Graphical Representation of *CP*-Nonconservation Parameters in K^0 Decay

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CP-nonconservation parameters in K^0 decay are plotted in a way which directly exhibits the conditions imposed by unitarity and *TCP* invariance. A similar construction shows the conditions under which the hypothesis of *T* invariance can be disproved.

INTRODUCTION

ALMOST all measurements of the amplitude ratios η_{+-} and η_{00} in $K^0 \to 2\pi$ decay are analyzed[1] in terms of the parametrization proposed by Wu and Yang,[2] which allows a simple geometrical interpretation. In this paper we describe an alternative and equally simple graphical representation which retains all the useful features of Wu-Yang plots while avoiding approximations usually made in drawing them.

I. ANALYSIS ASSUMING *TCP* INVARIANCE

The Wu-Yang triangles pictorially represent the equations[2,3]

$$\eta_{+-} = \epsilon + \epsilon' , \tag{1a}$$

$$\eta_{00} = \epsilon - 2\epsilon' , \tag{1b}$$

which are valid to lowest order in the parameters ϵ, ϵ', and $\Delta = 2^{-1/2}[(\mathrm{Re}A_2)/A_0]e^{i(\delta_2-\delta_0)}$. More general equations are[4,5]

$$\eta_{+-} = \epsilon + \epsilon'(1+\Delta)^{-1} , \tag{2a}$$

$$\eta_{00} = \epsilon - 2\epsilon'(1-2\Delta)^{-1} , \tag{2b}$$

where the smallness of ϵ and ϵ' can be directly demonstrated. That Δ should also be a small number, $|\Delta| \ll 1$, is expected from the general success of the $\Delta I = \frac{1}{2}$ rule. The measured value of the $\pi^+\pi^-/\pi^0\pi^0$ ratio C in $K_1^0 \to 2\pi$ decay, which is given by

$$C = 2\left|\frac{1+\Delta}{1-2\Delta}\right|^2 , \tag{3}$$

neglecting terms of relative order ϵ, is consistent with this hypothesis. However, this can be proved from measurements on K^0 decays only if one knows the values of C, η_{+-}, and η_{00} *as well as* the relevant $\pi\pi$ phase shifts.[6] In any case, the observed rate of $K^+ \to \pi^+\pi^0$ decay shows that, barring accidental cancellations, Δ is at least an order of magnitude larger than η_{+-} and η_{00}. This expectation is confirmed by a recent precise measurement of C.[7] Therefore, analyses which do not neglect Δ have eventual practical as well as logical interest. Another approximation which is frequently made in fitting Wu-Yang plots is to require ϵ to have the phase $\phi_w \approx \tan^{-1}[2(m_2-m_1)/\gamma_1]$, which it would have if $|\Delta| \ll 1$ and all η's were of the same order as η_{+-} and η_{00}.[8] Possible corrections to the phase of ϵ due to unknown *CP*-nonconserving amplitudes have been discussed before,[8] but could not be expressed in a simple way, in particular without the assumption $|\Delta| \ll 1$. The method we advocate is free from this difficulty.

Unitarity imposes the condition[4,9]

$$\langle K_1^0 | K_2^0 \rangle = [\tfrac{1}{2}(\gamma_1+\gamma_2) + i(m_2-m_1)]^{-1} \times \sum_f \langle f|T|K_2^0\rangle \langle f|T|K_1^0\rangle^* . \tag{4}$$

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[1] See, e.g., E. Yen, Phys. Rev. Letters **18**, 513 (1967).

[2] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).

[3] To conform to current practice, the structure parameter ϵ determines the composition of short-lived and long-lived kaon states through $K_{1,2}{}^0 = [2(1+|\epsilon|^2)]^{-1/2}[(1+\epsilon)K^0 \pm (1-\epsilon)\bar{K}^0]$. The relative phase of K^0 and $\bar{K}^0$ states is so chosen that their presumably dominant decay amplitudes to the $I=0$ $\pi\pi$ scattering eigenstate coincide both in magnitude, as required by *TCP* invariance, *and* in phase. $\epsilon' = 2^{-1/2}[(\mathrm{Im}A_2)/A_0]e^{i(\delta_2-\delta_0+\pi/2)}$. δ_2 and δ_0 are the *s*-wave $\pi\pi$ phase shifts for $I=2$ and $I=0$, respectively, at an energy equal to the kaon mass.

[4] J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965*, edited by T. Walsh *et al.* (Rutherford Laboratory, Chilton, England, 1966).

[5] We have written Eqs. (2) in the Wu-Yang phase convention.

[6] There are, in general, two solutions with equal and opposite phases for Δ in the two cases: M. Gourdin, Nucl. Phys. **B3**, 207 (1967); G. Field and P. Kabir, in *Proceedings of the International Conference on Particles and Fields*, edited by C. Hagen *et al.* (Wiley-Interscience, Inc., New York, 1967), p. 19. A simple geometrical construction which yields the values of ϵ, ϵ', and Δ from Eqs. (2a), (2b), and (3) will be given elsewhere.

[7] B. Gobbi, D. Green, W. Hakel, R. Moffett, and J. Rosen, Phys. Rev. Letters **22**, 682, (1969).

[8] L. Wolfenstein, Nuovo Cimento **42A**, 17 (1966).

[9] W. D. McGlinn and D. Polis, Phys. Rev. Letters **22**, 908 (1969), recently suggested that an *S*-matrix approach leads to a *different* unitarity relation. However, we have verified that there is no conflict between Eq. (4) and *S*-matrix unitarity.

With our choice of relative phases for K_1^0 and K_2^0 states, TCP invariance requires $\langle K_1^0|K_2^0\rangle$ to be purely real.[10] Thus, unitarity and TCP invariance require that

$$\mathrm{Im}(e^{-i\phi_w}\sum_f \eta_f\gamma_1{}^f)=0\,, \tag{5}$$

with

$$\phi_w=\tan^{-1}[2(m_2-m_1)/(\gamma_1+\gamma_2)]\approx 43°\,, \tag{6}$$

where $\gamma_1{}^f$ is the partial decay rate of K_1^0 into the channel f and

$$\eta_f=\langle f|T|K_2^0\rangle/\langle f|T|K_1^0\rangle\,.$$

It is convenient to write

$$(\gamma_1{}^{2\pi})^{-1}\sum_f \eta_f\gamma_1{}^f=Z+\sum_{f\neq 2\pi}\eta_f\left(\frac{\gamma_1{}^f}{\gamma_1{}^{2\pi}}\right), \tag{7}$$

where

$$Z=(C\eta_{+-}+\eta_{00})/(C+1) \tag{8}$$

is the contribution of 2π channels. Contributions of other channels can be taken into account explicitly if the corresponding partial decay rates $\gamma_1{}^f$ and amplitude ratios η_f are known. Where the complex parameters η_f are unknown, the partial decay rates $\gamma_1{}^f$ and $\gamma_2{}^f$ furnish the *magnitude* $(\gamma_1{}^f\gamma_2{}^f)^{1/2}$ of $\eta_f\gamma_1{}^f$ and thereby a limit on the absolute magnitude of the sum on the right-hand side of Eq. (7). Equation (5) thus leads to the inequality[11]

$$|\mathrm{Im}(e^{-i\phi_w}Z)|\leqslant M_\theta\,, \tag{9}$$

with

$$M_\theta=(\gamma_1{}^{2\pi})^{-1}[\sum_l(\gamma_1{}^l+\gamma_2{}^l)|\mathrm{Im}x_l|\cos\phi_w/(1+|x_l|^2)+\sum_{f\neq 2\pi,f\neq l}\lambda_f(\gamma_1{}^f\gamma_2{}^f)^{1/2}], \tag{10}$$

where the index l denotes leptonic channels, and x_l has its usual meaning; λ_f is a factor which can be approximated by $\cos\phi_w$ if the geometric mean of $\gamma_1{}^f$ and $\gamma_2{}^f$ greatly exceeds $|\langle K_1^0|K_2^0\rangle|$ times their arithmetic mean, and is unity otherwise.[12] If some partial decay *rates* are not known, e.g., those for K_1^0 decay to 3π channels, the experimental upper limits provide a corresponding upper bound. If M_θ could be neglected, Eq. (9) would require the complex vector Z to have the phase ϕ_w and therefore to lie along the straight line $\mathcal{L}_\theta$ through the origin making an angle $\phi_w\approx 43°$ with the positive x axis. For a finite value of M_θ, the condition (9), imposed by unitarity and TCP invariance, relaxes to the requirement that Z lie within a distance M_θ of $\mathcal{L}_\theta$, i.e., inside a band of width $2M_\theta$ whose median line is $\mathcal{L}_\theta$. The "allowed" strip is indicated by shading in Fig. 1. The distance of Z from $\mathcal{L}_\theta$ is a measure of the

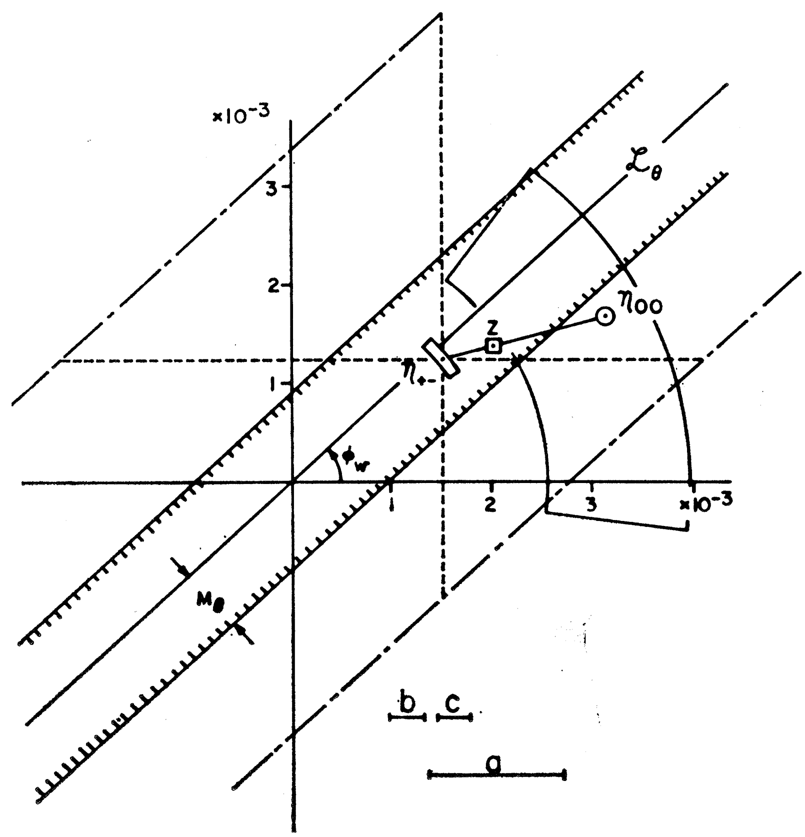

FIG. 1. The small rectangular box indicates the value η_{+-} given in Ref. 15. Unitarity and TCP invariance require Z to fall within the shaded strip, and the dot-dashed lines mark the corresponding boundaries within which η_{00} must lie, for the central value of η_{+-}. The large annular box represents the measured value of η_{00} reported in Ref. 16. (Instead of indicating experimental limits on ϕ_{00} and $|\eta_{00}|$ separately as represented by the annular box, it is preferable to show the correlation between the variations in these quantities corresponding to one or more standard deviations in an over-all χ^2 fit. The χ^2 contours quoted in Ref. 16 are given explicitly in Fig. 2.) The encased point is the value of Z corresponding to the central value of η_{+-} and the arbitrarily chosen value of η_{00} denoted by the encircled point. Values of Reϵ deduced from the charge asymmetry in K_2^0 leptonic decays are marked by a (Ref. 19), b (Ref. 20), and c (Ref. 15), respectively.

[10] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957).
[11] P. K. Kabir, Nature 220, 1310 (1968).
[12] S. L. Glashow and S. Weinberg, Phys. Rev. Letters 14, 835 (1965).

sum on the right-hand side of Eq. (7), i.e., of the magnitude of CP-nonconserving amplitudes in channels other than 2π.

If we neglect corrections of relative order α, arising from coupling of 2π channels with radiative channels [consistent with our neglect of radiative corrections in Eqs. (1)–(3)], another consequence of TCP invariance is[11]

$$\mathrm{Re}Z \approx \mathrm{Re}\epsilon \approx \tfrac{1}{2}\langle K_1^0 | K_2^0 \rangle . \quad (11)$$

TCP invariance also requires[10] that $\langle K_1^0|K_2^0\rangle$ be equal to the charge asymmetry A_l in K_2^0 leptonic decays, multiplied by a proportionality factor $C(x_l)$. If the $\Delta S = \Delta Q$ rule is valid, this factor is exactly unity, whereas if $\Delta S = -\Delta Q$ transitions also occur, $C(x_l)$ can be deduced from measurements of the time dependence of the corresponding leptonic decay mode of neutral kaons.[13] Such measurements therefore specify the ordinate (11) on which Z must lie.

The phase $\psi = \delta_2 - \delta_0(+\pi)$ of Δ is related to the argument χ of $(\eta_{+-} - \eta_{00})$ by[6,14]

$$\cot\psi = \mp \tan\chi \left(\frac{(2C-1)(C-2)\cot^2\chi + 2(C+1)^2}{9C} \right)^{1/2}, \quad (12)$$

where the negative sign gives the phase of the smaller root for Δ, and the positive sign corresponds to the other solution. The approximation $C \approx 2$ and the choice of the root $\Delta \approx 0$ yields the corresponding result of the Wu-Yang analysis, $\psi \approx \chi \pm \frac{1}{2}\pi$. According to Eq. (8), Z is the "centroid" of η_{+-} and η_{00}; therefore $(\eta_{+-} - Z)$ has the same phase as $(\eta_{+-} - \eta_{00})$ and the $\pi\pi$ phase-shift combination ψ can be determined as easily from the Z diagram as from the Wu-Yang plot.

II. COMPARISON WITH EXPERIMENT

The ingredients of Z which are best known experimentally are[7,15]

$$|\eta_{+-}| = (1.90 \pm 0.05) \times 10^{-3}, \quad (13a)$$

$$\arg\eta_{+-} \equiv \phi_{+-} = 39.8° \pm 6°, \quad (13b)$$

$$C = 2.285 \pm 0.055. \quad (13c)$$

There is disagreement on the magnitude of η_{00}, and further measurements of this quantity as well as of its phase are being made. All but one of the reported values of $|\eta_{00}|$ lie between the value of $|\eta_{+-}|$ (13a) and $2\times|\eta_{+-}|$. Chollet *et al.*[16] have also reported a preliminary result

$$\arg\eta_{00} \equiv \phi_{00} = 23° \pm 30°. \quad (13d)$$

The vector η_{+-} corresponding to the quoted values (13a) and (13b) lies within the small rectangular box in Fig. 1; the width of the shaded strip corresponds to the current best limit of 6.5×10^{-4} for M_θ.[11] If the $\Delta S = \Delta Q$ rule applies, this limit becomes 4.1×10^{-4}. Since, within the accuracy of the measurements, the phase of η_{+-} coincides with ϕ_w, the unitarity condition (9) hardly restricts the phase of η_{00} unless

$$|\eta_{00}| \geq (C+1)M_\theta . \quad (14)$$

If $|\eta_{00}|$ is appreciably greater than this value (which happens to be just about equal to $|\eta_{+-}|$), ϕ_{00} must fall approximately within $\pm\sin^{-1}[(C+1)M_\theta/|\eta_{00}|]$ of ϕ_w (modulo π) if Z is to fall within the allowed strip. The reported value (13d) obviously satisfies this condition. The largest contribution to M_θ comes from the poorly known limits on $K_1^0 \to 3\pi$ decay rates. If these rates could be shown to be significantly lower than the corresponding K_2^0 decay rates, as one expects theoretically, M_θ would become appreciably smaller and the restriction on ϕ_{00} strengthened accordingly. The limits between which η_{00} must lie, for a given η_{+-}, if Z is to fall within the unitarity-TCP strip, are straight lines parallel to $\mathcal{L}_\theta$ displaced by

$$C|\eta_{+-}|\sin(\phi_w - \phi_{+-}) \pm (C+1)M_\theta ,$$

respectively. These are shown by dot-dashed lines in Fig. 1 for the central value of η_{+-}. It is clear from the figure that the phase χ of $(\eta_{+-} - \eta_{00})$ will necessarily lie in the first or third quadrant only if $|\eta_{00}|$ is much smaller or much larger than $|\eta_{+-}|$. For values of η_{00} with magnitude comparable to $|\eta_{+-}|$, $\tan\chi$ can have either sign[17] and the values of η_{00} quoted in Ref. 16, shown by the annular box in Fig. 1, represent just such a situation. If we accept the negative values of $\tan\psi$ indicated by analyses of $\pi\pi$ scattering,[18] the condition $|\Delta| \ll 1$ would require $\tan\chi$ to be positive in order to satisfy Eq. (12). This requirement could not be fulfilled if the value of η_{00} fell into either the second or the fourth quadrant relative to η_{+-}.

A question of great interest is whether η_{+-} and η_{00} have identical values. If $\eta_{+-} = \eta_{00}$, Z coincides with η_{+-} and the small rectangular box in Fig. 1 also indicates the range of Z in that case. Unitarity would clearly be satisfied, but we have still to consider the additional TCP test (11). The only reported measurement of

[13] S. Bennett, D. Nygren, H. Saal, J. Sunderland, J. Steinberger, and K. Kleinknecht, Phys. Letters **27B**, 244 (1968).

[14] The magnitude of ψ is given more simply by

$$\sin^2\psi = [9C/2(C+1)^2]\cos^2\chi .$$

[15] These values are taken from the review by J. Steinberger, in *Proceedings of the Topical Conference on Weak Interactions*, edited by J. S. Bell (CERN, Geneva, 1969).

[16] J. Chollet, J. M. Gaillard, M. R. Jane, T. J. Ratcliffe, J. P. Repellin, K. R. Schubert, and B. Wolff, in Ref. 15, p. 309.

[17] The time dependence of the $\pi^+\pi^-/\pi^0\pi^0$ ratio in $K^0 \to 2\pi$ decay directly yields the phase χ. Measurement by B. Gobbi *et al.*, Phys. Rev. Letters **22**, 685 (1969), place restrictions on χ but do not yield a definite sign for $\tan\chi$.

[18] S. Marateck, V. Hagopian, W. Selove, L. Jacobs, F. Oppenheimer, W. Schultz, E. Marquit, D. Huwe, L. J. Gutay, D. H. Miller, J. Prentice, E. West, and W. D. Walker, Phys. Rev. Letters **21**, 1613 (1968).

charge-asymmetry in $K_2^0 \to \pi\mu\nu$ decays yielded[19]

$$A_\mu = (4.05 \pm 1.35) \times 10^{-3}, \qquad (15)$$

which is compatible with such a value of Z if the possible correction factor $C(x_\mu)$ is not too different from unity. The factor $C(x_e)$ has been measured[13] for $K^0 \to \pi e\nu$ decays. The latest value is[15]

$$C^{-1}(x_e) = 0.95 \pm 0.05, \qquad (16)$$

suggesting that there is at most a small correction due to possible violation of the $\Delta S = \Delta Q$ rule. Two measurements of the charge asymmetry in $K_2^0 \to \pi e\nu$ decay reported values[20]

$$A_e = (2.24 \pm 0.36) \times 10^{-3} \qquad (17)$$

and[15]

$$A_e = (3.15 \pm 0.3) \times 10^{-3}. \qquad (18)$$

It is difficult to decide whether these results confirm or disprove the hypothesis that $\mathrm{Re}\epsilon = \mathrm{Re}\eta_{+-}$. Under the hypothesis of TCP invariance, the values of $\mathrm{Re}\eta_{+-}$ and $\mathrm{Re}\epsilon$ determine the value of $\mathrm{Re}\eta_{00}$, which must differ from $\mathrm{Re}\eta_{+-}$ if $\mathrm{Re}\epsilon \neq \mathrm{Re}\eta_{+-}$.

III. TEST OF T INVARIANCE

If T invariance holds, the neutral kaon states with definite lifetimes can be written in the form[21,22]

$$|K_1^0\rangle = [2(1+|\zeta|^2)]^{-1/2} \times [(1+\zeta)|K^0\rangle + (1-\zeta)|\bar{K}^0\rangle], \qquad (19)$$

$$|K_2^0\rangle = [2(1+|\zeta|^2)]^{-1/2} \times [(1-\zeta)|K^0\rangle - (1+\zeta)|\bar{K}^0\rangle]. \qquad (20)$$

Then $\langle K_1^0 | K_2^0 \rangle$ is purely imaginary, so that the unitarity condition (4) yields

$$\mathrm{Re}(e^{-i\phi_w} \sum_f \eta_f \gamma_1{}^f) = 0. \qquad (21)$$

Proceeding exactly as in the TCP-invariant case, we obtain[11]

$$|\mathrm{Re}(e^{-i\phi_w} Z)| \leqslant M_T, \qquad (22)$$

with

$$M_T = (\gamma_1{}^{2\pi})^{-1} [\sum_l r_l \gamma_1{}^l \sin\phi_w + \sum_{f \neq 2\pi, f \neq l} \mu_f (\gamma_1{}^f \gamma_2{}^f)^{1/2}], \qquad (23)$$

where $r_\pm = \pm |1+x_\pm|^{-2}(1-|x_\pm|^2)$ and μ_f is a factor which can be approximated by $\sin\phi_w$ if

$$(\gamma_1{}^f \gamma_2{}^f)^{1/2} \gg \tfrac{1}{2} |\langle K_1^0 | K_2^0 \rangle| (\gamma_1{}^f + \gamma_2{}^f),$$

and by unity otherwise. Unitarity and T invariance thus require Z to lie within a distance M_T of the straight line $\mathcal{L}_T$ drawn through the origin at an inclination of $(\frac{1}{2}\pi + \phi_w) \approx 133°$ to the positive real axis. The present limit on M_T is[11] 4.9×10^{-4}; the allowed strip corresopnding to this value is shaded in Fig. 2. The contribution from leptonic channels, dependent on possible $\Delta S = -\Delta Q$ transitions, is estimated as 0.8×10^{-4}. Given the vector η_{+-}, which lies in the small rectangular box for the values quoted in (13a) and (13b), the condition (22), that Z fall within the allowed strip, requires η_{00} to lie within the band bounded by straight lines (dot-dashed in Fig. 2) parallel to $\mathcal{L}_T$ and displaced by

$$C|\eta_{+-}|\cos(\phi_w - \phi_{+-}) \pm (C+1)M_T$$

on the side of $\mathcal{L}_T$ opposite to η_{+-}. This will only be possible if $|\eta_{00}|$ exceeds the value

$$|\eta_{00}|_{\min} = C|\eta_{+-}|\cos(\phi_w - \phi_{+-}) - (C+1)M_T. \qquad (24)$$

For $\phi_{+-} \approx \phi_w$, and the quoted values of the other parameters, this minimum value is about 2.8×10^{-3}. This limit would increase if the limit on M_T could be reduced, e.g., by improving the limits on $K_1^0 \to 3\pi$ decay rates. If the magnitude of η_{00} exceeds this value, but is smaller than

$$C|\eta_{+-}|\cos(\phi_w - \phi_{+-}) + (C+1)M_T \approx 5.9 \times 10^{-3},$$

its phase can lie anywhere in the range

$$\pi + \phi_w - \alpha < \phi_{00} < \pi + \phi_w + \alpha,$$

with

$$\alpha = \cos^{-1}\{[C|\eta_{+-}|\cos(\phi_{+-} - \phi_w) - (C+1)M_T]/|\eta_{00}|\}. \qquad (25)$$

For $\phi_{+-} \approx \phi_w$, the permitted values of ϕ_{00} fall in the third quadrant for all values of $|\eta_{00}|$ less than 3.9×10^{-3}. If η_{00} lies on the same side of $\mathcal{L}_T$ as η_{+-}, i.e., if $|\phi_{00} - \phi_w| \leqslant \frac{1}{2}\pi$, the T-invariance condition (22) clearly cannot be satisfied for *any* value of $|\eta_{00}|$. Therefore, the reported[16] value (13d) of ϕ_{00} provides direct evidence against the hypothesis of T invariance.

There are indications of T noninvariance[23] even without referring to the measured value of ϕ_{00}. We have seen that if the condition (22) is to be satisfied, the magnitude of η_{00} must exceed the limit (24), whose value already exceeds some of the reported measurements of $|\eta_{00}|$. We shall now show that even if $|\eta_{00}|$

[19] D. Dorfan, J. Enstrom, D. Raymond, M. Schwartz, S. Wojcicki, D. H. Miller, and M. Paciotti, Phys. Rev. Letters **19**, 987 (1967).

[20] S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters **19**, 993 (1967).

[21] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

[22] Because of the difference in the expression of K_1^0 and K_2^0 states as superpositions of K^0 and $\bar{K}^0$ for the TCP-invariant case (Ref. 3) and in the case of T invariance [Eqs. (19 and (20)], interference experiments for the phase and magnitude of the η's must be interpreted as measuring slightly different quantities in the two cases. Since ϵ and ζ can both be shown to have modulus less than 10^{-2}, this will have no practical effect on our conclusions.

[23] R. C. Casella, Phys. Rev. Letters **21**, 1128 (1968); **22**, 554 (1969).

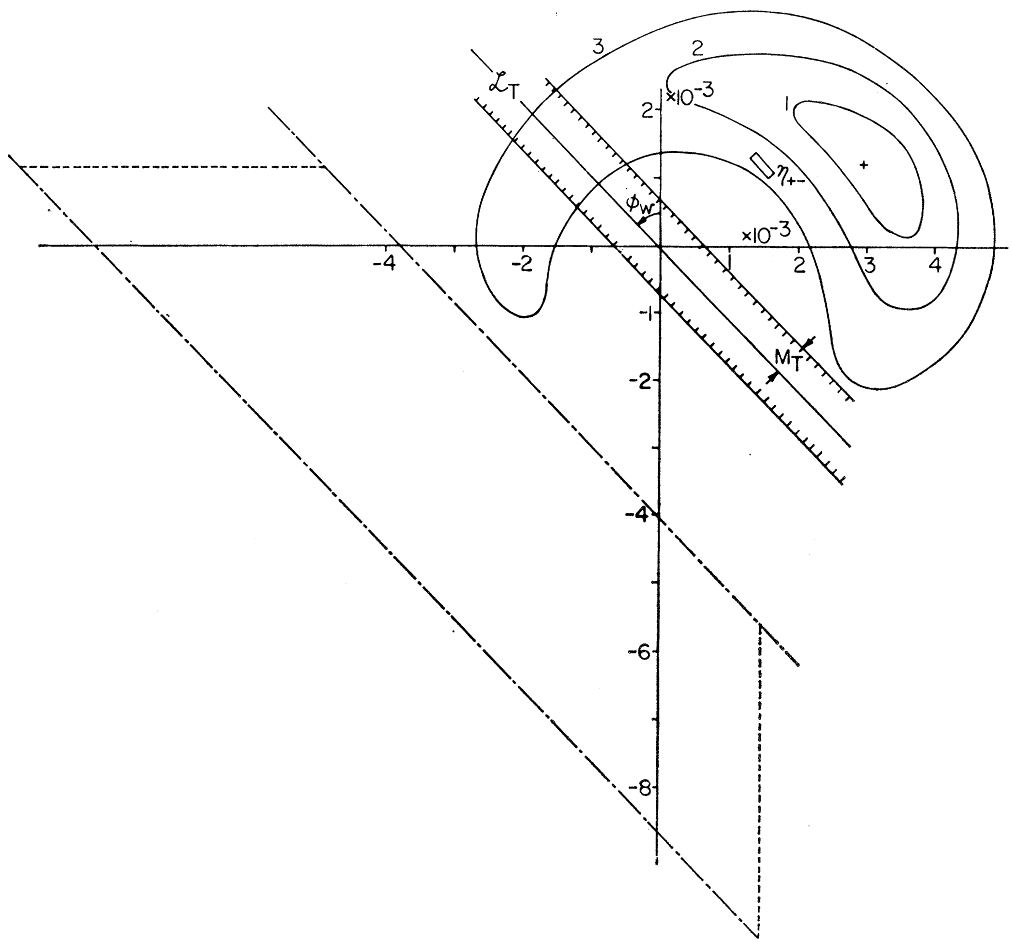

FIG. 2. Unitarity and T invariance require Z to fall within the shaded strip. For the value of η_{+-} enclosed by the small rectangular box, this requires η_{00} to lie in a "T-allowed" band whose boundary is shown by the dot-dashed lines. For illustrative purposes we have superimposed the quoted χ^2 contours for ($\mathrm{Re}\,\eta_{00}$, $\mathrm{Im}\,\eta_{00}$) [transcribed from contours in the ($\phi_{00}, |\eta_{00}|$) plane] corresponding to 1, 2, and 3 standard deviations from the central value reported in the preliminary measurements of Ref. 16 (shown by the cross). Values of η_{00} restricted to the region between the broken lines within the allowed η_{00} band all yield a phase χ in the first quadrant.

satisfies this constraint, the hypothesis of T invariance cannot be reconciled with $\pi\pi$ phase-shift information[18] if the $\Delta I=\frac{1}{2}$ rule is assumed. Referring to Fig. 2, we see that T invariance and the unitarity condition (22) require the vector

$$Z=\eta_{+-}+(\eta_{00}-\eta_{+-})/(C+1) \qquad (26)$$

to lie within the allowed strip. Equation (26) shows further that unless $|\eta_{00}|\gg|\eta_{+-}|$, the magnitudes of Z and η_{+-} are similar. It follows in that case that the phase χ of $\eta_{+-}-Z$ cannot be too different from that of η_{+-} as reported. The phase χ lies in the first quadrant for all values of η_{00} in the allowed band, between the limits shown by the broken lines. For $\phi_{+-}\approx\phi_w$ and the quoted values of the other parameters, this holds for all η_{00}'s satisfying (22) with magnitudes less than 5.4×10^{-3}. The formula analogous to Eq. (12) in the T-invariant case is

$$\cot\psi=\pm\cot\chi\left[\frac{(2C-1)(C-2)\tan^2\chi+2(C+1)^2}{9C}\right]^{1/2}, \qquad (27)$$

where the plus sign now corresponds to the small-$|\Delta|$ solution, and the minus sign to the other. A value of χ in the first quadrant cannot be reconciled with the negative values of $\tan\psi$ from $\pi\pi$ scattering[18] unless one is prepared to accept the solution with $|\Delta|$ of the order of unity, in contradiction to the $\Delta I=\frac{1}{2}$ rule. It may be remarked that such a solution requires the $\Delta I=\frac{3}{2}$ *and* $\Delta I=\frac{5}{2}$ components of $K\to2\pi$ decay amplitudes to be comparable in magnitude to the $\Delta I=\frac{1}{2}$ component. Only thus could one arrange for a large $I=2$ amplitude in $K^0\to2\pi$ decay while maintaining a relatively small

$K^+ \rightarrow \pi^+\pi^0$ decay rate. The requirement $|\Delta| \ll 1$ appears to be incompatible with solutions imposed by T invariance for all likely values of $|\eta_{00}|$, if $\tan\psi$ is a negative number.

IV. SUMMARY

An Argand diagram of the complex vectors η_{+-} and

$$Z = (C\eta_{+-} + \eta_{00})/(C+1)$$

exhibits the relations between CP-nonconservation parameters in K^0 decays. Under the hypothesis of TCP invariance, one has the following results:

(i) Unitarity requires Z to lie near the polar axis $\mathcal{L}_\theta$ at an inclination of $\phi_w = \tan^{-1}[2(m_2 - m_1)/(\gamma_1 + \gamma_2)] \approx 43°$ to the positive real axis. The distance of Z from the axis $\mathcal{L}_\theta$ is a measure of CP nonconservation in decay modes other than 2π channels, and can be bounded by a quantity M_θ determined by the partial decay rates of those modes.

(ii) The value of ReZ must equal the quantity Reϵ deduced from measurements of the charge asymmetry in K_2^0 leptonic decays.

(iii) The phase χ of $(\eta_{+-} - Z)$ determines the $\pi\pi$ phase shift $\psi = \delta_2 - \delta_0 (+\pi)$ through Eq. (12).

Statements analogous to (i) and (iii) can be made in the T-invariant case, and available data do not fit in easily with these conditions. If η_{+-} and η_{00} both lie in the first quadrant, unitarity and T invariance cannot be satisfied for any value of $|\eta_{00}|$.

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