ρ Meson in Low-Energy Pion-Nucleon Scattering*

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The low-energy, pion-nucleon scattering is studied by including the ρ-meson contribution to the axial-vector amplitude. By using a different treatment for the ρ contribution and the q^2 extrapolation, the results for the a_{1-}^- scattering length and b_{0+}^- effective range are improved while the results for the a_{0+}^- and a_{1+}^- scattering lengths are preserved. An explanation is also given of why there is no double counting of the ρ-meson contributions in pion-nucleon interactions when considering current-algebra and ρ-exchange models.

I. INTRODUCTION

In recent years, many low-energy pion-nucleon calculations[1–8] have been performed using the current algebra of the chiral $SU_2\otimes SU_2$ group, partial conservation of axial-vector current (PCAC), and an off-the-mass-shell extension of the Lehmann-Symanzik-Zimmermann reduction formula for the reduction of two pions. The usual method involves letting the pions go off the mass shell while leaving the nucleons on. It is assumed that the extrapolation back on the mass shell for the pions is smooth. The off-the-mass-shell amplitude can be defined by an expression of the form

$$R_{fi}=\frac{(q^2-\mu_\pi^2)(k^2-\mu_\pi^2)}{2\mu_\pi^4 f_\pi^2}\int d^4x d^4y \times e^{iqx}e^{-iky}\langle p_2|T\{\partial_\mu A_b{}^\mu(x)\partial_\nu A_a{}^\nu(y)\}|p_1\rangle, \quad (1.1)$$

while the on-the-mass-shell S-matrix element can be represented by

$$(S-1)_{fi}=\frac{(q^2-\mu_\pi^2)(k^2-\mu_\pi^2)}{2\mu_\pi^4 f_\pi^2}\left(\frac{m^2}{4p_{10}p_{20}q_0k_0}\right)^{1/2}\int d^4x d^4y \times e^{iqx}e^{-iky}\langle p_a|T\{\partial_u A_b{}^\mu(x)\partial_\nu A_a{}^\nu(y)\}|p_1\rangle. \quad (1.2)$$

In Eqs. (1.1) and (1.2), $A_b{}^\mu(x)$ represents the axial-vector density with isospin index b, f_π is the pion decay constant, and μ_π and m are the pion and nucleon masses, etc. Neglecting Schwinger terms and the σ term, the off-the-mass-shell expression can be written as

$$R_{fi}=\frac{(q^2-\mu_\pi^2)(k^2-\mu_\pi^2)}{2\mu_\pi^4 f_\pi^2}\int d^4x d^4y \times e^{iqx}e^{-iky}[q_\mu k_\nu\langle p_2|T\{A_b{}^\mu(x)A_a{}^\nu(y)\}|p_1\rangle +2(k+q)_\alpha\epsilon_{abc}\delta^4(x-y)\langle p_2|V_c{}^\alpha(x)|p_1\rangle], \quad (1.3)$$

where the vector-current–density-matrix element can be written as

$$\langle p_2|V_c{}^\alpha(0)p_1\rangle =\bar{u}(p_2)[\gamma^\alpha F_1(t)+i\sigma^{\alpha\nu}(p_2-p_1)_\nu F_2(t)]\tau_c u(p_1). \quad (1.4)$$

Here, $F_1(t)$ and $F_2(t)$ are the nucleon isovector form factors with $F_1(0)=\frac{1}{2}$ and $F_2(0)=1.85/2m$.

The earliest calculations[1–3] neglected the $q_\mu k_\nu$ term, but they enjoyed only limited success in reproducing the experimental scattering lengths and effective ranges. The axial-vector term was included in later calculations by authors such as Schnitzer,[6] Raman,[5,7] and Levin.[8] The N and N^* contributions were included by Schnitzer, while a more thorough study of the baryon resonances along with different extrapolation procedures was carried out by Raman. The results of these studies are listed in Table I and can be regarded, in general, as successful, except for the P-wave scattering length a_{1-}^- with Raman and the S-wave effective range b_{0+}^- with Schnitzer.

In this article, we wish to study the effect of the ρ meson in the pion-nucleon system at low energies. We begin with a non-current-algebra off-the-mass-shell extension of the LSZ reduction formula with the ρ meson included as an intermediate state. The basic equations for this model are presented in Sec. II.

In Sec. III, we return to the current-algebra model of Eq. (1.3) and study the ρ contribution to it. We find a relationship between the non-current-algebra model, the current-algebra model, and the concept of weak ρ dominance. A discussion is then presented on why a ρ-exchange Feynman diagram and a current-algebra model yield the same results for the S-wave scattering

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TABLE I. The results for the S- and P-wave scattering lengths and the S-wave effective range for the two different sets of form factors (dipole and HH) along with the contribution from the terms R_ρ and R_B (taken from Raman). For comparison, we include the results of Raman and of Schnitzer and the experimental results of Hamilton and Woolcock (Ref. 15) and of Roper, Wright, and Feld (Ref. 19).

	R_ρ (dipole)	R_ρ (HH)	R_B	Total (dipole)	Total (HH)	Raman	Schnitzer	HW	RWF
a_{0+}^-	0.079	0.079	0.001	0.080	0.080	0.100	...	0.086	0.086
b_{0+}^-	0.031	0.027	−0.020	0.011	0.007	...	−0.148[a]	0.010	...
a_{1+}^-	−0.005	−0.004	−0.084	−0.089	−0.088	−0.083	−0.075	−0.081	−0.081
a_{1-}^-	0.026	0.027	−0.029	−0.003	−0.002	0.012	−0.005	−0.021	−0.016

[a] We have used PCAC in determining h_A to calculate b_{0+}^- from Schnitzer's results.

lengths. An explanation is then given as to why there is no double counting of the ρ contributions.

In Sec. IV, we present our calculations of the S- and and P-wave scattering lengths and S-wave effective ranges using the q^2 extrapolation procedure presented in Sec. III.

The Bjorken-Drell metric and γ-matrix conventions along with the units of $\hbar=c=1$ will be used throughout this paper.

II. ρ CONTRIBUTIONS TO NON-CURRENT-ALGEBRA R_{fi}

We define the off-the-mass-shell three-particle reduction amplitude as

$$R_{fi}=i\int d^4xd^4yd^4z\; e^{iqx}e^{-iky}e^{-ip_1z} \times\langle p_2|T\{n_b(x)n_a(y)\lambda(z)\}|0\rangle u(p_1)\,, \quad (2.1)$$

where $n_j(x)$ and $\lambda(x)$ are the pion and nucleon source terms satisfying the equations

$$(\square^2+\mu_\pi^2)\varphi_j(x)=n_j(x)\,,\qquad (i\gamma^\mu\partial_\mu-m)\psi(x)=\lambda(x)\,. \quad (2.2)$$

Separating all the single-particle ρ, N, $N^*\ldots$ contributions correctly, R_{fi} can be separated into two parts,

$$R_{fi}=R_\rho+R_B\,, \quad (2.3)$$

where R_ρ represents the ρ contributions and R_B represents the baryon contributions. R_ρ can be expressed in terms of the $NN\rho$ and $\pi\pi\rho$ form factors which are defined by the equations[9]

$$\langle p_2|J_c^\alpha(0)|p_1\rangle =\bar{u}(p_2)[\gamma^\alpha F_{1NN\rho}(t)+i\sigma^{\alpha\beta}(p_2-p_1)_\beta F_{2NN\rho}(t)] \times\tau_c u(p_1) \quad (2.4)$$

and

$$\langle k_a|q_b,\rho_c^\lambda\rangle = -(2\pi)^4\delta^4(\rho-k+q)\epsilon_{abc}\epsilon_\alpha^\lambda(k+q)^\alpha F_{\pi\pi\rho}(t)\,, \quad (2.5)$$

where $t=(p_2-p_1)^2$ and $J_c^\alpha(z)$ is the ρ-meson source satisfying the equation

$$(\square^2+M_\rho^2)\Phi_c^\alpha(z)=J_c^\alpha(z)\,. \quad (2.6)$$

Using Eqs. (2.4)–(2.6), we have

$$R_\rho=2i(2\pi)^4\delta^4(p_1+k-p_2-q)F_{\pi\pi\rho}(t) \times\bar{u}(p_2)\frac{[\gamma\cdot QF_{1NN\rho}'(t)-(2m\nu+\tfrac{1}{2}t)F_{2NN\rho}'(t)]}{m_\rho^2-t-i\epsilon} \times\frac{[\tau_b,\tau_a]}{2}u(p_1)\,, \quad (2.7)$$

where $Q=\frac{1}{2}(k+q)$, $\nu=p_1k/m$,

$$F_{1NN\rho}'(t)=F_{1NN\rho}(t)+2mF_{2NN\rho}(t)\,,\qquad F_{2NN\rho}'(t)=F_{2NN\rho}(t)\,. \quad (2.8)$$

The baryon contributions can be expressed as

$$R_B=i(2\pi)^4\delta^4(p_1+k-p_2-q)(q^2-\mu_\pi^2)^2 \times\sum_j\left(\frac{\langle p_2|\varphi_b(0)|B_j\rangle\langle B_j|\varphi_a(0)|p_1\rangle}{s-m_{B_j}^2+i\epsilon}+\frac{\langle p_2|\varphi_a(0)|B_j'\rangle\langle B_j'|\varphi_b(0)|p_1\rangle}{u-m_{B_j}^2+i\epsilon}\right), \quad (2.9)$$

where j refers to the possible baryon states, m_{B_j} denotes the mass of each state, $B_j=p_2+q$, $B_j'=p_2-k$, $s=(p_2+q)^2$, $u=(p_2-k)^2$, and we have taken $k^2=q^2$ for convenience. We will return to R_B again in Sec. III.

III. ρ CONTRIBUTION TO NUCLEON–AXIAL-VECTOR SYSTEM

Let us rewrite Eq. (1.3) in the form

$$R_{fi}=R_{AA}+R_V\,, \quad (3.1)$$

where

$$R_{AA}=\frac{(q^2-\mu_\pi^2)(k^2-\mu_\pi^2)}{2f_\pi\mu_\pi^4}\int d^4xd^4y\; e^{iqx}e^{-iky}q_\mu k_\nu \times\langle p_2|T\{A_b^\mu(x)A_a^\nu(y)\}|p_1\rangle \quad (3.2)$$

and

$$R_V=\frac{(q^2-\mu_\pi^2)(k^2-\mu_\pi^2)}{f_\pi\mu_\pi^4}(k+q)_\alpha\epsilon_{abc} \times\int d^4x\langle p_2|V_c^\alpha(x)|p_1\rangle\,. \quad (3.3)$$

[9] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966).

The term R_V can be easily shown to be

$$R_V=2i(2\pi)^4\frac{\delta^4(p_2+q-p_1-k)}{\mu_\pi{}^4f_\pi{}^2}(q^2-\mu_\pi{}^2)(k^2-\mu_\pi{}^2)\times\bar{u}(p_2)[\gamma\cdot QF_1'(t)-(2m\nu+\tfrac{1}{2}t)F_2'(t)]\times\tfrac{1}{2}[\tau_b,\tau_a]u(p_1)\,, \quad (3.4)$$

where

$$F_1'(t)=F_1(t)+2mF_2(t)$$

and

$$F_2'(t)=F_2(t)\,.$$

Any direct ρ contribution would have to appear in R_{AA}. By reducing the incoming nucleon, we have

$$R_{AA}=-i\frac{(q^2-\mu_\pi{}^2)(k^2-\mu_\pi{}^2)}{\mu_\pi{}^4f_\pi{}^2}\int d^4xd^4yd^4z\, e^{iqx}e^{-iky}e^{-ip_1z}\times q_\mu k_\nu\langle p_2|T\{A_b{}^\mu(x)A_a{}^\nu(y)\lambda(z)\}|0\rangle u(p_1)\,. \quad (3.5)$$

Expanding the time-ordered product, we have

$$\begin{aligned}\langle p_2|T\{A_b{}^\mu(x)A_a{}^\nu(y)\lambda(z)\}|0\rangle &=\theta(z_0-x_0)\theta(x_0-y_0)\langle p_2|\lambda(z)A_b{}^\mu(x)A_a{}^\nu(y)|0\rangle\\&+\theta(z_0-y_0)\theta(y_0-x_0)\langle p_2|\lambda(z)A_a{}^\nu(y)A_b{}^\mu(x)|0\rangle\\&+\theta(x_0-y_0)\theta(y_0-z_0)\langle p_2|A_b{}^\mu(x)A_a{}^\nu(y)\lambda(z)|0\rangle\\&+\theta(y_0-x_0)\theta(x_0-z_0)\langle p_2|A_a{}^\nu(y)A_b{}^\mu(x)\lambda(z)|0\rangle\\&+\theta(x_0-z_0)\theta(z_0-y_0)\langle p_2|A_b{}^\mu(x)\lambda(z)A_a{}^\nu(y)|0\rangle\\&+\theta(y_0-z_0)\theta(z_0-x_0)\langle p_2|A_a{}^\nu(y)\lambda(z)A_b{}^\mu(x)|0\rangle\,. \quad (3.6)\end{aligned}$$

The matrix elements can be calculated as in Sec. II by correctly inserting the appropriate intermediate states ρ, N, $N^*\cdots$.

In order to evaluate the ρ contributions to R_{AA}, we must specify matrix elements of the form $\langle p_2|\lambda(0)|\rho_c{}^\lambda\rangle$, $\langle\rho_c{}^\lambda|A_b{}^\mu(0)|k_g\rangle$, and $\langle k_g|A_a{}^\nu(0)|0\rangle$. The matrix element $\langle k_g|A_a{}^\nu(0)|0\rangle$ can be expressed as

$$\langle k_g|A_a{}^\nu(0)|0\rangle=-\sqrt{2}k^\nu f_\pi(k^2)\delta_{g,a}\,, \quad (3.7)$$

where $f_\pi(\mu_\pi{}^2)=f_\pi$. We shall assume that $f_\pi(k^2)$ does not vary from $k^2=0$ to $\mu_\pi{}^2$. The matrix element $q_\mu\langle\rho_c{}^\lambda|A_b{}^\mu(0)|k_g\rangle$ can, with the help of the PCAC relationship

$$\partial_\mu A_b(x)=-i\sqrt{2}f_\pi\mu_\pi{}^2\varphi_b(x)\,, \quad (3.8)$$

be expressed as

$$q_\mu\langle\rho_c{}^\lambda|A_b(0)|k_g\rangle=\frac{-\mu_\pi{}^2f_\pi\sqrt{2}i}{q^2-\mu_\pi{}^2}\epsilon_{cbg}F_{\pi\pi\rho}(t)(k+q)^\alpha\epsilon_\alpha{}^\lambda\,. \quad (3.9)$$

The last term $\langle p_2|\lambda(0)|\rho_c{}^\lambda\rangle$ can be specified with the help of Eqs. (2.4) and (2.6). Using the above equations, we have

$$R_{AA}=R_{AA\rho}+R_{AAB}\,, \quad (3.10)$$

where $R_{AA\rho}$ represents the ρ contributions to R_{AA} and has the form

$$R_{AA\rho}=2i(2\pi)^4\delta^4(p_2+q-p_1-k)(q^2/\mu_\pi{}^2)F_{\pi\pi\rho}(t)\times\bar{u}(p_2)\left(\frac{\gamma\cdot QF_{1NN\rho}'(t)-(2m\nu+\frac{1}{2}t)F_{2NN\rho}'(t)}{m_\rho{}^2-t-i\epsilon}\right)\times\frac{[\tau_b,\tau_a]}{2}u(p_1)\,, \quad (3.11)$$

and R_{AAB} represents the baryon contributions to R_{AA} and has the form

$$R_{AAB}=i(2\pi)^4\delta^4(p_1+k-p_2-q)\frac{(k^2-\mu_\pi{}^2)(q^2-\mu_\pi{}^2)}{2\mu_\pi{}^4f_\pi{}^2}\times\sum_j\left(\frac{\langle p_2|q_\mu A_b{}^\mu(0)|B_j\rangle\langle B_j|k_\nu A_a{}^\nu(0)|p_1\rangle}{s-m_{B_j}{}^2+i\epsilon}+\frac{\langle p_2|k_\nu A_a{}^\nu(0)|B_j'\rangle\langle B_j'|q_\mu A_b{}^\mu(0)|p_1\rangle}{u-m_{B_j}{}^2+i\epsilon}\right), \quad (3.12)$$

where $B_j=p_2+q$, $B_j'=p_2-k$, and j refers to the possible baryon states. As an example of R_{AAB}, let us calculate the contribution of the nucleon with four-momentum N. Using the nucleon matrix element for the axial-vector current density

$$\langle p_2|A_b{}^\mu(0)|N\rangle=i\bar{u}(p_2)[\gamma^\mu g_A(q^2)+(N-p_2)^\mu h_A(q^2)]\gamma_5\tau_b u(N) \quad (3.13)$$

and the PCAC relationship of Eq. (3.8), we have

$$\begin{aligned}R_{AAN}=-i(2\pi)^4\delta^4(p_1+k-p_2-q)\times\Bigg\{&\frac{(q^2-\mu_\pi{}^2)^2}{\mu_\pi{}^4f_\pi{}^2}2g_A(q^2)[q^2h_A(q^2)-mg_A(q^2)]\delta_{b,a}\\&+\gamma\cdot Qg_{\pi NN}{}^2(q^2)\left(\frac{1}{s-m^2+i\epsilon}-\frac{1}{u-m^2+i\epsilon}\right)\delta_{b,a}\\&+\gamma\cdot Q\bigg[g_{\pi NN}{}^2(q^2)\left(\frac{1}{s-m^2+i\epsilon}+\frac{1}{u-m^2+i\epsilon}\right)\\&+\frac{(q^2-\mu_\pi{}^2)^2}{\mu_\pi{}^4f_\pi{}^2}g_A{}^2(q^2)\bigg]\frac{[\tau_b,\tau_a]}{2}\Bigg\}\,, \quad (3.14)\end{aligned}$$

where $g_{\pi NN}(q^2)$ is the pion-nucleon form factor defined by

$$\langle p_2|n_b(0)|N\rangle=i\bar{u}(p_2)\gamma_5\tau_b g_{\pi NN}(q^2)u(N) \quad (3.15)$$

and we have again taken $k^2=q^2$ for convenience. The above results agree with Schnitzer's[6] determination of the nucleon contribution to R_{AA}.

We can now write Eq. (3.1) as

$$R_{fi}=R_V+R_{AAB}+R_{AA\rho}\,. \quad (3.16)$$

In comparing the above equation with the work of Raman[7] and Schnitzer,[6] the difference is the term $R_{AA\rho}$. It is of interest to note that with the help of PCAC, the term R_{AAB} can be shown to be identical to the term R_B [Eq. (2.9)].

Let us next equate the two different methods for determining the amplitude R_{fi}. From Eqs. (2.3) and (3.16), we have

$$R_\rho+R_B=R_V+R_{AAB}+R_{AA\rho}, \qquad (3.17)$$

which reduces to

$$R_\rho=R_V+R_{AA\rho}. \qquad (3.18)$$

In the limit as q^2 approaches μ_π^2, we have the identity

$$\frac{[\gamma\cdot QF_{1NN\rho}'(t)-(2m\nu+\frac{1}{2}t)F_{2NN\rho}'(t)]}{m_\rho^2-t-i\epsilon}F_{\pi\pi\rho}(t) =\frac{[\gamma\cdot QF_{1NN\rho}'(t)-(2m\nu+\frac{1}{2}t)F_{2NN\rho}'(t)]}{m_\rho^2-t-i\epsilon}F_{\pi\pi\rho}(t). \qquad (3.19)$$

But in the limit as q^2 approaches zero, we have

$$\frac{[\gamma\cdot QF_{1NN\rho}'(t)-(2m\nu+t/2)F_{2NN\rho}'(t)]}{m_\rho^2-t-i\epsilon}F_{\pi\pi\rho}(t) =\frac{1}{f_\pi^2}[\gamma\cdot QF_1'(t)-(2m\nu+\tfrac{1}{2}t)F_2'(t)], \qquad (3.20)$$

which reduces to

$$F_1(t)=f_\pi^2F_{\pi\pi\rho}(t)\frac{F_{1NN\rho}(t)}{m_\rho^2-t-i\epsilon},$$
$$F_2(t)=f_\pi^2F_{\pi\pi\rho}(t)\frac{F_{2NN\rho}(t)}{m_\rho^2-t-i\epsilon}. \qquad (3.21)$$

If we make use of the assumption[10] that $F_{\pi\pi\rho}(t)$ varies very slowly over the range from $t=0$ to m_ρ^2 and can be replaced by the constant m_ρ/f_π which in turn can be rewritten as f_ρ, where f_ρ is the universal ρ coupling constant defined by the relationship

$$\langle 0|V_a(0)|\rho_c^\lambda\rangle=\epsilon_\lambda^\mu(m_\rho^2/f_\rho)\delta_{a,c}, \qquad (3.22)$$

then Eqs. (3.21) become

$$F_1(t)=\frac{m_\rho^2}{f_\rho}\frac{F_{1NN\rho}(t)}{m_\rho^2-t-i\epsilon},\quad F_2(t)=\frac{m_\rho^2}{f_\rho}\frac{F_{2NN\rho}(t)}{m_\rho^2-t-i\epsilon}. \qquad (3.23)$$

Equations (3.23) are just a statement of what is generally known as weak ρ dominance which can be derived from the more general statement that the isovector current density is proportional to the ρ field,[11,12]

$$V_a^\mu(x)=(m_\rho^2/f_\rho)\Phi_a^\mu(x). \qquad (3.24)$$

In deriving Eqs. (3.23) from Eq. (3.24), it is not necessary to have q^2 approach zero, so we may regard Eq. (3.23) as a general statement, true for all values of q^2.

Let us next neglect the term R_{AAB} as was done in the earlier calculations for the S-wave scattering lengths, but include the term $R_{AA\rho}$. Equation (3.16) becomes

$$R_{fi}=R_V+R_{AA\rho}. \qquad (3.25)$$

If we next make the assumption of weak ρ dominance in the form of Eqs. (3.23), we have

$$R_{fi}=\frac{(q^2-\mu_\pi^2)^2}{\mu_\pi^4}U(\nu,t,q^2)+\frac{q^2}{\mu_\pi^2}U(\nu,t,q^2), \qquad (3.26)$$

where

$$U(\nu,t,q^2)=\frac{2i(2\pi)^4}{f_\pi^2}\delta^4(p_2+q-p_1-k)\bar{u}(p_2)[\gamma\cdot QF_1'(t) -(2m\nu+\tfrac{1}{2}t)F_2'(t)]\tfrac{1}{2}[\tau_b,\tau_a]u(p_1). \qquad (3.27)$$

Next, let us compare the results of taking the limits as q^2 approaches zero and μ_π^2 in Eq. (3.26). In the limit as q^2 goes to zero, we have

$$\lim_{q^2\to 0} R_{fi}=U(\nu,t,0), \qquad (3.28)$$

while in the limit as q^2 goes to μ_π^2, we have

$$\lim_{q^2\to \mu_\pi^2} R_{fi}=U(\nu,t,\mu_\pi^2). \qquad (3.29)$$

A point to note here is that we have for simplicity of notation neglected to include in the form factors $F_{1,2NN\rho}(t)$, $F_{1,2}(t)$, and $F_{\pi\pi\rho}(t)$, a possible q^2 dependence, for example, $F_{\pi\pi\rho}(t,q^2)$. We now make the assumption that the q^2 dependence of the above form factors is negligible in the range $q^2=0$ to μ_π^2, for example,

$$F_{\pi\pi\rho}(t,0)\simeq F_{\pi\pi\rho}(t,\mu_\pi^2)=F_{\pi\pi\rho}(t). \qquad (3.30)$$

With this assumption on the form factors, we can see that

$$U(\nu,t,0)=U(\nu,t,\mu_\pi^2).$$

We can now understand in another way why current-algebra calculations give exactly the same results for the S-wave scattering lengths as the calculation using a Feynman diagram with a ρ exchange.[13] A ρ-exchange model is an on-the-mass-shell calculation and would give exactly Eq. (3.29), which comes originally from the term $R_{AA\rho}$, while the current-algebra calculations are performed at $q^2=0$ and would correspond to Eq. (3.28), which comes originally from the term R_V. The ambiguity of being able to add any term which is proportional

[10] From weak ρ dominance of the pion isovector form factor, we have $F_{\pi\pi\rho}(0)=m_\rho/f_\pi=5.64$. We also have (see Ref. 7)

$$F_{\pi\pi\rho}(m_\rho^2)=[4\pi(12m_\rho^2)\Gamma_\rho/(m_\rho^2-4\mu_\pi^2)^{3/2}]^{1/2}\simeq 5.53.$$

[11] M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 51 (1965).
[12] N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).
[13] J. Sakurai, Phys. Rev. Letters 17, 552 (1966).

to q^2 to the R_V contribution was first noticed by Tomozawa.[1] We have just observed that the ρ contribution to R_{AA} comes in naturally as a $q^2/\mu_\pi{}^2$ term and, together with the concept of weak ρ dominance, gives exactly the same results as the current-algebra calculations if we let $q^2=\mu_\pi{}^2$. Furthermore, the ρ-exchange model is identical to the $R_{AA\rho}$ term in the limit as q^2 approaches $\mu_\pi{}^2$. The amplitude is never twice as large[14] since one of the two terms is always zero at the limits of $q^2=0$ and $\mu_\pi{}^2$.

The procedure of calculating the scattering lengths and effective ranges by first letting $q^2\to 0$, then hoping that the scattering lengths and effective ranges extrapolate smoothly back to $q^2=\mu_\pi{}^2$ may be erroneous for the R_V part of the amplitude and may be the reason for the difficulties behind the calculations of $a_{1-}{}^-$ and $b_{0+}{}^-$. However, if we include the $R_{AA\rho}$ term along with R_V, the two terms from Eq. (3.18) can be replaced by R_ρ. From Eqs. (2.7), (2.8), and (3.21), R_ρ is expressible in terms of the form factors $F_1(t)$ and $F_2(t)$. With the assumption on the form factors of the form (3.30), the amplitude $R_\rho(\nu,t,q^2)$ has a very smooth q^2 dependence in the region from $q^2=0$ to $\mu_\pi{}^2$, not mattering if we let $q^2=0$ or $\mu_\pi{}^2$. The following procedure should then assure us of a smooth extrapolation for R_ρ. First we break R_ρ into the A and B amplitudes where

$$R_{fi}=i(2\pi)^4\delta^4(p_1+k-p_2-q)\bar{u}(p_2)\times[A(\nu,t,q^2)+\gamma\cdot QB(\nu,t,q^2)]u(p_1)\,. \quad (3.31)$$

We then extrapolate $A(\nu,t,q^2)$ and $B(\nu,t,q^2)$ to $q^2=0$. We extrapolate back on the mass shell by demanding that ν satisfies the equation

$$|\mathbf{k}|^2=m^2(\nu-\mu_\pi{}^2)/(2m\nu+m^2+\mu_\pi{}^2)\,. \quad (3.32)$$

From Eq. (3.32), we see that ν approaches $\mu_\pi{}^2$ as $|\mathbf{k}|^2$ approaches zero. The above procedure is of course equivalent to letting, $q^2=\mu_\pi{}^2$ directly, as can be seen from Eqs. (3.27)–(3.29). The procedure is different from that of Schnitzer[6] and Raman[7] and would, for example, avoid the large derivatives with respect to q^2 which gave rise to the difficulties that Schnitzer had with the S-wave effective range.

IV. CALCULATION OF SCATTERING LENGTHS AND EFFECTIVE RANGES

In this section we present our calculations of the S- and P-wave scattering lengths and the S-wave effective range, using the results presented in Secs. II and III.

The S- and P-wave scattering lengths and effective ranges are defined by[15]

$$\begin{aligned}\mathrm{Re}f_{0+}{}^\pm&=a_{0+}{}^\pm+b_{0+}{}^\pm|\mathbf{k}|^2\,,\\ \mathrm{Re}f_{1\pm}{}^\pm&=a_{1\pm}{}^\pm|\mathbf{k}|^2+b_{1\pm}{}^\pm|\mathbf{k}|^4\,,\end{aligned} \quad (4.1)$$

where $f_{l\pm}{}^\pm$ are the amplitudes of Chew, Goldberger, Low, and Nambu[16] for the even and odd crossing amplitudes.

The contributions of R_{AAB} to the S- and P-wave scattering lengths and S-wave effective range have already been determined in detail by others and need not be repeated here. We shall just adopt the values given by Raman[7] for our determination of the contribution of R_B to a_L and b_L.

The contribution of R_ρ can be easily calculated by using the relationship $F_\pi(t)\simeq m_\rho/f_\pi$, where we have used the value $f_\pi=0.94\mu_\pi$, and by using the weak ρ-dominance assumptions of Eq. (3.22) together with the empirical isovector form factor fits. Two different fits were used, one by Hofstader and Herman (HH)[17]:

$$F_1(t)=-0.10+\frac{0.60}{1-t/20}\,,\quad F_2(t)=\frac{3.70}{2m}F_1(t)\,, \quad (4.2)$$

and the simple dipole fit[18] for the electric and magnetic form factors:

$$G_E(t)=\frac{1}{2}\frac{1}{(1-t/36.2)^2}\,,\quad G_M(T)=4.70G_E(t)\,, \quad (4.3)$$

where t is in units of the pion mass square. $G_E(t)$ and $G_M(t)$ are just a linear combination of $F_1(t)$ and $F_2(t)$ with the form

$$\begin{aligned}G_E(t)&=F_1(t)+(t/2m)F_2(t)\,,\\ G_M(t)&=F_1(t)+2mF_2(t)\,.\end{aligned} \quad (4.4)$$

The final equations for the effective range and scattering lengths are relatively insensitive to the precise form of the form factors, depending only on $F_1(0)$, $F_2(0)$, and $\partial F_1(0)/\partial t$. Any form-factor fits with roughly the same derivative for $F_1(t)$ at $t=0$ should give similar results. Equations (4.2) give $\partial F_1(0)/\partial t=0.030$, while the dipole formulas (4.3) give $\partial F_1(0)/\partial t=0.018$ (t in $\mu_\pi{}^2$ units). They give similar results for the scattering lengths and effective range.

The numerical values for $a_{0+}{}^-$, $b_{0+}{}^-$, $a_{1+}{}^-$, and $a_{1-}{}^-$ for the two sets of form factors are listed in Table I along with the results of Raman and Schnitzer and the experimental results as given by Hamilton and Woolcock[15] and by Roper *et al.*[19]

The isospin-even results a^+ and b^+ were not listed since the R_ρ term does not contribute to them and our model would give the same results as those given by Raman. There is a possible contribution coming from

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the ϵ isoscalar meson[20] to the isospin-even amplitudes; however, since the a^+ and b^+ results of Raman which neglected the contributions of the ϵ meson were in excellent agreement with experiment, the contribution is probably negligible and was omitted in our calculation. As can be seen from Table I, the results are in agreement with experiment except perhaps for the a_{1-}^- scattering length. The difficulty which Schnitzer had with b_{0+}^- has been overcome, while a_{1-}^- is about the same as Schnitzer's calculation but is an improvement over Raman's value. We note that the σ term mentioned in Sec. I would not affect our calculation if we adopt the usual symmetric form[3] for it. It would then contribute only to the isospin-even amplitudes.

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Nucleon-Nucleon *T*-Violating Force from Electromagnetic Interaction

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A T-violating nucleon-nucleon potential is calculated with a model in which the basic violation is in the electromagnetic interaction. The conditions of Hermiticity and current conservation are applied to choose a T-violating matrix element of the nucleon electromagnetic current. Since current conservation requires that the T-violating term vanish on the mass shell, the longest-range force comes from diagrams corresponding to the exchange of one photon and one pion. In leading nonrelativistic order, there are three spin-dependent T-violating forces of range roughly equal to the Compton wavelength of the pion and of "strength" about 0.01% of the one-pion-exchange potential.

I. INTRODUCTION

IN 1964 Christenson *et al.*[1] found evidence for the 2π CP-violating decay of the K_2^0 meson. If the TCP theorem is valid, as is generally assumed, this experiment implies that T must also be violated. Various suggestions have been made that the violation is in the weak interaction,[2] in a "superweak" interaction,[3] or in the electromagnetic interaction.[4–6] Extensive experiments, especially with K mesons, have not been able so far to distinguish which of these theories is correct. Indeed, experiments involving searches for transverse polarization, polarized γ correlations, polarization-asymmetry inequality, static electric dipole moments, and violation of reciprocity in scattering have so far produced no direct evidence for T violation.

One of the more elegant theories of T violation is that of Lee and co-workers,[4–6] who propose that the violation is in the electromagnetic interaction. One basis for this suggestion is the fact that the ratio of the rate of the T-violating K decay to the normal decay is of the order of $\alpha/2\pi$. In Lee's theory, each fundamental interaction is separately invariant under its own T, C, and P operators. The product operator TCP is the same for every interaction. Violations can arise when the operator of one interaction is defined differently from that of another interaction. Thus, T (and C) violation arises because of the mismatch of the C operators for the strong (C_{strong}) and electromagnetic (C_γ) interactions, just as P is violated because of the mismatch of the parity operators of the weak (P_{weak}) and electromagnetic (P_γ) interactions. The mismatch of the operators could arise because of the existence of hypothetical "a" particles[5] which satisfy

$$C_\gamma a^+ C_\gamma^{-1} = a^-,$$

but

$$C_{\text{strong}} a^+ C^{-1}{}_{\text{strong}} = a^+,$$

or from the minimal electromagnetic interaction of a system of vector mesons.[6]

This electromagnetic violation of T will manifest itself in nuclear forces by time-reversal-violating (TRV)

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