storage of energy over a period of 9×10^{-9} sec. In their experiment the relative phases were doubtlessly random; they do not claim that the laser was operating in a single mode; also, the phase relations between the two beams could not be adequately controlled because of mechanical vibrations arising from the rotating-mirror system (private communication). In the Pfleegor-Mandel (PM) experiment the phases were also random. The required memory span in the PM case was some 15 times longer than in the JN experiment. We must conclude that in the PM experiment there was no cathode memory which could give rise to interference effects when the photons, *as particles*, arrived at the cathode an average 1.4×10^{-7} sec apart. Indeed, if there were such memory, the assumption of a Markovian process would be false. The concept of the photon as a particle here becomes manifestly untenable.[4] The semiclassical description gives an entirely adequate explanation, the only requirement being that the coherence length of the laser radiation shall exceed 3×10^{10} cm $\sec^{-1}\times1.4\times10^{-7}$ sec $=4200$ cm. That requirement explains the need for single-mode lasers in this PM experiment.

[4] Note the parenthetical remark in Ref. 2: "(insofar as particle concepts are applicable to photons)."

The distinction between (a) interference of two light beams from a small single source and (b) interference of beams from two wholly independent separate sources should be emphasized, for in (a) it is possible to think of each single photon as being "partly in both beams," i.e., in a superposition, whereas in (b) *two* wholly independent photons are involved and physically there is no ground for thinking that each such photon can partly impose its state on the other, especially when the two photons are *well separated in time*, i.e., superposition in the sense of (a) is impossible in (b). On the other hand, in the quantum-optical treatment of (a) and (b) it is essentially the *classical fields* which are superimposed, at least in that representation in which the basis states are eigenstates of the annihilation operators whose eigenvalues are just the Fourier components of the classical fields. Thus a superposition of classical fields enters the quantized treatment by the back door. It is not surprising, therefore, that the classical and quantized theories of these effects generally correspond very closely.[5]

I thank Dr. M. Jánossy who informed me of the experiment of Ref. 3 before publication.

[5] This point was brought out in the review article by L. Mandel and E. Wolf, Rev. Mod. Phys. **37**, 231 (1965).

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High-Energy Behavior of Chiral Lagrangians and the Process $\pi_a+\alpha\rightarrow\pi_b+\pi_c+\beta$†

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Weinberg's approach to the high-energy behavior of chiral Lagrangians is applied to the process $\pi+\alpha\rightarrow 2\pi+\beta$, and it is shown that the algebraic relations required to give good asymptotic behavior in this process are the same as those which give good behavior in the reaction $\pi+\alpha\rightarrow\pi+\beta$.

RECENTLY Weinberg has been looking at algebraic aspects of chiral symmetry.[1] He has examined the process $\pi_a+\alpha\rightarrow\pi_b+\beta$ from a chiral Lagrangian point of view and, by demanding the cancellation of terms in the amplitude for this process whose high-energy behavior is experimentally unacceptable, he has derived the following restrictions on the axial-vector coupling matrix $\mathbf{X}(\lambda)$, the vector coupling matrix $\mathbf{T}(\lambda)$, and the mass matrix m^2:

$$[X_a(\lambda),X_b(\lambda)]=i\epsilon_{abc}T_c, \tag{1}$$

$$[X_a(\lambda),[X_b(\lambda),m^2]]\propto\delta_{ab}, \tag{2}$$

where λ is the helicity.

The application of conservation of isospin allows him to write further that

$$[T_a,X_b(\lambda)]=i\epsilon_{abc}X_c(\lambda). \tag{3}$$

In this paper, Weinberg's[1] calculation is repeated for the process $\pi_a+\alpha\rightarrow\pi_b+\pi_c+\beta$, and it is shown that (1)–(3), which are required to give good high-energy behavior in the two-pion case, are also the relationships needed to ensure good asymptotic behavior in the three-pion case.

The process under consideration is

$$\pi(q,a)+\alpha(p,\lambda)\rightarrow\pi(q',b)+\pi(q'',c)+\beta(p',\lambda'),$$

where α and β label the type and isospin of the initial and final target particles; λ and λ' are their helicities; a, b, and c are the pion isovector indices; and q, q', q'', p, and p' are the respective four-momenta. The

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[1] S. Weinberg, Phys. Rev. **177**, 2604 (1969).

pion mass is neglected and only the forward direction is considered. To simplify matters, the coordinate system used is that in which all the momenta are collinear. Under these conditions angular-momentum conservation implies helicity conservation, and so the Feynman amplitude for the process can be written

$$M_{\lambda'\beta bc;\lambda\alpha a}(p'q'q'';pq)=\delta_{\lambda\lambda'}M_{\beta bc;\alpha a}(\omega\omega'\omega''\lambda).$$

This amplitude is now written

$$M=M^{(+)}+M^{(-)},$$

where both $M^{(+)}$ and $M^{(-)}$ are even under interchange of ω and $-\omega'$, i.e.,

$$M^{(+)}{}_{\beta bc;\alpha a}(\omega,\omega',\omega'',\lambda)=\tfrac{1}{2}[M_{\beta bc;\alpha a}(\omega,\omega',\omega'',\lambda)+M_{\beta ac;\alpha b}(\omega,\omega',\omega'',\lambda)],$$

$$M^{(-)}{}_{\beta bc;\alpha a}(\omega,\omega',\omega'',\lambda)=[1/2(\omega+\omega')][M_{\beta bc;\alpha a}(\omega,\omega',\omega'',\lambda)-M_{\beta ac;\alpha b}(\omega,\omega',\omega'',\lambda)].$$

$M^{(+)}$ and $M^{(-)}$ are now evaluated from a general chiral-invariant Lagrangian and the terms which go as ω^0 as $\omega\to\infty$ are picked out using the techniques of Sec. III of Ref. 1, with the additional assumption that the odd final pion is soft, i.e., $q''\to 0$.[2] The various isospin states are then projected out, giving the coefficients of ω^0 in $M^{(-)}(\omega,\lambda)$ [3] as follows:

$$-(8/3)EF_\pi^{-3}\{\epsilon_{dab}[T_d(\lambda),X_c(\lambda)]+\epsilon_{dca}[T_d(\lambda),X_b(\lambda)]+\epsilon_{dbc}[T_d(\lambda),X_a(\lambda)]\}\quad\text{for}\quad T=0,\tag{4}$$

$$2EF_\pi^{-3}\{(\delta_{bc}\epsilon_{dga}-\delta_{ac}\epsilon_{dgb})[T_d(\lambda),X_g(\lambda)]-2i[\delta_{bc}X_a(\lambda)-\delta_{ac}X_b(\lambda)]\}\quad\text{for}\quad T=1,\tag{5}$$

$$-\tfrac{2}{3}EF_\pi^{-3}\{2\epsilon_{dbc}[T_d(\lambda),X_a(\lambda)]+2\epsilon_{dca}[T_d(\lambda),X_b(\lambda)]-4\epsilon_{dab}[T_d(\lambda),X_c(\lambda)]+3\epsilon_{dgb}\delta_{ac}[T_d(\lambda),X_g(\lambda)]-3\epsilon_{dga}\delta_{bc}[T_d(\lambda),X_g(\lambda)]\}\quad\text{for}\quad T=2.\tag{6}$$

For the $T=3$ part of $M^{(+)}(\omega,\lambda)$, the coefficient of ω^0 is zero because of the symmetry properties of the projection operators and the Lagrangian. The $T=2$ part of $M^{(+)}(\omega,\lambda)$ has the following coefficient of ω^0:

$$F_\pi^{-3}\{2\epsilon_{dca}[[X_b(\lambda),m^2],T_d(\lambda)]+2\epsilon_{dcb}[[X_a(\lambda),m^2],T_d(\lambda)]+(\epsilon_{dbg}\delta_{ac}+\epsilon_{dag}\delta_{bc}-2\epsilon_{dcg}\delta_{ab})[[X_g(\lambda),m^2],T_d(\lambda)]\}.\tag{7}$$

The $T=1$ amplitude does not yield any algebraic information, for reasons that are given below, and so is not written down here.

If $P(T)$ represents the projection operator for an isospin state T, then Regge-pole theory predicts the following high-energy behavior for a particle of isospin T exchanged in the t channel:

$$P(T)M^{(-)}{}_{\beta bc;\alpha a}(\omega,\lambda)\propto\omega^{\alpha_T(0)-1},\tag{8}$$

$$P(T)M^{(+)}{}_{\beta bc;\alpha a}(\omega,\lambda)\propto\omega^{\alpha_T(0)},\tag{9}$$

where $\alpha_T(0)$ is the value of the dominant trajectory of isospin T at $t=0$.

Now the condition is imposed that the asymptotic behavior of M which has been calculated from tree graphs not be worse than would be predicted by Regge behavior.

Since $\alpha_T(0)<1$ for all Regge trajectories, Eq. (8) gives $P(T)M^{(-)}{}_{\beta bc;\alpha a}(\omega,\lambda)\to 0$ as $\omega\to\infty$. So the coefficients of ω^0 in $M^{(-)}(\omega,\lambda)$ [i.e., (4)–(6)] are all equal to zero. These conditions are all satisfied if Eq. (3) is true.

For $T=2$ there are indications that[4] $\alpha_2(0)<0$, so Eq. (9) gives $P(2)M^{(+)}{}_{\beta bc;\alpha a}(\omega,\lambda)\to 0$ as $\omega\to\infty$, which implies that the coefficient (7) is zero, which holds if (2) and (3) are true.

For $T=1$, $\alpha_1(0)>0$ (an A_1 particle can be exchanged), so that this component of $M^{(+)}$ does not yield any algebraic conditions.

So Eqs. (2) and (3), which were derived by considering the two-pion amplitude, are the conditions which must be imposed on the three-pion amplitude to ensure that its high-energy behavior not be worse than that predicted by Regge behavior.

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[2] In the t channel all pions are emitted pions. If $M^{(+)}$ and $M^{(-)}$ are defined such that they are even under the interchange of ω and $-\omega''$, then π_b must be taken as the soft pion.

[3] Since $\omega''=0$ and ω is related to ω' by conservation of energy, $M^{(+)}$ and $M^{(-)}$ are functions of ω and λ only.

[4] It is assumed that there are no Regge trajectories with $T=2$ and $\alpha(0)\geq 1$. See R. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters **21**, 576 (1968).