Field-Theoretic Model for High-Energy Scattering. II. Regge and Non-Regge Damping in πN Scattering*

H. M. FRIED
Physics Department, Brown University, Providence, Rhode Island 02912
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A field-theoretic formulation of πN scattering, combining soft virtual neutral vector-meson exchange with simple nucleon, 3-3 resonance, and ρ poles, is shown to produce damped amplitudes containing both fixed and Regge poles, yielding polarizations and differential cross sections with a qualitative resemblance to recent experiments.

I. INTRODUCTION

In a previous paper[1] a field-theoretic model of soft virtual neutral vector-meson (SVNVM) exchange between nucleons was used to provide a qualitative, few-parameter description of the strong damping observed in nucleon electromagnetic form factors and in pp scattering, at high momentum transfers and large energies. The analysis of I may be described by the sequence of operations: (i) Assume, in addition to the usual hadronic interactions, a NVM-baryon coupling (either a fundamental interaction or representing, e.g., a phenomenological nucleon-ρ_0 coupling), and rewrite every amplitude to make explicit the contributions coming from the exchange of the SVNVM which modify all the remaining nonsoft or "hard" interactions. (ii) Perform a dipolelike approximation, effectively decoupling the soft and hard parts of every amplitude. This approximation, which may be expected to be reasonable for large s, $-t$, yields amplitudes whose soft dependence occurs as a multiplicative factor. One drawback of this simple prescription is that an upper cutoff to the SVNVM momenta must be inserted by hand.

An alternative procedure, more complicated but probably more realistic, avoiding the temptation to decouple soft and hard effects, is to let the hard processes provide a natural upper cutoff for the SVNVM integrals. This paper deals with the results of using the simplest Born approximation graphs, corresponding to simple poles in the s, t, and u channels, for the hard portion of the πN elastic and charge-exchange amplitudes. In the absence of a t-channel exchange, the resulting amplitudes turn out to contain both fixed and Regge poles, the latter corresponding closely to Reggeized ρ exchange, unsignatured for $\pi^{\pm}$-p elastic, signatured for charge-exchange scattering. For any form of the πN interaction, the fixed-pole contribution to $d\sigma/dt$ turns out to be insignificant in the forward direction, leaving an effective Regge behavior. Polarizations are given by interference between the fixed and Regge amplitudes. Inclusion of a t-channel ρ exchange is possible but awkward in the present formalism and an argument based upon the methods of I is sketched to obtain an estimate for the asymptotic $\pi^{\pm}p$ total cross sections. In this way, a crude model with the correct qualitative features to fit recent experiments[2,3] can be constructed using only the direct-channel nucleon and 3-3 resonance poles, together with a t-channel ρ exchange. No detailed calculations have been attempted in this paper, and the need for a better calculation involving SVNVM emission by pions is made clear.

Although the results resemble, in part, those of Regge phenomenology, with the concept of duality appearing in an essential way, the formulation of this problem is within the context of conventional field theory. By considering that portion of every hadronic matrix element which contains SVNVM exchanges between nucleons, one transforms the fixed poles of the simple Born approximation into fixed plus Regge poles. This approach may be compared with recent eikonal approximations[4] where one effectively considers multiple soft-meson exchange, soft in the sense that any given exchange does not result in a significant momentum transfer to the colliding particles. The assignment of average values of initial and final momenta as eigenvalues of the four-momentum operators generates the leading terms in a relativistic eikonal expansion corresponding (roughly) to our soft exchanges surrounding a single hard exchange in a crossed channel. This does not produce Regge behavior in the first paper of Ref. 4 because the momentum transfer is held small, as it should be, in the absence of a hard exchange.

In the next section, an off-shell formula is derived which exhibits the SVNVM effects as they modify the hard part of any scattering amplitude. In Sec. III, application is made to πN scattering using a nucleon pole for the hard amplitude, while a crude model of the 3-3 resonance is employed in Sec. IV. A t-channel ρ-exchange pole is examined in Sec. V, and an amalga-

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[1] H. M. Fried and T. K. Gaisser, Phys. Rev. 179, 1491 (1969), hereafter called I.

[2] Recent polarization results have been given by R. J. Esterling *et al.*, Phys. Rev. Letters 21, 1410 (1968).

[3] See, e. g., the report of G. Bellettini, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 329, for a survey of recent cross-section and polarization measurements.

[4] H. D. I. Arbarbanel and C. Itzykson, Phys. Rev. Letters 23, 53 (1969); R. L. Sugar and R. Blankenbecler, Phys. Rev. 183, 1387 (1969). Forms similar to those obtained in the first of these papers were found, in another context, by G. W. Erickson and H. M. Fried, J. Math. Phys. 6, 414 (1965).

mation of these separate parts of the scattering amplitudes is discussed in Sec. VI.

II. DERIVATION

We began by writing down Eqs. (I-15) and (I-16), simplified to describe SVNVM exchange between nucleon legs only in the process $p_1(N)+p_2(\pi)\to p_1'(N)+p_2'(\pi)$:

$$\delta(p_1+p_2-p_1'-p_2')\tilde{M}(p_1,p_1',p_2,p_2')$$
$$=(2\pi)^{-4}\int dx_1 e^{ip_1x_1}\int dy_1 e^{-ip_1'y_1}$$
$$\times\int dx_2 e^{ip_2x_2}\int dy_2 e^{-ip_2'y_2}e^{\Phi(x_1-y_1)}$$
$$\times M_H(x_1,y_1,x_2,y_2)\,,\quad(1)$$

with

$$\Phi(z)=ig_0^2(p_1\cdot p_1')\int_0^\infty d\xi\int_0^\infty d\eta\;\Delta_c(z-\xi p_1-\eta p_1')\,,\quad(2)$$

where $\delta_{\mu\nu}\Delta_c$ denotes the nongradient part of the NVM propagator with mass μ, and g_0 is the NVM coupling to the nucleon. If this interaction is considered to be that of an elementary NVM field coupled to a conserved baryonic current, then (1) is complete as it stands. However, if one thinks of a phenomenological description of soft, neutral resonance exchange between nucleons, there is nothing to prevent such exchanges from occurring between pion and nucleon legs, in all possible combinations, and a formula akin to that of the pp situation in I would be more appropriate. Two arguments will later be put forth to support this second point of view, but for simplicity, only the form (1) will be treated in this and the following two sections.

Equation (1) is incomplete in the sense that the soft exchanges must still be renormalized, and the procedure to be followed in this paper is to subtract from the t-dependent $\Phi(x_1-y_1)$ of (1) its value at that momentum transfer corresponding to a ρ exchange in the crossed channel, at $t=+\mu_\rho^2$. That is, the subtraction constant, or renormalization point, is adjusted such that the complete amplitude has a pole in the crossed channel at the ρ mass, with the same coupling as used for the hard amplitude ρ exchange. This procedure serves to introduce the ρ mass as a parameter in the Regge function which appears when s- and u-channel poles are used for the hard part of the amplitude. It is not a unique method of renormalization, but it is physically intuitive, and adequate if form factors are not numerically sensitive to variations of low-lying renormalization points. When soft exchanges are contemplated between more than one pair of legs in an amplitude where the mass of the hard particle exchanged is larger than the sum of the masses of the emitting legs, this form of renormalization will mean multiplication by a complex number, an expression of the decay possibility of the resonance used for the hard exchange.

Inserting a representation for Δ_c and performing the parametric integrations of (2), one can find a representation for $\Phi(z)$ in the form

$$\Phi(z)=i\frac{g_0^2}{4\pi^2}(p_1\cdot p_1')\int_0^1\frac{dx}{-\bar{p}^2}\int_0^\infty da\;e^{-ia+i\mu^2z^2/4a}$$
$$\times\int_0^\infty cdc\;e^{-iac^2-ic\mu z\bar{p}\sqrt{(-\bar{p}^2)}}\,,\quad(3)$$

where $\bar{p}=xp_1+(1-x)p_1'$, m is the nucleon mass, and $-\bar{p}=m^2+x(1-x)(-t)$ is positive for $t<4m^2$. For later convenience we introduce an approximation to (3) whose significance is most easily understood by first rewriting the parametric integration over the variable x in terms of the familiar integral over (mass)^2:

$$\int_0^1 dx(-\bar{p}^2)^{-1}e^{-ic\mu z\cdot\bar{p}/\sqrt{(-\bar{p}^2)}}=2\int_{4m^2}^\infty dt'(t')^{-1}(t'-t)^{-1}$$
$$\times\left(1-\frac{4m^2}{t'}\right)^{-1/2}\exp\left[-i\frac{c\mu}{2m}\frac{z\cdot(p+p')}{(1-t/t')^{1/2}}\right]$$
$$\times\cos\left[\frac{c\mu}{2m}z\cdot(p_1-p_1')\left(\frac{t'-4m^2}{t'-t}\right)^{1/2}\right].\quad(4)$$

For small $-t$, $p_1-p_1'\sim0$, and (4) becomes

$$\sim2e^{-ic(\mu/m)z\cdot p_1}\int_{4m^2}^\infty\frac{dt'}{t'}\frac{1}{t'-t}\frac{1}{(1-4m^2/t')^{1/2}},\quad(5)$$

with the z dependence separated from the details of the t' integration. Such separation is so convenient in subsequent manipulations that we replace (4) by an approximate form obtained by evaluating the t' dependence of the phase terms at $t'=4m^2$:

$$2e^{-ic\mu z\cdot(p_1+p_1')/2M}\int_{4m^2}^\infty\frac{dt'}{t'}\frac{1}{t'-t}\frac{1}{\sqrt{(1-4m^2/t')}},\quad(6)$$

where $M\equiv(m^2-\frac{1}{4}t)^{1/2}$. This approximation is not valid for $t\sim4m^2$, and hence our later Regge functions can only be correct for small $t>0$. In terms of the parametric x integral of (3) this approximation corresponds to evaluating the phase $\exp[ic\mu z\cdot\bar{p}/(-\bar{p}^2)^{1/2}]$ at $x=x_0=\frac{1}{2}$, and it is not difficult to see that the precise value of $x_0(t)$ becomes significant only when $t\sim4m^2$; one finds, in subsequent usage, that a factor of $z\cdot p_1$ in the exponential of (6) is essentially equivalent to a factor of $z\cdot p_1'$, or of $\frac{1}{2}z\cdot(p_1+p_1')$. We shall make use of (6) as a specific and simple form for

$$\Phi=\Phi(\mu(z/2M)\cdot(p_1+p_1'),\,\mu^2z^2)\,,$$

expected to be valid for small t, and not too unreasonable for larger values of $-t>0$.

In the asymptotic scattering regions we will be interested in $\Phi((\mu/2M)z\cdot(p_1+p_1'),\, \mu^2z^2)$ for small values of its arguments, and it is not difficult to demonstrate that it is sufficient to consider $\Phi((\mu/2M)z\cdot(p_1+p_1'),\, 0) \equiv \mathfrak{F}((\mu/2M)z\cdot(p_1+p_1'))$; this step is not crucial, but does simplify the analysis to a considerable extent. Introducing the representation (6) into (3), one finds

$$\mathfrak{F}(\lambda)=\gamma_0 f(t)\int_0^\infty \frac{c\,dc}{1+c^2}e^{-ic\lambda}, \tag{7}$$

where $f(t)=-1+F(t)$, in the notation of I, and

$$F(t)=t\int_{4m^2}^\infty \frac{dt'}{t'}\frac{1}{t'-t}\left(1-\frac{2m^2}{t'}\right)\left(1-\frac{4m^2}{t'}\right)^{-1/2}, \tag{8}$$

with $\gamma_0=g_0^2/4\pi^2$. For small t, $F\sim t/3m^2$, while $F\sim -\ln(|t|/m^2)$ for $-t\gg m^2$. Near $\lambda\sim 0+$, (7) has the form

$$\mathfrak{F}(\pm\lambda)\sim -\gamma_0 f\ln\lambda \mp \tfrac{1}{2}i\pi\gamma_0 f+\cdots. \tag{9}$$

Renormalization may be imposed, in (9), by subtracting from $\mathfrak{F}((\mu/2M)z\cdot(p_1+p_1'))$ its value at $t=+\mu_\rho^2$. We simplify matters by neglecting the variation of $M(t)$, retaining only the subtracted form in which $f(t)$ in (9) is understood to be replaced by

$$f(t)-f(\mu_\rho^2)=F(t)-F(\mu_\rho^2)\approx F(t)-\mu_\rho^2/3m^2.$$

With the representation

$$\exp\left[\mathfrak{F}\left(\frac{\mu}{2M}z\cdot(p_1+p_1')\right)\right]=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\lambda$$
$$\times\exp\mathfrak{F}(\lambda)\int_{-\infty}^{+\infty}da\,\exp(-ia\lambda+ia\mu z\cdot\mathcal{P}),$$
$$\mathcal{P}\equiv(1/2M)(p_1+p_1'), \tag{10}$$

the Fourier transforms indicated in (1) may be computed to give

$$\tilde{M}(p_1p_1'p_2p_2')=\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)$$
$$\times\int da\,e^{-ia\lambda}\tilde{M}_H(p_1+a\mu\mathcal{P},\, p_1'+a\mu\mathcal{P},\, p_2,\, p_2'), \tag{11}$$

which is our working expression of the SVNVM effects, as they modify the hard part of the scattering amplitude. It should be noted that this is an off-mass-shell effect, with a pair of the mass variables of M_H, $(p_1+a\mu\mathcal{P})^2=(p_1'+a\mu\mathcal{P})^2$ carried away from their usual values of $p_1^2=p_1'^2=-m^2$. From (11) it is clear why the present process of soft exchanges between protons only is relatively simple, compared to the situation where the SVNVM are exchanged between pions and nucleons, and where our subsequent manipulations would become fearfully complicated. From the derivation of I it is also clear that the replacements $p_1\to p_1+a\mu\mathcal{P}$, $p_1'\to p_1'+a\mu\mathcal{P}$ inside $\tilde{M}_H$ are not to be carried out inside spinor $\bar{u}_\alpha(p_1')$ and $u_\beta(p_1)$ factors, but only in the invariant amplitudes A and B, here written in the form $T=A+i\gamma\cdot p_2 B$, where

$$\langle p_1'p_2'|S|p_1p_2\rangle=\delta_{fi}-i(2\pi)^{-2}\delta(p_1+p_2-p_1'-p_2')$$
$$\times\left(\frac{m^2}{4E_1E_1'\omega_1\omega_2'}\right)^{1/2}\bar{u}(p_1')\cdot T\cdot u(p_1).$$

Thus,

$$A(p_1p_1'p_2p_2')=\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)$$
$$\times\int da\,e^{-ia\lambda}A_H(p_1+a\mu\mathcal{P},\, p_1'+a\mu\mathcal{P},\, p_2,\, p_2'), \tag{12}$$

with

$$A(s,t,u)=A(p_1p_1'p_2p_2')\,|_{p_1^2=p_1'^2=-m^2,\,p_2^2=p_2'^2=-m_\pi^2},$$

and similarly for $B(p_1p_1'p_2p_2')$. In all calculations to follow, we set $m_\pi=0$. Crossing symmetry of the complete amplitudes under $s\leftrightarrow u$ interchange[5] is guaranteed, as long as it is satisfied by the A_H and B_H, because of the property $\mathfrak{F}^*(\lambda)=\mathfrak{F}(-\lambda)$, which follows from the representation (7) for physical $-t\geqslant 0$.

For the $s\gg -t,\, m^2$ situation, experimental quantities are given by

$$P\cdot\frac{d\sigma}{dt}=\frac{|t|^{1/2}}{8\pi s}\,\mathrm{Im}(AB^*) \tag{13}$$

and

$$\frac{d\sigma}{dt}=\frac{1}{4\pi M^2}\left(\frac{m^2}{s}\right)^2\left(\left|\left(\frac{M}{m}\right)^2A-\frac{s}{2m}B\right|^2+\frac{|t|}{4m^2}\left|\frac{s}{2m}B\right|^2\right),$$

or, for $-t\ll 4m^2$,

$$\frac{d\sigma}{dt}\approx\frac{1}{16\pi}\left(\left|B-\frac{2m}{s}A\right|^2+\frac{|t|}{4m^2}\left|\frac{2m}{s}A\right|^2\right). \tag{14}$$

Total cross sections are given by

$$\sigma_T\approx\mathrm{Im}\left(B-\frac{2m}{s}A\right)_{t=0}. \tag{15}$$

III. NUCLEON-POLE CONTRIBUTIONS

We now examine the results of using the simplest possible expressions for M_H, the nucleon-pole terms pictured in Figs. 1 and 2 for $\pi^\pm p$ elastic scattering. The form of the πN vertex still remains to be specified and is chosen to be of phenomenological axial-vector form, described by an effective interaction $\mathcal{L}'=iG\bar{\psi}\gamma_5\gamma_\mu\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau}\psi$, $(2mG)^2/4\pi<15$, in order to demonstrate an extra-

[5] Here, the complex conjugation associated with this operation, and frequently omitted in the literature, is crucial; see, e. g., S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), p. 367.

ordinary diminution in the asymptotic behavior of the fixed-pole part of the amplitude; this effect occurs automatically in the model, yielding an asymptotic forward scattering amplitude of the same form as that generated by a simple γ_5 coupling. With

$$T_H(p_1p_1'p_2p_2')=G^2\gamma_5\gamma\cdot p_2'\tilde{S}_c(p_1+p_2)\gamma_5\gamma\cdot p_2\,, \tag{16}$$

one has for the π^-p amplitude

$$A(s,t)=\frac{G^2}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}m(s-m^2)\left(2+\frac{a\mu}{M}\right)\times\left[m^2-s-(a\mu)^2-\frac{a\mu}{M}(s+m^2)-i\epsilon\right]^{-1}, \tag{17}$$

$$B(s,t)=\frac{G^2}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}\times\left[s\left(1+\frac{a\mu}{M}\right)+m^2\left(3+\frac{a\mu}{M}\right)\right]\times\left[m^2-s-(a\mu)^2-\frac{a\mu}{M}(s+m^2)-i\epsilon\right]^{-1}, \tag{18}$$

while the corresponding nucleon u-channel contributions to the π^+p amplitude are given by

$$A(u,t)=\frac{G^2}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}m(u-m^2)\left(2+\frac{a\mu}{M}\right)\times\left[m^2-u-(a\mu)^2-\frac{a\mu}{M}(u+m^2)-i\epsilon\right]^{-1}, \tag{19}$$

$$B(u,t)=-\frac{G^2}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}\times\left[u\left(1+\frac{a\mu}{M}\right)+m^2\left(3+\frac{a\mu}{M}\right)\right]\times\left[m^2-u-(a\mu)^2-\frac{a\mu}{M}(u+m^2)-i\epsilon\right]^{-1}. \tag{20}$$

In the absence of soft effects, $\mathfrak{F}=0$, the λ integration produces a factor $\delta(a)$, and one obtains the amplitudes

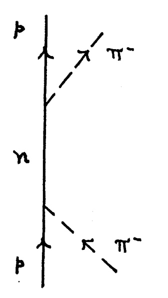

FIG. 1. The s-channel nucleon-pole contribution to π^-p scattering.

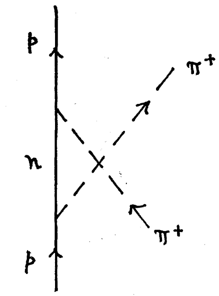

FIG. 2. The u-channel nucleon-pole contribution to π^+p scattering.

corresponding to the simple tree graphs of Figs. 1 and 2; such amplitudes each behave as s^0 in the asymptotic limit, and generate a constant contribution to $d\sigma/dt$. With $\mathfrak{F}\neq0$, it is simplest to first perform the parametric a integrations. For the s-channel terms one obtains

$$\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}\left\{1,\frac{a\mu}{M}\right\}\times\left[m^2-s-(a\mu)^2-\frac{a\mu}{M}(s+m^2)-i\epsilon\right]^{-1}$$
$$=\frac{i}{\mu^2}[Q^{(+)}-Q^{(-)}]^{-1}\int_0^\infty d\lambda\times\exp[\mathfrak{F}(\lambda)+i\lambda Q^{(-)}]\left\{1,-\frac{\mu}{M}Q^{(-)}\right\}+\exp[\mathfrak{F}^*(\lambda)-i\lambda Q^{(+)}]\left\{1,-\frac{\mu}{M}Q^{(+)}\right\}, \tag{21}$$

where

$$Q^{(\pm)}=\frac{s+m^2}{2M\mu}\left\{1\pm\left[1-\frac{4M^2(s-m^2)}{(s+m^2)^2}\right]^{1/2}\right\}.$$

Further evaluation of (21) depends upon the specification of $\mathfrak{F}(\lambda)$. For the asymptotic situation $s\gg m^2$, $-t$, μ^2, $Q^{(+)}\sim s/M\mu$, and the behavior of the $Q^{(+)}$ integrals of (21) is determined by the small-λ form of $\mathfrak{F}(\lambda)$, as in (9). Further, $Q^{(-)}\sim M/\mu$, and the same approximation to $\mathfrak{F}^*(\lambda)$ may be used in the $Q^{(-)}$ integrals if $M/\mu>1$, with this estimate becoming better for larger values of this ratio; these terms generate the fixed poles, which do not significantly contribute to the forward scattering cross section for any value of M/μ, although they do enter into the polarizations. If $M\geqslant m$, and we imagine that the NVM's are $\rho_0 s$, $\mu\sim\mu_\rho$, then $M/\mu>1$; in this case the approximation of using (9) for $\mathfrak{F}^*(\lambda)$ is not unreasonable, since the corresponding integral of (21) will be effectively cut off for $\lambda<1$.

With these replacements, (21) becomes

$$\left(\frac{M}{\mu}\right)^{\gamma_0 f}\frac{\Gamma(1-\gamma_0 f)}{s}\left[\left(\frac{s}{M^2}\right)^{\gamma_0 f-1}e^{i\pi\gamma_0 f}\left\{1,-\frac{\mu}{M}Q^{(+)}\right\}-e^{-i\pi\gamma_0 f}\left\{1,-\frac{\mu}{M}Q^{(-)}\right\}\right] \tag{22}$$

and the removal of the dominant part of the fixed-pole contribution to the forward scattering amplitude is now apparent. This term is generated by the numerator s dependence of $B(s,t)$, in (18), which appears there multiplied by the factor $1+a\mu/M$; from (22), such a term enters in the combination $s[1-(\mu/M)Q^{(-)}]\sim 2m^2-M^2=m^2+\frac{1}{4}t$ for large s, and hence the fixed-pole portion of $B(s,t)$ is reduced by a factor of s compared to its behavior in the absence of the SVNVM. In essence, the $\gamma_0=0$ fixed pole has been split into two parts, a moving Regge pole visible in the $Q^{(+)}$ terms, and a fixed pole with an asymptotic behavior resembling that of the gentler γ_5 interaction. This same feature occurs if higher-spin baryons are used in place of the virtual nucleon.

The amplitude $C=B-2mA/s$ has, from (17) and (18), the representation

$$C(s,t)=\frac{G^2}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}s\left(1+\frac{a\mu}{M}\right)\times\left[m^2-s-(a\mu)^2-\frac{a\mu}{M}(s+m^2)-i\epsilon\right]^{-1}\quad(23)$$

for large s; from (22) this is evaluated as

$$C(s,t)\sim-\Omega\left[\left(\frac{s}{M^2}\right)^{\gamma_0 f}e^{i\pi\gamma_0 f}+\left(\frac{m^2+\frac{1}{4}t}{s}\right)e^{-i\pi\gamma_0 f}\right],$$

$$\Omega\equiv G^2(M/\mu)^{\gamma_0 f}\Gamma(1-\gamma_0 f),\quad(24)$$

while

$$A(s,t)\sim-m\Omega\left[\left(\frac{s}{M^2}\right)^{\gamma_0 f}e^{i\pi\gamma_0 f}+e^{-i\pi\gamma_0 f}\right].\quad(25)$$

In a similar fashion, u-channel quantities may be obtained using

$$\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}\left\{1,\frac{a\mu}{M}\right\}\times\left[m^2+|u|-(a\mu)^2+\frac{a\mu}{M}(|u|-m^2)-i\epsilon\right]^{-1}$$

$$\cong\left(\frac{M}{\mu}\right)^{\gamma_0 f}\frac{\Gamma(1-\gamma_0 f)}{|u|}\left[\left(\frac{|u|}{M^2}\right)^{\gamma_0 f-1}\left\{1,\frac{|u|}{M^2}\right\}+e^{i\pi\gamma_0 f}\left\{1,-\left(1+\frac{m^2+\frac{1}{4}t}{|u|}\right)\right\}\right],\quad(26)$$

where the same approximations used in arriving at (22) have been followed, and where $|u|=-u\gg m^2,-t$; from now on we shall replace $|u|$ by s. Note that the absence of phase in the moving-pole terms of (26) is a consequence of crossing symmetry. The remarks made above concerning the asymptotic behavior of the fixed- and Regge-pole terms are valid here also. With $C(u,t)=B(u,t)-(2m/|u|)A(u,t)$, we obtain

$$C(u,t)\sim\Omega\left[\left(\frac{s}{M^2}\right)^{\gamma_0 f}-\left(\frac{m^2+\frac{1}{4}t}{s}\right)e^{i\pi\gamma_0 f}\right]\quad(27)$$

and

$$A(u,t)\sim-m\Omega[(s/M^2)^{\gamma_0 f}+e^{i\pi\gamma_0 f}].\quad(28)$$

As they stand, (28) and (27) lead to a π^+p polarization dominated by the term

$$\mathrm{Im}[A(u,t)C^*(u,t)]\sim m\Omega^2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\sin(\pi\gamma_0|f|),\quad(29)$$

while (24) and (25) would predict a π^-p polarization proportional to

$$\mathrm{Im}[A(s,t)C^*(s,t)]\sim-m\Omega^2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\times[-\sin(2\pi\gamma_0|f|)].\quad(30)$$

Near the forward direction, the cross sections of (14) are controlled by

$$|C(u,t)|^2\approx\Omega^2\left[\left(\frac{s}{M^2}\right)^{2\gamma_0 f}+\left(\frac{m^2+\frac{1}{4}t}{s}\right)^2-2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\left(\frac{m^2+\frac{1}{4}t}{s}\right)\cos(\pi\gamma_0|f|)\right]\quad(31)$$

and

$$|C(s,t)|^2\approx\Omega^2\left[\left(\frac{s}{M^2}\right)^{2\gamma_0 f}+\left(\frac{m^2+\frac{1}{4}t}{s}\right)^2+2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\left(\frac{m^2+\frac{1}{4}t}{s}\right)\cos(2\pi\gamma_0|f|)\right]\quad(32)$$

in the two cases, while the total cross sections would be given by

$$\sigma_T(+)\approx\mathrm{Im}C(u,0)\sim\Omega(m^2/s)\sin(\pi\gamma_0|f|)|_{t=0}\quad(33)$$

and

$$\sigma_T(-)\approx\mathrm{Im}C(s,0)\sim\Omega(s/M^2)^{\gamma_0 f}\sin(\pi\gamma_0|f|)|_{t=0}.\quad(34)$$

At $t=0$, $\gamma_0|f|=\gamma_0[|F(t)|+F(\mu_\rho^2)]\sim\gamma_0\mu_\rho^2/3m^2$, with this constant specified by the strength of the nucleon-NVM coupling, while for $-t\ll 4m^2$, $\gamma_0|f|\sim(\gamma_0/3m^2)\times[\mu_\rho^2+|t|]$. The π^+p polarization given by (29) will be positive, for small $|t|$, if $\gamma_0|f|<1$, with a zero occurring when $\gamma_0|f|=1$. Experimentally, this happens for $-t\cong 0.6$, and we now use this number to determine γ_0, which comes out to be 2.2, corresponding to a $g_0^2/4\pi\simeq 7$. On the basis of the crude analysis of I, had we coupled the pion legs to the SVNVM, excluding interactions between pions and nucleons, there would result a "doubling" of the soft effect, with the pion contribution serving to replace γ_0 by $(1/4\pi^2)(g_{\rho NN}^2+g_{\rho\pi\pi}^2)$. If $g_{\rho\pi\pi}\simeq 2g_{\rho NN}$, we would now find $g_{\rho\pi\pi}^2/4\pi\simeq 5.6$.

Finally, if all possible SVNVM exchanges were permitted, we would expect a further reduction in the value of the coupling, with our crude estimate of $g_{\rho\pi\pi}^2$ less than double its experimental value. This is our first reason for associating ρ_0 exchange with the SVNVM.

The above determination of γ_0 now has as its consequence the property that $(\gamma_0|f|)_{t=0}\cong\frac{1}{2}$, which means that the forward scattering of (31) and (32) is dominated by the Regge terms. Writing these cross sections as proportional to $s^{2[\alpha(t)-1]}$, we have derived an effective Regge behavior, with an $\alpha(t)=1+\gamma_0 f\simeq\frac{1}{2}+(2.2)F(t)$. Since F is linear for small t, the intercept $\alpha(0)$ and slope $\alpha'(0)$ are simultaneously determined by the vanishing of the polarization. Of course, the numerical value of the ρ mass has already been inserted in α by the renormalization of Sec. II. Since our analysis is not valid for large positive t, no statement can be made about rising trajectories. For large, positive $-t$ this trajectory is dropping logarithmically, giving asymptotic power-law behavior in the physical scattering region.

In the range $0\leqslant -t\leqslant 0.6$, $\frac{1}{2}\leqslant\gamma_0|f|\leqslant 1$, and the π^-p polarization of (30) is negative and also vanishes at $|t|=0.6$. These fits are similar to the older Regge parametrization of the small momentum transfer data,[6] and can be realized with about a third of the number of parameters. In addition, the charge-exchange polarization, calculated in terms of the amplitudes

$$A^{\text{ex}}=(1/\sqrt{2})[A(u,t)-A(s,t)],$$
$$B^{\text{ex}}=(1/\sqrt{2})[B(u,t)-B(s,t)],$$
$$C^{\text{ex}}=B^{\text{ex}}-(2m/s)A^{\text{ex}},$$

is proportional to

$$\operatorname{Im}[A^{\text{ex}}C^{\text{ex}*}]\sim 2m\Omega^2(s/M^2)^{\gamma_0 f}\times\sin(\pi\gamma_0|f|)\cos^2(\tfrac{1}{2}\pi\gamma_0|f|),\quad(35)$$

and is positive in this $-t$ region. The forward charge-exchange scattering is given by

$$|C^{\text{ex}}|^2\sim 2\Omega^2\left\{\left(\frac{s}{M^2}\right)^{2\gamma_0 f}+4\left(\frac{m^2+\frac{1}{4}t}{s}\right)\times\left[\frac{m^2+\frac{1}{4}t}{s}-\left(\frac{s}{M^2}\right)^{\gamma_0 f}\right]\sin^2(\tfrac{1}{2}\pi\gamma_0 f)\right\}\times\cos^2(\tfrac{1}{2}\pi\gamma_0 f),\quad(36)$$

while the amplitudes, written in terms of the α introduced above, are

$$C^{\text{ex}}\sim\frac{\pi G^2}{\sqrt{2}}\left(\frac{M}{\mu}\right)^{\alpha-1}\frac{1-\alpha}{\Gamma(\alpha)}\times\left\{-2i\left(\frac{m^2+\frac{1}{4}t}{s}\right)+\left(\frac{s}{M^2}\right)^{\alpha-1}[1-e^{i\pi\alpha}]\frac{1}{\sin\pi\alpha}\right\}\quad(37)$$

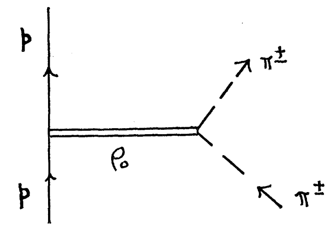

FIG. 3. The t-channel p-meson-pole contribution to π^+p scattering.

and

$$A^{\text{ex}}\sim -m\frac{\pi G^2}{\sqrt{2}}\left(\frac{M}{\mu}\right)^{\alpha-1}\frac{1-\alpha}{\Gamma(\alpha)}\times\left\{-2i+\left(\frac{s}{M^2}\right)^{\alpha-1}[1+e^{i\pi\alpha}]\frac{1}{\sin\pi\alpha}\right\}.\quad(38)$$

It is interesting to note that neither amplitude has a t-channel pole at $\alpha=1$; this avoids any question of double counting, since our original renormalization assumed that there is a ρ pole in M_H, arising from the graph of Fig. 3, which has yet to be considered. These signatured amplitudes have t-channel poles at alternating integers.

The differential cross sections given by (31) and (32) contain a dip/bump structure, decreasing in amplitude as $-t$ increases, with the position of successive extrema weakly dependent on the values of s/m^2. As they stand, they are not capable of reproducing the very sharp falloff of the experimental $d\sigma/dt$ curves away from the forward direction. The charge-exchange cross section given by (36) has pronounced dips, vanishing whenever $\gamma_0|f|=2n+1$. It is clear that these simple expressions are successful only in superimposing a dip/bump structure upon a mixture of a diffractive, or Regge low $-t$ contribution, plus a fixed-pole part becoming significant at larger momentum transfers.[7] Finally, the total cross sections given by (33) and (34) are both positive, with $\sigma_T(-)>\sigma_T(+)$, but are falling off too rapidly with energy to compare with the data.

IV. NUCLEON-RESONANCE CONTRIBUTIONS

The polarizations predicted by (29) and (30) have the unfortunate property of reversing sign as $-t$ increases past 0.6, while the latest experiments show that the elastic polarizations retain the sign of their low momentum transfer values. In this section, we would like to make essentially an arithmetical observation to produce amplitudes whose polarizations follow the experiments more closely. One way of understanding such forms is to consider them as contributions following from the

[6] C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. 153, 1485 (1967).

[7] Were it not for the s dependence of the fixed-pole terms, this model would be a partial realization of the form suggested by H. D. I. Arbarbanel, S. D. Drell, and F. Gilman, Phys. Rev. Letters 20, 280 (1968). The similarity increases when use is made of (54).

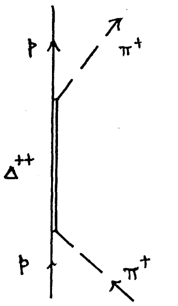

FIG. 4. The s-channel Δ^{++}-pole contribution to π^+p scattering.

propagation of the 3-3 resonance, in particular, the Δ^{++}, used in addition to the nucleon pole above. This is not an unambiguous matter, since we are going to use the resonance propagator at energies far from the actual resonance. Aside from a possible awkwardness of too large an asymptotic growth (of the Regge part of the amplitude only, since the fixed-pole contribution again shows the special cancellations of the previous section), the one new—and, for this discussion, essential—feature of the 3-3 resonance is that it permits s-channel contributions (Fig. 4) to elastic π^+p and u-channel contributions (Fig. 5) to elastic π^-p scattering. The question of evading a too large growth has been met with before in the literature,[8] and will not be considered here; rather, since the motivation is to mix crossed-channel quantities at high energies, we take the simplest route of assuming that the amplitudes constructed from the Δ^{++} propagator can be crudely expressed in terms of the previous nucleon-pole amplitudes, and write for the now complete elastic amplitudes

$$\begin{aligned} A(\pi^+p)&=A(u,t)+\xi_1A(s,t)\,,\\ B(\pi^+p)&=B(u,t)+\eta_1B(s,t)\,,\\ A(\pi^-p)&=A(s,t)+\xi_2A(u,t)\,,\\ B(\pi^-p)&=B(s,t)+\eta_2B(u,t)\,, \end{aligned} \tag{39}$$

where $\xi_{1,2}$ and $\eta_{1,2}$ are four real parameters. Crossing symmetry then requires that $\xi_1=\xi_2\equiv\xi$ and $\eta_1=\eta_2\equiv\eta$,

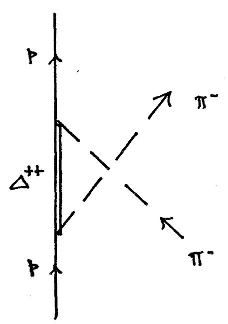

FIG. 5. The u-channel Δ^{++}-pole contribution to π^-p scattering.

[8] For example, E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

and the new C amplitudes are given by

$$\begin{aligned} C(\pi^+p)&=C(u,t)+\eta C(s,t)+(2m/s)(\eta-\xi)A(s,t)\,,\\ C(\pi^-p)&=C(s,t)+\eta C(u,t)+(2m/s)(\eta-\xi)A(u,t)\,. \end{aligned} \tag{40}$$

The new polarizations are proportional to

$$\begin{aligned} &\mathrm{Im}[A(\pi^+p)C^*(\pi^+p)]\sim m\Omega^2\\ &\quad\times\{(s/M^2)^{\gamma_0 f}[1-\xi+2\xi\eta\cos(\pi\gamma_0|f|)]\sin(\pi\gamma_0|f|)\\ &\quad\quad+(m^2/s)(\xi-\eta)(3+t/4m^2)\sin(2\pi\gamma_0|f|)\} \end{aligned} \tag{41}$$

and

$$\begin{aligned} &\mathrm{Im}[A(\pi^-p)C^*(\pi^-p)]\sim -m\Omega^2\\ &\quad\times\{(s/M^2)^{\gamma_0 f}[\eta-2\cos(\pi\gamma_0|f|)]\sin(\pi\gamma_0|f|)\\ &\quad+(m^2/s)[2(\xi-\eta)-\eta(1+t/4m^2)]\sin(2\pi\gamma_0|f|)\}\,, \end{aligned} \tag{42}$$

and a mechanism to preserve the signs of these quantities as $-t$ increases past 0.6 is now apparent. Suppose that the coefficients of $\sin(\pi\gamma_0|f|)$ and $\sin(2\pi\gamma_0|f|)$ in (41) and (42) are all positive. Since $|t|=0.6$ corresponds to $\gamma_0|f|=1$, for $|t|<0.6$ the $(s/M^2)^{\gamma_0|f|}$ terms in each expression are dominant, providing a situation similar to that in (29) and (30). On the other hand, for $|t|>0.6$ the m^2/s terms are the most important, and these act to preserve the signs of the lower $-t$ values. To guarantee that the coefficients are positive we require the conditions

$$\begin{aligned} &\text{(i)}\quad \xi>\eta\,, &&\text{(iii)}\quad \eta>0\,,\\ &\text{(ii)}\quad \xi(1+2\eta)<1\,, &&\text{(iv)}\quad 2(\xi-\eta)\cong0.83\eta\,, \end{aligned} \tag{43}$$

which limit η to lie between 0 and 0.3, with ξ bounded by $1/(1+2\eta)>\xi\cong\sqrt{2}\eta$.

The total cross sections are given by

$$\sigma_T(\pi^-p)\sim\tfrac{1}{2}G^2(\sqrt{\pi})(m^2/s)^{1/2}\,, \tag{44}$$

$$\sigma_T(\pi^+p)\sim\tfrac{1}{2}\eta G^2(\sqrt{\pi})(m^2/s)^{1/2}\,, \tag{45}$$

where both now have the same-order asymptotic falloff (but still too fast), with $\sigma_T(-)>\sigma_T(+)$.

That portion of the differential cross sections important for near-forward scattering will be given by

$$\begin{aligned} \left|\frac{C(\pi^+p)}{\Omega}\right|^2 &\approx\left(\frac{s}{M^2}\right)^{2\gamma_0 f}[1+\eta^2-2\eta\cos(\pi\gamma_0|f|)]\\ &+2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\left(\frac{m^2}{s}\right)\{a[1-\eta\cos(\pi\gamma_0|f|)]\\ &\times\cos(\pi\gamma_0|f|)+b\eta\sin^2(\pi\gamma_0|f|)\}\\ &+\left(\frac{m^2}{s}\right)^2[a^2\cos^2(\pi\gamma_0|f|)+b^2\sin^2(\pi\gamma_0|f|)] \end{aligned} \tag{46}$$

and

$$\left|\frac{C(\pi^-p)}{\Omega}\right|^2 \approx \left(\frac{s}{M^2}\right)^{2\gamma_0 f}\left[1+\eta^2-2\eta\cos(\pi^{\gamma_0}|f|)\right]$$
$$+2\left(\frac{s}{M^2}\right)^{\gamma_0 f}\left(\frac{m^2}{s}\right)\{a[\eta-\cos(\pi\gamma_0|f|)]$$
$$\times\cos(\pi\gamma_0|f|)-b\sin^2(\pi\gamma_0|f|)\}$$
$$+\left(\frac{m^2}{s}\right)^2\left[a^2\cos^2(\pi\gamma_0|f|)+b^2\sin^2(\pi\gamma_0|f|)\right], \quad (47)$$

with

$$a=2(\xi-\eta)-(1+\eta)(1+t/4m^2),$$
$$b=2(\xi-\eta)+(1-\eta)(1+t/4m^2).$$

These forms display the previous property of a function gradually changing from a Regge to a fixed-pole regime, as $-t$ increases. However, the introduction of the ξ, η parameters changes the previous dip/bump structure, and rough estimates suggest that the fully amalgamated amplitudes of Sec. VI can be made to yield the experimental wiggles of $d\sigma/dt$, especially near the dramatic dip at $|t|\sim 3$ BeV^2 in π^-p scattering. It hardly needs to be emphasized that the use of these ξ, η parameters is quite crude, and that a better estimate of the Δ^{++} contributions is needed.

V. ρ-EXCHANGE CONTRIBUTION

In this section we briefly discuss the situation of (*un*-Reggeized) ρ exchange, as the t-channel contribution to M_H (Fig. 3). For elastic scattering there are only the invariant $B_{\rho,H}{}^{\pm}$ amplitudes, with $B_{\rho,H}{}^{+}=-B_{\rho,H}{}^{-}$, to be inserted under the integrals of (12):

$$B_\rho{}^{\pm}(p_1p_1'p_2p_2')=\frac{1}{2\pi}\int d\lambda\,\exp\mathfrak{F}(\lambda)\int da\,e^{-ia\lambda}$$
$$\times B_{H,\rho}{}^{(\pi^\pm p)}(p_1+a\mu\mathcal{P},\,p_1'+a\mu\mathcal{P},\,p_2,\,p_2'). \quad (48)$$

Because

$$\tfrac{1}{2}\gamma\cdot(p_2+p_2')B_{\rho,H}{}^{\pm}(p_1p_1'p_2p_2')$$
$$=\Gamma_\mu(p_1^2,p_1'^2,(p_1-p_1')^2)\Delta_c{}^{\mu\nu}(p_1-p_1')$$
$$\times\Gamma^{\pm}(p_2^2,p_2'^2,(p_1-p_1')^2)(p_2+p_2')_\nu,$$

where $\Gamma_\mu\Delta_c{}^{\mu\nu}\Gamma^{\pm}K_\nu$ denote the (ρNN vertex)⊗(the ρ propagator)⊗($\rho\pi\pi$ vertex), inclusion of the off-mass-shell p_1^2, $p_1'^2$ dependence of Γ_μ is essential if an overly damped, zero result for (48) is to be avoided; this is due to the circumstance that any hard amplitude dependence on the variable p_1-p_1' will be free from soft effects, so that the parametric a integral produces a factor of $\delta(\lambda)$, and $|\exp\mathfrak{F}(\lambda)|\to\lambda^{-\gamma_0 f}\to 0$. Use of a more exact representation for $\mathfrak{F}(\mu z\cdot\mathcal{P},\mu^2z^2)$ does not change this situation. Within the present formalism, which neglects pion SVNVM exchange, this hard t-channel amplitude does not provide the natural cutoff for the virtual NVM momenta, and one must either resort to the methods of I or introduce off-mass-shell structure for the ρNN vertex.

We discuss the second possibility first, and in the simplest context, where one sets

$$\Gamma_\mu(p_1^2,p_1'^2,(p_1-p_1')^2)$$
$$=\gamma_\mu\Lambda^2(\Lambda^2+p_1^2-i\delta)^{-1/2}(\Lambda^2+p_1'^2-i\delta)^{-1/2}|_{\delta\to 0},$$

and finds

$$B_\rho{}^{\pm}\sim(\Lambda/\mu)^{\gamma_0 f}\Gamma(1-\gamma_0 f)b_\rho{}^{\pm}(t)\cos(\tfrac{1}{2}\pi\gamma_0 f)e^{\frac{1}{2}\pi\gamma_0 f\epsilon(\delta)}, \quad (49)$$

where $b_\rho{}^{\pm}(t)$ are the ordinary Born approximation t exchanges, with $b_\rho{}^{+}=-b_\rho{}^{-}$. The cutoff Λ, which has been assumed large, $\Lambda/\mu\gg 1$, in arriving at (49), could now be evaluated in terms of the constant asymptotic limits of the total cross sections, provided that opposite phases are assigned to π^+p and π^-p; but it is hard to understand and justify this assignment, since the ρ propagator is supposed to separate the details of the ρNN vertex from those of the $\rho\pi\pi$ vertex. Assigning the same phase to both $B_\rho{}^{\pm}$ removes this objection, but makes one of the two total cross sections come out negative, and is also hard to reconcile with crossing symmetry, which involves a complex conjugation and suggests that these phases are opposite. One might imagine that a better calculation of the exchanged ρ, taking into account its structure, would generate an answer in which the term $(N\mu)^{\gamma_0 f}$ of (49) is replaced by an energy-dependent factor such as $(s/\mu^2)^{\gamma_0 f}$, thereby removing the crossing objections to the same phases; but then one would find an asymptotically decreasing contribution to σ_T.

In contrast, it is easy to imagine how the inclusion of SVNVM exchange between nucleons and pions can produce the necessary phases, and this is our second reason for associating meson resonances with the NVM exchange. To see this we now revert to a crude type-I calculation in which the NVM momenta are cut off by hand; the amplitude corresponding to Fig. 3 would then be written as

$$B_\rho{}^{\pm}(t)=e^{\gamma_{NN}f(t)}b_\rho{}^{\pm}(t), \quad (50)$$

with $\gamma\cong(g_0^2/8\pi^2)\ln(1+\mu_c^2/\mu^2)$ and μ_c the effective cutoff. This $B_\rho{}^{\pm}$ is a real quantity. In order to obtain phases, we imagine SVNVM exchange in all possible combinations; in the spirit of this approximation, one would write

$$B_\rho{}^{\pm}\sim b_\rho{}^{\pm}(t)e^{\gamma_{NN}[F_{NN}(t)-F_{NN}(\mu_\rho^2)]+\gamma_{\pi\pi}[F_{\pi\pi}(t)-F_{\pi\pi}(\mu_\rho^2)]}$$
$$\times e^{\pm 2\gamma_{NN}[F_{\pi N}(u)-F_{\pi N}(s)]}, \quad (51)$$

where $F_{\pi\pi}$ and $F_{\pi N}$ denote two-particle "phase-space" integrals analogous to the F_{NN} of (8), but containing both pion and nucleon mass in $F_{\pi N}$ and only the pion mass in $F_{\pi\pi}$. The signs of the $\gamma_{\pi N}$ terms in the exponent of (51) are opposite, because π^+ and π^- couple to ρ_0 with opposite signs. Equation (51) is merely the transcription of the results of the pp calculation of I to this somewhat more complicated case, using a different γ for ρ_0 exchange between NN, $N\pi$, and $\pi\pi$. Note that the calculation is independent of the renormalization point of the s- and u-channel SVNVM, because these soft effects appear in the combination $f_{\pi N}(u)-f_{\pi N}(s)=F_{\pi N}(u)-F_{\pi N}(s)$.

We may reasonably take a minimum value of γ_{NN} from the nucleon-form-factor estimates of I, $\gamma_{NN}\sim 2.5$. Neglecting very small contributions coming from the structure of the ρ resonance itself, the phases of (51) result from $\mathrm{Im}F_{\pi\pi}(\mu_\rho^2)\sim\pi$ and $\mathrm{Im}F_{\pi N}(s)\sim\pi$ for $s\gg m^2$, so that

$$\mathrm{Im}B_\rho^{\pm}\sim -\sin(\pm 2\pi\gamma_{\pi N}+\pi\gamma_{\pi\pi})b_\rho^{\pm} \times e^{\gamma_{NN}[F_{NN}(t)-F_{NN}(\mu_\rho^2)]}e^{\gamma_{\pi\pi}[F_{\pi\pi}(t)-\mathrm{Re}F_{\pi\pi}(\mu_\rho^2)]} \times e^{\pm 2\gamma_{\pi N}[F_{\pi N}(u)-\mathrm{Re}F_{\pi N}(s)]}, \quad (52)$$

and we must now fix the relative magnitude of $\gamma_{\pi N}$ and $\gamma_{\pi\pi}$. These numbers reflect the ease with which virtual ρ_0's are emitted by pions and nucleons in this crude cutoff calculation, and one has the intuitive feeling that this effect should be far more appropriate for nucleons, or $\gamma_{NN}\gg\gamma_{\pi N}\gg\gamma_{\pi\pi}$. In this case, $\mathrm{Im}B_\rho^{\pm}$ will be essentially proportional to $\sin(2\pi\gamma_{\pi N})$, and $B_\rho^{\pm}$ will be almost imaginary if $\gamma_{\pi N}=\frac{1}{4}+\frac{1}{2}n$. To keep the inequality strong, $\gamma_{NN}\gg\gamma_{\pi N}$, we take the smallest $\gamma_{\pi N}$ value of $\frac{1}{4}$.

Recent experiments on the pion's electromagnetic form factor[9] suggest that $\gamma_{\pi\pi}$ is small. This is because a factor $e^{\gamma_{\pi\pi}F_{\pi\pi}(t)}\sim 1+(\gamma_{\pi\pi}/3m_\pi^2)t$ would modify any calculation of the hard part of the form factor near $-t\sim 0$, producing a correction of order $[1+(\gamma_{\pi\pi}m_\rho^2/18m_\pi^2)]^{1/2}$ to previous ρ-dominance estimates of the pion rms radius. Since the experimental upper bounds are only slightly larger ($\lesssim 10\%$) than the estimates, we conclude that $\gamma_{\pi\pi}$ is very small indeed. One expression of this difference in magnitude, which takes into account the different emission probabilities in a crude way, is given by a "factorization" hypothesis, $\gamma_{\pi N}^2\sim\gamma_{\pi\pi}\gamma_{NN}$. Combining such a guess with the previous numbers leads to $\gamma_{\pi\pi}\sim 1/40$, which is sufficiently small to keep $[1+(\gamma_{\pi\pi}m_\rho^2/18m_\pi^2)]^{1/2}$ from exceeding unity by about 5%. A larger $\gamma_{\pi\pi}\sim 0.1$ can be accommodated in the same way. While the pion-form-factor experiments are suggestive of a small rms radius, they are by no means conclusive, and it may be that a larger $\gamma_{\pi\pi}$ is appropriate. For the σ_T estimate of this section, $\gamma_{\pi\pi}$ has been assumed zero.

In the asymptotic region, $F_{\pi N}(u)\sim\mathrm{Re}F_{\pi N}(s)$, so that

$$B_\rho^{\pm}\sim\mp ib_\rho^{\pm}(t) \times e^{\gamma_{NN}[F_{NN}(t)-F_{NN}(\mu_\rho^2)]+\gamma_{\pi\pi}[F_{\pi\pi}(t)-\mathrm{Re}F_{\pi\pi}(\mu_\rho^2)]}. \quad (53)$$

Since $\sigma_T(\pm)\sim\mathrm{Im}C(\pm)|_{t=0}$, in the true asymptotic region we pick up only the contribution of $B_\rho^{\pm}$, acting as our Pomeranchuk exchange.[10] If

$$b_\rho^{\pm}(t)\simeq\mp g_{\rho\pi\pi}^2/(m_\rho^2-t),$$

this means that

$$B_\rho^{\pm}\sim +i\frac{\sigma_T(\pm)}{1-t/m_\rho^2}e^{\gamma_{NN}F_{NN}(t)+\gamma_{\pi\pi}F_{\pi\pi}(t)}, \quad (54)$$

where, with $g_{\rho\pi\pi}^2/4\pi\sim 2.2$, σ_T is determined as

$$\sigma_T(\pm)\sim\frac{g_{\rho\pi\pi}^2}{m_\rho^2}e^{-\gamma_{NN}F_{NN}(\mu_\rho^2)}\sim 10\text{ mb}, \quad (55)$$

which compares favorably with the experimental values of $\sim$25 mb. In effect, the SVNVM have provided the phase which then permits this Born approximation estimate of the (maximum) σ_T. Of course, the preceding steps really need to be justified before one can attach numerical significance to (55), but it is gratifying that the order of magnitude is correct. It is amusing to note that a change of renormalization point, from $t=+\mu_\rho^2$ to $t=0$, raises the estimate of (55) to $\sim$17 mb. The inclusion of other hard-resonance exchanges would have the same effect, as would a nonzero value of $\gamma_{\pi\pi}$ in (51) and (52); a $\gamma_{\pi\pi}\sim 0.4$ yields a $\sigma_T\sim$25 mb.

VI. AMALGAMATION AND SUMMARY

The next step in this analysis is to combine the separate parts of the scattering amplitude discussed in the previous sections, and choose a handful of parameters in order to make a comparison with the data. Attempts at detailed fits using the exact kinematical expressions for P and $d\sigma/dt$ as well as the exact forms of (22) and (26) are in progress and will be reported separately. Here one follows the procedure of using a type-I calculation for SVNVM exchange between pions and between pions and protons in the nucleon direct-channel terms and everywhere in the t-channel exchange as in Sec. V. For simplicity, we here omit dependence on $\gamma_{\pi\pi}$ and γ_{NN}, except that these constants provide the identical phase contribution to every term, as in (52). Thus, with $e^{\mp 2i\gamma_{\pi N}\pi}\sim\mp i$, we have

$$A(\pi^+p)\sim -i[A(u,t)+\xi A(s,t)], \quad (56)$$

$$A(\pi^-p)\sim +i[A(s,t)+\xi A(u,t)], \quad (57)$$

$$B(\pi^+p)\sim -i[b_\rho^{(+)}(t)e^{\gamma_{NN}[F_{NN}(t)-F_{NN}(m_\rho^2)]}+B(u,t)+\eta B(s,t)], \quad (58)$$

$$B(\pi^-p)\sim +i[b_\rho^{(-)}(t)e^{\gamma_{NN}[F_{NN}(t)-F_{NN}(m_\rho^2)]}+B(s,t)+\eta B(u,t)]. \quad (59)$$

[9] An excellent review of the experimental situation has been given by K. Kang and M. Widgoff (unpublished). Recent estimates of the pion rms radius may be found in W.-S. Lam, Ph.D. thesis, Brown University, 1969 (unpublished).

[10] A corresponding ρ-exchange $I=1$ contribution appears in the charge-exchange amplitude defined by $B_\rho^{\text{ex}}=(1/\sqrt{2})[B_\rho^+-B_\rho^-]$, but has the nice property of vanishing in the forward direction; this is simply due to the structure of (53) and (54), with $F(u)-\mathrm{Re}F(s)\sim|t|/s$ for $s\gg -t$, m^2, so that $B_\rho^{\text{ex}}\sim(i/\sqrt{2})\sigma_T(1-t/m_\rho^2)^{-1}\,4\gamma_{\pi N}(|t|/s)\,\exp[\gamma_{NN}F_{NN}(t)+\gamma_{\pi\pi}F_{\pi\pi}(t)]$.

The common $-i$ factors of (56) and (58) will have no effect in the $d\sigma/dt$ and P computation of π^+p scattering, while the $+i$ factors of (57) and (59) have no effect on the π^-p processes; the total, constant, asymptotic cross sections are still equal and given by (55).

One should note that all of the preceding operations and considerations can be carried through if a phenomenological pseudoscalar-pseudoscalar pion-nucleon interaction, $\mathcal{L}'=ig\bar{\psi}\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi$, is used in place of or in addition to the axial-vector interaction of Sec. III, and this introduces one more parameter, g, into the amplitudes (56)–(59). Having chosen $\mu^2\sim\mu_\rho^2$, $\gamma_0\sim2.2$, $g_{\rho\pi\pi}^2/4\pi\sim2.2$, and with $\sigma_T\sim25$ mb, we then have five parameters to specify: g, $2mG\equiv g_A$, ξ, η, and γ_{NN}. Speaking qualitatively, the γ_{NN} dependence acts to damp out the t-channel exchange contribution as one moves away from the forward direction, just as the Regge behavior dominant in the direct and resonance nucleon terms provides damping as $|t|$ increases; the difference between these styles of damping is that the latter displays shrinkage while the former does not, and whether or not the complete amplitude shrinks depends upon the relative weighting of the individual pieces. The comparative flattening out of the observed π^-p elastic cross section, after the minimum at $|t|\sim3$ BeV^2 is in this model given by the emergence of the fixed-pole parts of the amplitudes, with order of magnitude given by the g, g_A dependence (multiplied by the previously neglected $\gamma_{\pi N}$, $\gamma_{\pi\pi}$ non-Regge damping, common to all parts of the amplitude). Crude estimates suggest that the dips and bumps in this relatively flat portion of $d\sigma/dt$ are in part controlled by the ξ and η parameters, while the u-channel part of the fixed-pole dependence generates a small backwards peak. The question of polarization sign preservation is now far more complicated than in Sec. IV, and we defer further comments until the detailed calculations have been completed.

Whether this model can be taken seriously, and, in particular, the effects of a better treatment of resonances and vertices, is clearly a matter for more detailed computation. There are certainly many more hard amplitudes which should be considered, but it would be a pleasant economy if just the simple Born terms written here were sufficient to give a good fit to the elastic and charge-exchange data. An economy of another sort has been demonstrated in this paper, in the sense that effective Regge poles in πp scattering have been generated using a slightly refined calculation based upon the same physical model which gave, in I, a single-parameter qualitative fit to the pp data.

Note added in proof. It should have been stressed that the method of renormalization used in this paper was chosen because it produces something resembling a ρ trajectory. The alternate method used in I would have retained $f=-1+F$ in the s- and u-channel amplitudes, and required multiplication by a constant $\exp\gamma$, which these calculations estimated to be ~10. The present renormalization goes beyond the ordinary multiplication by constants, and is an s-dependent effect which changes the output Regge parameters into those of the phenomenological ρ exchange.

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