Origin of the Weak-Interaction Angle. II

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We present a theory of the origin of the weak-interaction angle θ, based on a self-consistency condition linking weak, strong, and electromagnetic symmetry-breaking effects. The value of θ is related to the observed pattern of breaking of $SU(3)\otimes SU(3)$, and depends upon the value of a new parameter ξ, which gives the strength of the weak corrections to strong processes. A characteristic result of the theory is the prediction of a nonelectromagnetic isospin breaking. This solves the $\eta\rightarrow 3\pi$ puzzle and accounts for the observed deviation from the Dashen sum rule: $m_{K^+}{}^2-m_{K^0}{}^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2$. Although we are not able to give a theoretical evaluation of ξ in a realistic model of weak interactions, a phenomenological determination of ξ is proposed, based on the observed isospin-breaking mass splittings among pseudoscalar mesons. In this way we get the prediction $\theta\approx 0.25$, which is in good agreement with the experimental value.

I. INTRODUCTION AND OUTLINE

RECENTLY, we presented a theory of the weak-interaction angle θ.[1] In this paper we intend to discuss this theory in greater detail and to present some of those aspects which were only outlined in I, as well as further developments of our ideas.

Our theory is based on the idea that the angle θ, which determines the relative orientation of the weak forces and of the $SU(3)$ breaking, should arise from a dynamical interplay of weak, electromagnetic (e.m.), and strong forces.

To enforce such a philosophy we assume that $SU(3)$ breaking arises from a theory where weak and e.m. forces are the only explicit sources of breaking. This is essential: Since we want to obtain the direction of the breaking as an output, we cannot include informations about the breaking itself as an input.

First attempts to determine θ along these lines[2,3] considered a $SU(3)$-symmetric theory of strong interactions giving rise to asymmetric solutions, on which the effects of weak and e.m. interactions were superimposed as small perturbations. In the absence of such perturbations, nonsymmetric solutions of the symmetric theory appear in families of physically equivalent solutions, formally obtainable one from the other by $SU(3)$ rotations. However, only a finite number among these solutions are a good starting point for the actual solution in the presence of the small perturbations. The first effect of the e.m. and weak corrections thus consists in choosing the direction of the breaking.

Unfortunately, in the actual solutions, the direction of breaking is such that $\theta=0°$ or $90°$, and it has been argued[2] that this result follows quite generally if weak interactions are treated as a small perturbation.

In I a new approach was proposed in which weak, e.m., and strong breakings are treated on the same footing, and are required to satisfy a consistency condition which leads to a definite value of θ, in good agreement with the observed value.

The starting point of the work in I is the consideration that higher-order weak corrections to hadron processes can, in principle, give rise to large effects, and cannot be treated in a purely perturbative framework. The existence of such large effects is suggested by the formal appearance of divergences of increasing order when the usual Feynman-diagram technique is applied in the evaluation of such corrections. These divergences should only be taken as an indication of the failure of the usual treatment and of the fact that in a better computational scheme large, but finite, results will be obtained. This point has been especially stressed by Lee,[4] who also showed in a particular example that the summing up of all the cutoff perturbation series, before letting the cutoff parameter go to infinity, leads to sensible results.

We have based our work on a phenomenological model of strong interactions which has been studied in particular by Glashow and Weinberg,[5] and by Gell-Mann, Oakes, and Renner,[6] in which the Hamiltonian which describes the strong interactions of hadrons has the form

$$H=H_0+h\,, \tag{1.1}$$

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[1] N. Cabibbo and L. Maiani, Phys. Letters **28B**, 131 (1968), hereafter referred to as I.

[2] N. Cabibbo, in *Hadrons and Their Interactions*, edited by A. Zichichi (Academic Press Inc., New York, 1968); N. Cabibbo and G. De Franceschi, in Proceedings of the Eighth Nobel Symposium on Elementary Particles Theory, Aspenäsgärden, 1968 (to be published).

[3] L. Michel and L. Radicati, in *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy*, edited by A. Pearlmutter, C. A. Hurst, and B. Kursunoglu (W. H. Freeman and Co., San Francisco, 1968).

[4] T. D. Lee, Nuovo Cimento **59A**, 579 (1969).

[5] S. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[6] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

where H is invariant under the chiral $SU(3)\otimes SU(3)$ group and the breaking term h transforms as a $(3,\bar{3})\oplus(\bar{3},3)$ under the same group. The use of such a Hamiltonian, which explicitly includes breaking terms, is not in contradiction with our previous requirement, since we shall assume that the structure of h is determined by a self-consistency condition.

In order to specify completely the symmetry breaking contained in such a theory, one should also have some knowledge of the symmetry and symmetry breaking in the vacuum. According to the investigations of Refs. 5 and 6, this is a very important portion of the breaking, and accounts in particular for the large mean masses of baryon multiplets. In the limit $h\rightarrow 0$, one would have a spontaneously broken $SU(3)\otimes SU(3)$, with particles organized in multiplets of $SU(3)$, and the conservation of axial currents would be achieved through zero-mass pseudoscalar mesons. The finite masses of pseudoscalar mesons are thus directly connected with the perturbation term h.

If one includes effects of the weak forces, one finds that the strongest self-energy corrections (i.e., those corresponding to the leading divergences at each order) amount to a modification of h, δh^W, which depends upon h and belongs to the same $(3,\bar{3})\oplus(\bar{3},3)$ representation.

Electromagnetic corrections can be divided in a tadpole part $\delta h^{\text{e.m.}}$ plus residual (nontadpole) terms. In a theory exactly invariant under $SU(3)\otimes SU(3)$ (i.e., with $h=0$ and symmetric vacuum), one would expect that $\delta h^{\text{e.m.}}=0$, since the minimal e.m. interaction is still formally invariant under the chiral $U(2)\otimes U(2)$ subgroup corresponding to U spin.[7] In general, one expects $\delta h^{\text{e.m.}}$ to be different from zero both because of the explicit breaking due to the h term, and of the noninvariance of the vacuum under chiral transformations. Since the latter causes the largest deviations from exact symmetry, we expect $\delta h^{\text{e.m.}}$ to be mainly determined by the breaking of the vacuum, and to be, to a good extent, independent of h. The arising of e.m. tadpole contributions will be discussed in greater detail in Secs. II and III.

We can now introduce our main hypothesis, which links dynamically the strong, weak, and e.m. interactions. This consists in requiring that H give a complete description of the hadron dynamics (mass spectra, etc.), which is not further modified by inclusion of weak and e.m. tadpole corrections. In I we interpreted this condition, as implying the equation

$$\delta h^W+\delta h^{\text{e.m.}}=0. \tag{1.2}$$

In order to enforce this condition one had to rely on the fact that nondiagonal terms in δh^W (i.e., parity- and strangeness-violating terms) could be discarded, since they can be reexpressed as divergences of currents, by the use of the equations of motion implied by Eq. (1.1). It has since become clear to us that this procedure, although justified in second-order calculations of the weak corrections, is generally not valid; nondiagonal terms must be retained and properly interpreted. This led us to a new interpretation of the self-consistency requirement.

We will give here a brief outline of the new formulation; Sec. V contains a more detailed and formal treatment.

We first note that the violation of parity and strangeness caused by δh^W is only apparent. In fact, after inclusion of weak corrections, the breaking term in the Hamiltonian h is changed into $\tilde{h}$:

$$\tilde{h}=h+\delta h^W(h),$$

and $\tilde{h}$ can always be rediagonalized by a change of frame of reference which is equivalent to a $SU(3)\otimes SU(3)$ transformation and to a redefinition of both parity and strangeness. On the other hand, the vacuum state cannot fail to be affected by the weak corrections embodied in the transition from h to $\tilde{h}$, and, barring pathological situations, the new vacuum will be invariant under the parity and strangeness operations defined by $\tilde{h}$. This is made plausible by the following argument. In the absence of any explicit breaking term, if the vacuum is not symmetric, we would have an infinity of possible degenerate vacua, corresponding to as many different definitions of parity and, if $SU(3)$ as well as $SU(3)\otimes SU(3)$ is broken, of strangeness. When the explicit breaking term is turned on, the degeneracy should disappear and one solution only be chosen, most reasonably the one which points in the same direction as the breaking h or $\tilde{h}$. This situation has been explored in Refs. 2 and 3 in the case of a spontaneous $SU(3)$ breaking.

The e.m. correction $\delta h^{\text{e.m.}}$ has then to be computed in terms of the dynamics defined by $\tilde{h}$ and the new vacuum state, and will therefore conserve the same parity and strangeness. We thus see that, although we expect the strength of $\delta h^{\text{e.m.}}$ to be roughly independent of $\tilde{h}$, its direction via that of the vacuum state is determined by $\tilde{h}$.

In the light of these remarks we may formulate our condition as the requirement that $\tilde{h}+\delta h^{\text{e.m.}}$ in the new frame of reference be equal to h in the old frame. If this requirement is met, the net effect of the inclusion of weak and e.m. tadpole corrections to H simply consists in a change of frame in the $SU(3)\otimes SU(3)$ space. This condition will be seen to determine the value of θ, both in the old and in the new frame, which will be identified with the physical one.

The value of θ is found to depend upon a new parameter ξ related to the strength of the weak corrections to h. A characteristic result of our theory is the appear-

[7] In a recent paper S. Adler [Institute for the Advanced Studies, Princeton, report (unpublished)] casts some doubts on the consistency of such formal manipulations, indicating that, in order to obtain a consistent theory, one might be forced to introduce nonminimal couplings (e.g., allowing for $\pi^0\rightarrow\gamma\gamma$) which would spoil the expected symmetry.

ance of an isospin breaking which is not of an e.m. origin, whose strength directly depends upon ξ.

Although we have not been able to compute the value of ξ in the frame of a realistic model of weak interactions the appearence of this new source of isospin breaking opens the possibility of a phenomenological determination of ξ. In this respect, a very favorable process appears to be the $\eta \to 3\pi$ decay which in the soft-pion limit cannot proceed via a conventional e.m. interaction.[8] Moreover, Dashen[9] has recently pointed out that a purely e.m. interaction should give rise to the mass formula

$$m_{K^+}{}^2 - m_{K^0}{}^2 = m_{\pi^+}{}^2 - m_{\pi^0}{}^2 .$$

We will show that the discrepancy, which is quite large, can be explained by this new isospin breaking, leading to an approximate evaluation of ξ.

We close this introduction with a brief summary of the contents of the following sections.

Sections II and III are devoted to the study of symmetry breaking in the vacuum and to the emergence and structure of e.m. tadpoles. This will be done with the aid of a simple generalization of the σ model.[10] Section IV contains an analysis of the structure of δh^W. We will argue, using different models, that the form of δh^W is essentially unique, apart from the scale parameter ξ. In Sec. V we apply our consistency condition to determine the value of θ.

In Sec. VI we discuss the problem of isospin breaking. Finally, Sec. VII is devoted to a discussion both of the present state of our program and of lines of future development.

II. SYMMETRY BREAKING IN THE VACUUM. GENERAL EQUATIONS

Following the authors of Refs. 5 and 6, we consider a model Lagrangian of the form

$$L = L_0 + L' , \tag{2.1}$$

where L_0 is formally invariant under $SU(3)\otimes SU(3)$ and L' is a breaking term. The model will be based on two nonets of scalar and pseudoscalar fields σ_i and π_i ($i=0, \ldots, 8$), transforming as $(3,\bar{3})\oplus(\bar{3},3)$. These fields can be put together to form a 3×3 matrix:

$$\mathfrak{M} = \sum_{i=0}^{8} (\sigma_i + i\pi_i)\frac{\lambda_i}{\sqrt{2}} . \tag{2.2}$$

L' will be assumed to be a linear combination of these fields

$$L' = \mathrm{Tr}(\mathfrak{M} h^\dagger + \mathfrak{M}^\dagger h) . \tag{2.3}$$

A transformation of $SU(3)\otimes SU(3)$ can be characterized by a pair of unitary, unimodular matrices (U,V). The transformation law of $\mathfrak{M}$ is

$$\mathfrak{M} \to U\mathfrak{M}V^\dagger , \quad \mathfrak{M}^\dagger \to V\mathfrak{M}^\dagger U^\dagger . \tag{2.4}$$

The subgroup with $U=V$ generates the usual $SU(3)$ transformations; pure chiral transformations correspond to $V^\dagger = U$.

The natural definition of parity, according to Eq. (2.2), is

$$P\mathfrak{M}P^{-1} = \mathfrak{M}^\dagger , \quad P(U,V)P^{-1} = (V,U) . \tag{2.5}$$

Actually, there are infinitely many possible definitions of parity, all equivalent up to a group conjugation. These parity operations are defined in terms of a unitary, unimodular matrix X, as

$$\begin{aligned} P_X \mathfrak{M} P_X{}^{-1} &= X\mathfrak{M}^\dagger X , \\ P_X(U,V)P_X{}^{-1} &= (XVX^\dagger, X^\dagger UX) . \end{aligned} \tag{2.6}$$

This parity operation leaves invariant the $SU(3)$ subgroup whose elements are $(U,X^\dagger UX)$. Under conjugation with an element (U,V) of the group, we have

$$X \to \tilde{X} = UXV^\dagger . \tag{2.7}$$

If $SU(3)\otimes SU(3)$ is an exact symmetry, all these definitions are physically equivalent; we could not tell a σ from a π. If the symmetry is broken to a definite $SU(3)$, defined as above by a given X, P_X is singled out as the only possible parity operation.

We also introduce a spinor field ψ, which for simplicity we take to belong to the quark representation $(3,1)\oplus(1,3)$. This transforms according to

$$\psi \to (Va_+ + Ua_-)\psi , \tag{2.8}$$

where

$$a_\pm = \tfrac{1}{2}(1\pm\gamma_5) .$$

To be definite, we shall occasionally use a specific model for the invariant Lagrangian L_0, constructed, for the meson fields, out of the bilinear invariant $\mathrm{Tr}(\mathfrak{M}\mathfrak{M}^\dagger)$, the trilinear forms $\det\mathfrak{M}$ and $\det\mathfrak{M}^\dagger$ (which should appear symmetrically if L_0 has to be invariant under parity), the quadrilinear forms $\mathrm{Tr}(\mathfrak{M}\mathfrak{M}^\dagger)^2$ and $(\mathrm{Tr}\mathfrak{M}\mathfrak{M}^\dagger)^2$. We will then write

$$\begin{aligned} L_0 = \bar{\psi} i\gamma\cdot\partial\psi + g\bar{\psi}(\mathfrak{M}a_+ + \mathfrak{M}^\dagger a_-)\psi \\ -\tfrac{1}{2}\mathrm{Tr}(\partial^\mu\mathfrak{M}\partial_\mu\mathfrak{M}^\dagger) + \nu\,\mathrm{Tr}(\mathfrak{M}\mathfrak{M}^\dagger) - \lambda[\mathrm{Tr}(\mathfrak{M}\mathfrak{M}^\dagger)]^2 \\ -\mu\,\mathrm{Tr}(\mathfrak{M}\mathfrak{M}^\dagger)^2 + \tau(\det\mathfrak{M} + \det\mathfrak{M}^\dagger) . \end{aligned} \tag{2.9}$$

If $\tau=0$, L_0 is actually invariant under the larger group $U(3)\otimes U(3)$ (i.e., we may drop the condition $\det U = \det V = 1$, and at the same time the condition $\det X = 1$ in the definition of P_X). We can for the moment consider λ, μ, ν, and τ as free parameters. One might wish to impose the stability conditions

$$\lambda > 0 , \quad \mu > -\lambda , \tag{2.10}$$

which ensure that the Hamiltonian derived from L_0 is bounded from below.

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[8] D. G. Sutherland, Phys. Letters **23**, 384 (1966).
[9] R. Dashen, Phys. Rev. **183**, 1245 (1969).
[10] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

The symmetry properties of the theory are characterized by the matrix h appearing in Eq. (2.3), as well as by the symmetry properties of the vacuum, described by a matrix η:

$$\eta=\langle 0|\mathfrak{M}|0\rangle. \qquad (2.11)$$

Even in the limit $h=0$, η can be different from zero. This situation is usually referred to as spontaneous breaking.

The general conditions to be imposed on η have been discussed in Ref. 11. One first introduces a displaced field

$$M(x)=\mathfrak{M}(x)-\eta.$$

In terms of M, the Lagrangian will have the form

$$L=L_F(M,\eta)+L_{\rm int}(M,\eta)+{\rm Tr}[M^\dagger(G_0(\eta)+h)+{\rm H.c.}]+S_0(\eta)+{\rm Tr}(\eta h^\dagger+\eta^\dagger h), \qquad (2.12)$$

where $L_F(M,\eta)$ is the free-field part, with masses depending upon η. $L_{\rm int}(M,\eta)$ contains terms of third order or more in M as well as the interaction of M with the fermion fields, with coefficients again depending upon η. If L_0 is the one given in Eq. (2.9), fourth-order terms will in fact be the same as in the original Lagrangian. We have written explicitly the terms linear in M and the c-number terms; the matrix $G_0(\eta)$ is related to the c number $S_0(\eta)$ through the equation

$$[G_0(\eta)]_{ij}=\frac{\partial}{\partial\eta_{ij}}S_0(\eta) \quad (i,j=1,2,3). \qquad (2.13)$$

From the Lagrangian (2.12), one then constructs the complete, proper, connected vacuum-to-vacuum amplitude $S(\eta,h)$. In computing $S(\eta,h)$, one has not to include contributions from graphs where an M line ends in the vacuum because Eq. (2.11) implies that

$$\langle 0|M|0\rangle=0. \qquad (2.14)$$

We will then have

$$S(\eta,h)=S_0(\eta)+{\rm Tr}(\eta h^\dagger+\eta^\dagger h)+S'(\eta), \qquad (2.15)$$

where $S'(\eta)$ arises from graphs containing loops and does not depend upon h (in fact, h appears only in the term linear in M, which contributes to the graphs that we have previously excluded).

Equation (2.14) implies the vanishing of the proper, connected one-meson–to–vacuum amplitude, and leads therefore to the equation[11]

$$\frac{\partial S}{\partial\eta_{ij}}=[G_0(\eta)]_{ij}+h_{ij}+\frac{\partial}{\partial\eta_{ij}}S'(\eta)=0, \qquad (2.16)$$

which gives the desired condition on η.

With the same method one can study the effect of the introduction of minimal e.m. interactions. For the Lagrangian equation (2.1), with L_0 given by Eq. (2.9), this can be done via the substitution

$$\partial_\mu\psi\to\partial_\mu\psi-ieA_\mu Q\psi,$$
$$\partial_\mu\mathfrak{M}\to\partial_\mu\mathfrak{M}-ieA_\mu[Q,\mathfrak{M}],$$
$$\partial_\mu\mathfrak{M}^\dagger\to\partial_\mu\mathfrak{M}^\dagger-ieA_\mu[Q,\mathfrak{M}^\dagger],$$

where Q is the 3×3 matrix corresponding to the charge operator. The new term in the Lagrangian is then

$$L^{\rm e.m.}=eA_\mu\bar\psi\gamma_\mu Q\psi+\tfrac{1}{2}i\,{\rm Tr}\{[Q,\mathfrak{M}]\partial^\mu\mathfrak{M}^\dagger+\partial^\mu\mathfrak{M}[Q,\mathfrak{M}^\dagger]\}-e^2A_\mu A^\mu\,{\rm Tr}\{[Q,\mathfrak{M}][Q,\mathfrak{M}^\dagger]\}. \qquad (2.17)$$

This term is still formally invariant under the $[U(2)\otimes U(2)]_U$ subgroup whose elements are pairs (U,V) such that

$$[U,Q]=[V,Q]=0,$$

and is invariant under a parity operation P_X, provided that

$$[X,Q]=0.$$

The effect of $L^{\rm e.m.}$ will be to add new terms to the vacuum-vacuum amplitude $S(\eta,h)$, which can be written now as

$$S(\eta,h)=S_0(\eta)+{\rm Tr}(\eta h^\dagger+n^\dagger h)+S'(\eta)+S^{\rm e.m.}(\eta,Q).$$

The new term (obviously of order e^2) must be a function of η, invariant under the $[U(2)\otimes U(2)]_U$ group defined above, i.e., it will contain terms of the kind

$$a\,{\rm Tr}(\eta\eta^\dagger)+b\,{\rm Tr}(\eta Q\eta^\dagger)+\cdots. \qquad (2.18)$$

Equation (2.16), accordingly, will be modified into

$$[G_0(\eta)]_{ij}+\frac{\partial}{\partial\eta_{ij}}S'(\eta)+\frac{\partial}{\partial\eta_{ij}}S^{\rm e.m.}(\eta,Q)+h_{ij}=0. \qquad (2.19)$$

The new term will cause a shift in the value of η, and if we put

$$\eta=\bar\eta+\delta\eta, \quad \delta\eta=O(e^2), \qquad (2.20)$$

where $\bar\eta$ is a solution of Eq. (2.16), the new solution η will be then determined to order e^2 by the equation

$$[G_0(\eta)]_{ij}+\frac{\partial}{\partial\eta_{ij}}S'(\eta)+\frac{\partial}{\partial\eta_{ij}}S^{\rm e.m.}(\eta,Q)_{\eta=\bar\eta}+h_{ij}=0. \qquad (2.21)$$

Equation (2.21) shows that the effect of electromagnetism is equivalent to adding a new term $\delta h^{\rm e.m.}(\bar\eta)$ to h.

III. STRUCTURE OF VACUUM BREAKING; THE ARISING OF e.m. TADPOLES

In this section we study the structure of solutions to Eq. (2.19). The nature of the stable solutions (i.e., those yielding real masses, etc.) in general will depend upon the values of the free parameters of the model [i.e., ν, λ, μ, τ for the model Lagrangian equations (2.1) and (2.9)]. However, instead of attempting a general study of all possible solutions, we will accept

[11] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

the indications given by phenomenological analyses of the experimental situation.[6]

The suggestion one derives from these analyses is that the vacuum breaking η is essentially a $SU(3)$ singlet, and accounts for the major part of $SU(3) \otimes SU(3)$ breaking. This causes the appearance of large values for hadron masses. The pseudoscalar mesons, in the limit $h=0$, would act as an octet of Goldstone bosons. The smallness of pseudoscalar octet masses as well as the success of the hypothesis of partially conserved axial-vector current (PCAC) can be taken as an indication that the effect of h is relatively small, and can thus be treated as a perturbation. We therefore focus our attention on solutions of Eq. (2.19) of the form

$$\eta=\eta_0+\delta\eta\,,$$

where η_0 is a $SU(3)$ singlet solution ($\eta_0=\eta_0\mathbf{1}$) of the equation

$$G_0(\eta)_{ij}+(\partial/\partial\eta_{ij})S'(\eta)=0\,. \tag{3.1}$$

Equation (3.1) admits, in general, nonzero solutions, which appear in degenerate families because of the underlying $SU(3)\otimes SU(3)$ symmetry: Given a solution η_0, $U\eta_0V^\dagger$ will also satisfy Eq. (3.1). This degeneracy will be removed at least partially in the solutions of the complete Eq. (2.19).

The removal will be complete, i.e., we will have only isolated solutions (possibly only a single one), if the addition of the e.m. perturbation and of the h term reduces the symmetry of the Lagrangian to that of the solution, i.e., for the physically interesting solutions, to the combined gauge groups of charge and hypercharge.

In exploring the structure of the solutions to Eqs. (2.19) and (3.1), one can use the model Lagrangian (2.9) in the phenomenological approximation,[12] which consists in neglecting the loop term $S'(\eta)$. This can be justified by noting that the Lagrangian (2.9) already has enough complexity for the phenomenological approximation to cover the range of solutions one might have to deal with in the exact treatment. In fact, many properties of the solutions we shall prove in this context can be shown to hold in general, as will be discussed elsewhere. Equation (3.1) then becomes[13]

$$\begin{aligned}G_0(\eta)=\nu\eta-2\lambda\eta\,\mathrm{Tr}(\eta\eta^\dagger)-2\mu\eta\eta^\dagger\eta\\ +\tau[\eta^\dagger\eta^\dagger+\tfrac{1}{2}(\mathrm{Tr}\eta^\dagger)^2-\eta^\dagger\,\mathrm{Tr}\eta^\dagger-\tfrac{1}{2}\mathrm{Tr}(\eta^\dagger)^2]=0\,.\end{aligned} \tag{3.2}$$

Equation (3.2) is covariant under $SU(3)\otimes SU(3)$ transformations of η, so that we may, without loss of generality, assume η to be diagonal.[14] Furthermore, upon multiplying Eq. (3.2), by $\eta^\dagger$ one sees[13] that $\det\eta$ is a real number, so that η can be chosen as a real diagonal matrix.[14] This means that any solution of Eq. (3.2) allows for a definition of parity, i.e., the theory will be parity-conserving. Moreover, from Eq. (3.2) one sees that the eigenvalues of η satisfy a quadratic equation, with coefficients which are functions of $\mathrm{Tr}\eta$, $\mathrm{Tr}\eta^2$, and $\det\eta$.[15] It follows, then, that we cannot have three different eigenvalues, i.e., either η is a multiple of the unit matrix corresponding to exact $SU(3)$, or it preserves an $SU(2)$ symmetry.[16]

As stated above, we shall discuss briefly only the case where η is $SU(3)$-invariant.

We then put

$$\eta_0=\eta_0\cdot\mathbf{1}\,,\quad \eta_0\ \text{real}.$$

We have only two possibilities:

(1) $\eta_0=0$, i.e., no spontaneous breakdown. All the mesons have degenerate masses, equal to $(-2\nu)^{1/2}$, and the quarks remain massless. This solution requires $\nu\leq 0$.

(2) $\eta_0\neq 0$. Equation (3.2) then implies that

$$\eta_0=\frac{\tau\pm[\tau^2+4\nu(6\lambda+2\mu)]^{1/2}}{2(6\lambda+2\mu)}\,.$$

In this case, the pseudoscalar octet becomes massless, i.e., these mesons act as Goldstone bosons.[17] The scalar and pseudoscalar singlets and the scalar octet acquire different masses and the quarks acquire a common mass $-g\eta_0$. The parameters λ, μ, τ, and ν are restricted by the requirement that all the boson masses are real. One finds

$$\nu\geq 2\tau^2/(6\lambda+2\mu)\,,$$

which implies that ν is positive by virtue of Eq. (2.10), consistently with real η_0.

Moreover,

$$\eta_0\tau\geq 0\,,$$

which, for a given sign of τ, chooses one of the two possible solutions for η_0. If $\mu\leq 0$, one must also require that

$$\eta_0\tau\geq -2\mu\eta_0^2\,.$$

We note, in conclusion, that for any given set of values for ν, λ, μ, and τ, there exists at most one solution.

[12] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969).

[13] We have made use of the following identity, valid for any 3×3 matrix: $\det\mathfrak{M}=\mathfrak{M}^3-\mathfrak{M}^2\,\mathrm{Tr}\mathfrak{M}+\frac{1}{2}\mathfrak{M}[(\mathrm{Tr}\mathfrak{M})^2-\mathrm{Tr}(\mathfrak{M}^2)]$, which can be easily derived from the characteristic equation of $\mathfrak{M}$ [see S. Coleman, in *Hadrons and their Interactions*, edited by A. Zichichi (Academic Press Inc., New York, 1968)].

[14] This follows from the fact that any matrix η can be written as $\eta=W_1KW_2e^{i\varphi}$, where K is real and diagonal, W_1 and W_2 are unitary, unimodular matrices, and $3\varphi=\arg\det\eta$. A proof of this theorem, which is certainly buried in the mathematical literature, can be easily obtained following the arguments in N. Cabibbo, R. Gatto, and C. Zemach, Nuovo Cimento **16**, 168 (1960). Transforming η with the $SU(3)\otimes SU(3)$ element $(W_1^\dagger,W_2)$, we can then put η into a diagonal form.

[15] To obtain this result, one has to reduce the η^3 term in Eq. (3.2) with the aid of the identity reported in Ref. 13.

[16] This is a particular case of the general rule that spontaneous breaking of $SU(3)$ tends to preserve a $SU(2)$ subgroup. See R. E. Cutkosky, in *Particle Symmetries*, edited by M. Chrétien and S. Deser (Gordon and Breach Science Publishers, Inc., New York, 1966).

[17] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961); see also J. Goldstone, A. Salam, and S. Weinberg, quoted in Ref. 11.

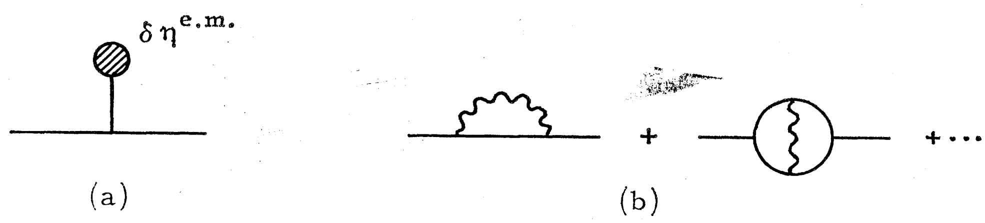

FIG. 1. Electromagnetic contributions to mass splitting: (a) tadpole; (b) nontadpole.

We consider now the effect of the introduction of e.m. interactions as well as of the explicit breaking term h. We shall impose the conditions

$$\det h \text{ is real}, \quad [h,Q]=0.$$

The first condition ensures[14] that, by a suitable $SU(3)\otimes SU(3)$ transformation, h can be diagonalized to a real matrix, so that the Lagrangian conserves parity. The second condition guarantees charge conservation.

The effect of e.m. interactions is displayed in Eqs. (2.19) and (2.21), and consists in adding a new term $\delta h^{\text{e.m.}}(\eta,Q)$ to h:

$$\delta h^{\text{e.m.}}(\eta,Q)_{ij}=\frac{\partial}{\partial\eta_{ij}}S^{\text{e.m.}}(\eta,Q). \tag{3.3}$$

If we treat both h and $\delta h^{\text{e.m.}}$ as perturbations, i.e., neglect terms of order e^4, e^2h, h^2, etc., then $\delta h^{\text{e.m.}}$ will be uniquely determined by η_0 and will have the form [see Eq. (2.18)]

$$\delta h^{\text{e.m.}}=A+BQ, \tag{3.4}$$

A and B being real functions of η_0. We see then that $\delta h^{\text{e.m.}}$ is in this approximation U-spin-invariant. The new solution η can be now easily computed to first order in e^2 and h, in terms of η_0, A, B, and h, and will turn out to be of the form

$$\eta=\eta_0+\delta\eta^{\text{e.m.}}+\delta\eta^h,$$

where $\delta\eta^{\text{e.m.}}$ and $\delta\eta^h$ will have the form

$$\delta\eta^{\text{e.m.}}=A'+B'Q, \quad \delta\eta^h=C'+D'h.$$

Let us now discuss the e.m. corrections to the masses of the various particles.

The e.m. shift to η, $\delta\eta^{\text{e.m.}}$ will cause e.m. mass differences among both fermions and bosons. This shift corresponds exactly to the contribution of tadpole graphs, as shown in Fig. 1(a).

Further contributions come from other kind of graphs, e.g., those represented in Fig. 1(b), and will be referred to as the nontadpole e.m. contributions. In the case of scalar and pseudoscalar mesons, both tadpole and nontadpole contributions can be derived from the vacuum-vacuum amplitude (in the soft-meson limit).

In fact, the inverse propagator for these mesons at zero four-momentum, i.e., the mass-squared matrix extrapolated at zero four-momentum, is given by second derivatives[11] of the total vacuum-vacuum amplitude $S(\eta)$. Symbolically, we have

$$M^2=\partial^2S/\partial\eta\partial\eta. \tag{3.5}$$

The e.m. corrections thus have two effects.

First, they introduce a new term in $S(\eta)$, i.e., $S^{\text{e.m.}}(\eta,Q)$; secondly, the presence of this new term causes a shift $\delta\eta^{\text{e.m.}}$. The net effect can be symbolically written as

$$(\delta M^2)^{\text{e.m.}}=\delta\eta^{\text{e.m.}}\left(\frac{\partial^3S(\eta_0)}{\partial\eta\partial\eta\partial\eta}\right)+\frac{\partial^2S^{\text{e.m.}}(\eta_0,Q)}{\partial\eta\partial\eta}. \tag{3.6}$$

The first term on the right-hand side is the tadpole contribution [in fact, the third derivative of $S(\eta)$ is connected[11] to the vertex function], and the second term is the effect of nontadpole graphs.

Note that since the total vacuum-vacuum amplitude including the e.m. contributions is formally invariant under chiral U-spin transformations, the neutral pseudoscalar mesons would still act, were it not for the presence of h, as Goldstone bosons, so that their e.m. mass shifts vanish. One can check explicitly that the two terms in Eq. (3.6) cancel for π^0, η^0, K^0, and $\bar{K}^0$. Moreover, due to the same symmetry, the e.m. shifts to π^+ and K^+, which lie in the same U-spin multiplet, must be the same. One then expects the following relation among the e.m. contributions to isospin-breaking mass shifts to hold:

$$(m_{K^+}{}^2-m_{K^0}{}^2)_{\text{e.m.}}=(m_{\pi^+}{}^2-m_{\pi^0}{}^2)_{\text{e.m.}}. \tag{3.7}$$

This result has been first obtained by Dashen[9] and should hold if one neglects terms of order e^2h. We note that terms of this order have been neglected in deriving the very successful Coleman-Glashow relation for baryon mass differences. The failure of Eq. (3.7) to match the experimental mass differences could then only be understood by introducing a non-e.m. isospin breaking in h.

As we shall see in Sec. V, such a term arises naturally in our theory; in Sec. VI we shall refer again to Eq. (3.7).

In the next section we conjecture that the main effect of weak corrections to hadron physics, as described by the Lagrangian equations (2.1) and (2.3), consists in a modification of the explicit breaking term, $h\rightarrow\tilde{h}$.

The effect of this change will reflect on the vacuum breaking η through Eq. (2.19). As we shall see, if we start from a diagonal h, $\tilde{h}$ will in general not be diagonal, i.e., will not conserve the same parity and strangeness operations as h. We will find, however, that $\tilde{h}$ conserves charge and has a real determinant, if h has these properties. $\tilde{h}$ can then be diagonalized through a $[U(2)\otimes U(2)]_U$ transformation. The previous analysis for the solutions of Eq. (2.19) applies unchanged in this new frame. The new starting solution η_0 has to be chosen diagonal and $SU(3)$-symmetric in the new frame, and as a consequence $\delta h^{\text{e.m.}}$, a function of η_0, will also be diagonal. In other words, if $\tilde{h}_D$ is the diagonal form of $\tilde{h}$ in the new frame, all the arguments of this section apply unchanged, replacing h with $\tilde{h}_D$. This change of frame will be studied explicitly in Sec. V.

IV. WEAK CORRECTIONS TO SYMMETRY BREAKING; THE PARAMETER ξ

This section is devoted to the study of the weak self-energy corrections. We assume a Lagrangian of the form (2.1), where the breaking L' will be specified with respect to its transformation properties under $SU(3)\otimes SU(3)$, i.e., we shall assume L' to belong to a $(3,\bar{3})\oplus(\bar{3},3)$ representation. We can again formally write L' as in Eq. (2.3), without necessarily associating with physical particles the local fields contained in $\mathfrak{M}$. For example, in a pure quark model, one would identify σ_i and π_i with the usual scalar and pseudoscalar densities. The matrix h will be parametrized by its eigenvalues α, β, and γ. In a pure quark model α, β, and γ would correspond to the masses of p, n, and λ quarks. This picture, mnemonically very useful, can, however, be completely misleading if taken literally.

For example, in the more realistic model previously discussed, quark masses are directly related to the vacuum breaking η, and only indirectly to h.

Let us consider the weak corrections to an hadronic process. At each order in perturbation theory, one meets with different kinds of divergent terms. The most divergent ones are proportional to $(G\Lambda^2)^n$ and are followed by terms the $G(G\Lambda^2)^{n-1}$, $G(G\Lambda^2)^{n-1}\ln\Lambda$, etc. According to the philosophy proposed by Lee,[4] one should sum all the divergent terms of the same kind, before letting the cutoff go to infinity.

In this way the sum of the most divergent terms should give a definite function $f(G\Lambda^2)$ which, if the procedure is consistent, has a finite limit as $\Lambda\to\infty$. In this limit, the sum is independent of the weak coupling constant G. On the other hand, the next divergent terms should add up to something like $GM^2g(G\Lambda^2)$, M being some finite mass, and when $\Lambda\to\infty$, tend to $GM^2g(\infty)$. Logarithmic divergences should be treated separately and resemble the divergences found in electrodynamics. We thus see that only the leading divergences appear to give corrections competitive with strong-interaction effects, and are precisely those which will enter our self-consistency requirement, as discussed in the Introduction. We will present different arguments suggesting the general validity of a conjecture advanced in I.

Conjecture: If the hadron Lagrangian has the form (2.1), the effect of the leading weak corrections to strong processes consists in adding a further breaking term δh^W, represented, as in Eq. (2.3), by a matrix of the form

$$\delta h^W = -\xi\begin{bmatrix}\alpha & 0 & 0\\ 0 & \beta\cos^2\theta_0 & \frac{1}{2}\beta\sin 2\theta_0\\ 0 & \frac{1}{2}\gamma\sin 2\theta_0 & \gamma\sin^2\theta_0\end{bmatrix}, \tag{4.1}$$

where ξ is a real parameter, related to the value of the limit of $f(G\Lambda^2)$ as $\Lambda\to\infty$.

Equation (4.1) is valid in a frame where h is diagonal, and θ_0 is the weak-interaction angle in this frame. As discussed in the Introduction, θ_0 is not the observed weak-interaction angle, which is instead the angle of the weak currents in the frame where $\tilde{h}\equiv h+\delta h^W$ is diagonal. An immediate consequence of this conjecture is that $\tilde{h}$ is a real matrix, i.e., an $SU(3)\otimes SU(3)$ frame can be chosen, such that $\tilde{h}$ is a real and diagonal matrix. This result means that, at the strong level, weak interactions do not cause breakdown of parity and strangeness.

This conjecture will be proved to be true for the following cases:

(i) second-order calculation,

(ii) self-masses of weak interacting free quarks (to all orders),

(iii) models with weak interactions mediated by a neutral vector boson.

The first case has already been considered in I and in Ref. 18, and gives $\xi=G\Lambda^2$. In this case, when $\Lambda\to\infty$ one would have $\xi=\infty$. As we have stated before, the second order by itself does not give a meaningful result. However, until now we have not been able to go beyond a second-order computation in a realistic theory, i.e., a theory which allows for strong interactions and where weak interactions are mediated by charged vector bosons.

The conjecture can be proved[19] for the case of the weak self-energy corrections to free quarks. The argument runs as follows.

We start from a Lagrangian L defined as

$$L=\bar{\psi}[i\gamma\cdot\partial-(ha_+ + h^\dagger a_-)]\psi + 2g[\bar{\psi}\lambda^+\gamma_\mu a_+\psi W^\mu + \text{H.c.}]. \tag{4.2}$$

This Lagrangian has the structure of Eq. (2.1), with the breaking appearing as a quark mass term. λ^+ is a 3×3 matrix of the form

$$\lambda^+ = \begin{bmatrix}0 & \cos\theta_0 & \sin\theta_0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \tag{4.3}$$

[18] R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **28B**, 128 (1968).

[19] This argument has been presented by one of us (N. C.) in a seminar given at CERN on October 23, 1968. We would like to thank B. Touschek for an enlightening discussion on this subject.

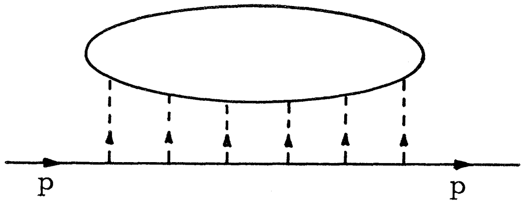

FIG. 2. Weak contributions to the proper self-energy part $\Sigma(p)$.

We consider the amplitude $-i\Sigma(p)$, defined as the sum of all the proper and connected self-energy graphs of the kind shown in Fig. 2, where the bubble indicates a set of connections of boson lines, possibly including closed fermion loops. Because of the triplet structure of the quark multiplet, the charges of the boson lines must alternate along the quark line.

Since we are interested in the most divergent part of this amplitude, we can write down immediately the most general form of $\Sigma(p)$:

$$\Sigma(p)=\zeta\gamma\cdot p a_+ Z\,, \tag{4.4}$$

where ζ is a (cutoff-dependent) number, and

$$Z=\{\lambda^+,\lambda^-\}\,,\quad \lambda^-=(\lambda^+)^\dagger\,. \tag{4.5}$$

The $SU(3)$ structure of Eq. (4.4) follows from the fact that each diagram with n positive and n negative boson lines leaving the main quark line contributes a term proportional to $(\lambda^+\lambda^-)^n+(\lambda^-\lambda^+)^n=\{\lambda^+,\lambda^-\}$. The $\gamma\cdot pa_+$ term arises from the structure of the interaction, and ζ depends only upon $G\Lambda^2$ since we are considering only the most divergent contributions.

The effect of $\Sigma(p)$ is equivalent to modifying the free term of L to

$$\tilde{L}_{\text{free}}=\bar{\psi}[i\gamma\cdot\partial(1-\zeta Z a_+)-(ha_+ + h^\dagger a_-)]\psi\,. \tag{4.6}$$

At the same time, we have to consider the higher-order corrections to the weak vertex. The most divergent part of the proper vertex will be written as

$$\Gamma_\mu{}^+=2\gamma_\mu a_+\lambda^+ + \Lambda_\mu{}^+\,, \tag{4.7}$$

$$\Lambda_\mu{}^+=2\gamma_\mu a_+\lambda^+\chi\,, \tag{4.8}$$

and χ is again a cutoff-dependent number. We then get

$$\tilde{L}_{\text{int}}=2g(1+\chi)(\bar{\psi}\lambda^+\gamma_\mu a_+\psi W^\mu+\text{H.c.})\,. \tag{4.9}$$

If we transform the field ψ as[20]

$$\psi=(Ra_+ + a_-)\psi'\,, \tag{4.10}$$

where

$$R=(1-\xi Z)\,,\quad \xi=1-1/(1-\zeta)^{1/2}\,, \tag{4.11}$$

the Lagrangian $\tilde{L}=\tilde{L}_{\text{free}}+\tilde{L}_{\text{int}}$ is brought into the form

$$\tilde{L}=\bar{\psi}'[i\gamma\cdot\partial-(\tilde{h}a_+ + \tilde{h}^+ a_-)]\psi' + 2g[(1+\chi)/(1-\zeta)] \times(\bar{\psi}'\lambda^+\gamma_\mu a_+\psi' W^\mu+\text{H.c.})\,,$$

where

$$\tilde{h}=hR=h-\xi hZ\,.$$

[20] N. Cabibbo, R. Gatto and C. Zemach, quoted in Ref. 14.

If we start from a diagonal h,

$$h=\begin{bmatrix}\alpha & 0 & 0\\ 0 & \beta & 0\\ 0 & 0 & \gamma\end{bmatrix}\,, \tag{4.12}$$

we get the result of Eq. (4.1).

It is interesting to remark that if $\Sigma(p)$ diverges, i.e., $\zeta\to\infty$, we get $\xi=1$.

The exact calculation of even the most divergent part of weak corrections looks a formidable task in the realistic case of weak interactions mediated by charged W's. The main difficulty consists in the non-Abelian character of the algebra generated by the corresponding currents. Even in the very simple case of free quarks, we have not been able to evaluate ξ.

Furthermore, we observe that apart from the mass renormalization $h\to\tilde{h}$, there is also a renormalization of the weak vertex by a factor

$$(1+\chi)/(1-\zeta)\,,$$

which also cannot be computed in this model.

The situation is completely different in a model of weak interactions transmitted by a neutral vector boson.

In I we have considered a situation of this kind, with the neutral W coupled to a current $J_\mu{}^3$, corresponding to the third component of weak isospin. In quark notation,

$$J_\mu{}^3=\bar{\psi}\gamma_\mu(1+\gamma_5)\lambda^3\psi\,,\quad \lambda^3=[\lambda^+,\lambda^-]\,. \tag{4.13}$$

This interaction, in the context of a free-quark model treated along the lines indicated above, leads to the relation $\chi=-\zeta$, which implies no renormalization for the weak vertex. In fact, the most divergent parts of the proper vertex $\Lambda_\mu{}^3$ and of the self-energy part $\Sigma(p)$ are related by a Ward identity of the form

$$(p'-p)^\mu\Lambda_\mu{}^3(p',p)=-[\Sigma(p')-\Sigma(p)]2\lambda^3\,.$$

This identity arises from the fact that, in this model, the neutral weak current is conserved to all orders apart from mass terms, which are, however, irrelevant for the most divergent contributions.

In a model of nonleptonic weak interactions mediated by a neutral vector boson, the explicit summation of the most divergent terms can be carried out. This result can be done with current-algebra techniques without any reference to the detailed structure of the strong-interaction Hamiltonian, except the requirement that it has the forms (2.1) and (2.3). For the sake of brevity we will not report here this general proof (to be contained in a further communication) and will restrict ourselves to strong interactions described by the generalized σ model introduced in Sec. II. We will choose the Lagrangian

$$\begin{aligned}L=&L_0+L'+gW^\mu[\bar{\psi}2\lambda^3\gamma_\mu a_+\psi\\ &+i\,\mathrm{Tr}(\mathfrak{M}\lambda^3\partial_\mu\mathfrak{M}^\dagger-\partial_\mu\mathfrak{M}\lambda^3\mathfrak{M}^\dagger)]\\ &-2g^2W_\mu W^\mu\,\mathrm{Tr}[\mathfrak{M}(\lambda^3)^2\mathfrak{M}]+L_{\text{free}}{}^W\\ =&L_0+L'+L_{\text{int}}(W)+L_{\text{free}}{}^W\,,\end{aligned} \tag{4.14}$$

where L_0 and L' are the same as in Eqs. (2.9) and (2.3). Using the Stückelberg formalism,[4,21] it is possible to express the weak-interaction part in terms of two fields $\tilde{W}_\mu$ and θ such that the propagator of $\tilde{W}_\mu$ is simply

$$-ig_{\mu\nu}/(q^2-m_W^2)\,,$$

and that of θ is

$$[i/(q^2-m_W^2)](q_\mu q_\nu/m_W^2)\,.$$

This corresponds to writing

$$W_\mu=\tilde{W}_\mu+(1/m_W)\partial_\mu\theta$$

in the interaction term in Eq. (4.14) and to a suitable modification of $L_{\text{free}}{}^W$. The most divergent contributions to weak corrections arising from θ exchange can be formally extracted by means of a Dyson transformation[4] on the fields $\mathfrak{M}$ and ψ. If we let

$$\mathfrak{M}=\mathfrak{M}'e^{(2ig/m_W)\theta\lambda^3}\,,\quad \psi=e^{(2ig/m_W)a_+\theta\lambda^3}\psi'\,,\qquad(4.15)$$

then L goes into

$$L_0(\psi',\mathfrak{M}')+L_{\text{int}}(\tilde{W},\psi',\mathfrak{M}')+L_{\text{free}}(\tilde{W})+L_{\text{free}}(\theta)$$
$$+\mathrm{Tr}(\mathfrak{M}'e^{(2ig/m_W)\theta\lambda^3}h^\dagger+\text{H.c.})\,.\qquad(4.16)$$

The dangerous field θ is now only present in the explicit breaking term.

Following the analysis of Ref. 4, the effect of the most divergent θ corrections is obtained by the substitution

$$h\to h\langle e^{-(2ig/m_W)\theta\lambda^3}\rangle_0=h+(e^{-G\Lambda^2}-1)h(\lambda^3)^2\,.\qquad(4.17)$$

This takes care of all insertions in Feynman graphs of θ loops starting and ending at the same point. In the limit $\Lambda\to\infty$, Eq. (4.17) reduces to Eq. (4.1) with $\xi=1$, since $(\lambda^3)^2=Z$.

It is interesting to compare this result with the previous analysis of the free-quark model. The Dyson transformation carries with it a wave-function renormalization. In fact, we can read Eq. (4.15), in the limit $\Lambda^2\to\infty$, as

$$\psi=[1-(\lambda^3)^2a_+]\psi'$$
$$+(e^{-(2ig/m_W)\theta\lambda^3}-\langle e^{-(2ig/m_W)\theta\lambda^3}\rangle_0)\psi'\,,\qquad(4.18)$$

and we see that the first term is equivalent to the wave-function renormalization (4.10), with $\xi=1$.

It must be observed that the models (4.14) and (4.16) contain (apart from the leading divergences because of θ loops, which we have discussed) quadratic divergences due to $\tilde{W}$ exchange. The latter are in fact similar to those which appear in the electrodynamics of spinless bosons and their origin is connected with the failure of the Bjorken limit. The model therefore does not satisfy all the requirements we impose on the theory, which include the validity of the Bjorken limit. The use of this model here is only intended as an illustration of the treatment of leading divergences due to θ exchange.

[21] E. C. G. Stückelberg, Helv. Phys. Acta **11**, 225 (1938); **11**, 299 (1938).

V. EFFECT OF WEAK INTERACTIONS AS A CHANGE OF FRAME; DETERMINATION OF θ

In Sec. IV we have seen how the weak corrections induce on a Lagrangian, which includes a breaking term characterized by a matrix h, a modification $h\to\tilde{h}$:

$$\tilde{h}=hR=h-\xi hZ\,.\qquad(5.1)$$

The physical content of the modified Lagrangian is best described in a frame where $\tilde{h}$ is diagonal. The new frame can be obtained from the old one through an $SU(3)\otimes SU(3)$ rotation, which transforms $\tilde{h}$ into $\tilde{h}_D$:

$$\tilde{h}_D=U\tilde{h}V^\dagger\,.\qquad(5.2)$$

In the new frame the weak current is described by a matrix $\tilde{\lambda}^+$:

$$\tilde{\lambda}^+=V\lambda^+V^\dagger\,.\qquad(5.3)$$

This transformation will cause a change of the weak-interaction angle, from θ_0 to a new value θ, which we will identify with the angle measured in semileptonic processes.

The explicit form of $\tilde{h}$ is

$$\tilde{h}=\begin{bmatrix}\alpha(1-\xi) & 0 & 0\\ 0 & \beta(1-\xi\cos^2\theta_0) & \tfrac12\beta\xi\sin2\theta_0\\ 0 & \tfrac12\gamma\xi\sin2\theta_0 & \gamma(1-\xi\sin^2\theta_0)\end{bmatrix}.\qquad(5.4)$$

This matrix can obviously be diagonalized with matrices U and V with the same block structure. We can then restrict ourselves to the relevant 2×2 submatrix $\tilde{H}$ of $\tilde{h}$, which we can rewrite in terms of Pauli matrices σ_i as

$$\tilde{H}=\tfrac14[\delta(2-\xi)+\xi\epsilon\cos2\theta_0]+i\sigma_2\,\tfrac14\xi\epsilon\sin2\theta_0$$
$$-\sigma_3\,\tfrac14[\epsilon(2-\xi)+\xi\delta\cos2\theta_0]-\sigma_1\,\tfrac14\xi\delta\sin2\theta_0\,,\qquad(5.5)$$

where

$$\delta=\gamma+\beta\,,\quad \epsilon=\gamma-\beta\,.\qquad(5.6)$$

By the transformation (5.2), $\tilde{H}$ can be put in its diagonal form

$$\tilde{H}_D=\tfrac12(\tilde{\delta}-\tilde{\epsilon}\sigma_3)\,,\qquad(5.7)$$

where

$$\tilde{\delta}=\tfrac12\{[\delta(2-\xi)+\xi\epsilon\cos^2\theta_0]^2+\xi^2\epsilon^2\sin^22\theta_0\}^{1/2}\,,\qquad(5.8)$$

$$\tilde{\epsilon}=\tfrac12\{[\epsilon(2-\xi)+\xi\delta\cos2\theta_0]^2+\xi^2\delta^2\sin^22\theta_0\}^{1/2}\,.\qquad(5.9)$$

To achieve this, it is sufficient to choose

$$V=\begin{pmatrix}1 & 0\\ 0 & e^{i\psi\sigma_2}\end{pmatrix}=e^{i\psi\lambda_7}\,,\qquad(5.10)$$

$$U=e^{i\varphi\lambda_7}\,.\qquad(5.11)$$

The angle ψ, which is relevant to the evaluation of θ, can be easily computed by noticing that $\tilde{h}^\dagger\tilde{h}$ is diagonalized by the unitary transformation:

$$\tilde{h}^\dagger\tilde{h}\to V\tilde{h}^\dagger\tilde{h}V^\dagger\,,\qquad(5.12)$$

as a consequence of Eq. (5.2). One finds

$$\psi=\tfrac{1}{2}\tan^{-1}\left(\frac{\xi(2-\xi)(\delta^2+\epsilon^2)\sin 2\theta_0+\xi^2\delta\epsilon\sin 4\theta_0}{\delta\epsilon(2-\xi)^2+\xi(2-\xi)(\delta^2+\epsilon^2)\cos 2\theta_0+\delta\epsilon\xi\cos 4\theta_0}\right),\tag{5.13}$$

and from Eq. (5.3) one sees that the physical value of θ is

$$\theta=\theta_0-\psi.\tag{5.14}$$

For $\xi=1$, one obtains $\psi=\theta_0$, i.e., $\theta=0$.[22] We have already seen for the free-quark case that $\xi=1$ implies that the weak self-energy term $\Sigma(p)$ diverges. The result $\theta=0$ is then not surprising; weak interactions become infinitely strong in the limit $\xi=1$, so as to drive the breaking completely along their own direction.

In fact $\xi=1$ is a limiting case. It is possible to prove that the substitution $\xi\rightarrow 2-\xi$ leaves unchanged the physical content of the theory, so that we may always choose $\xi\leq 1$. This circumstance is suggested by Eq. (4.11), which shows that in the free-quark model, the substitution $\xi\rightarrow 2-\xi$ amounts only to choosing the opposite sign for $(1-\xi)^{1/2}$.

To prove this symmetry in general, we note, looking at the block form (5.4), that under this substitution,

$$\alpha(1-\xi)\rightarrow-\alpha(1-\xi),\quad \tilde{H}(\xi)\rightarrow\tilde{H}(2-\xi),$$

and

$$\det\tilde{H}(\xi)=-\det\tilde{H}(2-\xi),$$
$$\mathrm{Tr}\tilde{H}^\dagger(\xi)\tilde{H}(\xi)=\mathrm{Tr}\tilde{H}(2-\xi)^\dagger\tilde{H}(2-\xi).$$

These equations imply that, by a suitable $SU(3)\otimes SU(3)$ transformation, $\tilde{h}(2-\xi)$ can be brought into the same diagonal matrix $\tilde{h}_D$ as $\tilde{h}(\xi)$. Furthermore, the final value of θ obtained after this transformation can be proved (with some tedious algebra) to coincide with the one obtained in the diagonalization of $\tilde{h}(\xi)$. In the following we shall then be able to assume $\xi\leq 1$, without loss of generality.

As discussed in Sec. III, $\delta h^{\mathrm{e.m.}}$ is expected to be diagonal in the same frame as $\tilde{h}$ and, furthermore, to a good extent, to be U-spin-invariant. We shall then write

$$\delta h^{\mathrm{e.m.}}=\begin{bmatrix}\alpha^{\mathrm{e.m.}} & 0 & 0\\ 0 & \beta^{\mathrm{e.m.}} & 0\\ 0 & 0 & \beta^{\mathrm{e.m.}}\end{bmatrix}.\tag{5.15}$$

Our self-consistency condition requires the combined effects of weak and tadpole e.m. corrections on hadron dynamics to cancel—i.e., we shall require

$$\tilde{h}_D+\delta h^{\mathrm{e.m.}}=h.\tag{5.16}$$

Equation (5.16) is equivalent to the equations

$$\alpha\xi=\alpha^{\mathrm{e.m.}},\tag{5.17}$$

$$\tilde{\delta}+2\beta^{\mathrm{e.m.}}=\delta,\tag{5.18}$$

$$\tilde{\epsilon}=\epsilon.\tag{5.19}$$

From Eqs. (5.18) and (5.19) and (5.8) and (5.9), one easily gets

$$\sin^2\theta_0=\frac{\delta-\epsilon}{2\delta}\,\frac{1+\xi(\delta-\epsilon)/4\epsilon}{1-\frac{1}{2}\xi},\tag{5.20}$$

$$\beta^{\mathrm{e.m.}}=\tfrac{1}{2}\delta\{1-[1-\xi(\delta^2-\epsilon^2)/\delta^2]^{1/2}\}.\tag{5.21}$$

We shall consider α, δ, and ϵ, i.e., α, β, and γ as input parameters to be obtained from the hadron mass spectra. As explained in I, to this purpose we have used the analysis of Ref. 6 (which in this respect gives results in substantial agreement with those of Ref. 5), as well as the tadpole analysis for "electromagnetic" mass splitting of hadrons.[23] This analysis indicates that α and β are actually much smaller than γ (this will be discussed in more detail in Sec. VI). We note that Eq. (5.20) is consistent only for values of ξ such that

$$\xi\geq-2(\gamma-\beta)/\beta.\tag{5.22}$$

On the other hand, Eq. (5.21) states that $\beta^{\mathrm{e.m.}}$ is a monotonically increasing function of $|\xi|$, which for small values of $|\xi|$, i.e., $|\xi|\ll 2(\gamma-\beta)/\beta$, gives

$$\beta^{\mathrm{e.m.}}\sim\xi\beta\tag{5.23}$$

and suggests $|\xi|$ to be small. We shall restrict ourselves to this situation, where, neglecting terms of order $(\beta/\gamma)^2$, we can write

$$\psi=\xi\theta_0,\tag{5.24}$$

$$\theta=(\beta/\gamma)^{1/2}(1-\xi)(1-\tfrac{1}{2}\xi)^{-1/2}.\tag{5.25}$$

While Eqs. (5.17) and (5.23) essentially coincide with the corresponding ones given in I, Eq. (5.25) gives θ as a function of ξ.

We shall show in Sec. VI that ξ can be obtained from a phenomenological analysis of isospin-breaking effects.

We simply note here that for small values of $|\xi|$, Eq. (5.25) gives the result

$$\theta=(\beta/\gamma)^{1/2}\approx 0.22,\tag{5.26}$$

which is in good agreement with the observed value.

VI. NONELECTROMAGNETIC ISOSPIN BREAKING

In this section we discuss the nature of isospin-breaking effects in our theory.

[22] This is true with the exception of the cases $\delta=\epsilon$, $\theta_0=90°$ or $\delta=-\epsilon$, $\theta_0=0°$. Both of these situations correspond to the pathological cases $\tilde{h}=0$, where ψ and φ are completely undetermined.

[23] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964); R. Socolow, *ibid.* **148**, B1221 (1965).

The key parameters, from the point of view of hadron dynamics, are the eigenvalues of the breaking matrix h:

$$h=\begin{pmatrix}\alpha & 0 & 0\\ 0 & \beta & 0\\ 0 & 0 & \gamma\end{pmatrix}\equiv\frac{1}{\sqrt{6}}(\alpha+\beta+\gamma)\lambda_0+\frac{\alpha+\beta-2\gamma}{2\sqrt{3}}\lambda_8+\tfrac{1}{2}(\alpha-\beta)\lambda_3,$$

where $\lambda_0=(\sqrt{\tfrac{2}{3}})\mathbf{1}$. This corresponds [see Eq. (2.3)] to a breaking Hamiltonian

$$h=(1/\sqrt{3})(\alpha+\beta+\gamma)\sigma_0+[(\alpha+\beta-2\gamma)/\sqrt{6}]\sigma_8+[(\alpha-\beta)/\sqrt{2}]\sigma_3. \quad (6.1)$$

If $\alpha\neq\beta$, h includes a breaking of isospin. From the arguments presented in Sec. III, as well as in Refs. 2, 3, and 16, one would not expect this term to arise from a purely symmetric strong-interaction bootstrap. However, this term is needed in order to satisfy our consistency condition, and, in particular, Eqs. (5.17) and (5.21). In the following we shall assume $|\xi|$ to be small with respect to $2(\gamma-\beta)/\beta$ (which, according to the phenomenological analysis we are going to present, is about 40), so that Eq. (5.23) can be used instead of Eq. (5.21), allowing us to write

$$\frac{1}{\sqrt{2}}(\alpha-\beta)\approx\frac{1}{\xi}\frac{\alpha^{\text{e.m.}}-\beta^{\text{e.m.}}}{\sqrt{2}}. \quad (6.2)$$

This equation shows how the isospin breaking in h arises from a combined effect of e.m. and weak interactions. For $\xi=1$, the effect is actually of a purely e.m. origin.

However, this value of ξ would lead to an incorrect prediction for θ, i.e., $\theta=0$. On the other hand, if $|\xi|\ll1$, then Eq. (6.2) indicates that a substantial portion of the experimentally observed isospin breaking is not of e.m. origin.

The computational scheme in any given situation of isospin breaking which emerges from this picture consists in adding to the effect contained in h, the contribution of the nontadpole e.m. effects, i.e., those effects not contained in $\delta h^{\text{e.m.}}$.[24]

An alternative procedure to compute isospin breaking is to add the complete contribution of the purely e.m. correction to that part of isospin-breaking term in h which cannot be ascribed to electromagnetism. The two procedures can be described by the following symbolical equations:

$$\text{isospin breaking}=[(1/\sqrt{2})(\alpha-\beta)\sigma_3]+(\text{nontadpole e.m. corrections}), \quad (6.3)$$

$$\text{isospin breaking}=[(1/\sqrt{2})(\alpha-\beta)(1-\xi)\sigma_3]+(\text{total e.m. corrections}). \quad (6.4)$$

[24] In this connection, we note that if one were able to compute separately $(\alpha^{\text{e.m.}}-\beta^{\text{e.m.}})$, one could obtain, via Eqs. (5.16) and (5.22), an estimate of the very important parameter ξ.

The first procedure closely corresponds to the tadpole picture of Coleman and Glashow.[23] The first term of the right-hand side corresponds to their tadpole contribution.[23] The same authors gave a successful fitting of baryon mass differences. From this fit, one can obtain a measure of the ratio:

$$R_1=\frac{\alpha-\beta}{\alpha+\beta-2\gamma}=\frac{1}{2}\frac{(\Sigma^--\Sigma^+)_{\text{tad}}}{\Xi-N}=\frac{1}{2}\frac{[(\Xi^--\Xi^0)+(p-n)]_{\text{tad}}}{\Sigma-\Lambda}=\frac{1}{2}\frac{[(n-p)+(\Xi^0-\Xi^-)]_{\text{tad}}}{\Xi+N-2\Sigma}. \quad (6.5)$$

The first two determinations give for this ratio, respectively, (i) 0.0091, (ii) 0.010. The third determination gives a much larger value, i.e., 0.045. However, this discrepancy can be ascribed to a near cancellation in the denominator, which makes this determination less reliable than the others. We will therefore choose

$$R_1=(\alpha-\beta)/(\alpha+\beta-2\gamma)\approx0.010. \quad (6.6)$$

It is clear that this determination is subject to an error, which might be quite large, especially due to the uncertainties in the separation of nontadpole contributions. We shall also use the analysis of Ref. 6, which determines the independent ratio:

$$R_2=\frac{1}{\sqrt{2}}\frac{\alpha+\beta-2\gamma}{\alpha+\beta+\gamma}. \quad (6.7)$$

Finally, we can combine these two ratios to obtain up to an arbitrary normalization (to be reabsorbed in the arbitrary normalization of the operators σ_i):

$$\alpha\approx0.087,\quad \beta\approx0.140,\quad \gamma\approx2.77. \quad (6.8)$$

Coleman and Glashow and Socolow also made an analysis of pseudoscalar mass differences which could be used [according to the picture outlined in Eq. (6.3)] as an alternative way of determining the parameter R_1. A better scheme for using these data is suggested by the result obtained by Dashen,[9] which we have discussed in Sec. III, Eq. (3.7).

In view of this results, using Eq. (6.4) we find the following equation for masses squared:

$$\frac{(K^+-K^0)-(\pi^+-\pi^0)}{2(K-\pi)}=R_1(1-\xi). \quad (6.9)$$

Using the experimental data, we find

$$R_1(1-\xi)\approx1.2\times10^{-2},$$

which, compared with Eq. (6.6), gives ξ the value

$$\xi\approx-0.2.$$

In view of the uncertainties in the determination of R_1, this can only be taken as an indication that $|\xi|$ is small compared to 1, which leads to a good prediction for the angle θ. In particular, the value of ξ given above leads to

$$\theta \approx 0.25.$$

An interesting case that can be treated by the same procedure is the $\eta \to 3\pi$ decay. Sutherland has shown[8] that, in the soft-pion limit, the e.m. contribution to such decay vanishes because of current-algebra commutators. This has been considered as a baffling result, in view of the fact that similar hypotheses give excellent results for nonleptonic K-meson decays, and prompted various authors[25] to propose the possibility of nonelectromagnetic isospin-breaking interactions as the source of the $\eta \to 3\pi$ decay.

As we have seen, such an additional $SU(2)$ breaking is provided quite naturally by our theory. In the light of Sutherland's theorem, Eq. (6.4) states that

$$\langle 3\pi | T | \eta \rangle = [(\alpha-\beta)/\sqrt{2}](1-\xi)\langle 3\pi | \sigma_3(0) | \eta \rangle . \quad (6.10)$$

When treated with standard soft-pion techniques, Eq. (6.10) is known to reproduce the correct slope of the Dalitz-plot distribution[26] as well as the correct order of magnitude for the rate. We shall give a more complete analysis of $\eta \to 3\pi$ decay in our theory elsewhere.

It is interesting to note that the σ_3 interaction, which was introduced before as an *ad hoc* assumption,[26] is here derived directly from our self-consistency requirement.

VII. CONCLUSION AND FINAL REMARKS

In this paper we have presented the state of our theory of the angle θ of which an outline was given in I. In many ways the ideas discussed here represent an evolution in respect to those contained in I. The first difference consists in a more thorough treatment of the large violations of both parity and strangeness induced by weak corrections, which we have shown to be only apparent. In fact, the main effect of the corresponding terms consists in a redefinition, via an $SU(3)\otimes SU(3)$ rotation, of both parity and strangeness. The necessity of such a rotation is now at the core of our theory. The amount of rotation necessary to reinstate parity and strangeness conservation depends upon the important parameter ξ, whose introduction is the second new aspect of this paper.

This rotation modifies the actual value of the weak-interaction angle θ. In particular, if $\xi=1$, the physically observed value of θ would be zero. This result is independent of our consistency requirement. In fact, $\xi=1$ was shown to result from any theory where weak interactions are mediated by a neutral vector boson coupled to the current corresponding to the third component of weak isospin. It is unfortunate that we are not able to compute the correct value of ξ in a realistic theory, with charged (or with both charged and neutral) bosons, so that ξ has, at least at this stage, to remain a phenomenological parameter. In this respect, however, a happy circumstance is the emergence, in our theory, of nonelectromagnetic isospin breaking, which strongly depends upon the value of ξ, and allows a phenomenological determination of this parameter. As we have discussed in Sec. VI, an estimate for ξ can be obtained through an analysis of isospin-breaking mass differences for pseudoscalar mesons. In this respect, it is encouraging that in this way we have found a value for ξ in substantial agreement with the experimental value of θ.

This nonelectromagnetic isospin breaking can provide also the basis for an understanding of the $\eta \to 3\pi$ puzzle.

A very delicate point for the enforcement of our program is the knowledge of the structure of $\delta h^{\text{e.m.}}$. As we have indicated in I, we expect $\delta h^{\text{e.m.}}$ to have the $SU(3)$ structure dictated by U-spin invariance, apart from the relatively small effects of the $SU(3)$ breaking. In Sec. III we discussed how a large vacuum breaking, suggested by the phenomenological analyses of symmetry breaking, plays a crucial role in enforcing this condition, and actually allows such a situation to apply in reality. In any case, a reliable computation of $\delta h^{\text{e.m.}}$ and the evaluation of ξ remain a main challenge for the future development of our program.

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[25] See D. G. Sutherland, Nucl. Phys. **B2**, 433 (1967), and references contained therein.

[26] Y. T. Chiu, I. Schechter, and Y. Ueda, Phys. Rev. **161**, 1612 (1967).