Effect of Annihilation on Matter-Antimatter Separation

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The effect of the annihilation of nucleon-antinucleon pairs upon the process of matter-antimatter separation in blackbody radiation is considered. It is shown to be negligible if annihilation is correctly described by the statistical model.

It has been proposed recently that a phase transition could exist in blackbody radiation at very high temperatures.[1] This effect could have very important astrophysical consequences. The phenomenon follows from the assumption that mesons are bound states consisting of nucleon-antinucleon pairs.

When treating this problem, it was necessary to make several assumptions, of which we shall mention only the ones that are critical:

(1) The effect of the mesons as bound states was not included in the calculation of the virial coefficients after they had been included in the calculation of the free energy as free particles. This exclusion was made in order to avoid double counting, but had no fundamental justification.

(2) The annihilation reactions were neglected when treating the equilibrium of nucleons and antinucleons. This approximation was extremely difficult to accept, since it is known that the annihilation cross section is very large and the elastic cross section almost purely diffractive.

(3) The phase transition was studied by using a virial expansion cut off at the second term.

In the present note, we want to show that approximation (2) is indeed justified, at least in a well-defined model for annihilation. Our discussion is based on an important recent work by Dashen, Ma, and Bernstein,[2] who have written the grand canonical potential Ω in terms of the S matrix as

$$\Omega-\Omega_0=-kT\int dE\frac{e^{-\beta E}}{4\pi i}\left(\mathrm{Tr}AS^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right)_c . \qquad (1)$$

Here Ω_0 is the free-particle grand potential, E the total energy, and S the on-energy-shell S matrix

$$S_{fi}=\delta_{fi}-2\pi i\delta(E_f-E_i)T_{fi}. \qquad (2)$$

A is a permutation operator, so that the trace operation may be applied to unsymmetrized states if necessary. Finally, the index c means that only connected operators should be retained. We refer to the original work for more details.

Let us now review the main effects which determine the second virial coefficient, a factor of the product $N\bar{N}$ of nucleon and antinucleon numbers. According to (2), one has to compute

$$\mathrm{Tr}_2\left(-i\frac{\partial}{\partial E}(T+T^\dagger)+T^\dagger\frac{\overleftrightarrow{\partial}}{\partial E}T\right)_c , \qquad (3)$$

where the index 2 relates to nucleon-antinucleon pair states.

(1) The effect of N-$\bar{N}$ states with angular momentum $J\geq 1$ in the second term of (3) is zero if the corresponding amplitude is purely diffractive, i.e., pure imaginary. It was already shown before that the real phase shifts introduced by nuclear forces have negligible effects.

(2) It is well known that the statistical model correctly describes the spectrum and multiplicities of annihilation, and is, in fact, the only model at our disposal. In this model, the amplitudes for annihilation into a well-defined final quantum state are real and depend only upon energy. Furthermore, they are the same for different orthogonal states.

In the framework of this model, it is obvious that the second term of (3) is zero. Accordingly, the contributions of annihilation to the second virial coefficient could come only from refined phase effects which will be difficult to observe experimentally.

(3) Finally, where there is a bound state, there will be an effect of a decreasing real phase shift, in spite of the annihilation, but it will be multiplied by $e^{-2\delta_I}$ for the first term of (3) and by $e^{-4\delta_I}$ in the second term, where $e^{-2\delta_I}$ is the inelasticity. Our approximation is, therefore, only that the inelasticity *in the S waves* is not too large in a region of c.m. energies around 300 MeV.

Furthermore, let us recall that Dashen, Ma, and Bernstein have completely justified the first approximation we mentioned.

This simple analysis allows us to recover the results of Ref. 1 with only very slight corrections.

Note. Let us mention that it is possible to compute the size of the condensations after a time t by using the fluctuation of velocity as given by the Maxwell distribution. A subsystem of nucleons of radius R and

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[1] R. Omnès, Phys. Rev. Letters 23, 38 (1969).
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density ρ will be displaced by a distance R in the time t if

$$R^5=\rho^{-1}kTt^2, \tag{4}$$

leaving and creating nucleon-antinucleon separation. In the Gamow universe, $kT\simeq 400$ MeV, $\rho\simeq 10^{15}$ g/cm^3, and $t\simeq 10^{-5}$ sec, so that $R\simeq 10^{-6}$ cm.

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Asymptotic Properties of Planar Duality Amplitudes*

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Planar duality amplitudes are shown to have Regge asymptotic behavior when $|s|\to\infty$ at any non-vanishing angle to the real axis with t fixed. The asymptotic behavior of the amplitudes in the same s limit with u fixed is also investigated.

A PROCEDURE has been formulated,[1] based on the extension of the Veneziano formula[2] to N-point functions,[3] for incorporating duality into a perturbation expansion of scattering amplitudes. In this work the authors showed that each of the N-loop four-point planar diagrams has Regge asymptotic behavior in the limit $\mathrm{Re}s\to-\infty$ at fixed t, and that the sum of planar diagrams has Regge behavior, controlled by a complex output trajectory function, in the same limit. In this paper we establish the validity of two additional asymptotic properties of the planar diagrams, which were left unsettled in Ref. 1.[4] The first is a demonstration that Regge behavior continues to hold at fixed t in the half-plane $\mathrm{Re}s>0$. The second property we suggest is that each planar diagram decreases faster than any power as $|s|\to\infty$ at fixed u, the channel free from Regge singularities.

The methods used to study asymptotic limits in this paper are best illustrated with the Veneziano (zero-loop planar) amplitude, for which one has the possibility of using Stirling's formula as a simple check. Therefore, we consider first

$$I_0(s,t)=\int_0^1 dx\, x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1} \tag{1}$$

for fixed t and large s.[5] Introducing the variable $\eta=-\ln x$, we have

$$I_0(s,t)=\int_0^\infty e^{\eta\alpha(s)}(1-e^{-\eta})^{-\alpha(t)-1}d\eta. \tag{2}$$

Since $\alpha(s)$ is a straight-line trajectory and the intercept does not matter at large s, we set $\alpha(s)=|s|e^{i\phi}$. Setting

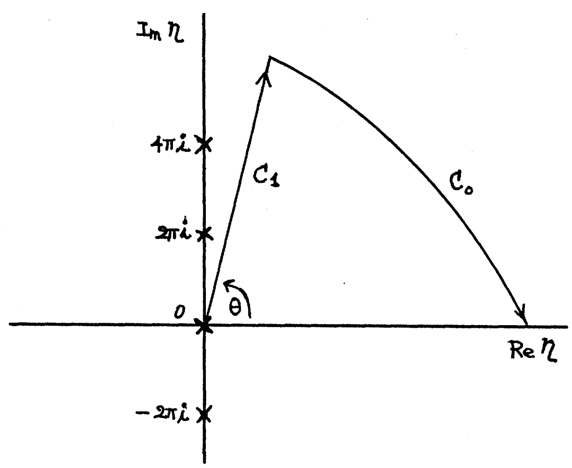

FIG. 1. Integration contour for the fixed-t asymptotic analysis. The singularities of the integrand in Eq. (2) are displayed.

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[4] This implies that we include only the same finite set of factors as in Ref. 1. Possible modifications of the reasoning required to include infinite products are not considered here.

[5] Similar techniques for studying asymptotic properties of integrals were used by S. Mandelstam, Phys. Rev. Letters **21**, 1724 (1968); M. Suzuki, *ibid.* **23**, 205 (1969).