'^2-p^2)^2.$$

Then the whole F_3 term vanishes on the mass shell as expected from Ward's identity. It must be emphasized that the choices of the G_j's above represent only the behavior close to the mass shell.[10] In fact, these choices lead to divergence difficulties in the potential which will be discussed in Sec. III. The three candidates are now

$$(1)\ \ iF_1'(q^2)(\gamma^\mu P\cdot q-\boldsymbol{q}P^\mu),$$
$$(2)\ \ -(p'^2-p^2)F_2'(q^2)\sigma^{\mu\nu}q_\nu, \qquad (8)$$
$$(3)\ \ i(p'^2-p^2)^2F_3'(q^2)(q^\mu-q^2P^\mu/P\cdot q).$$

Each term has something to recommend it. The first arises naturally in Lee's theory of T-violating *a* particles[5] (see Ref. 12). The second has appealing simplicity and was chosen by Lipshutz.[10] The third turns out to be easiest to use in calculations, since it never involves component sums of Dirac matrices across two spinor products (like $[\bar{u}(p_1')\gamma^\mu u(p_1)][\bar{u}(p_2')\gamma^\mu u(p_2)]$). We let simplicity in calculation be the overriding factor and choose the term (3).

Finally, notice that we have not kept strictly to the three terms of Eq. (3) in the choice of our complete T-violating current. The F_1 counterterm $\boldsymbol{q}q^\mu$ is actually one of the other 12 terms[10,11] of the full off-the-mass-shell current. The TRV counterterm P^μ, on the other hand, is related to the γ^μ and $\sigma^{\mu\nu}q_\nu$ terms through the well-known Gordon decomposition of the current.

III. SUMMARY OF CALCULATION

In this section we discuss features of the calculation of the nucleon-nucleon TRV potential and summarize the results.

There are four possible graphs of pion-photon exchange with the TRV vertex on the first leg and four more with the TRV vertex on the second leg (Fig. 1). There are also eight other graphs with the final particles exchanged which must be subtracted from the first eight for the proper Fermi-Dirac symmetry. The net result can be expressed as the operator $1-P_{\text{ex}}$ times the result obtained from the first eight where P_{ex} exchanges the particles in the final state as in OPEP.[7]

In addition to the graphs enumerated above, there are other graphs in which there is a TRV vertex in the electromagnetic self-mass corrections on the nucleon legs and in the correction to the pion vertex part. For simplicity, all diagrams of these types have been omitted from the calculations and only π-γ exchange diagrams are included. The calculation is still gauge-invariant, since the TRV verticies Γ' satisfy $q_\mu\Gamma'^\mu=0$.

Since all possible TRV potentials[14] contain the momentum variable $\mathbf{p}$, it is necessary to ascertain how this dependence arises in $M(q)$. Referring to Fig. 1 in the c.m. frame, we have $\mathbf{p}_1=-\mathbf{p}_2\equiv\mathbf{p}$ and $\mathbf{p}_1'=-\mathbf{p}_2'=\mathbf{p}+\mathbf{q}$. The amplitude $M(q)$ in general will be a function of all four momenta $\mathbf{p}_1$, $\mathbf{p}_1'$, $\mathbf{p}_2$, and $\mathbf{p}_2'$, since the spinor products are reduced to their nonrelativistic limits. When $M(q)$ is summed over the four diagrams of Fig. 1, the expression can be written in terms of the variables

$$\mathbf{p}_1'-\mathbf{p}_1=\mathbf{p}_2-\mathbf{p}_2'=\mathbf{q}\,,$$
$$\mathbf{p}_1'+\mathbf{p}_1=-(\mathbf{p}_2+\mathbf{p}_2')=2\mathbf{p}+\mathbf{q}\equiv\mathbf{P}.$$

(The second equality holds only in the c.m. system.) Then $\mathbf{P}$ is the variable in $M(q)$ to represent $\mathbf{p}$ in the potential $V(\mathbf{v})$. The $\mathbf{q}$ term is needed for Hermiticity of the potential. Thus, for the "simplest" TRV potential,[14] $V(\mathbf{r})$ can be written

$$\begin{aligned}V(\mathbf{r})&=\mathbf{p}\cdot\mathbf{r}F(r)+F(r)\mathbf{r}\cdot\mathbf{p}\\&=-2iF(r)\mathbf{r}\cdot\boldsymbol{\nabla}-i[\boldsymbol{\nabla}\cdot(\mathbf{r}F(r))].\end{aligned}$$

In the last form of this equation, the second term is the $\mathbf{q}$ term, i.e., the gradient acts on the potential only; the first term is the $2\mathbf{p}$ term, i.e., the gradient acts only beyond the potential (on the state functions).

The T violation appears through a remarkable cancellation of the fermion propagator in each graph. This cancellation is most easily seen for the second TRV term of (8):

$$(2)\quad -(p'^2-p^2)F_2'(q^2)\sigma^{\mu\nu}q_\nu.$$

For one particle on the mass shell, the factor p'^2-p^2 equals p'^2-m^2 exactly, canceling the propagator for the internal leg. Such a factor (or the propagator itself) equals

$$(p'^2-p^2)=(p'-p)\cdot(p'+p)\rightarrow\mathbf{r}\cdot\mathbf{p}\quad\text{in the potential}$$

(for the vector parts). A factor like this occurs in every TRV term and shows how the violating potential arises. It is noted that in the set (8), term (3), one power of $(p'^2-p^2)^2$ cancels the propagator and one power is left over; however, it is easy to show that the factor q^μ itself acts to cancel the other propagator, leaving an odd number of $\mathbf{r}\cdot\mathbf{p}$-like terms. In evaluating the TRV diagrams, we must therefore be careful not to throw away the momentum dependence in making the standard nonrelativistic approximation to the propagators,

$$\frac{1}{(p_1+k)^2-m^2+i\epsilon}\approx\frac{1}{2m\omega},$$

where ω is the fourth component of k^μ. This term actually cancels in the TRV diagrams when the diagram with the photon and pion crossed is added.

With this summary in mind, we proceed to the actual results of the calculation. One great simplification is that in all the diagrams of Fig. 1, the P^μ term gives contributions of the order of μ^2/m^2 relative to the q^μ terms. We can see how this comes about as follows: Note that the factors $P\cdot q$ and p'^2-p^2 both equal the propagator p'^2-m^2, which will be denoted by $\mathcal{P}_1$, where the subscript indicates the first nucleon line. From a q^μ term we get a factor like

$$\mathcal{P}_1^2\mathcal{P}_2/\mathcal{P}_1\mathcal{P}_2=\mathcal{P}_1.$$

(The $\mathcal{P}_2$ in the numerator comes from the q^μ factor itself.) From a P^μ term the same factor is

$$\frac{\mathcal{P}_1^2}{\mathcal{P}_1}\frac{1}{\mathcal{P}_1\mathcal{P}_2}=\frac{1}{\mathcal{P}_2}.$$

We have mentioned that the propagators $\mathcal{P}$ consist of a large term $2m\omega$ which cancels in the cross diagram and a small momentum-dependent term which forms the T-violation factor. It is seen that in the q^μ term, the large part of $\mathcal{P}$ cancels in the numerator, but in the P^μ term it remains as an extra factor of m^{-2} in the denominator.

If these terms of order μ^2/m^2 are neglected, there is only one term in the complete current

$$J^\mu=F_1\gamma^\mu+iF_2\sigma^{\mu\nu}q_\nu+(1-F_1)\frac{qq^\mu}{q^2}+iF_3(p'^2-p^2)^2\left(q^\mu-\frac{q^2P^\mu}{P\cdot q}\right)$$

which contributes to T violation in the four diagrams of Fig. 1, the term in which the TRV vertex is $\propto F_3q^\mu$ and the T-normal vertex is $\propto F_1\gamma^\mu$. The potential resulting from this term for the four diagrams of Fig. 1 in which the TRV vertex is on the first nucleon leg is

$$V(\mathbf{r})=-\frac{g^2}{(2\pi)^7}\int d^3kd^3k'\,e^{i(\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}')}\int d\omega\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}-\frac{\omega}{2m}\boldsymbol{\sigma}_1\cdot\mathbf{P}\right)\frac{\boldsymbol{\sigma}_2\cdot\mathbf{q}}{2m}\times\frac{[2(k^2-q\cdot k)F_3{}^{(s)}F_1{}^{(v)}(-i)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z+2P\cdot kF_3{}^{(v)}F_1{}^{(v)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2-\tau_1{}^z\tau_2{}^z)]}{(\omega^2-\omega_k{}^2+i\epsilon)(\omega^2-k'^2+i\epsilon)}.\quad(9)$$

[14] P. Herczeg, Nucl. Phys. **75**, 655 (1966).

Here $\omega_k^2=k^2+\mu^2$ and we have used the standard "trick" as in TPEP[9] of letting $(\mathbf{q}-\mathbf{k})\cdot\mathbf{r}\equiv\mathbf{k}'\cdot\mathbf{r}\to\mathbf{k}'\cdot\mathbf{r}'$, where the limit $\mathbf{r}'\to\mathbf{r}$ is to be taken after doing the integral. The factor in square brackets can be written symbolically as

$$(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z(T\text{-even factors})$$
$$+P\cdot k(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2-\tau_1^z\tau_2^z)(T\text{-even factors}). \quad (10)$$

The denominator of the integrand is even in ω, so in the $P\cdot k$ term the vector part goes with $\boldsymbol{\sigma}_1\cdot\mathbf{k}$ to form $\mathbf{p}\cdot\mathbf{r}$ terms, whereas the time part $\approx 2m\omega$ goes with the $(-\omega/2m)\boldsymbol{\sigma}_1\cdot\mathbf{P}$ to form $(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p})$ terms.

What of the first term of (10)? The factor $(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z$ is itself odd under T[14] (because of the imaginary τ^Y's), so these terms also violate T. In fact, however, when diagrams with the violating vertex on the other leg are added on, these terms are cancelled; this is as it should be, since they have the wrong symmetry under the exchange of particles 1 and 2 (the other factors are symmetric in $1\leftrightarrow 2$).

The entire effect of the four diagrams with the TRV vertex on the second nucleon line can be added to the previous result by making the correspondence shown in Fig. 1. It is easy to see that we can get the result with the TRV vertex on the second nucleon line by making the following substitutions in Eq. (9):

$$1\to 2,\quad 1'\to 2',\quad \mathbf{k}\to-\mathbf{k},$$
$$\mathbf{q}=\mathbf{p}_1'-\mathbf{p}_1=\mathbf{p}_2-\mathbf{p}_2'\to-\mathbf{q},$$
$$\mathbf{P}=\mathbf{p}_1+\mathbf{p}_1'=-(\mathbf{p}_2+\mathbf{p}_2')\to-\mathbf{P}\quad(\text{in c.m. system only}),$$
$$P^0\cong 2m\to P^0.$$

Then all the terms with $(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z$ vanish, since they are antisymmetric under $1\to 2$. This is true even though one term is $(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{q})$ and the other is $(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{k})$, since $\mathbf{q}$ essentially equals $2\mathbf{k}$ when the gradient operators are pulled out of the integral. The terms like $(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{r})$ are symmetrized in 1 and 2. Terms like $\mathbf{r}\cdot\mathbf{p}$ are merely multiplied by 2.

The integrals which determine the radial dependence of the potential are

$$I(r,r')=\int d^3k\,d^3k'\,e^{i\mathbf{k}\cdot\mathbf{r}+\mathbf{k}'\cdot\mathbf{r}'}\times\int\frac{d\omega}{(\omega^2-\omega_k^2+i\epsilon)(\omega^2-k'^2+i\epsilon)},\quad(11)$$

and its various derivatives with respect to r and r'. By using complex-variable techniques, the expression above can be reduced to

$$I(r,r')=\frac{(2\pi)^4 i}{2rr'}\left[\int_0^\mu d\omega\,\sin\omega r'\exp[-(\mu^2-\omega^2)^{1/2}r]+\int_\mu^\infty d\omega\,\sin[(\omega^2-\mu^2)^{1/2}r+\omega r']\right].$$

This integral does not converge and the divergence is worse when several derivatives are taken with respect to r and r' as required by the form (9). The reason why this integral diverges and the similar TPEP integral does not is the extra factor of $(p'^2-p^2)^2$ introduced for T violation in Sec. II, which becomes a factor of ω^2 in the integral [Eq. (9)]. We know that an F_3 form factor with the dependence $(p'^2-p^2)^2$ may be correct near the mass shell; however, F_3 is not expected to blow up at large p^2, since the form factors satisfy dispersion relations in p^2.[11] A possible choice which approaches $(p'^2-p^2)^2$ near the mass shell is

$$\frac{m^4(p'^2-p^2)^2}{[m^4+\frac{1}{4}(p'^2-p^2)^2]^2}\sim\frac{(p'^2-p^2)^2}{(m^2+\omega^2)^2}.\quad(12)$$

This factor effectively cuts the integral off at $\omega\simeq m$, where the nonrelativistic approximation and potential theory become invalid, and allows us to take up to three derivatives with respect to r or r' without divergence difficulties. The corrected integral,

$$J(r,r')=\frac{(2\pi)^4 i}{2rr'}\left[\int_0^\mu\frac{d\omega\,\sin\omega r'\exp[-(\mu^2-\omega^2)^{1/2}r]}{(m^2+\omega^2)^2}+\int_\mu^\infty\frac{d\omega\,\sin[(\omega^2-\mu^2)^{1/2}r+\omega r']}{(m^2+\omega^2)^2}\right],$$

which must be evaluated numerically, is asymptotic to $e^{-\mu r}/r^2$, as expected.

Letting $\mathbf{k}\to-i\boldsymbol{\nabla}$, $\mathbf{k}'\to-i\boldsymbol{\nabla}'$, as in a TPEP calculation, and performing the required operations, we obtain the result

$$V(r)=(\mu^2/m)[g^2/(2\pi)^3]F_1^{(v)}F_3^{(v)}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2-\tau_1^z\tau_2^z)\times[(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\hat{r}\cdot\mathbf{p})Q_1(x)+Y_{12}Q_2(x)+Z_{12}Q_3(x)]+\text{H.c.},\quad(13)$$

where

$$x=\mu r,$$
$$Y_{12}=(\boldsymbol{\sigma}_1\cdot\hat{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p})+(\boldsymbol{\sigma}_2\cdot\hat{r})(\boldsymbol{\sigma}_1\cdot\mathbf{p})-\tfrac{2}{3}\hat{r}\cdot\mathbf{p}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2),$$
$$Z_{12}=(\boldsymbol{\sigma}_1\cdot\hat{r})(\boldsymbol{\sigma}_2\cdot\hat{r})(\hat{r}\cdot\mathbf{p})-\tfrac{1}{5}[(\boldsymbol{\sigma}_1\cdot\hat{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p})+(\boldsymbol{\sigma}_2\cdot\hat{r})(\boldsymbol{\sigma}_1\cdot\mathbf{p})+(\hat{r}\cdot\mathbf{p})(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)],$$

and Q_i are certain combinations of the dimensionless integrals

$$P(n,m)=\lim_{x'\to x}\left(\frac{d}{dx}\right)^n\left(\frac{d}{dx'}\right)^m\times\left[\int_0^1 d\sigma\frac{\sin\sigma x'\exp[-(1-\sigma^2)^{1/2}x]}{(49+\sigma^2)^2}+\int_1^\infty d\sigma\frac{\sin[(\sigma^2-1)^{1/2}x+\sigma x']}{(49+\sigma^2)^2}\right].\quad(14)$$

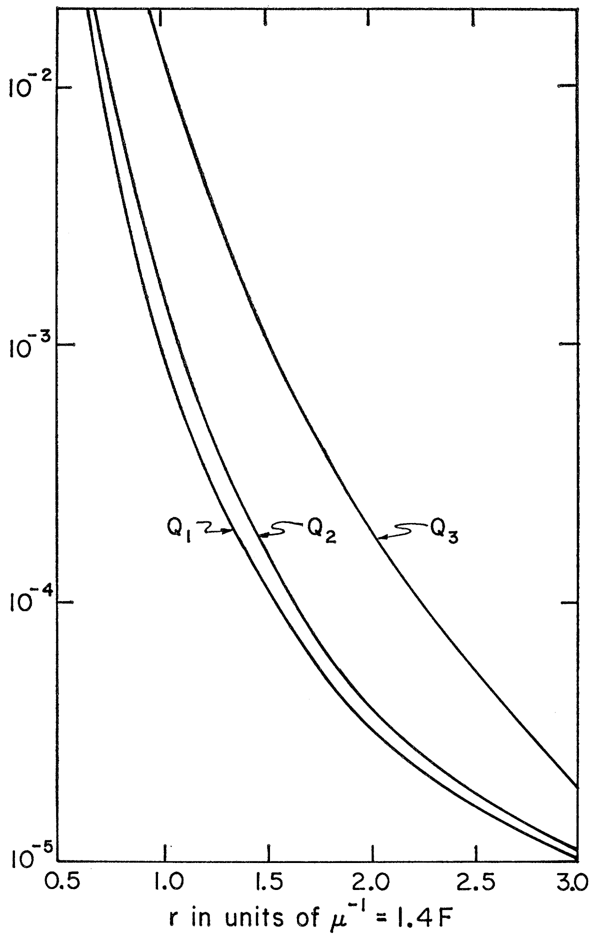

FIG. 2. Radial dependence of the T-odd potential form factors Q_1, Q_2, and Q_3.

The explicit forms of the Q_i in terms of the $P(n,m)$ are

$$Q_1=(4/3x^5)\{P(0,0)-x[P(1,0)+P(0,1)]+x^2[P(1,1)+P(2,0)+P(0,2)]\}-(2/3x^2)\times[P(1,2)+P(0,3)+P(3,0)+P(2,1)],$$

$$Q_2(x)=(1/5x^5)\{P(0,0)-x[P(0,1)+P(1,0)]+x^2[4P(2,0)+P(1,1)+10P(0,2)]-x^3[5P(0,3)+5P(1,2)+2P(3,0)+2P(2,1)]\},$$

$$Q_3(x)=(1/x^5)\{36P(0,0)-x[36P(1,0)+6P(0,1)]+x^2[6P(1,1)+14P(2,0)]-x^3[2P(3,0)+2P(2,1)]\}.$$

Graphs of the Q_i are shown in Fig. 2.

IV. DISCUSSION

The potential derived in this work, Eq. (13), consists of three of the six possible types of nucleon-nucleon parity-even TRV potentials.[14] Each of the three terms is spin-dependent, as might have been expected from the spin-dependent pion-nucleon coupling. The terms Y_{12} and Z_{12} are traceless tensors of ranks 1 and 3, respectively.

If the TRV force, Eq. (13), is written in the approximate form

$$V_{\mathrm{TRV}}=\frac{g^2}{(2\pi)^3}\frac{\mu^2}{2m}F_1^{(v)}F_3^{(v)}\frac{\mathbf{r}\cdot\mathbf{p}}{r}Q(\mu r),$$

its strength can be compared to the spin-spin part of OPEP,[7]

$$V_{\mathrm{OPEP}}(r)=\frac{g^2}{3\pi}\left(\frac{\mu}{2m}\right)^2\tfrac{1}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{e^{-\mu r}}{r}.$$

The spin and isospin factors have expectation values of the order of 1. Furthermore, these dependences of OPEP and the Q_1 term of the TRV potential are quite similar. The expectation value of $\mathbf{r}\cdot\mathbf{p}$ is also approximately 1 by the uncertainty principle (at least for a tightly bound state). The T-normal form factor may be approximated by its value at $q^2=0$: $F_1^{(v)}(0)=(4\pi\alpha)^{1/2}$, where α is the fine-structure constant $[=(137)^{-1}]$. The TRV form factor $F_3^{(v)}$ is completely unknown, but a reasonable guess is $F_3\approx(4\pi\alpha)^{1/2}m^{-1}$, to be compared with the anomalous magnetic moment term $F_2(0)=(4\pi\alpha)^{1/2}(\kappa/2m)$. These approximations give the ratio of the forces, F, as

$$F=\frac{V_{\mathrm{TRV}}}{V_{\mathrm{OPEP}}}\approx\frac{(g^2/4\pi)(\mu^2/m^2)(\alpha/\pi)}{\frac{1}{3}(g^2/4\pi)\frac{1}{4}(\mu^2/m^2)}\frac{Q(\mu r)/\mu r}{e^{-\mu r}/\mu r}.$$

The "strength" of the radial dependences can be compared by means of the area of the potentials

$$\int_b^\infty V(r)dr.$$

The integration should be started somewhere near $r=\mu^{-1}$ because neither of these potentials is valid very far inside this distance. For three values of the starting point b in the normalization integral above, the ratios of the forces for the Q_1 term of Eq. (13) are

$$b=1.0\mu^{-1},\quad F=0.004\%;$$
$$b=0.75\mu^{-1},\quad F=0.012\%;$$
$$b=0.5\mu^{-1},\quad F=0.10\%.$$

This critical dependence on the starting point of the normalization arises from factors up to $(\mu r)^5$ in the denominators of the Q's, and makes an evaluation of the strength of V_{TRV} difficult. Inside $r\simeq\mu^{-1}$ there are short-range TRV forces coming from the exchange of a photon and two or more pions or a heavier meson. These effects will be at least partially masked by repulsive-core effects, but the radius at which the longest-range forces become secondary in importance or the calculation becomes invalid is ill defined. For definiteness in the discussion of the following paper, we have fixed on the value of F for $b=0.75\mu^{-1}\simeq 1$ F, $F=0.01\%$. The Q_2 term is about 0.65 times smaller and the Q_3 term is about 8.5 times larger. In considering the reliability of this estimate of the strength of V_{TRV}, it should be emphasized that the cutoff factor [Eq. (11)] and $F_3^{(v)}$ are totally unknown quantities, and that certain graphs (e.g., corrections to the pion vertex part) have been omitted from this calculation.

At this point we may discuss the estimate of the strength of V_{TRV} in Sec. I a little further. Let us start with a T-odd vertex of the second type, since it is somewhat simpler:

$$T\text{-odd vertex}\sim\alpha^{1/2}\frac{(p'^2-p^2)}{m^2}\frac{\sigma^{\mu\nu}q_\nu}{m}.$$

For the p'^2-p^2 factor,

$$p'^2-p^2\sim P\cdot q\sim \mathbf{p}\cdot\frac{\mu^2\mathbf{r}}{(\mu r)^2}\sim\mu^2.$$

The momentum transfer q is of order μ, so the T-odd vertex contributes a factor of $\alpha^{1/2}(\mu^2/m^2)(\mu/m)$. Thus the off-the-mass-shell character of the force contributes a factor of μ^2/m^2. Notice that this factor is larger for heavier mesons in shorter-range forces. The other vertices,

$$T\text{-normal vertex}\sim\alpha^{1/2},$$
$$\text{pion vertices}\sim g\mu/2m \text{ each},$$

yield the estimate of Sec. I,

$$V_{\text{TRV}}/V_{\text{OPEP}}\sim\alpha(\mu/m)^3,$$

which is about half the value of F for $b=1.0\mu^{-1}$. For a T-odd vertex of the third type as was used in the present calculation,

$$T\text{-odd vertex}\sim\alpha^{1/2}\frac{(p'^2-p^2)^2}{m^4}\frac{q^\mu}{m}.$$

If we estimate as before

$$(p'^2-p^2)^2\sim\mu^4,$$

we obtain

$$V_{\text{TRV}}/V_{\text{OPEP}}\sim\alpha(\mu/m)^5.$$

The reason why the calculation gives an answer two orders of magnitude larger than this originates in factors like

$$(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})(\mathbf{p}\cdot\boldsymbol{\nabla}).$$

For instance, in the quantity

$$(\mathbf{A}\cdot\boldsymbol{\nabla})(\mathbf{B}\cdot\boldsymbol{\nabla})(\mathbf{C}\cdot\boldsymbol{\nabla})e^{-\mu r}/\mu r,$$

the coefficient of $(\mathbf{A}\cdot\mathbf{r})(\mathbf{B}\cdot\mathbf{r})(\mathbf{C}\cdot\mathbf{r})$ at $r=\mu^{-1}$ is about 40.

A final interesting point is the isospin dependence of the potential, Eq. (13). Both the F_1 and F_3 form factors are assumed to have isoscalar and isovector parts [Eq. (2)], but only the isovector parts contribute (to this order in μ^2/m^2). For instance, one form of Lee's theory[5] of a particles assumes that the a's are unitary singlets, in which case the T violation is purely isoscalar. The results of our calculation for an F_3 T-violating form factor show that the nucleon-nucleon TRV force would be reduced by an additional factor of μ^2/m^2 in this case.

Also note that

$$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2-\tau_1^z\tau_2^z=\tfrac{1}{2}(\tau_1^-\tau_2^++\tau_1^+\tau_2^-)$$

and

$$(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)^z=(1/2i)(\tau_1^-\tau_2^+-\tau_1^+\tau_2^-),$$

so the TRV forces are charge-exchange forces, a fact also true of the parity-violating force.[15]

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[15] R. J. Blin-Stoyle, Phys. Rev. 118, 1605 (1960).