APPENDIX

$$g_1^{\alpha\beta\gamma}=g(f_{\alpha\beta\gamma}L_\gamma+gf_{\alpha\epsilon\sigma}d_{\sigma\beta\gamma}\eta_\epsilon\xi_{p\gamma})Z_{p\beta}{}^{1/2}Z_{p\gamma}{}^{1/2},$$

$$g_2^{\alpha\beta\gamma}=f_{\alpha\beta\gamma}(\tfrac{1}{2}g\xi_{p\beta}\xi_{p\gamma}+\tfrac{1}{2}hL_\beta L_\gamma)Z_{p\beta}{}^{1/2}Z_{p\gamma}{}^{1/2},$$

$$g_3^{\alpha\beta\gamma}=-g^2(d_{\alpha\delta\lambda}f_{\lambda\beta\gamma}\eta_\delta-d_{\alpha\gamma\lambda}f_{\beta\delta\lambda}\eta_\delta)Z_{p\gamma}{}^{1/2},$$

$$g_4^{\alpha\beta\gamma}=-f_{\alpha\beta\gamma}(g\xi_{p\gamma}+ghL_\gamma\zeta_\beta)Z_{p\gamma}{}^{1/2},$$

$$g_5^{\alpha\beta\gamma}=(gf_{\alpha\beta\gamma}\xi_{p\gamma}+\tfrac{1}{2}ghf_{\sigma\beta\epsilon}\eta_\epsilon d_{\alpha\gamma\sigma}L_\gamma)Z_{p\gamma}{}^{1/2},$$

$$g_6=-\tfrac{1}{2}g^2(M_{KA}/M_\rho{}^2)(F_\pi/M_{K^*}+F_{SK}/M_{A1}),$$

$$g_7=-\tfrac{1}{2}g^2(M_{A1}/M_\rho{}^2)(F_K/M_{K^*}-F_{SK}/M_{KA}),$$

$$g_8=\tfrac{1}{2}g^2(M_{K^*}/M_\rho{}^2)(F_K/M_{A1}+F_\pi/M_{KA}),$$

$$g_9=\left\{\frac{3C_1}{\sqrt{2}}-C_2[\eta_0-(\surd\tfrac{4}{3})\eta_8]\right\}Z_\pi{}^{1/2}Z_K{}^{1/2}Z_{SK}{}^{1/2},$$

where

$$L_\beta=1-\zeta_\beta\xi_\beta.$$

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Universal Isovector Current with Many 1^- Poles*

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A simple prescription is suggested for preserving universality of the isovector current in the presence of many 1^- poles ρ_i. By identifying $g_{\rho_i-\gamma}$ as proportional to $g_{\rho_i\pi\pi}$, predictions are made regarding (a) the modification of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation, (b) charge radii, (c) $\pi\pi$ resonance production in colliding beam experiments, (d) high-energy ρ meson photoproduction and photoabsorption, (e) photoproduction of ρ' and ρ'', and (f) asymptotic behavior of form factors.

In this paper we suggest a simple modification of the hypothesis of ρ dominance[1] which preserves the universality of the isovector current. Our starting point is that low-energy pion scattering, while purely hadronic, also obeys a form of universality.[2]

In a resonance approximation one may write the isovector "electric" form factor of any particle x in the form

$$F_{xx}{}^\gamma(t)=\sum_{n=1}^{\infty}\beta_n g_{nx\bar{x}}/(m_n{}^2-t). \quad (1)$$

The β_n represent the couplings of the photon to the intermediate states ρ_n.[3]

A similar "form factor" may be defined in the case of the $I_t=1$ amplitude for π-x elastic scattering at threshold:

$$M_t{}^{(1)}(\nu,t)=2(q_1+q_2)^\mu[(p_1+p_2)_\mu F_{xx}{}^\pi(t)]=4\nu F_{xx}{}^\pi(t), \quad (2)$$

where $\nu\equiv\frac{1}{2}(s-u)$, q_1 and q_2 are the momenta of the incoming and outgoing pion, and p_1 and p_2 are the momenta of the incoming and outgoing x (spinless here for convenience). Since π-x is a hadronic scattering, all spin exchanges are possible in the t channel, and in particular the t-channel resonance expansion (or the t-channel partial-wave expansion) blows up at the various s- (and u-) channel singularities. The rather remarkable consequence of partial conservation of axial-vector current (PCAC) and current algebra[2] is that at threshold and at $t=0$, the $I_t=1$ crossing-odd amplitude is given in the soft-pion limit by a conserved vector interaction. This means that in this particular kinematic region only $J=1^-$ isovector t-channel exchanges are relevant.[4] Those are precisely the same states contributing to (1), and we have

$$F_{xx}{}^{\pi(\text{soft})}(t=0)=\sum_{n=1}^{\infty}\frac{g_{n\pi\pi}g_{nx\bar{x}}}{m_n{}^2}. \quad (3)$$

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‡ Supported in part by the U. S. Atomic Energy Commission under Contract No. AT-(11-1)-1764.

[1] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 79 (1962); 8, 191(E) (1962); M. Gell-Mann, D. Sharp, and W. Wagner, *ibid.* 8, 261 (1962).

[2] S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[3] In the case of ρ dominance, for example, $\beta_1=m_\rho{}^2/2\gamma_\rho=m_\rho{}^2/g_{\rho\pi\pi}$.

[4] We suggest, therefore, that the sum over all *spin-1* exchanges gives $I=1$ exchange in $\pi x\rightarrow\pi x$ at threshold. This is equivalent to taking only pole contributions to the dispersion relation in t for the $J=1$ partial wave.

The form factor (1) must obey

$$F_{xx}{}^{\gamma}(0)=\sum_{n=1}^{\infty}\frac{\beta_n g_{nx\bar{x}}}{m_n{}^2}=I_x{}^{(3)}\,. \qquad (4)$$

If Eq. (1) needed a subtraction, this condition could be achieved via a subtraction constant. We shall assume that this is not the case. Equation (4) then represents a nontrivial set of constraints on the β_n and $g_{nx\bar{x}}$, since there are *many* targets x for which Eq. (4) must hold with the same set of β_n.

A universality condition similar to Eq. (4) also holds for π-x scattering.[5] The familiar result of current algebra is that[6]

$$\partial M_t{}^{(1)}(\nu,0)/\partial\nu|_{\nu=0}=2I_x{}^{(3)}/f_\pi{}^2\,, \qquad (5)$$

where $f_\pi=94$ MeV is the pion decay constant. This implies that

$$F_{xx}{}^{\pi}(0)=I_x{}^{(3)}/2f_\pi{}^2\,. \qquad (6)$$

Again, if Eq. (3) requires no subtractions, Eq. (6) implies an infinite set of constraints on $g_{n\pi\pi}$ identical to the β_n constraints of Eq. (4). Obviously the set of equations (4) could possess many or even an infinite number of solutions.[7] A particular case is when each ρ_n is separately coupled universally[8]:

$$g_{nx\bar{x}}\sim I_x{}^{(3)}\,, \qquad (7)$$

in which case Eq. (4) is trivially satisfied for all β_n. In the following we assume that only one solution exists. Indeed, since there is only one conserved isovector current both in electromagnetic and weak interactions, there seems to be a "lack of sufficient reason" why the strong interaction parameters $g_{nx\bar{x}}/m_n{}^2$ should be over-constrained so as to allow several different conserved currents. Having made this uniqueness assumption, we readily find

$$\beta_n=2g_{n\pi\pi}f_\pi{}^2\,. \qquad (8)$$

Let us now turn to the specific implications of these relations.

(*a*) *Modified KSRF relation.* If we assume ρ dominance, Eqs. (4) and (8) imply the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation.[9] In general, one has the sum rule

$$(2f_\pi{}^2)^{-1}=\sum g_{n\pi\pi}{}^2/m_n{}^2\,. \qquad (9)$$

Taking the values predicted by the Veneziano $\pi\pi$ formula, the first five terms give $m_\rho{}^2/(2f_\pi{}^2g_{\rho\pi\pi}{}^2)=1.19$, and an estimate of the remainder based on the high-n behavior of $g_{n\pi\pi}{}^2$ [see Sec. (f), below] changes this number to 1.23. Either figure agrees well with experiment.

(*b*) *Charge radii.* For the pion, we find

$$F_\pi{}'(0)\leq 1/m_\rho{}^2\,. \qquad (10)$$

The experimental value for this number is about $1/m_\rho{}^2$,[10] so that bounds on the contributions of higher states to $F_\pi(t)$ may be placed. These require more information than is currently available. [The Veneziano model[11] predicts only a 14% reduction in $F_\pi{}'(0)$, in a calculation similar to that of Sec. (a).]

Note that Eq. (10) depends only on the narrow-resonance approximation and the positivity of residues in the π form factor. The expression for $\langle r^2\rangle$,

$$\tfrac{1}{6}\langle r^2\rangle=F_\pi{}'(0)=\int\frac{dm^2\rho(m^2)}{(m^2)^2}\,,$$

weighs heavily the low m^2 contributions. Deviations from the narrow-resonance approximation *below* the ρ pole may be important, and would increase $\langle r^2\rangle$—improving the agreement with experiment. The smaller relative contribution of higher states to $F_\pi{}'(0)$ as compared with contributions to Eq. (9) allows some freedom in saturating Eq. (9).

For the nucleon, the fact that $G_E{}^{V\prime}(0)$ exceeds $1/m_\rho{}^2$ indicates that some couplings $g_{n\bar{N}N}$ must differ in sign from $g_{\rho\pi\pi}$. The form of $g_{n\pi\pi}g_{n\bar{N}N}$ is model-dependent and is now being studied in the Veneziano model.[12]

(*c*) *Colliding-beam experiments.* Near the mass of a resonance,

$$F_\pi(t\simeq m_n{}^2)\simeq 2f_\pi{}^2g_{n\pi\pi}{}^2/(m_n{}^2-t-im_n\Gamma_n)\,, \qquad (11)$$

where Γ_n is the total width of the resonance n. (We neglect energy-dependent corrections for simplicity.)[13] Defining

$$\eta_{\pi\pi}(n)\equiv\Gamma(n\to\pi\pi)/\Gamma_n\,, \qquad (12)$$

we then find

$$|F_\pi(t=m_n{}^2)|=12\pi f_\pi{}^2m_n\eta_{\pi\pi}(n)/k_n{}^3\,, \qquad (13)$$

where k_n is the magnitude of the pion's three-momentum in the rest frame of n.

Equation (13) gives $|F_\pi(t=m_\rho{}^2)|\simeq 5.5$ to be compared with the experimental values $|F_\pi(t=m_\rho{}^2)|=6.5\pm0.7$ (Novosibirsk)[14] and 7.6 ± 0.4 (Orsay).[15] This is not bad for a rule of thumb that gives *all* the values

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[5] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966).

[6] S. Adler, Phys. Rev. **140**, 736 (1965); W. Weisberger, *ibid.* **143**, 1302 (1966).

[7] See, e.g., D. Atkinson and M. B. Halpern, Phys. Rev. **166**, 1724 (1968). We assume that the coefficient $g_{nx\bar{x}}/M_n{}^2$ decreases sufficiently quickly not to allow infinitely many solutions. One of us (S. N.) thanks D. Atkinson for a discussion of this point.

[8] We thank Professor P. G. O. Freund for raising this point.

[9] K. Kawarabayashi and M. Suzuki, Phys. Letters **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[10] A. Akerlof *et al.*, Phys. Rev. **163**, 1482 (1967); C. Mistretta *et al.*, Phys. Rev. Letters **20**, 1523 (1968).

[11] J. Yellin and J. Shapiro, University of California Lawrence Radiation Laboratory Report No. UCRL-18500 (unpublished); C. Lovelace, Phys. Letters **28B**, 264 (1968); G. Veneziano, Nuovo Cimento **57A**, 190 (1968); J. Shapiro, Phys. Rev. **179**, 1345 (1969).

[12] S. Fenster and K. C. Wali (private communication).

[13] See G. J. Gounaris and J. J. Sakurai, Phys. Rev. Letters **21**, 244 (1968).

[14] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967). We use the reanalyzed values of these authors [R. Arnowitt and J. J. Sakurai (private communication)].

[15] J. E. Augustin *et al.*, Phys. Letters **28B**, 508 (1969).

of F_π at higher poles in terms of hadronic parameters, especially when threshold corrections can modify the result somewhat upward.[13]

For $\rho'(1400)$ and $\rho''(1670)$ [see Sec. (e)], we predict

$$|F_\pi(t=m_{\rho'}^2)|\simeq 1.4\eta_{\pi\pi}(\rho')$$

and

$$|F_\pi(t=m_{\rho''}^2)|\simeq 1.0\eta_{\pi\pi}(\rho'').$$

It will be interesting to test these predictions at higher-energy storage ring facilities.

(*d*) *High-energy ρ photoproduction and photoabsorption.* Photoproduction experiments on nuclei suggest that the coupling of the photon to the ρ meson, $em_\rho^2/2\gamma_\rho$, may be smaller than expected from ρ dominance alone,[16] with $\gamma_\rho^2/4\pi\simeq 1.0$. Our result, $\beta_\rho=2f_\pi^2 g_{\rho\pi\pi}$, suggests $\gamma_\rho^2/4\pi\simeq 0.8$ in agreement with this value within errors. (We use $\Gamma_\rho=112$ MeV [15] to obtain $g_{\rho\pi\pi}$.)

The presence of higher-mass ρ states is then essential to explain the experimental value of $\sigma_T(\gamma p)$, as has been suggested previously, and can be tested by measurement of $\sigma_T(\gamma A)$ for nuclei of mass A.[17]

(*e*) *Photoproduction of higher-mass $\pi\pi$ resonances.* Searches for higher-mass resonances in pion pair photoproduction have been performed at Cornell[18] and DESY.[19] Comparison of corrected bump heights[18] yields the experimental quantity

$$R_n\equiv\frac{\gamma_n^2}{\gamma_\rho^2}\frac{\Gamma_n}{\Gamma(n\to\pi\pi)}\frac{\Gamma_n}{\Gamma_\rho}.$$

In our approach, $\gamma_n=m_n^2/(2g_{n\pi\pi}f_\pi^2)$, so that

$$\eta_{\pi\pi}(n)=(m_n/m_\rho)(k_n/k_\rho)^{3/2}R_n^{-1/2}. \tag{14}$$

We apply this expression to the two observed bumps.[18] (i) $\rho'(1400)$. Ref. 18 implies $R_{\rho'}\gtrsim 200$, leading to $\eta_{\pi\pi}(\rho')\lesssim 0.3$. This prediction of an inelastic ρ' can be tested via the four-pion final state in colliding beam and photoproduction experiments. As $\Gamma(\rho')\lesssim 100$ MeV,[18] we predict $\Gamma(\rho'\to\pi\pi)\lesssim 30$ MeV, at least a factor of 3 below the prediction of the Veneziano model. (ii) $\rho''(1670)$. With $R_{\rho''}\lesssim 250$ (Ref. 18), expression (14) gives $\eta_{\pi\pi}(\rho'')\lesssim 0.5$. With $\Gamma(\rho'')\simeq 50$ MeV,[20] this bound is consistent with the Veneziano-model prediction $\Gamma(\rho''\to\pi\pi)=14$ MeV.

We have assumed with Ref. 18 that $\sigma_T(\rho N)\simeq\sigma_T(\rho' N)\simeq\sigma_T(\rho'' N)$, and that higher-spin $\pi\pi$ resonances are not as strongly produced. Regardless of these details, the difference of γ_n^2 and γ_ρ^2 by at least two orders of magnitude may in our view reflect merely the relatively low mass of the ρ and the moderate inelasticity of higher $\pi\pi$ resonances.

(*f*) *Asymptotic behavior of form factors.* Recent muon pair production experiments[20] could perhaps provide information about the form factor of the pion for timelike t.[21] In that case, preliminary results would suggest a rather slow falloff as $t\to\infty$. On the other hand, the nucleon electric isovector form factor $G_E^V(t)$ falls off more rapidly for timelike t. These different asymptotic behaviors have a natural interpretation. We write

$$F_\pi(t)=\sum g_{n\pi\pi}^2/(m_n^2-t) \tag{15}$$

and

$$G_E^V(t)=\sum g_{n\pi\pi}g_{nN\bar{N}}/(m_n^2-t), \tag{16}$$

and assume that for large m_n^2, $g_{n\pi\pi}^2$ (or $g_{n\pi\pi}g_{nN\bar{N}}$) is given by the $J=1^-$ partial-wave projection of the appropriate $\pi\pi\to\pi\pi$ (or $\pi\pi\to N\bar{N}$) amplitude. Provided this amplitude has the asymptotic behavior $A\sim B(s)t^{\alpha(s)}$, one then finds[22,23] $g_{n\pi\pi}^2\sim(m_n^2)^{\alpha_\rho(0)-1}$ and $g_{n\pi\pi}g_{nN\bar{N}}\sim(m_n^2)^{\alpha_\Delta(0)-1-1/2}$. The $-\frac{1}{2}$ in the second exponent results from the behavior of the singularity-free amplitudes A and B in $\pi\pi\to N\bar{N}$ as $t^{\alpha_\Delta(s)-1/2}$.[24] With $\alpha_\rho(0)\simeq\frac{1}{2}$ and $\alpha_\Delta(0)\simeq 0.1$,[25] we then find $F_\pi(t)\sim t^{-1/2}$ and $G_E^V(t)\sim t^{-1.4}$. [If the effect of the nucleon is in fact dominant, we obtain instead $G_E^V(t)\sim t^{-1.8}$.] We can therefore see at least qualitatively why $G_E^V(t)$ might fall off faster than $F_\pi(t)$.[26]

To conclude, we have given a simple way of preserving universality of the isovector current when vector mesons in addition to the ρ are important. Our predictions include a modified KSRF relation, a bound for the charge radius of the pion, an expression for $|F_\pi(t)|^2$ near any resonance, a value of $\gamma_\rho^2/4\pi$ in accord with the Cornell and SLAC data, upper bounds on the elasticities of higher $\pi\pi$ resonances, and predictions for asymptotic behavior of form factors.

Note added in proof. As has been emphasized [see discussion following Eq. (2)], the only justification for restricting the sum over the states in Eq. (3) to those

[16] G. McClellan *et al.*, Phys. Rev. Letters **22**, 377 (1968); F. Bulos *et al.*, *ibid.* **22**, 490 (1969).

[17] S. Brodsky and J. Pumplin, Phys. Rev. **182**, 1794 (1969).

[18] G. McClellan *et al.*, Phys. Rev. Letters **23**, 718 (1969).

[19] M. Krammer (private communication).

[20] S. Christiansen, G. Hicks, L. Lederman, P. Liaman, B. Pope, and E. Zavattini, Brookhaven National Laboratory Report (unpublished). One of us (S. N.) thanks L. Lederman and E. Zavattini for an enlightening discussion of the experimental results prior to publication.

[21] This has been brought to our attention by E. Zavattini. It should be emphasized that these experiments can be given other interpretations, as noted, e.g., by S. Berman [L. Lederman and E. Zavattini (private communication)].

[22] We would like to thank Dr. M. Kugler for emphasizing this important point.

[23] Taking α_ρ rather than α_P as the leading trajectory in $\pi\pi\to\pi\pi$ is suggested by the Veneziano formula for the $\pi\pi$ amplitude. If the Pomeranchukon is generated by the peripheral production of two resonances (as suggested by Veneziano and Freund), then its asymptotic projection on $J=1$ waves may be negligible.

[24] See, e.g., V. Singh, Phys. Rev. **129**, 1889 (1963).

[25] F. Hayot (private communication).

[26] Similar results were obtained in 1966 using Fubini sum rules [R. Dashen and I. Muzinich (unpublished)]. Closed expressions for form factors based on the Veneziano model have been suggested by di Vecchia and Drago, Y. Oyanagi, H. Sugawara, P. H. Frampton, Rosner and Suura, Suura, and others. There is some freedom in the form taken for the axial-current-3π vertex in Oyanagi's analysis [R. Arnowitt (private communication)]. We believe our process is more straightforward and unique (though it, too, is subject to unknown hard pion corrections). Another approach [I. Gerstein, K. Gottfried, and K. Huang (unpublished)] derives many of our results by assuming that the photon couples to hadrons exclusively via the $\pi\pi$ intermediate state.

of spin 1 is the Weinberg analysis itself. It is amusing to note that if the coupling constants are defined as the coefficients of Feynman propagators, threshold factors would arise naturally so as to eliminate the contribution of $J>1$ spin states.

Thus, for example, the g meson ($J^P=3^-$) would contribute a term

$$M(\nu,t)=g_{g\pi\pi}g_{g\pi\bar{\pi}}[(-2\nu)^3+6\nu t(t-4m_\pi^2)/5]/(m_g^2-t)$$

to $M_t^{(1)}(\nu,t)$ in the soft-pion limit. The term proportional to ν in this expression vanishes at $t=0$ and does not contribute in Eq. (5).

Insofar as this recipe differs from the constant residues prescribed in the dispersive approach, it is obviously *ad hoc*, as it allows in the residues only the threshold factors coming from the Feynman propagators, but no other t dependence.

Another point which should be emphasized is that our use of the $g_{\rho_i\pi\pi}$ obtained from the Veneziano-Lovelace formula has been for illustrative purposes only.

Indeed, in that simple model it is not true that only 1^- states contribute in Eq. (3). Thus, while the Veneziano formula can be easily made to yield the right magnitude of the $I_t=1$ scattering lengths, it does not satisfy the full restrictions of PCAC and current algebra —in particular, the requirement that only 1^- states contribute in Eq. (3).

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Baryon Spectral-Function Sum Rules*

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Necessary and sufficient conditions for baryon spectral-function sum rules are obtained under the assumptions that (1) the equal-time commutator of the axial charges $Q_5{}^a(x_0)$ $(a=1,2,3)$ and the nucleon field $\bar\psi(y)$ is given by $[Q_5{}^a(y_0),\bar\psi(y)]=-r_A\bar\psi(y)\gamma_5\tau^a+(\Delta I=\frac{3}{2}$ terms) and that (2) the axial-vector current $A_\mu{}^a(x)$ is conserved. For each of these sum rules (enumerated by $n=1,2,3\ldots$), the equivalence to $\int d^3z\langle[Q_5{}^a(y_0),[(\partial/\partial y_0)^{2n-1}\psi(y),\bar\psi(z)]_+]|_{y_0=z_0}\rangle_0=0$ is actually shown under weaker conditions: assumption (1) and, instead of (2), $\sum_{j=0}^{2n-2}\langle[\{(\partial/\partial y_0)^j[\int d^3x\,\partial^\mu A_\mu{}^a(y_0\mathbf{x}),(\partial/\partial y_0)^{2n-2-j}\psi(y)]\},\bar\psi(z)]_+|_{y_0=z_0}\rangle_0=0$. Further equivalences are given. The sum rules connect the $(I=\frac{1}{2},J=\frac{1}{2}^+)$ and $(I=\frac{1}{2},J=\frac{1}{2}^-)$ baryon spectrum and include (for $n=1$) a sum rule, obtained independently by Rothleitner and (in the one-particle approximation) by Sugawara. In our derivation we make no assumptions on high-energy behavior and we use an identity of the Jacobi type. Assuming the first two sum rules to be valid, the model then predicts a $P_{11}(m>1470$ MeV) resonance [which may be identified as the observed $P_{11}(1750)$] from the existence of the four nucleon resonances $P_{11}(940)$, $P_{11}(1470)$, $S_{11}(1550)$, and $S_{11}(1710)$.

THE spectral-function sum rules, derived by Weinberg[1] for the chiral $SU(2)\otimes SU(2)$ currents, have been extended by several authors[2–4] and various proofs have been given.[1–6] Among these, Glashow, Schnitzer, and Weinberg[3] have described a derivation of the first Weinberg sum rule using the Jacobi identity, and Jackiw[5] has used the Jacobi identity in order to derive a condition for the second Weinberg sum rule. The main difference between Weinberg's[1] original proof of the second sum rule and the one given by Jackiw lies in the replacement of the assumption on high-energy behavior, made in Ref. 1, by the assumption that a certain vacuum expectation value of a triple commutator vanishes.

Among the extensions of the Weinberg sum rules, Rothleitner[4] has derived a sum rule for baryon spectral functions, assuming that[7]

$$\lim_{p^2\to\infty}\lim_{q_\mu\to 0}\int d^4x d^4y\, e^{-iqx+ipy}\times\langle iT\{(q_\mu+\partial_\mu)A_\mu{}^a(x),\psi(y),\bar\psi(0)\}\rangle_0=0\,,\quad(1)$$

* Supported by the DAAD through a NATO grant.
[1] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).
[2] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); P. A. Cook and G. C. Joshi, Nucl. Phys. **B10**, 253 (1969).
[3] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).
[4] J. Rothleitner, Nucl. Phys. **B3**, 89 (1967).
[5] R. Jackiw, Phys. Letters **27B**, 96 (1968).
[6] W. Bierter and K. M. Bitar, Nuovo Cimento Letters **1**, 192 (1969).

[7] Depending on how the pion mass is treated, either of the two terms vanishes trivially: For massless pions and conserved axial currents, the $[\partial^\mu A_\mu{}^a(x)]$ term vanishes trivially (not the q_μ term, since it has a pion pole at $q_\mu=0$). For massive pions and PCAC, there is no pion pole at $q_\mu=0$, and the $(q^\mu A_\mu{}^a)$ term vanishes trivially. In order to leave room for both interpretations, we will not specify Eq. (1) further.