Model of Spontaneous Breakdown of $SU(3)\otimes SU(3)$ Symmetry. I

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Strong interactions of spin-zero and spin-one mesons are studied in a model in which the Lagrangian is approximately invariant under coordinate-dependent $SU(3)\otimes SU(3)$ gauge transformations. The scalar and pseudoscalar nonets are assigned to the $(3,3^*)\oplus(3^*,3)$ representation, whereas the vector and axial-vector octets are introduced as gauge fields belonging to the $(1,8)\oplus(8,1)$ representation. Spontaneous symmetry breaking to the isospin-hypercharge level is introduced by giving nonzero vacuum expectation values to some scalar fields. The effective Lagrangian obtained after considering field mixings and renormalizations is used in a phenomenological manner to study the masses and couplings of various particles. With some of the experimental meson masses as input, the predicted masses of remaining observed mesons are in reasonable agreement with experiment. Some scalar-meson masses are also predicted ($\mu_{S_K}=527$ MeV and $\mu_{S_\pi}=771$ MeV). The widths for $V\to PP$ and $A\to VP$ decays are calculated and compare well with experiment. The axial-vector currents and the strangeness-changing vector currents satisfy partial conservation equations. Decay constants for spin-zero mesons are calculated. We get $F_K/F_\pi=1.17$ and $F_{S_K}/F_\pi=-0.59$. The K_{l3} form factors are calculated, the effect of the scalar kaon being included; we get $f_+(0)=0.86$, $\xi=-0.197$, $\lambda_+=0.023$, and $\lambda_-=0.013$.

I. INTRODUCTION

MANY papers have recently appeared which consider the chiral group $SU(2)\otimes SU(2)$ or $SU(3)\otimes SU(3)$ as the basic symmetry of the dynamics of elementary particles. Most of these investigations have employed the so-called nonlinear realization[1] of the symmetry. It has been pointed out by some authors[2] that results obtained from phenomenological Lagrangians involving nonlinear realization of the symmetry can also be obtained from the conventional Lagrangians involving linear representations by breaking the symmetry spontaneously. This latter procedure has its own advantage of allowing a simpler and more compact formulation of dynamics.

In this paper we report on a study of the strong interactions of spin-zero and spin-one mesons in a model which employs[3] fields belonging to linear representations of $SU(3)\otimes SU(3)$. We assign the scalar and pseudoscalar nonets to the $(3,3^*)\oplus(3^*,3)$ representation and the vector and the axial-vector octets to the $(1,8)\oplus(8,1)$ representation, the latter being an obvious choice in view of the fact that we introduce the vector and axial-vector fields as gauge fields. The symmetry is broken spontaneously to the level of $SU(2)\otimes U(1)$ corresponding to isospin and hypercharge by giving nonzero vacuum expectation values to some scalar fields. The resulting pattern of particle masses and couplings exhibits a systematically broken symmetry and is well in accord with experiment.

The idea of combining gauge invariance of Yang-Mills type with the spontaneous breakdown of symmetry is a very attractive one and has been proposed by Higgs[4] and Kibble[5] as a simultaneous cure for the masslessness of the Goldstone bosons and the gauge particles. In the procedure adopted by these authors, the Goldstone bosons completely disappear from dynamics and provide for the longitudinal modes of vector mesons. This procedure, if followed faithfully, would be disastrous in the present context because the whole octet of pseudoscalar mesons which appear as Goldstone bosons in this model will have to be eliminated. We therefore do not carry out the polar decomposition of fields along the lines of Kibble, but adopt the simpler procedure[3] of subtracting directly from the scalar fields their nonzero vacuum expectation values.

A price for retaining the Goldstone bosons has to be paid, i.e., to introduce an explicit symmetry breaking to ensure nonzero mass for these particles. So we cannot afford the luxury of starting with a fully symmetric Lagrangian and having a purely spontaneous breakdown of the symmetry. The explicit symmetry-breaking term that we employ is a linear function of the scalar fields. This choice of symmetry breaking guarantees[6] that the currents corresponding to broken-symmetry components satisfy partial conservation equations.

Since the mechanism of spontaneous breakdown does not generate mass for the gauge particles corresponding to unbroken-symmetry components, it is essential to introduce, as done by several authors,[7,8] a common mass term for the spin-one mesons which is invariant only under constant parameter $SU(3)\otimes SU(3)$ transformations. The mass term arising from the Higgs-Kibble mechanism then accounts for the mass splittings.

[1] See, for example, the review article by S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN ,Geneva, 1968), p. 253.

[2] S. Gasiorowicz and D. A. Geffen, Argonne Laboratory Report No. ANL/HEP 6809 (unpublished).

[3] Our approach is similar to that of M. Lévy, Nuovo Cimento **52A**, 23 (1967).

[4] P. W. Higgs, Phys. Rev. **145**, 1156 (1966).

[5] T. W. B. Kibble, Phys. Rev. **155**, 1554 (1967); also *Proceedings of the 1967 International Conference on Particles and Fields, Rochester, 1967* (Wiley-Interscience, Inc., New York, 1968); Y. S. Kim and F. L. Markley, Nuovo Cimento **63A**, 60 (1969).

[6] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).

[7] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[8] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967). Our treatment of vector and axial-vector mesons is the same as in this paper.

Apart from the terms mentioned in the two preceding paragraphs, the rest of our Lagrangian is completely gauge-invariant. One would like to stick to minimal couplings for the vector particles; however, it turns out that at least at the phenomenological level at which our Lagrangian is being considered, it is essential to introduce nonminimal couplings to account for the decay properties of axial-vector mesons.

The plan of the paper is as follows: In Sec. II, the basic Lagrangian is written down and after introducing the symmetry breaking, the field mixings and renormalizations are considered. Particle masses are discussed in Sec. III and couplings and decay rates in Sec. IV. Section V is devoted to a study of the vector and axial-vector currents and their divergences. The decay constants of spin-zero mesons and the K_{l3} form factors are discussed in this section. Section VI contains some concluding remarks.

II. LAGRANGIAN

We start by introducing pseudoscalar and scalar nonets and vector and axial-vector octets

$$P=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}\lambda_i p_i\,,\qquad \Phi=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}\lambda_i\phi_i\,,$$
$$V_\mu=\frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i v_{\mu i}\,,\quad A_\mu=\frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i a_{\mu i}\,, \tag{2.1}$$

where the λ_i's are the usual unitary spin matrices. The representations of $SU(3)\otimes SU(3)$ assigned to these are $(3,3^*)\oplus(3^*,3)$ for scalar and pseudoscalar mesons and $(1,8)\oplus(8,1)$ for vector and axial-vector mesons. The basic Lagrangian is

$$\mathfrak{L}=\mathfrak{L}_1(P,\Phi,V,A)+\mathfrak{L}_2(P,\Phi)+\mathfrak{L}_3(V,A)+\mathfrak{L}_4(\Phi)\,, \tag{2.2}$$

where

$$\mathfrak{L}_1=-\tfrac{1}{2}\{(D_\mu\Phi)^2+(D_\mu P)^2\}\,,$$

$$\mathfrak{L}_2=-\tfrac{1}{2}\mu_0^2W_2-\tfrac{1}{2}C_1W_3-\tfrac{1}{2}C_2W_4-\tfrac{1}{2}C_3(W_2)^2\,,$$

$$\mathfrak{L}_3=-\tfrac{1}{4}\{F_{\mu\nu}F_{\mu\nu}+G_{\mu\nu}G_{\mu\nu}\}-\tfrac{1}{2}m_0^2\{V_\mu V_\mu+A_\mu A_\mu\}\,,$$

$$\mathfrak{L}_4=\{\epsilon\Phi\}\,,$$

$$D_\mu\Phi=\partial_\mu\Phi+\frac{ig}{\sqrt{2}}[V_\mu,\Phi]_-+\frac{g}{\sqrt{2}}[A_\mu,P]_+\,,$$

$$D_\mu P=\partial_\mu P+\frac{ig}{\sqrt{2}}[V_\mu,P]_--\frac{g}{\sqrt{2}}[A_\mu,\Phi]_+\,,$$

$$W_2=\{\Phi^2+P^2\}\,,$$

$$W_3=\tfrac{1}{2}[\det(\Phi+iP)+\text{H.c.}]\,,$$

$$W_4=\{[(\Phi+iP)(\Phi-iP)]^2\}\,,$$

$$F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu+\frac{ig}{\sqrt{2}}[V_\mu,V_\nu]_-+\frac{ig}{\sqrt{2}}[A_\mu,A_\nu]_-\,,$$

$$G_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+\frac{ig}{\sqrt{2}}[V_\mu,A_\nu]_--\frac{ig}{\sqrt{2}}[V_\nu,A_\mu]_-\,,$$

$$\epsilon=\frac{1}{\sqrt{2}}\sum_{i=0}^{8}\epsilon_i\lambda_i\,.$$

The symbol { } means that trace is to be taken.

The Lagrangian (2.2) is invariant under coordinate-dependent $SU(3)\otimes SU(3)$ gauge transformations except for the m_0^2 term and $\mathfrak{L}_4$. The vector-meson mass term, which is invariant only under coordinate-independent $SU(3)\otimes SU(3)$ transformations, is included to ensure a nonzero mass of the gauge particles coupled to conserved currents (i.e., ρ and ω mesons) as explained in the Introduction. The symmetry-breaking term $\mathfrak{L}_4$ is necessary to ensure nonzero masses of the Goldstone particles (i.e., the eight pseudoscalar mesons and the scalar kaon) in spite of the fact that actual contributions to these masses come from $\mathfrak{L}_2$. This point will be made more clear in Sec. VI.

The spontaneous symmetry breaking is introduced, as usual, by giving a nonzero vacuum expectation value to some scalar fields; we assume that

$$\eta\equiv\langle\Phi\rangle_0=\frac{1}{\sqrt{2}}(\eta_0 I+\eta_8\lambda_8)\,. \tag{2.3}$$

The η_0 breaks the symmetry from $SU(3)\otimes SU(3)$ to $SU(3)$, whereas η_8 breaks it down to $SU(2)\otimes U(1)$ corresponding to isospin and hypercharge. We write

$$\Phi=\eta+S\,. \tag{2.4}$$

The substitution of (2.4) in $\mathfrak{L}_1$ gives terms like $A_\mu\cdot\partial_\mu P$ and $V_\mu\cdot\partial_\mu S$. To remove these, we write

$$a_{\mu\alpha}=\hat{a}_{\mu\alpha}+\xi_{p\alpha}\partial_\mu p_\alpha\,,$$
$$v_{\mu\alpha}=\hat{v}_{\mu\alpha}+\xi_{s\alpha\beta}\partial_\mu s_\beta\,, \tag{2.5}$$

where the coefficients $\xi_{p,s}$ are to be determined by the requirement that terms of the type $\hat{a}_\mu\cdot\partial_\mu p$ and $\hat{v}_\mu\cdot\partial_\mu s$ are absent from the Lagrangian. This gives

$$\xi_{p\alpha}=g\zeta_\alpha/[m_0^2+(g\zeta_\alpha)^2+\tfrac{2}{3}(g\eta_8)^2\delta_{\alpha8}]\,,$$
$$\xi_{s\alpha\beta}=g\eta_8 f_{8\alpha\beta}/(m_0^2\delta_{\alpha\beta}+g^2\eta_8^2 f_{8\alpha\gamma}f_{8\beta\gamma})\,, \tag{2.6}$$

where

$$\zeta_\alpha=\eta_0+\eta_8 d_{\alpha\alpha8}\,. \tag{2.7}$$

The substitution (2.5) in $\mathfrak{L}_1$ modifies the kinetic energy term of pseudoscalar and scalar mesons. To renormalize these, we write

$$p_\alpha=Z_{p_\alpha}^{1/2}\hat{p}_\alpha\,,$$
$$s_\alpha=Z_{s_\alpha}^{1/2}\hat{s}_\alpha\,, \tag{2.8}$$

and determine Z's such that the physical fields $\hat{p}_\alpha$ and $\hat{s}_\alpha$ satisfy the canonical commutation relations. This gives

$$Z_{p_\alpha}=1+(g\zeta_\alpha)^2/[m_0^2+\tfrac{2}{3}(g\eta_8)^2\delta_{\alpha 8}],\qquad Z_{s_\alpha}=1+(g\eta_8)^2 f_{\alpha 8\gamma}f_{\gamma 8\alpha}/m_0^2. \tag{2.9}$$

After the substitution (2.4) in (2.2) there should be no terms linear in S fields present in the Lagrangian. This requires that only ϵ_0 and ϵ_8 are nonzero and satisfy the two conditions

$$\eta_0\mathbf{u}_0^2+\frac{3}{\sqrt{2}}C_1\left(\eta_0^2-\frac{\eta_8^2}{3}\right)+C_2\left(\eta_0^3+\frac{6\eta_0\eta_8^2}{3}-\frac{2\eta_8^3}{3\sqrt{3}}\right)+(\sqrt{\tfrac{2}{3}})\epsilon_0=0 \tag{2.10}$$

and

$$\eta_8\left[\mathbf{u}_0^2-\frac{3C_1}{\sqrt{2}}\left(\eta_0+\frac{\eta_8}{\sqrt{3}}\right)+3C_2\left(\eta_0^2-\frac{\eta_0\eta_8}{\sqrt{3}}+\frac{\eta_8^2}{3}\right)\right]-\epsilon_8=0, \tag{2.11}$$

where

$$\mathbf{u}_0^2=\mu_0^2+C_3(3\eta_0^2+2\eta_8^2). \tag{2.12}$$

More is said of Eqs. (2.10) and (2.11) in Sec. VI.

When the Lagrangian (2.2) is expressed in terms of the physical fields $\hat{S}$, $\hat{P}$, $\hat{V}$, and $\hat{A}$, one finds that the symmetry is broken in a systematic manner. The resulting pattern of particle masses and couplings is explored in the following sections.

III. PARTICLE MASSES

A. Masses of Vector and Axial-Vector Mesons

The mass matrices of vector and axial-vector mesons are given by

$$(M_V^2)_{\alpha\beta}=m_0^2\delta_{\alpha\beta}+g^2\eta_8^2 f_{8\alpha\gamma}f_{8\beta\gamma}$$

and

$$(M_A^2)_{\alpha\beta}=[m_0^2+g^2(\zeta_\alpha^2+\tfrac{2}{3}\eta_8^2\delta_{8\alpha})]\delta_{\alpha\beta}, \tag{3.1}$$

where ζ_α is given by (2.7). As expected, the η_0 splits the axial-vector multiplet from the vector multiplet, whereas η_8 causes splitting among different isospin multiplets. The symmetry breaking alters, in fact, increases,[8] the masses of only those spin-one mesons that correspond to the broken components of the symmetry.[5] We identify the vector octet $\hat{V}$ with the $\rho(760)$, $K^*(890)$, and $\omega(780)$. The experimental situation regarding the axial-vector mesons is not clear as yet. We tentatively identify the axial octet $\hat{A}$ with the $A_1(1070)$, $K_A(1335)$, and $A_8(1420)$ resonances.[9] From the observed A_1 and K_A masses, we get

$$(g\eta_0)^2=(982.7\text{ MeV})^2 \tag{3.2}$$

and

$$\xi=\eta_8/\eta_0=-0.40. \tag{3.3}$$

[9] Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

Inserting these values in (3.1) we get

$$M_{K^*}=834\text{ MeV},\quad M_{A_8}=1467\text{ MeV}. \tag{3.4}$$

B. Masses of Pseudoscalar and Scalar Mesons

The mass matrix for pseudoscalar mesons is given by

$$(\mu_p^2)_{\alpha\beta}=[\mathbf{u}_0^2\delta_{\alpha\beta}+(C_1/\sqrt{2})T_{\alpha\beta}^1+(C_2/\sqrt{2})(T_{\alpha\beta}^2-T_{\alpha\beta}^3)]Z_{p_\alpha}^{1/2}Z_{p_\beta}^{1/2}, \tag{3.5}$$

where $\mathbf{u}_0^2$ is given by (2.12) and

$$T_{\alpha\beta}^1=(3\eta_0-6\eta_8 d_{8\alpha\alpha})\delta_{\alpha\beta}-6\eta_0\delta_{\alpha0}\delta_{\beta0}+2\sqrt{3}\eta_8\delta_{\alpha0}\delta_{\beta8},$$

$$T_{\alpha\beta}^2=(2\eta_0^2+\tfrac{4}{3}\eta_8^2)\delta_{\alpha\beta}+4\eta_0\eta_8 d_{\alpha\beta8}-(4/\sqrt{3})d_{\alpha\beta8}\eta_8^2,$$

$$T_{\alpha\beta}^3=(\eta_0^2-\tfrac{2}{3}\eta_8^2)\delta_{\alpha\beta}+2\eta_0\eta_8 d_{\alpha\beta8}+\eta_8^2\Big(\tfrac{4}{3}\delta_{8\alpha}\delta_{8\beta}+\sum_{\gamma=1}^{8}2d_{\alpha\beta\gamma}d_{8\gamma\beta}-(1/\sqrt{3})d_{\alpha\beta8}\Big).$$

We take account of the mixing between $\hat{p}_0$ and $\hat{p}_8$ in the usual manner by introducing the physical fields $p_{\eta'}$ and p_η through the equations

$$\hat{p}_8=p_\eta\cos\theta+p_{\eta'}\sin\theta,\qquad \hat{p}_0=-p_\eta\sin\theta+p_{\eta'}\cos\theta. \tag{3.6}$$

The requirement that there be no mixed terms $p_\eta p_{\eta'}$ gives

$$\tan 2\theta=(\mu_p^2)_{08}/(\mu_{p_0}^2-\mu_{p_8}^2). \tag{3.7}$$

The p_η and $p_{\eta'}$ masses are given by

$$\mu_{p_\eta}^2=\tfrac{1}{2}\{\mu_{p_0}^2+\mu_{p_8}^2-[(\mu_{p_0}^2-\mu_{p_8}^2)^2+\mu_{p_{08}}^4]^{1/2}\},\qquad \mu_{p_{\eta'}}^2=\tfrac{1}{2}\{\mu_{p_0}^2+\mu_{p_8}^2+[(\mu_{p_0}^2-\mu_{p_8}^2)^2+\mu_{p_{08}}^4]^{1/2}\}. \tag{3.8}$$

The scalar-meson mass matrix is

$$(\mu_s^2)_{\alpha\beta}=[\mathbf{u}_0^2-(C_1/\sqrt{2})T_{\alpha\beta}^1+\tfrac{1}{2}C_2(T_{\alpha\beta}^2+T_{\alpha\beta}^3)+\tfrac{1}{2}C_3(4\eta_8^2\delta_{8\alpha}\delta_{8\beta}+6\eta_0^2\delta_{0\alpha}\delta_{0\beta}+4(\sqrt{6})\eta_0\eta_8\delta_{\alpha0}\delta_{8\beta})]Z_{s_\alpha}^{1/2}Z_{s_\beta}^{1/2}. \tag{3.9}$$

We determine the parameters $\mathbf{u}_0^2$, C_1, and C_2 from the experimental masses of $\pi(140)$, $K(494)$, and $\eta'(960)$. Since $\mu_{p_{\eta'}}$ is quadratic in $\mathbf{u}_0^2$, C_1, and C_2, we get two solutions for the masses of the remaining particles and other parameters:

	Solution I	Solution II
μ_{p_η}	530 MeV	5×10^8 MeV
μ_{S_π}	771 MeV	3×10^7 MeV
μ_{S_K}	527 MeV	527 MeV
θ	27°	25°
$\mathbf{u}_0^2$	$4.13\times10^5(\text{MeV})^2$	$1.26\times10^{15}(\text{MeV})^2$
C_1/g	-113.8 MeV	-3.25×10^{11} MeV
C_2/g^2	-0.098	-4.79×10^8

(3.10)

TABLE I. Decay widths (in MeV) of vector and axial-vector mesons calculated for $g^2/4\pi=4$ and various values of δ.

δ	0	0.05	0.1	0.2	0.3	0.4	0.5	Experimental values[a]
$\Gamma(\rho\to\pi\pi)$	115	119	123	131	139	148	157	125±20
$\Gamma(K^*\to K\pi)$	41.2	42	43.2	45.3	47.5	49.7	52	49.7±1.1
$\Gamma(A_1\to\rho\pi)$	121	94	69	32	9	0.17	5.7	80±35
$\Gamma(K_A\to\rho K)$	76	62	50	29	15	7	4 }	70±10
$\Gamma(K_A\to K^*\pi)$	369	292	225	117	44	6.5	4.6 }	
$\Gamma(A_8\to K^*\bar{K})$	126	93	65	24	3.2	2.4	21	35±1

[a] See Ref. 9.

The second solution which corresponds to large masses and couplings for scalar particles[10] is extraneous (in fact, it disappears in the limit $g\to 0$) and we discard it. The mass of the scalar kaon is consistent with the Glashow-Weinberg inequality $\mu_{S_K}\leqslant 670$ MeV for $F_K/F_\pi>0$. Moreover, it is below the $K\pi$ threshold so that the scalar kaon can decay only electromagnetically by $S_K\to K+2\gamma$. The existence of such a particle is not excluded by experiments.[11]

After considering the mixing of $\hat{S}_8$ and $\hat{S}_0$, equations analogous to (3.8) are obtained for the S_η and $S_{\eta'}$ masses where, however, $\mathbf{u}_0^2$ and C_3 do not occur in the same combination as they occur in other masses and therefore it is not possible to predict these masses. For $C_3=0$ we find that $\mu_{S_\eta}^2<0$, $\mu_{S_{\eta'}}=335$ MeV for solution I. C_3 therefore cannot be zero. This is in contrast to the result of Ref. 3 where even for $C_3=0$ these masses were positive. One can easily trace back the source of this difference to the nontrivial renormalization of spin-zero fields arising due to coupling with gauge fields.

A comment is in order in connection with the choice of input masses. The choice of M_{K^*} instead of M_{K_A} as input gave $\mu_{S_K}=471$ MeV which means that the $K\to S_K+2\gamma$ could be allowed contrary to experiment. Similarly the choice of μ_{p_η} instead of $\mu_{p_{\eta'}}$ as input gives $\mu_{p_{\eta'}}=501$ MeV and 10^7 MeV in the two solutions. Since there is no observed pseudoscalar meson around 500 MeV, one would perhaps prefer in this case solution II, implying thereby that the ninth pseudoscalar meson and some scalar mesons are large-mass objects.[10] We, however, prefer to adopt the following attitude: Since the predictions of this model are not expected to be very accurate, it is desirable to choose input masses from both the lower and higher ends of a multiplet to obtain a balanced empirical fit.

Expanding meson masses in powers of η_8 and keeping only first order terms in η_8, we see that meson (squared) masses satisfy the Gell-Mann–Okubo mass formula.

[10] In this connection it is interesting to recall the work of W. A. Bardeen and B. W. Lee [Phys. Rev. 177, 2389 (1968)], who advocate that only the pseudoscalar octet and the strange scalar meson are the low-energy excitations, whereas their remaining chiral partners are some large-mass objects. To achieve this, they make some parameters infinite with appropriate restrictions so that the masses of the physical particles are finite. In our model, the situation of Bardeen and Lee is automatically realized if we take the $\eta(550)$ mass as input and pick up the solution II instead of I.

[11] L. Kirsch, Phys. Rev. 175, 1733 (1968).

IV. COUPLINGS

The various coupling constants are to be read off from the effective Lagrangian obtained by substituting (2.4), (2.5), and (2.8) in (2.2). The V-P-P and A-V-P interaction Lagrangians are of the following form:

$$\mathfrak{L}(VPP)=g_1^{\alpha\beta\gamma}\hat{v}_{\mu\alpha}\hat{p}_\beta\partial_\mu\hat{p}_\gamma+g_2^{\alpha\beta\gamma}(\partial_\mu\hat{v}_{\nu\alpha}-\partial_\nu\hat{v}_{\mu\alpha})\partial_\mu\hat{p}_\beta\partial_\nu\hat{p}_\gamma,\quad(4.1)$$

$$\mathfrak{L}(AVP)=g_3^{\alpha\beta\gamma}\hat{a}_{\mu\alpha}\hat{v}_{\mu\beta}\hat{p}_\gamma+g_4^{\alpha\beta\gamma}(\partial_\mu\hat{v}_{\nu\alpha}-\partial_\nu\hat{v}_{\mu\alpha})\hat{a}_{\nu\beta}\partial_\mu\hat{p}_\gamma+g_5^{\alpha\beta\gamma}(\partial_\mu\hat{a}_{\nu\alpha}-\partial_\nu\hat{a}_{\mu\alpha})\hat{v}_{\nu\beta}\partial_\mu\hat{p}_\gamma.\quad(4.2)$$

The various coupling constants in these equations are those given in the Appendix with $h=0$. After performing an integration by parts on the last term of (4.1), we get, for the effective V-P-P coupling constant,

$$G(\hat{v}_\alpha\hat{p}_\beta\hat{p}_\gamma)=g_1^{\alpha\beta\gamma}+q_\alpha^2g_2^{\alpha\beta\gamma},\quad(4.3)$$

where q_α is the four-momentum of the vector meson. For $q_\alpha^2=0$, Eq. (4.3) gives the universal $SU(3)$ value $gf_{\alpha\beta\gamma}$ for the effective coupling of vector mesons corresponding to the conserved symmetry components, namely, ρ and ω, whereas the coupling of K^* gets modified by renormalizations and mixings.

The decay of axial-vector mesons according to (4.2) is of pure S-wave type, whereas in a more realistic calculation[12] one needs two coupling constants, corresponding to S wave and D wave, defined through the equation

$$\langle v(q)p(p)|a(Q)\rangle=G_S\epsilon^v\cdot\epsilon^a+G_D\epsilon^v\cdot Q\epsilon^a\cdot q,\quad(4.4)$$

where the ϵ's are the polarization vectors. We can include the D-wave coupling constants by adding a nonminimal coupling term[13] of the form

$$\mathfrak{L}=(h/\sqrt{2})\{-\tfrac{1}{2}iF_{\mu\nu}[D_\mu P,D_\nu P]_-+\tfrac{1}{2}G_{\mu\nu}[D_\mu P,D_\nu\Phi]_+\}.\quad(4.5)$$

One of the predictions of our model is that

$$\Gamma(K_A\to\omega K)=\Gamma(K_A\to\rho K),$$

which is expected because $M_\rho=M_\omega$ and the ϕ meson is

[12] D. A. Geffen, Phys. Rev. Letters 19, 770 (1967); S. G. Brown and G. B. West, *ibid.* 18, 812 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* 19, 1067 (1967); H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[13] This is the $SU(3)\otimes SU(3)$ analog of the κ term of J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967).

absent from the model, mixing of which with ω suppresses the ω mode.

There are two parameters so far undetermined, i.e., g and h. The decay widths of the allowed two-particle modes of vector and axial-vector mesons, namely, $\rho\to\pi\pi$, $K^*\to K\pi$, $A_1\to\rho\pi$, $K_A\to(\rho K,K^*\pi)$, and $A_8\to K^*\bar{K}$ as functions of g and $\delta=-hM_\rho^2/g$ are given by[14]

$$\begin{aligned}\Gamma(\rho\to\pi\pi)&=(g^2/4\pi)[28.8+19.3\delta+3.2\delta^2],\\ \Gamma(K^*\to K\pi)&=(g^2/4\pi)[10.3+5.1\delta+0.6\delta^2],\\ \Gamma(A_1\to\rho\pi)&=(g^2/4\pi)[30.4-147\delta+179\delta^2],\\ \Gamma(K_A\to\rho K)&=(g^2/4\pi)[18.9-72\delta+73\delta^2],\\ \Gamma(K_A\to K^*\pi)&=(g^2/4\pi)[92.3-404\delta+444\delta^2],\\ \Gamma(A_8\to K^*\bar{K})&=(g^2/4\pi)[31.4-177\delta+250\delta^2].\end{aligned}\tag{4.6}$$

The best over-all agreement of calculated widths with experiment is obtained with $g^2/4\pi\approx4$ and $\delta\approx0.05$. Widths for this value of $g^2/4\pi=4$ and various values of δ are given in Table I.

V. CURRENTS AND THEIR DIVERGENCES

The infinitesimal transformations for the fields ϕ_α, p_α, $v_{\mu\alpha}$, and $a_{\mu\alpha}$ under $SU(3)$ are given by

$$\begin{aligned}\delta_V\phi_\alpha&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^V\phi_\gamma,\quad \delta_Vp_\alpha=-f_{\alpha\beta\gamma}\epsilon_\beta{}^Vp_\gamma,\\ \delta_Vv_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^Vv_{\mu\gamma}-(1/g)\partial_\mu\epsilon_\alpha{}^V,\\ \delta_Va_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^Va_{\mu\gamma},\end{aligned}\tag{5.1}$$

whereas under the axial $SU(3)$ transformations these fields transform as

$$\begin{aligned}\delta_A\phi_\alpha&=d_{\alpha\beta\gamma}\epsilon_\beta{}^Ap_\gamma,\quad \delta_Ap_\alpha=-d_{\alpha\beta\gamma}\epsilon_\beta{}^A\phi_\gamma,\\ \delta_Av_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^Aa_{\mu\gamma},\\ \delta_Aa_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^Av_{\mu\gamma}-(1/g)\partial_\mu\epsilon_\alpha{}^A.\end{aligned}\tag{5.2}$$

For physical fields, the corresponding transformations are

$$\begin{aligned}\delta_V\hat{s}_\alpha&=-f_{\alpha\beta\gamma}\frac{\epsilon_\beta{}^V}{Z_{s\alpha}{}^{1/2}}[Z_{s\gamma}{}^{1/2}\hat{s}_\gamma+\eta_\gamma],\\ \delta_V\hat{p}_\alpha&=-f_{\alpha\beta\gamma}\frac{\epsilon_\beta{}^V}{Z_{p\alpha}{}^{1/2}}Z_{p\gamma}{}^{1/2}\hat{p}_\gamma,\\ \delta_V\hat{v}_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^V[\hat{v}_{\mu\gamma}+Z_{s\sigma}{}^{1/2}\partial_\mu\hat{s}_\sigma\xi_{s\gamma\sigma}]\\ &\quad+f_{\lambda\beta\gamma}\epsilon_\beta{}^V\xi_{s\alpha\lambda}Z_{s\gamma}{}^{1/2}\partial_\mu\hat{s}_\gamma\\ &\quad-\partial_\mu\epsilon_\beta{}^V[(1/g)\delta_{\alpha\beta}-f_{\lambda\beta\sigma}\xi_{s\alpha\lambda}(Z_{s\sigma}{}^{1/2}\hat{s}_\sigma+\eta_\sigma)],\\ \delta_V\hat{a}_{\mu\alpha}&=-f_{\alpha\beta\gamma}\epsilon_\beta{}^V[\hat{a}_{\mu\gamma}+(\xi_{p\gamma}-\xi_{p\alpha})Z_{p\gamma}{}^{1/2}\partial_\mu\hat{p}_\gamma]\\ &\quad+\partial_\mu\epsilon_\beta{}^Vf_{\alpha\beta\gamma}\xi_{p\alpha}Z_{p\gamma}{}^{1/2}\hat{p}_\gamma,\end{aligned}$$

$$\begin{aligned}\delta_A\hat{s}_\alpha&=d_{\alpha\beta\gamma}\epsilon_\beta{}^A\frac{Z_{p\gamma}{}^{1/2}}{Z_{s\alpha}{}^{1/2}}\hat{p}_\gamma,\\ \delta_A\hat{p}_\alpha&=-d_{\alpha\beta\gamma}\frac{\epsilon_\beta{}^A}{Z_{p\alpha}{}^{1/2}}[Z_{s\gamma}{}^{1/2}\hat{s}_\gamma+\eta_\gamma],\\ \delta_A\hat{v}_{\mu\alpha}&=-\epsilon_\beta{}^A[f_{\alpha\beta\gamma}(\hat{a}_{\mu\gamma}+\xi_{p\gamma}Z_{p\gamma}{}^{1/2}\partial_\mu\hat{p}_\gamma)\\ &\quad-d_{\lambda\beta\gamma}\xi_{s\alpha\lambda}Z_{p\gamma}{}^{1/2}\partial_\mu\hat{p}_\gamma]-d_{\lambda\beta\gamma}\partial_\mu\epsilon_\beta{}^A\xi_{s\alpha\lambda}Z_{p\gamma}{}^{1/2}\hat{p}_\gamma,\\ \delta_A\hat{a}_{\mu\alpha}&=-\epsilon_\beta{}^A[f_{\alpha\beta\gamma}(\hat{v}_{\mu\alpha}+\xi_{s\alpha\sigma}Z_{s\sigma}{}^{1/2}\partial_\mu\hat{s}_\sigma)\\ &\quad-d_{\alpha\beta\gamma}\xi_{p\alpha}Z_{s\gamma}{}^{1/2}\partial_\mu\hat{s}_\gamma]\\ &\quad+\partial_\mu\epsilon_\beta{}^A[-(1/g)\delta_{\alpha\beta}+d_{\alpha\beta\gamma}\xi_{p\gamma}(Z_{s\gamma}{}^{1/2}\hat{s}_\gamma+\eta_\gamma)].\end{aligned}\tag{5.3}$$

Now the vector and axial-vector currents can be calculated from the Lagrangian.[15] The vector current is given by

$$J_{\mu\alpha}{}^V=-(m_0{}^2/g)(\hat{v}_{\mu\alpha}+\xi_{s\alpha\lambda}Z_{s\lambda}{}^{1/2}\partial_\mu\hat{s}_\lambda)+K_{\mu\alpha}{}^V,\tag{5.4}$$

where

$$\begin{aligned}K_{\mu\alpha}{}^V=-(1/g)\partial_\nu F_{\mu\nu,\alpha}-f_{\lambda\beta\gamma}\partial_\nu[\xi_{s\alpha\lambda}F_{\mu\nu,\beta}(Z_{s\gamma}{}^{1/2}\hat{s}_\gamma+\eta_\gamma)\\ +\delta_{\alpha\lambda}\xi_{p\lambda}G_{\mu\nu,\beta}Z_{p\gamma}{}^{1/2}\hat{p}_\gamma].\end{aligned}$$

As expected, the $\partial_\mu\hat{s}$ term in Eq. (5.4) is nonzero only for the strangeness-changing current. Defining the decay constant of the S_K meson through the relation

$$\langle0|J_{\mu K^+}{}^V(0)|S_K{}^+(q)\rangle=-F_{SK}q_\mu,\tag{5.5}$$

we get

$$F_{SK}=\tfrac{1}{2}\sqrt{3}\eta_8/Z_{SK}{}^{1/2}.\tag{5.6}$$

The axial-vector current is given by

$$J_{\mu\alpha}{}^A=-(m_0{}^2/g)(\hat{a}_{\mu\alpha}+\xi_{p\alpha}Z_{p\alpha}{}^{1/2}\partial_\mu\hat{p}_\alpha)+K_{\mu\alpha}{}^A,\tag{5.7}$$

where

$$\begin{aligned}K_{\mu\alpha}{}^A=-(1/g)\partial_\nu G_{\mu\nu,\alpha}\\ -d_{\lambda\beta\gamma}\partial_\nu[\xi_{p\alpha}G_{\mu\nu,\beta}(Z_{s\gamma}{}^{1/2}\hat{s}_\gamma+\eta_\gamma)\delta_{\alpha\lambda}\\ +\xi_{s\alpha\lambda}F_{\mu\nu,\beta}Z_{p\gamma}{}^{1/2}\hat{p}_\gamma].\end{aligned}$$

The decay constants for the pseudoscalar mesons are defined through the relation

$$\langle0|J_{\mu\alpha}{}^A|p_\alpha(q)\rangle=-iF_{p\alpha}q_\mu.$$

Equation (5.7) gives

$$F_{p\alpha}=(m_0{}^2/g)\xi_{p\alpha}Z_{p\alpha}{}^{1/2}.\tag{5.8}$$

[14] For all calculations we have used the particle masses as predicted by our model. If, for example, we use the physical K^* mass in the phase space, the decay width of $K^*\to K\pi$ will be increased whereas widths of $K_A\to K^*\pi$ and $A_8\to K^*\bar{K}$ will decrease.

[15] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960). A straightforward application of the Gell-Mann–Lévy prescription $J_\mu=-\delta\mathcal{L}/\delta\partial_\mu\epsilon$ would give the vector current without the K_μ terms in (5.4) and (5.7). However, when the field variations involve derivatives of the group parameters, this procedure gives a wrong result, as is clear from the fact that the equations so obtained are inconsistent with the equations of motion for the gauge fields. A suitably modified procedure [T. Dass, Nuovo Cimento Letters **2**, 584 (1969)] gives Eqs. (5.4) and (5.7), which are, in fact, nothing but the equations of motion of the gauge fields. The prescription for calculating current divergences remains unchanged.

The meson masses, their decay constants, and renormalization constants satisfy the Glashow-Weinberg relations[6]

$$F_\pi Z_\pi^{1/2}=F_K Z_K^{1/2}+F_{S_K}Z_{S_K}^{1/2},$$
$$\mu_\pi^2 F_\pi Z_\pi^{-1/2}=\mu_K^2 F_K Z_K^{-1/2}+\mu_{S_K}^2 F_{S_K}Z_{S_K}^{-1/2}. \qquad (5.9)$$

Incidentally, since μ_{S_K} is related to μ_π and μ_K through (5.9), it explains why we got the same value for μ_{S_K} for both the solutions in Sec. III.

The quantities F_π, M_{A_1}, and M_ρ are related to each other through the relation

$$F_\pi^2=(M_\rho^2/g)^2(1/M_\rho^2-1/M_{A_1}^2), \qquad (5.10)$$

which is precisely Weinberg's first sum rule[16] in the single-particle approximation. Similarly for the strange vector and axial-vector currents, we have

$$F_K^2-F_{S_K}^2=(M_\rho^2/g)^2(1/M_{K^*}^2-1/M_{K_A}^2). \qquad (5.11)$$

From (5.6) and (5.8) we have

$$F_\pi=535/g\ \text{MeV}=76.4\ \text{MeV}\quad(\text{for } g^2/4\pi=4.0),$$
$$F_K/F_\pi=1.17, \qquad (5.12)$$
$$F_{S_K}/F_\pi=-0.59.$$

From experiment we have[2]

$$(F_K\tan\theta_A/F_\pi)^2=0.075. \qquad (5.13)$$

Substituting the value of F_K/F_π from (5.12) in (5.13), we get

$$\tan\theta_A=0.23. \qquad (5.14)$$

It is interesting to compare this with an alternative estimate of θ_A. Recently, an expression for the axial-vector Cabibbo angle was obtained[17] by requiring that the second-order quadratically divergent part of the weak self-masses of hadronic states should vanish; it is

$$\tan^2\theta_A=\frac{1}{2}\frac{F_\pi}{F_K}\frac{\mu_\pi^2}{\mu_K^2}\frac{Z_K^{1/2}}{Z_\pi^{1/2}}. \qquad (5.15)$$

An estimate of the right-hand side from our model gives

$$\tan\theta_A=0.20, \qquad (5.16)$$

which agrees well with (5.14).

Now we use Eq. (5.4) and the effective Lagrangian to determine the K_{l3} form factors $f_\pm(q^2)$ which are defined through the relation

$$\langle\pi^0(p)|J_\mu^{VK^+}(0)|K^+(k)\rangle$$
$$=-\tfrac{1}{2}[(p+k)_\mu f_+(q^2)+(k-p)_\mu f_-(q^2)]. \qquad (5.17)$$

These are normalized such that in the $SU(3)$ limit $f_+(0)=1$. In Eq. (5.4) we consider only the first two terms corresponding to the K^* and S_K poles and neglect the K_μ^V term.[18] We make use of the effective K^* couplings from Eqs. (4.1) and (4.5) and the effective S_K couplings:

$$\mathcal{L}(K^+S_K^-\pi^0)=g_6K^+\partial_\mu S_K^-\partial_\mu\pi^0+g_7\partial_\mu K^+\partial_\mu S_K^-\pi^0$$
$$+g_8\partial_\mu K^+S_K^-\partial_\mu\pi^0+g_9K^+S_K^-\pi^0. \qquad (5.18)$$

The coupling constants are given in the Appendix. We get

$$f_+(q^2)=(m_0^2/g)[2/(q^2+M_{K^*}^2)]$$
$$\times(g_1^{K^*K\pi}+\tfrac{1}{2}q^2g_2^{K^*K\pi}), \qquad (5.19)$$

$$f_-(q^2)=-(m_0^2/g)[2/(q^2+M_{K^*}^2)]$$
$$\times(\mu_K^2-\mu_\pi^2)\left(\frac{g_1^{K^*K\pi}}{M_{K^*}^2}-\tfrac{1}{2}g_2^{K^*K\pi}\right)$$
$$+[F_{S_K}/(q^2+\mu_{S_K}^2)][2g_9-\mu_K^2(-g_6+g_7+g_8)$$
$$-\mu_\pi^2(g_6-g_7+g_8)-\mu_{S_K}^2(g_6+g_7-g_8)]. \qquad (5.20)$$

We see that f_+ is governed by K^* pole, whereas f_- is governed by both K^* and S_K poles. For small q^2, we write

$$f_\pm(q^2)=f_\pm(0)(1-\lambda_\pm q^2/\mu_\pi^2). \qquad (5.21)$$

Using the values[14] of the various masses and coupling constants, we get

$$f_+(0)=0.86. \qquad (5.22)$$

From experiment,[19] we have

$$F_K/(F_\pi f_+(0))=1.28. \qquad (5.23)$$

Using the values of F_K/F_π from Eq. (5.12), we get

$$f_+(0)=0.91, \qquad (5.24)$$

which agrees reasonably well with (5.22). The other quantities $f_-(0)$, λ_+, and λ_-, which are functions of δ, are almost constant for small values of δ. We have (for $\delta=0$)

$$\lambda_+=0.023,\quad \lambda_-=0.013,$$
and
$$\xi=f_-(0)/f_+(0)=-0.197. \qquad (5.25)$$

From experiment, we have[20]

$$\lambda_+(K^+)=0.023\pm0.008.$$

The experimental situation about ξ and λ_- is not clear as yet.[20] As in most other theoretical analyses,[21] we have small values for these parameters which are in agree-

[16] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967).

[17] R. Gatto, G. Sartori, and M. Tonin, Phys. Letters **28B**, 128 (1968). See, also, N. Cabibbo and L. Maiani, *ibid.* **28B**, 131 (1968).

[18] Note that the K_μ^V term contributes only to f_-.

[19] N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. **B6**, 255 (1968).

[20] Rapporteur talk by W. Willis, in *Proceedings of the Heidelberg International Conference on Elementary Particles and High-Energy Physics, 1967* (North-Holland Publishing Co., Amsterdam, 1968); S. H. Aronson and K. W. Chen, Phys. Rev. **175**, 1708 (1969).

[21] B. W. Lee, Phys. Rev. Letters **20**, 617 (1968); L. N. Chang and Y. C. Leung, *ibid.* **21**, 122 (1968); I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968); R. Dashen and M. Weinstein, Phys. Rev. Letters **22**, 1337 (1969); L. K. Pande, *ibid.* **23**, 353 (1969).

ment with the $K_{\mu3}/K_{e3}$ branching-ratio (R) experiments. Substituting our values into the expression for the branching ratio,[22]

$$R=0.646+0.48\xi\lambda_-+1.40\lambda_++0.127\xi+0.019\xi^2,$$

we obtain

$$R=0.652,$$

which agrees with the experimental value[23]

$$R=0.648\pm0.03$$

within experimental errors.

The divergence of the vector and axial-vector currents can be calculated from the Lagrangian with the usual prescription

$$\partial_\mu J_{\mu\alpha}{}^{V,A}=-\delta\mathcal{L}/\delta\epsilon^{V,A}. \qquad (5.26)$$

This gives

$$\partial_\mu J_{\mu K^+}{}^V=iF_{S_K}\mu_{S_K}{}^2S_K{}^+$$

and

$$\partial_\mu J_{\mu\alpha}{}^A=F_{p_\alpha}\mu_{p_\alpha}{}^2p_\alpha. \qquad (5.27)$$

We see that both the axial-vector currents as well as the strangeness-changing vector current satisfy the partial conservation equations. As is clear from the work of Glashow and Weinberg,[6] such partial conservation equations would hold in any model in which the symmetry-breaking term is a linear function of fields belonging to some linear representation of the basic symmetry group, as is the case with the term $\mathcal{L}_4$ in Eq. (2.2).

VI. CONCLUDING REMARKS

(i) The strength of the $SU(3)$ breaking relative to the $SU(3)\otimes SU(3)$ breaking is expressed by

$$\xi=\eta_8/\eta_0=-0.40,$$
$$C=\epsilon_8/\epsilon_0=-1.24. \qquad (6.1)$$

The parameter ξ gives the relative strength of the spontaneous breaking of $SU(3)$ and $SU(3)\otimes SU(3)$, whereas the quantity C is a measure of the relative strengths of the corresponding intrinsic breaking. The latter is close to the $SU(2)\otimes SU(2)$ value ($-\sqrt{2}$) and is also in agreement with the value obtained by Gell-Mann, Oakes, and Renner,[24] who get $C=-1.25$. The value of ξ, however, is nowhere close to the $SU(2)\otimes SU(2)$ value (i.e., $\xi=-\sqrt{3}$); but it is close to the value obtained by Glashow, Schnitzer, and Weinberg[8] [i.e., $(\sqrt{\frac{2}{3}})\xi=-0.3$] and somewhat larger than the value of Lévy[3] [i.e., $(\sqrt{\frac{2}{3}})\xi=-0.22$], presumably because the latter author did not take into account the vector-meson couplings. If the spontaneous breakdown of $SU(2)\otimes SU(2)$ were also negligible, the masses of the ρ and A_1 and of π and S_π should be equal. The deviation of ξ from the $SU(2)\otimes SU(2)$ value is therefore a measure of the ρ-A_1 mass difference, whereas the deviation of C from the $SU(2)\otimes SU(2)$ value is a measure of the pion mass; comparing the experimental value of the ρ-A_1 mass difference ($\sim$310 MeV) with the experimental pion mass ($\sim$140 MeV), we see that the former deviation is expected to be larger on empirical grounds also.

(ii) We have treated the quantities η_0 and η_8 as if they were free parameters. In principle, they can be calculated in terms of other parameters in the theory; in fact, we already have two equations, namely, (2.10) and (2.11), to determine these parameters. However, these equations involve ϵ_0 and ϵ_8 which do not appear in the coupling constants and masses directly. It is therefore convenient for us to use η_0 and η_8, instead of ϵ_0 and ϵ_8, as free parameters.

(iii) It is interesting to analyze the consequences of Eqs. (2.10) and (2.11) with $\epsilon_0=\epsilon_8=0$. These equations can be written in the following form[25]:

$$(\sqrt{\tfrac{2}{3}})\epsilon_0+(1/\sqrt{3})\epsilon_8=\mu_\pi{}^2F_\pi Z^{-1/2}, \qquad (6.2)$$

$$(\sqrt{\tfrac{2}{3}})\epsilon_0-(1/2\sqrt{3})\epsilon_8=\mu_K{}^2F_K Z_K{}^{-1/2}. \qquad (6.3)$$

We also have the relation

$$\epsilon_8=(2/\sqrt{3})\mu_{S_K}{}^2F_{S_K}Z_{S_K}{}^{-1/2}. \qquad (6.4)$$

Let us first put $\epsilon_8=0$. Equation (6.4) then implies that at least one of the three quantities on the right must vanish. The case $\mu_{S_K}{}^2=0$ corresponds to the prediction of the Goldstone theorem in the case of a purely spontaneous breakdown of $SU(3)$ to the isospin-hypercharge level. Noting the relation

$$F_{S_K}=\tfrac{1}{2}\sqrt{3}\eta_8Z_{S_K}{}^{-1/2}, \qquad (6.5)$$

we see that $F_{S_K}=0$ with $|Z_{S_K}|<\infty$ implies $\eta_8=0$, which corresponds to the situation when the $SU(3)$ symmetry remains unbroken and the strangeness-changing vector current along with the isospin and hypercharge current is conserved. Finally, from the relation

$$Z_{S_K}=M_{K^*}{}^2/M_\rho{}^2=[1+\tfrac{3}{4}(g(\eta_8/m_0))^2], \qquad (6.6)$$

we see that the case $|Z_{S_K}|=\infty$ corresponds to $m_0=0$ (assuming that $g\eta_8$ is nonzero and finite). In this case there will be no kinetic energy term for the S_K meson and this particle is eliminated from the theory. This is precisely what one expects from the work of Higgs[4] and Kibble[5]; when in a fully gauge-invariant Lagrangian the symmetry is spontaneously broken, the Goldstone fields get eliminated and become the longitudinal modes of the massive gauge fields.

Putting $\epsilon_0=0$, similar conclusions are reached for the pseudoscalar mesons.

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One of us (AKB) thanks Dr. Aditya Kumar for many useful discussions.

[22] R. C. Field and P. B. Jones, Phys. Rev. Letters **21**, 327 (1968).
[23] G. R. Evans *et al.*, Phys. Rev. Letters **23**, 427 (1969).
[24] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1969).
[25] Equations (6.2)–(6.4) are analogous to Eq. (16) in Ref. 6.

APPENDIX

$$g_1{}^{\alpha\beta\gamma}=g(f_{\alpha\beta\gamma}L_\gamma+gf_{\alpha\epsilon\sigma}d_{\sigma\beta\gamma}\eta_\epsilon\xi_{p\gamma})Z_{p\beta}{}^{1/2}Z_{p\gamma}{}^{1/2},$$

$$g_2{}^{\alpha\beta\gamma}=f_{\alpha\beta\gamma}(\tfrac{1}{2}g\xi_{p\beta}\xi_{p\gamma}+\tfrac{1}{2}hL_\beta L_\gamma)Z_{p\beta}{}^{1/2}Z_{p\gamma}{}^{1/2},$$

$$g_3{}^{\alpha\beta\gamma}=-g^2(d_{\alpha\delta\lambda}f_{\lambda\beta\gamma}\eta_\delta-d_{\alpha\gamma\lambda}f_{\beta\delta\lambda}\eta_\delta)Z_{p\gamma}{}^{1/2},$$

$$g_4{}^{\alpha\beta\gamma}=-f_{\alpha\beta\gamma}(g\xi_{p\gamma}+ghL_\gamma\zeta_\beta)Z_{p\gamma}{}^{1/2},$$

$$g_5{}^{\alpha\beta\gamma}=(gf_{\alpha\beta\gamma}\xi_{p\gamma}+\tfrac{1}{2}ghf_{\sigma\beta\epsilon}\eta_\epsilon d_{\alpha\gamma\sigma}L_\gamma)Z_{p\gamma}{}^{1/2},$$

$$g_6=-\tfrac{1}{2}g^2(M_{K_A}/M_\rho{}^2)(F_\pi/M_{K^*}+F_{SK}/M_{A1}),$$

$$g_7=-\tfrac{1}{2}g^2(M_{A1}/M_\rho{}^2)(F_K/M_{K^*}-F_{SK}/M_{KA}),$$

$$g_8=\tfrac{1}{2}g^2(M_{K^*}/M_\rho{}^2)(F_K/M_{A1}+F_\pi/M_{KA}),$$

$$g_9=\left\{\frac{3C_1}{\sqrt{2}}-C_2[\eta_0-(\surd\tfrac{4}{3})\eta_8]\right\}Z_\pi{}^{1/2}Z_K{}^{1/2}Z_{SK}{}^{1/2},$$

where

$$L_\beta=1-\zeta_\beta\xi_\beta.$$

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Universal Isovector Current with Many 1^- Poles*

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A simple prescription is suggested for preserving universality of the isovector current in the presence of many 1^- poles ρ_i. By identifying $g_{\rho_i-\gamma}$ as proportional to $g_{\rho_i\pi\pi}$, predictions are made regarding (a) the modification of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation, (b) charge radii, (c) $\pi\pi$ resonance production in colliding beam experiments, (d) high-energy ρ meson photoproduction and photoabsorption, (e) photoproduction of ρ' and ρ'', and (f) asymptotic behavior of form factors.

IN this paper we suggest a simple modification of the hypothesis of ρ dominance[1] which preserves the universality of the isovector current. Our starting point is that low-energy pion scattering, while purely hadronic, also obeys a form of universality.[2]

In a resonance approximation one may write the isovector "electric" form factor of any particle x in the form

$$F_{x\bar{x}}{}^\gamma(t)=\sum_{n=1}^{\infty}\beta_n g_{nx\bar{x}}/(m_n{}^2-t). \quad (1)$$

The β_n represent the couplings of the photon to the intermediate states ρ_n.[3]

A similar "form factor" may be defined in the case of the $I_t=1$ amplitude for π-x elastic scattering at threshold:

$$M_t{}^{(1)}(\nu,t)=2(q_1+q_2)^\mu[(p_1+p_2)_\mu F_{x\bar{x}}{}^\pi(t)] =4\nu F_{x\bar{x}}{}^\pi(t), \quad (2)$$

where $\nu\equiv\frac{1}{2}(s-u)$, q_1 and q_2 are the momenta of the incoming and outgoing pion, and p_1 and p_2 are the momenta of the incoming and outgoing x (spinless here for convenience). Since π-x is a hadronic scattering, all spin exchanges are possible in the t channel, and in particular the t-channel resonance expansion (or the t-channel partial-wave expansion) blows up at the various s- (and u-) channel singularities. The rather remarkable consequence of partial conservation of axial-vector current (PCAC) and current algebra[2] is that at threshold and at $t=0$, the $I_t=1$ crossing-odd amplitude is given in the soft-pion limit by a conserved vector interaction. This means that in this particular kinematic region only $J=1^-$ isovector t-channel exchanges are relevant.[4] Those are precisely the same states contributing to (1), and we have

$$F_{x\bar{x}}{}^{\pi\,(\mathrm{soft})}(t=0)=\sum_{n=1}^{\infty}\frac{g_{n\pi\pi}g_{nx\bar{x}}}{m_n{}^2}. \quad (3)$$

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[1] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 79 (1962); 8, 191(E) (1962); M. Gell-Mann, D. Sharp, and W. Wagner, *ibid.* 8, 261 (1962).

[2] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[3] In the case of ρ dominance, for example, $\beta_1=m_\rho{}^2/2\gamma_\rho=m_\rho{}^2/g_{\rho\pi\pi}$.

[4] We suggest, therefore, that the sum over all *spin-1* exchanges gives $I=1$ exchange in $\pi x\to\pi x$ at threshold. This is equivalent to taking only pole contributions to the dispersion relation in t for the $J=1$ partial wave.