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Three-Meson Problem in Dispersion Theory

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The addition of a meson to the $2V$ sector of the Lee model provides an opportunity to study collision problems involving two composite particles formed by the exchange of one and two such mesons. Following Amado, we give a dispersion treatment which avoids five-particle intermediate states, but previously excluded four-particle states now make their appearance. It is found, as in the recent three-meson example of Bronzan, that the dynamics reduces to the solution of a Fredholm integral equation in one variable brought about by a factorization property of the connected S matrix in the $2V$ sector. Similarities and differences between the two cases are pointed out. Dispersion-theoretic investigations of the one- and two-meson exchange interactions of two nucleons in crossing-symmetric static models, and the scattering of a meson by these nucleons are indicated for future consideration.

I. INTRODUCTION

IN preceding papers[1–3] we have presented, from several viewpoints, the complete solution of the $2V$ sector of the Lee model with boson sources at zero separation. Hereafter, we refer to this subspace as the two-meson sector.[4] With this solution at hand, it is natural to inquire into this system after adding another meson. Thus this paper is concerned with a three-meson sector. Similar higher-sector work has been carried out by Bronzan[5] who introduces into the Lee model a third static source U which couples to the original V particle together with a meson. In fact, our case simulates the dynamical situation considered by that author without the introduction of the third particle. However, we are forced into mathematical details of greater complexity which tend to thwart our desire to simplify the final results as much as possible. It is of some interest that these studies implicate many-particle intermediate states. For example, the two- and three-meson sectors contain four- and five-particle intermediate states, respectively. Admittedly, in each of these states there is a considerable simplification since two of the particles are of the N type. Nevertheless, these investigations may provide insight into less tractable composite particle problems more closely descriptive of the physical world.

We wish to examine the three-meson sector within the framework of dispersion theory. For this purpose, we adopt the computational scheme devised by Amado[6] in his calculation of the elastic scattering of a meson by the V particle. Previously,[1] we applied this approach to the elastic scattering of a meson by the VN system and found that it was necessary to contract the composite particle representing the VN bound state. That problem was solved by operating with the product of a V and an N operator in the usual asymptotic definition of a state. In this way, and with the help of certain knowledge gained earlier in the Tamm-Dancoff formalism,[2] we achieved a dispersion solution of the $(VN)\theta$ elastic scattering amplitude which circumvented intermediate states containing four particles. In the case at hand, which deals with the transition amplitude for $(2V)\theta$ elastic scattering, we contract the $2V$ composite particle by operating with the product of two V operators in the asymptotic state. This time we avoid intermediate states carrying five particles, but the previously excluded four-particle states now make their appearance. From this, one might conclude that the dynamical description of the three-meson sector is given by integral equations in two variables. It turns out, however, that the fundamental equation is an integral equation of the Fredholm type in one variable. The reason for this is found in a factorization property of the S matrix in the two-meson sector. This property also emerges in the two-meson solution of the charged-scalar theory[7] and in the $V\theta$ sector of the Lee model.[5,8] A detailed discussion can be found in these references.

As already mentioned, it is desirable to capitalize on the general relevance of our work to other composite particle problems more closely approximating the real world. In this direction it is of interest to consider static models with crossing symmetry. In a separate paper we plan to study the role of this principle in the charged-scalar theory of two source particles corresponding roughly to the neutron and the proton. We would first seek the one- and two-meson solution of the associated "deuteron" problem. In light of the present paper, we could then go on to explore the process of elastic meson-"deuteron" scattering.

In Sec. II we reexamine the dispersion approach to the one- and two-meson sectors. It is shown that the solution in the latter case reduces to a consideration of the transition amplitude Y for the elastic scattering of a meson by the VN system. Having already secured this amplitude using Amado's procedure, we now choose to follow Ref. 5, which leads us to a Low-type equation

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[2] L. M. Scarfone, J. Math. Phys. **9**, 977 (1968).
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[4] The VN subspace will be called the one-meson sector.
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[6] R. D. Amado, Phys. Rev. **122**, 696 (1961).
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[8] J. B. Bronzan, M. Cassandro, and M. Vaughn, Nuovo Cimento **46**, 128 (1966).

for V. The solution to this equation is obtained via the analytic properties of an auxiliary function constructed from V and the amplitude for $(2N)\theta$ elastic scattering. In Sec. III we undertake a study of the three-meson sector, in particular, the S-matrix elements characteristic of the $(2V)\theta$ channel. These elements are presented in terms of two associated amplitudes which, in turn, are determined by the solution of the fundamental equation in the theory. The contents of this section include a brief account of the diagonalization of the S matrix in the $2V$ sector. The final section contains some remarks on the relation between the calculations performed in this paper and those in Ref. 5. It concludes with speculations on future problems involving two static sources.

II. ONE- AND TWO-MESON SECTORS

Of the various methods[9] applicable to solution of the Lee model, it is particularly characteristic of dispersion theory that one utilizes amplitudes previously derived in lower sectors in order to discuss higher ones. For this reason, we first include a brief review of the dispersion analysis of the one- and two-meson sectors before proceeding to the three-meson case under consideration in this paper. A similar situation occurs in the dispersion formulation of the three-meson example recently treated by Bronzan[5]. Indeed, it is hardly necessary to point out that two V particles are equivalent to his U particle insofar as they both communicate with states involving two mesons.

We begin with the S-matrix element describing the collision between one meson and two N particles. By definition, and with one contraction, we obtain[10]

$$\langle 2N\theta_{k'}|S|2N\theta_k\rangle = \langle 2N\theta_{k'},\text{ out}|2N\theta_k,\text{ in}\rangle = \delta_{kk'}+2\pi i\delta(\omega-\omega')X^2(\omega)M(\omega)\,, \quad (1)$$

where the transition amplitude $M(\omega)$ is given by

$$M(\omega)=X^{-1}(\omega)\langle 2N|j|2N\theta_k,\text{ in}\rangle\,. \quad (2)$$

This amplitude satisfies the Low equation

$$M(\omega)=-\frac{2(gZ_0)^2}{\omega-\omega_0}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')|M(\omega')|^2d\omega'}{\omega'-\omega-i\epsilon}\,, \quad (3)$$

which is readily established by contracting the remaining in-state meson. The Chew-Low solution of this equation, that is, the one without Castillejo-Dalitz-Dyson (CDD) poles,[11] may be written as

$$M(\omega)=-2g^2/G(\omega)\,, \quad (4)$$

where the denominator function $G(\omega)$ is defined by

$$G(\omega)=2\omega[1-\beta(\omega)]+Z\delta m_V-Z\omega\,. \quad (5)$$

The function $\beta(\omega)$ has the integral representation

$$\beta(\omega)=-\frac{g^2\omega}{\pi}\int_\mu^\infty\frac{\rho(\omega')d\omega'}{\omega'^2(\omega'-\omega-i\epsilon)}\,, \quad (6)$$

while the V-particle self-energy and field operator renormalization constants are known to be

$$\delta m_V=\frac{g^2}{\pi Z}\int_\mu^\infty\frac{\rho(\omega)d\omega}{\omega}\,,\quad Z=1-\frac{g^2}{\pi}\int_\mu^\infty\frac{\rho(\omega)d\omega}{\omega^2}\,. \quad (7)$$

The VN bound-state parameters ω_0 and Z_0 are determined by the conditions $G(\omega_0)=0$ and $G'(\omega_0)=Z_0^{-2}$, respectively; the prime denotes differentiation. Upon subtracting $G(\omega_0)$ from Eq. (5), we can give $G(\omega)$ the useful form

$$G(\omega)=(\omega-\omega_0)\alpha(\omega)\,, \quad (8)$$

where

$$\alpha(\omega)=Z_0^{-2}+\frac{2g^2}{\pi}(\omega-\omega_0)\int_\mu^\infty\frac{\rho(\omega')d\omega'}{(\omega'-\omega_0)^2(\omega'-\omega-i\epsilon)}\,. \quad (9)$$

In this way the root of $G(\omega)$ is factored out and we encounter the function $\alpha(\omega)$ which has a cut for $\mu<\omega<\infty$ and no zeros or poles in the cut plane. It can be shown by contour integrations that

$$\alpha(\omega)=Z+\frac{1}{\pi}\int_\mu^\infty\frac{\mathrm{Im}\alpha(\omega')d\omega'}{\omega'-\omega-i\epsilon}\,, \quad (10)$$

while

$$G^{-1}(\omega)=\frac{Z_0^2}{\omega-\omega_0}+\frac{1}{\pi}\int_\mu^\infty\mathrm{Im}[G^{-1}(\omega')]\frac{d\omega'}{\omega'-\omega-i\epsilon}\,. \quad (11)$$

The amplitude M is related to the corresponding phase shift η for this scattering by

$$e^{2i\eta(\omega)}=1+2i\rho(\omega)M(\omega)\,. \quad (12)$$

Next, we turn to the two-meson sector. In analogy with $V\theta$, this case embraces two elastic scattering amplitudes, a production amplitude, and a bound-state problem, all of which have now been studied by the Lehmann-Symanzik-Zimmermann (LSZ) formalism,[3] the Tamm-Dancoff approximation,[2] and the methods of dispersion theory.[1] It remains for future investigations to apply such approaches as the N-quantum approximation[12] and the algebraic technique of Bolsterli[13] to the dynamics of the two-meson sector. In all of these applications one must incorporate the necessary modifications required by the introduction of two nontrivial source particles.

The S-matrix in the two-meson sector can be determined in terms of the transition amplitude V. We recall

[9] A comparison of three such methods has been given by M. S. Maxon, Phys. Rev. **149**, 1273 (1966).

[10] We follow, as closely as possible, the notation of Ref. 1; $\rho(\omega)$ is used as an abbreviation for $kf^2(\omega)/4\pi$.

[11] L. Castillejo, R. Dalitz, and F. Dyson, Phys. Rev. **101**, 458 (1956).

[12] A. Pagnamenta, Ann. Phys. (N. Y.) **39**, 453 (1966); A. Halprin, Phys. Rev. **172**, 1495 (1968).

[13] M. Bolsterli, Phys. Rev. **166**, 1760 (1968).

that

$$\langle B_0\theta_{k'}|S|B_0\theta_k\rangle=\langle B_0\theta_{k'},\text{ out}|B_0\theta_k,\text{ in}\rangle$$
$$=\delta_{kk'}+2\pi i\delta(\omega-\omega')X^2(\omega)Y(\omega),\quad(13)$$

where

$$Y(\omega)=X^{-1}(\omega)\langle B_0|j|B_0\theta_k,\text{ in}\rangle.\quad(14)$$

The bound state of the VN system will be treated as a stable particle, called B_0, with energy $2m+\omega_0$. To fill out the scattering matrix we must also have the elements describing the remaining elastic collision and the production process. These are readily found to be

$$\begin{aligned}\langle 2N\theta_{k_4}\theta_{k_3}|S|2N\theta_{k_2}\theta_{k_1}\rangle&=\langle 2N\theta_{k_4}\theta_{k_3},\text{ out}|2N\theta_{k_2}\theta_{k_1},\text{ in}\rangle\\&=\tfrac{1}{2}\langle 2N\theta_{k_3},\text{ out}|2N\theta_{k_2},\text{ in}\rangle\langle 2N\theta_{k_4},\text{ out}|2N\theta_{k_1},\text{ in}\rangle\\&\quad+\tfrac{1}{2}\langle 2N\theta_{k_3},\text{ out}|2N\theta_{k_1},\text{ in}\rangle\langle 2N\theta_{k_4},\text{ out}|2N\theta_{k_2},\text{ in}\rangle\\&\quad+(2\pi i/\sqrt{2})\delta(\omega_4+\omega_3-\omega_2-\omega_1)X(\omega_4)X(\omega_3)X(\omega_2)\\&\qquad\times X(\omega_1)e^{2i\eta(\omega_3)}\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)\end{aligned}\quad(15)$$

and

$$\begin{aligned}\langle 2N\theta_{k_3}\theta_{k_2}|S|B_0\theta_{k_1}\rangle&=\langle B_0\theta_{k_1}|S|2N\theta_{k_2}\theta_{k_3}\rangle\\&=\langle 2N\theta_{k_3}\theta_{k_2},\text{ out}|B_0\theta_{k_1},\text{ in}\rangle\\&=(4\pi i/\sqrt{2})\Omega\delta(\omega_1+\omega_0-\omega_2-\omega_3)X(\omega_3)X(\omega_2)X(\omega_1)\\&\qquad\times e^{2i\eta(\omega_2)}N_0(\omega_2,\omega_1).\end{aligned}\quad(16)$$

A comparison between the above Eq. (15) and Eq. (102) of Ref. 1 shows that

$$\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)=\mathcal{Q}(\omega_3,\omega_2,\omega_1)-(1/\sqrt{2})X^{-2}(\omega_2)M(\omega_1)\delta_{k_2k_3}-(1/\sqrt{2})X^{-2}(\omega_1)M(\omega_2)\delta_{k_1k_3},\quad(17)$$

where the amplitude $\mathcal{Q}$ differs from an ordinary transition amplitude by having in-states on both sides, that is,

$$\mathcal{Q}(\omega_3,\omega_2,\omega_1)=X^{-1}(\omega_3)X^{-1}(\omega_2)X^{-1}(\omega_1)\times\langle 2N\theta_{k_3},\text{ in}|j|2N\theta_{k_2}\theta_{k_1},\text{ in}\rangle,\quad(18)$$

while the "associated" amplitude $\mathcal{Q}_0$ is obtained from $\mathcal{Q}$ by removing the disconnected parts as in Eq. (17). In Eq. (98) of Ref. 1, the production amplitude $\mathcal{P}(\omega_2,\omega_1)$, defined as

$$\mathcal{P}(\omega_2,\omega_1)=[X^{-1}(\omega_2)X^{-1}(\omega_1)/2\Omega]\times\langle 2N\theta_{k_2},\text{ out}|j|B_0\theta_{k_1},\text{ in}\rangle,\quad(19)$$

is related to the amplitude $N(\omega_2,\omega_1)$ which has in-states on both sides,

$$N(\omega_2,\omega_1)=[X^{-1}(\omega_2)X^{-1}(\omega_1)/2\Omega]\times\langle 2N\theta_{k_2},\text{ in}|j|B_0\theta_{k_1},\text{ in}\rangle,\quad(20)$$

according to

$$\mathcal{P}(\omega_2,\omega_1)=e^{2i\eta(\omega_2)}N(\omega_2,\omega_1).\quad(21)$$

When the disconnected part of $N(\omega_2,\omega_1)$ is separated out, we arrive at the "associated" amplitude

$$N_0(\omega_2,\omega_1)=N(\omega_2,\omega_1)+(g\sqrt{2}Z_0/2\Omega)X^{-2}(\omega_1)\delta_{k_1k_2},\quad(22)$$

appearing in Eq. (16). On substituting $N_0(\omega_2,\omega_1)$ for $N(\omega_2,\omega_1)$ in Eq. (16), one must note that the second term on the right-hand side of Eq. (22) makes no contribution because of energy conservation. It is straightforward to show that the associated amplitudes $N_0(\omega_2,\omega_1)$ and $\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)$ satisfy the coupled Omnès-type integral equations

$$\begin{aligned}N_0(\omega_2,\omega_1)&=\frac{g\sqrt{2}}{2\Omega}Z_0[Y(\omega)-M(\omega)]\left(\frac{1}{\omega_2-\omega_0}+\frac{1}{\omega_1-\omega_2+i\epsilon}\right)\\&\quad+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega)}\sin\eta(\omega)N_0(\omega,\omega_1)\\&\quad\times\left(\frac{1}{\omega+\omega_2-\omega_1-\omega_0-i\epsilon}+\frac{1}{\omega-\omega_2+i\epsilon}\right)d\omega\end{aligned}\quad(23)$$

and

$$\begin{aligned}\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)&=-2gZ_0\Omega N_0^*(\omega_1,\,\omega_1+\omega_2-\omega_0-i\epsilon)\\&\quad\times\left(\frac{1}{\omega_0-\omega_3}+\frac{1}{\omega_3+\omega_0-\omega_2-\omega_1-i\epsilon}\right)\\&\quad+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega)}\sin\eta(\omega)\mathcal{Q}_0(\omega,\omega_2,\omega_1)\\&\quad\times\left(\frac{1}{\omega-\omega_3+i\epsilon}+\frac{1}{\omega+\omega_3-\omega_2-\omega_1-i\epsilon}\right)d\omega.\end{aligned}\quad(24)$$

In arriving at Eq. (24) we have employed the fact that the function $R(\omega_2,\omega_1)$, defined in Ref. 1 by

$$R(\omega_2,\omega_1)=X^{-1}(\omega_2)X^{-1}(\omega_1)\langle B_0|j|2N\theta_{k_2}\theta_{k_1},\text{ in}\rangle,\quad(25)$$

can be replaced in the theory by

$$\sqrt{2}\Omega N_0^*(\omega_1,\,\omega_1+\omega_2-\omega_0-i\epsilon).$$

This relationship is analogous to that appearing in Eq. (11) of Ref. 5.

Using the results found in Ref. 1 for $N(\omega_2,\omega_1)$ and $\mathcal{Q}(\omega_3,\omega_2,\omega_1)$, or else solving Eqs. (23) and (24), we can write expressions for $\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)$ and $N_0(\omega_2,\omega_1)$ which exhibit their dependence on $Y(\omega)$. These are

$$N_0(\omega_2,\omega_1)=\frac{g\sqrt{2}G(\omega_1)[Y(\omega_1)-M(\omega_1)]}{2\Omega Z_0G^*(\omega_2)G(\omega_1+\omega_0-\omega_2)}\quad(26)$$

and

$$\mathcal{Q}_0(\omega_3,\omega_2,\omega_1)=\frac{g\sqrt{2}G(\omega_1+\omega_2-\omega_0)R(\omega_2,\omega_1)}{Z_0G^*(\omega_3)G(\omega_1+\omega_2-\omega_3)}=\frac{g^2\sqrt{2}G^2(\omega_1+\omega_2-\omega_0)[Y(\omega_1+\omega_2-\omega_0)-M(\omega_1+\omega_2-\omega_0)]}{Z_0{}^2G^*(\omega_3)G(\omega_2)G(\omega_1)G(\omega_1+\omega_2-\omega_3)}.\quad(27)$$

This shows that the complete solution of the two-meson scattering matrix reduces to the solution for $Y(\omega)$. Let us recall that in using Amado's procedure for obtaining this amplitude we encountered the scalar product $\langle 2_V|B_0\theta_k, \text{in}\rangle$, where the bra vector denotes the normalized bare state of two V particles. Since we had previously solved for the bare-state expansion of $|B_0\theta_k, \text{in}\rangle$ by the Tamm-Dancoff method, we were able to evaluate this product at once without studying it any further via the contraction technique which would have only complicated the situation by implicating intermediate states that we were trying to avoid. Instead of following Amado and contracting B_0 from the left in Eq. (14), we could have chosen to contract the in-state meson. This procedure leads to the Low integral equation

$$Y(\omega)=\frac{2g^2Z_0^2}{\omega-\omega_0}-\frac{\Gamma_0^2}{\omega-\Delta}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')|Y(\omega')|^2d\omega'}{\omega-\omega'-i\epsilon}+\left(\frac{g}{\pi Z_0}\right)^2\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega')\rho(\omega'')|G(\omega'+\omega''-\omega_0)|^2|Y(\omega'+\omega''-\omega_0)-M(\omega'+\omega''-\omega_0)|^2d\omega'd\omega''}{|G(\omega')|^2|G(\omega'')|^2(\omega'+\omega''-\omega-\omega_0-i\epsilon)}, \quad (28)$$

which can be solved by the analytic method introduced by Bronzan in the two-meson solution of the charged-scalar theory. In developing Eq. (28), we have substituted into the integrand of the double integral an expression for the square of the magnitude of $R(\omega',\omega'')$ which may be read off from Eq. (27).

The vertex $\Gamma_0=\langle B_0|j|B\rangle$ was found in Ref. 1 to be

$$\Gamma_0=-(g\sqrt{2}Z_B/Z_0)[1-Z_0^{-2}G(\Delta)A(\Delta)]^{-1}, \quad (29)$$

where Z_B is the normalization constant of the physical $2V$ bound state $|B\rangle$. We let $\Delta=\omega_B-\omega_0$, where ω_B is the energy of interaction associated with this state. It is also useful to recall the eigenvalue condition

$$D(\omega_B)\equiv Z^2(\omega_B-2\delta m_V)[1-Z_0^{-2}G(\Delta)A(\Delta)]+Z_0^{-2}G(\Delta)=0. \quad (30)$$

The integral function $A(\omega)$ is given by

$$A(\omega)=\frac{1}{\pi}\int_\mu^\infty \text{Im}\left(\frac{1}{G(\omega')}\right)\frac{d\omega'}{G(\omega+\omega_0-\omega')}. \quad (31)$$

As in the case of $G(\omega)$, it may be convenient to factor out the root in $D(\omega+\omega_0)$ by subtracting $D(\omega_B)$.

To solve Eq. (28), let us introduce the function

$$F(\omega)=g^{-2}(\omega-\omega_0)[Y^{-1}(\omega)-M^{-1}(\omega)]^{-1}, \quad (32)$$

the analytic properties of which follow from Eqs. (3) and (28). It is found that $F(\omega)$ has the discontinuity

$$F(\omega+i\epsilon)-F(\omega-i\epsilon)=8ig^2(\omega-\omega_0)E(\omega) \quad (33)$$

across the cut beginning at 2μ-ω_0 and that there is no cut beginning at μ. The integral $E(\omega)$ has the form

$$E(\omega)=\frac{g^2}{\pi Z_0^2}\int_\mu^{\omega+\omega_0-\mu}\frac{\rho(\omega')\rho(\omega+\omega_0-\omega')d\omega'}{|G(\omega')|^2|G(\omega+\omega_0-\omega')|^2}. \quad (34)$$

The right-hand side of Eq. (33) vanishes at the high-energy limit. At the values ω_0 and Δ, we have

$$\begin{aligned}F(\omega)&=Z_0^2, && \omega=\omega_0\\ &=2(\Delta-\omega_0)G^{-1}(\Delta), && \omega=\Delta.\end{aligned} \quad (35)$$

Under these conditions, together with the assumption that $F(\omega)$ approaches a constant at the high-energy limit, we obtain the representation

$$F(\omega)=Z_0^2-\frac{(\omega-\omega_0)(ZZ_0)^{-2}}{\omega+\omega_0-2\delta m_V}+(\omega-\omega_0)C(\omega), \quad (36)$$

where $C(\omega)$ is defined to be

$$C(\omega)=\frac{4g^2}{\pi}\int_{2\mu-\omega_0}^\infty\frac{E(\omega')d\omega'}{\omega'-\omega-i\epsilon}. \quad (37)$$

It is immediately obvious that this expression for $F(\omega)$ yields $F(\omega_0)=Z_0^2$ and Eq. (33), while $F(\Delta)$ can also be verified at sight with the help of the connection between the known functions $A(\omega)$ and $C(\omega)$. From Eqs. (11) and (31), we find

$$1+Z_0^{-2}G(\omega)A(\omega)=Z_0^2G(\omega)/(\omega-\omega_0)+G(\omega)C(\omega). \quad (38)$$

Using this relation and the definition of $D(\omega+\omega_0)$, we see that

$$F(\omega)=\frac{Z_0^2D(\omega+\omega_0)-2Z_0^{-2}(\omega-\omega_0)+(\omega-\omega_0)C(\omega)D(\omega+\omega_0)}{D(\omega+\omega_0)-Z_0^{-2}G(\omega)}. \quad (39)$$

Thus $F(\Delta)$, as given by Eq. (35), follows at once from this form of $F(\omega)$. It remains only to unite Eqs. (32) and (39) to secure the amplitude

$$Y(\omega)=\frac{g^2M(\omega)[Z_0^2D(\omega+\omega_0)-2Z_0^{-2}(\omega-\omega_0)+(\omega-\omega_0)C(\omega)D(\omega+\omega_0)]}{(\omega-\omega_0)M(\omega)[D(\omega+\omega_0)-Z_0^{-2}G(\omega)]+g^2[Z_0^2D(\omega+\omega_0)-2Z_0^{-2}(\omega-\omega_0)+(\omega-\omega_0)C(\omega)D(\omega+\omega_0)]}. \quad (40)$$

We can eliminate $C(\omega)$ from Eq. (40) by means of Eq. (38) to obtain the more convenient expression

$$Y(\omega)=M(\omega)\left(\frac{2Z_0^{-2}G(\omega)}{D(\omega+\omega_0)[1-Z_0^{-2}G(\omega)A(\omega)]}-\frac{1+Z_0^{-2}G(\omega)A(\omega)}{1-Z_0^{-2}G(\omega)A(\omega)}\right), \quad (41)$$

which agrees with that found by other methods of solution. On multiplying both sides of Eq. (41) by the factor $(\omega-\omega_0)$ and taking the limit as $\omega\rightarrow\omega_0$, we arrive at the residue $2g^2Z_0^2$ as expected from Eq. (28). Similarly, the residue at the pole $\omega=\Delta$ is calculated to be

$$\lim_{\omega\rightarrow\Delta} Y(\omega)=-\left(\frac{2g}{Z_0}\right)^2\{Z_0^2[1-Z_0^{-2}G(\Delta)A(\Delta)]+Z_0^{-2}G'(\Delta)+Z_0^{-4}G^2(\Delta)A'(\Delta)\}^{-1}. \quad (42)$$

This quantity is derived from the first term on the right-hand side of Eq. (41) by factoring out the root in $D(\omega+\omega_0)$. In Ref. 2, Eq. (54), we have shown that the inverse factor in Eq. (42) is equal to the inverse of $2Z_B^{-2}[1-Z_0^{-2}G(\Delta)A(\Delta)]^2$ and thus, in accordance with Eq. (29), it follows that the residue of $Y(\omega)$ at $\omega=\Delta$ is equal to $-\Gamma_0^2$, in agreement with Eq. (28).

Having completed the two-meson scattering matrix, we now go on to study the three-meson sector—in particular, the S-matrix elements involving the $(2V)\theta$ channel.

III. THREE-MESON SECTOR

In this section we concentrate on a dispersion formulation of three collision processes, namely, the elastic scattering of a meson by two V-particles in static interaction at zero separation, and the production of one and two mesons by these particles. The two V sources will be treated as a stable particle, called B, with energy $E_B=2m+\omega_B$. Owing to unwieldy mathematics, we shall restrict ourselves to a somewhat formal consideration and shall not attempt to express the final results in their most simplified forms.

Since our calculations proceed along the lines prescribed by Amado, it follows that we must contract the B particle in order to make progress beyond the initial meson contraction in the elastic scattering matrix element. For this purpose we introduce the product operator[14] $\psi_B=\psi_V\psi_V/Z_B\sqrt{2}$ and its corresponding current operator

$$f_B(t)=[H,\psi_B(t)]+(2m+\omega_B)\psi_B(t)=(\omega_B-2\delta m_V)\psi_B(t)-(g\sqrt{2}/ZZ_B)\psi_V(t)\psi_N(t)\sum_k X(\omega)a_k(t). \quad (43)$$

The definition of ψ_B is such that the matrix elements $\langle 0|\psi_B|B\rangle=1$ and $\langle 0|f_B|B\rangle=0$. The former property is seen at once in Eq. (64) of Ref. 1, whereas the latter follows from Eq. (43) and the bare-state expression for $|B\rangle$ given in Eq. (42) of Ref. 2. We find

$$\langle 0|f_B|B\rangle =Z^{-1}[Z(\omega_B-2\delta m_V)-g\sqrt{2}\sum_k X(\omega)\varphi_1(\omega)], \quad (44)$$

where $\varphi_1(\omega)$ is the expansion coefficient in $|B\rangle$ associated with the bare states containing all three particles V, N, and θ. The vanishing of this expression is predicted in the Tamm-Dancoff treatment by Eq. (43a) of Ref. 2.

Let us now consider the S-matrix element

$$\langle B\theta_{k'}|S|B\theta_k\rangle=\langle B\theta_{k'},\text{ out}|B\theta_k,\text{ in}\rangle =\delta_{kk'}+2\pi i\delta(\omega-\omega')X^2(\omega)\mathcal{B}(\omega), \quad (45)$$

where the transition amplitude $\mathcal{B}(\omega)$ is defined by

$$\mathcal{B}(\omega)=X^{-1}(\omega)\langle B|j|B\theta_k,\text{ in}\rangle. \quad (46)$$

Using the usual asymptotic definition of a state, we contract the B particle on the left to get

$$\mathcal{B}(\omega)=iX^{-1}(\omega)\int_{-\infty}^{\infty}e^{iE_Bt}\langle 0|[f_B(t),j]\theta(t)|B\theta_k,\text{ in}\rangle dt. \quad (47)$$

The equal-time commutator $[\psi_B,j]$ resulting from the differentiation of the step function vanishes. In the usual way, we introduce intermediate states and make time translations to obtain

$$\mathcal{B}(\omega)=\frac{\Gamma_0V_1(\omega)}{\omega-\Delta}+\frac{1}{\pi\sqrt{2}}\int_\mu^\infty\frac{\rho(\omega')P(\omega')V_2(\omega',\omega)d\omega'}{\omega'+\omega-\Delta-\omega_0}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')V_1(\omega')\mathcal{B}_1(\omega',\omega)d\omega'}{\omega'-\Delta}+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega_1)\rho(\omega_2)V_2(\omega_1,\omega_2)\mathcal{B}_2(\omega_1,\omega_2,\omega)d\omega_1d\omega_2}{\omega_1+\omega_2-\Delta-\omega_0}. \quad (48)$$

Here we have introduced the definitions

$$V_1(\omega)=X^{-1}(\omega)\langle 0|f_B|B_0\theta_k,\text{ in}\rangle, \quad (49a)$$

$$V_2(\omega_1,\omega_2)=V_2(\omega_2,\omega_1)=X^{-1}(\omega_1)X^{-1}(\omega_2)\langle 0|f_B|2N\theta_{k_1}\theta_{k_2},\text{ in}\rangle, \quad (49b)$$

$$\mathcal{B}_1(\omega_1,\omega_2)=X^{-1}(\omega_1)X^{-1}(\omega_2)[\langle B_0\theta_{k_1},\text{ in}|j|B\theta_{k_2},\text{ in}\rangle-\Gamma_0\delta_{k_1k_2}], \quad (49c)$$

$$\mathcal{B}_2(\omega_1,\omega_2,\omega_3)=\mathcal{B}_2(\omega_2,\omega_1,\omega_3)=X^{-1}(\omega_1)X^{-1}(\omega_2)X^{-1}(\omega_3)[\langle 2N\theta_{k_1}\theta_{k_2},\text{ in}|j|B\theta_{k_3},\text{ in}\rangle-(\delta_{k_1k_3}/\sqrt{2})\langle 2N\theta_{k_2},\text{ in}|j|B\rangle-(\delta_{k_2k_3}/\sqrt{2})\langle 2N\theta_{k_1},\text{ in}|j|B\rangle]. \quad (49d)$$

[14] The factor of $1/\sqrt{2}$ in the definition of ψ_B was inadvertently omitted in the concluding section of Ref. 1.

The matrix element $\langle 2N\theta_k,\text{in}|j|B\rangle$ is defined in Ref. 1 as $X(\omega)P(\omega)$. There it was found that $P(\omega)$ satisfies an inhomogeneous integral equation, with the same kernel as in Eq. (23), having the solution

$$P(\omega)=g\Gamma_0\sqrt{2}G(\Delta)/Z_0G^*(\omega)G(\Delta+\omega_0-\omega). \tag{50}$$

Thus the amplitude $\mathcal{B}_2$ has disconnected parts to be removed in its definition, whereas the corresponding function $A_{24}{}^3$ of Ref. 5 has no such terms.

The vertex functions $V_1(\omega)$ and $V_2(\omega_1,\omega_2)$ can be calculated at once by appealing to the bare-state expansions for $|B_0\theta_k,\text{ in}\rangle$ and $|2N\theta_{k_1}\theta_{k_2},\text{ in}\rangle$, which are given in Ref. 2. In this way we obtain

$$V_1(\omega)=2\sqrt{2}g(\Delta-\omega)/Z_0Z_BD(\omega+\omega_0), \tag{51}$$

and

$$V_2(\omega_1,\omega_2)=\frac{2\sqrt{2}g^2(\Delta-\omega_1-\omega_2+\omega_0)G(\omega_1+\omega_2-\omega_0)}{Z_0{}^2Z_BG(\omega_1)D(\omega_1+\omega_2)G(\omega_2)}. \tag{52}$$

It is also of interest to consider a pure dispersion-theoretic derivation of these functions as this leads to the problem of diagonalizing the connected scattering matrix of the two-meson sector. In fact, we shall need this matrix later on. At the same time, this approach provides a check on Eqs. (51) and (52). Application of the contraction technique to mesons in V_1 and V_2 yields the following pair of coupled singular integral equations:

$$V_1(\omega)=-\frac{gZ_0\sqrt{2}}{ZZ_B}+\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega')[Y^*(\omega')-M^*(\omega')]}{\omega'-\omega-i\epsilon}V_1(\omega')d\omega' +\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega_1)\rho(\omega_2)R^*(\omega_1,\omega_2)V_2(\omega_1,\omega_2)d\omega_1d\omega_2}{\omega_1+\omega_2-\omega-\omega_0-i\epsilon}+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\eta(\omega')}\sin\eta(\omega')V_1(\omega')d\omega'}{\omega'-\omega-i\epsilon} \tag{53}$$

and

$$V_2(\omega_1,\omega_2)=\frac{M(\omega_1)}{ZZ_B\sqrt{2}}+\frac{gZ_0V_1(\omega_1)}{\omega_2-\omega_0}+\frac{\Omega\sqrt{2}}{\pi}\int_\mu^\infty\frac{\rho(\omega')N_0{}^*(\omega_1,\omega')V_1(\omega')d\omega'}{\omega'+\omega_0-\omega_1-\omega_2-i\epsilon} +\frac{1}{\pi^2\sqrt{2}}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega')\rho(\omega'')\mathcal{A}_0{}^*(\omega_1,\omega',\omega'')V_2(\omega',\omega'')d\omega'd\omega''}{\omega'+\omega''-\omega_1-\omega_2-i\epsilon}+\frac{1}{\pi}\int_\mu^\infty\frac{e^{-i\eta(\omega')}\sin\eta(\omega')V_2(\omega_1,\omega')d\omega'}{\omega'-\omega_2-i\epsilon}. \tag{54}$$

The first of these equations has been modified by the addition and subtraction of the last term on the right-hand side. This term and the last one on the right-hand side of the second equation can be removed by treating the remaining terms in each equation as some unknown function. Standard methods then give

$$V_1(\omega)=-\frac{gZ_0\sqrt{2}}{Z_B\alpha(\omega)}+\frac{1}{\pi\alpha(\omega)}\int_\mu^\infty\frac{\rho(\omega')[Y^*(\omega')-M^*(\omega')]V_1(\omega')\alpha^*(\omega')d\omega'}{\omega'-\omega-i\epsilon} +\frac{1}{\pi^2\alpha(\omega)}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega')\rho(\omega'')R^*(\omega',\omega'')V_2(\omega',\omega'')\alpha^*(\omega'+\omega''-\omega_0)d\omega'd\omega''}{\omega'+\omega''-\omega-\omega_0-i\epsilon} \tag{55}$$

and

$$V_2(\omega_1,\omega_2)=\frac{M(\omega_1)}{Z_B\alpha(\omega_2)\sqrt{2}}+\frac{gV_1(\omega_1)}{Z_0G(\omega_2)}+\frac{\Omega\sqrt{2}}{\pi\alpha(\omega_2)}\int_\mu^\infty\frac{\rho(\omega')N_0{}^*(\omega_1,\omega')V_1(\omega')\alpha^*(\omega'+\omega_0-\omega_1)d\omega'}{\omega'+\omega_0-\omega_1-\omega_2-i\epsilon} +\frac{1}{\pi^2\sqrt{2}\alpha(\omega_2)}\int_\mu^\infty\int_\mu^\infty\frac{\rho(\omega')\rho(\omega'')\mathcal{A}_0{}^*(\omega_1,\omega',\omega'')V_2(\omega',\omega'')\alpha^*(\omega'+\omega''-\omega_1)d\omega'd\omega''}{\omega'+\omega''-\omega_1-\omega_2-i\epsilon}. \tag{56}$$

On substituting Eq. (55) into the second term on the right-hand side of Eq. (56), we are led to the relation

$$V_2(\omega_1,\omega_2)=\frac{gG(\omega_1+\omega_2-\omega_0)V_1(\omega_1+\omega_2-\omega_0)}{Z_0G(\omega_1)G(\omega_2)}, \tag{57}$$

which shows that V_1 and V_2 are not independent functions. Of course, this result is already present in Eqs. (51) and (52), and reduces the problem to finding one unknown function of a single variable. We let that function be

$$V(\omega)=\alpha(\omega)V_1(\omega), \tag{58}$$

and rewrite Eq. (55) as

$$V(\omega)=-\frac{gZ_0\sqrt{2}}{Z_B}+\frac{1}{\pi}\int_{\mu}^{\infty}\frac{\rho(\omega')[Y^*(\omega')-M^*(\omega')]e^{2i\eta(\omega')}V(\omega')d\omega'}{\omega'-\omega-i\epsilon}+\frac{1}{\pi}\int_{2\mu-\omega_0}^{\infty}\frac{d\omega'[G^*(\omega')]^2[Y^*(\omega')-M^*(\omega')]E(\omega')V(\omega')}{\omega'-\omega-i\epsilon}. \quad (59)$$

It will now be shown that Eq. (59) is actually an Omnès-type singular integral equation. To achieve this, we must diagonalize the connected S matrix S_C of the two-meson sector. This matrix has the factorized form $S_C=S_DSS_D$, where S_D is the diagonalized, unitary, disconnected S matrix, defined in terms of its matrix elements

$$\langle B_0\omega'|S_D|B_0\omega\rangle=\delta(\omega-\omega'),\quad \langle 2N\omega_1\omega_2|S_D|B_0\omega\rangle=0,$$
$$\langle 2N\omega_1\omega_2|S_D|2N\omega_3\omega_4\rangle=\tfrac{1}{2}e^{-i\eta(\omega_3)-i\eta(\omega_4)} \times[\delta(\omega_1-\omega_3)\delta(\omega_2-\omega_4)+\delta(\omega_1-\omega_4)\delta(\omega_2-\omega_3)], \quad (60)$$

where the states $|B_0\omega\rangle$ and $|2N\omega_1\omega_2\rangle$ have the representation

$$|B_0\omega\rangle=\left(\frac{2\pi^2}{k\omega\Omega}\right)^{1/2}\sum_{k'}\delta(\omega-\omega')|B_0\theta_{k'}\rangle,$$
$$|2N\omega_1\omega_2\rangle=\left(\frac{2\pi^2}{k_1\omega_1\Omega}\frac{2\pi^2}{k_2\omega_2\Omega}\right)^{1/2}\times\sum_k\sum_{k'}\delta(\omega_1-\omega)\delta(\omega_2-\omega')|2N\theta_k\theta_{k'}\rangle; \quad (61)$$

these states are normalized to δ functions of energy. The problem of finding the eigenvalues of S_C requires its matrix elements with respect to the states in Eq. (61). When written out, these elements express a factorization property which enables us to show that the eigenvalue equation

$$S_C|\omega,\lambda\rangle=\lambda|\omega,\lambda\rangle, \quad (62)$$

with energy ω, and with $|\omega,\lambda\rangle$ given by the form

$$|\omega,\lambda\rangle=C_1(\omega,\lambda)|B_0\omega\rangle+\int_{\mu}^{\omega+\omega_0-\mu}C_2(\omega',\omega,\lambda)|2N\omega',\omega+\omega_0-\omega'\rangle d\omega', \quad (63)$$

has precisely two eigenvalues, say, λ_1 and λ_2, distinct from unity, these being the roots of

$$\lambda^2-2\lambda\{1+i\rho(\omega)Y(\omega)+iG^2(\omega)[Y(\omega)-M(\omega)]E(\omega)\}+2iG^2(\omega)[Y(\omega)-M(\omega)]\times e^{2i\eta(\omega)}E(\omega)+2i\rho(\omega)Y(\omega)+1=0. \quad (64)$$

Noting that the product of these roots is equal to the λ-independent terms in Eq. (64), while also equaling the determinant of S_C, we conclude, after some minor manipulations, that

$$\det S_C=\lambda_1\lambda_2=\left(\frac{Y(\omega)}{M(\omega)}-1\right)\left(\frac{Y^*(\omega)}{M^*(\omega)}-1\right)^{-1}=\frac{D^*(\omega+\omega_0)}{D(\omega+\omega_0)}. \quad (65)$$

It is convenient to define $\lambda_i=e^{2i\theta_i}$ so that

$$(\lambda_1\lambda_2e^{-2i\eta}-1)/2i=e^{i(\theta_1+\theta_2-\eta)}\sin(\theta_1+\theta_2-\eta)=[Y(\omega)-M(\omega)][\rho(\omega)e^{-2i\eta}+G^2(\omega)E(\omega)]. \quad (66)$$

This result proves that $V(\omega)$ satisfied the Omnès equation

$$V(\omega)=-\frac{gZ_0\sqrt{2}}{Z_B}+\frac{1}{\pi}\int_{\mu}^{\infty}e^{-i[\theta_1(\omega')+\theta_2(\omega')-\eta(\omega')]}\times\sin[\theta_1(\omega')+\theta_2(\omega')-\eta(\omega')]\times V(\omega')d\omega'/(\omega'-\omega-i\epsilon). \quad (67)$$

Since the integral in Eq. (67) vanishes at the high-energy limit, it is routine to obtain the solution

$$V(\omega)=-\frac{gZ_0\sqrt{2}}{Z_B}\exp\left(\frac{1}{\pi}\int_{\mu}^{\infty}\frac{[\theta_1(\omega')+\theta_2(\omega')]d\omega'}{\omega'-\omega-i\epsilon}-\frac{1}{\pi}\int_{\mu}^{\infty}\frac{\eta(\omega')d\omega'}{\omega'-\omega-i\epsilon}\right). \quad (68)$$

In view of Eq. (65), and the relation $\ln\lambda_1\lambda_2=2i(\theta_1+\theta_2)$, we can replace $\theta_1+\theta_2$ in Eq. (68) by the quantity

$$-\frac{1}{2i}\ln\left(\frac{D(\omega'+\omega_0)}{\omega'-\Delta}\Big/\frac{D^*(\omega'+\omega_0)}{\omega'-\Delta}\right).$$

We then write the resulting integral in the complex ω' plane over a contour which runs from ∞ to μ infinitesimally below the real axis, encircling the point μ, and then back to ∞ infinitesimally above the real axis. When $\omega\rightarrow\infty$, we find that $D(\omega+\omega_0)/(\omega-\Delta)$ approaches $2ZZ_0^{-2}$. The function $G(\omega)/(\omega-\omega_0)$ plays a similar role in the second integral in Eq. (68) and has the asymptotic value Z. Thus by the residue theorem, we find

$$V(\omega)=2\sqrt{2}g(\Delta-\omega)G(\omega)/Z_0Z_B(\omega-\omega_0)D(\omega+\omega_0). \quad (69)$$

It remains to combine Eqs. (58) and (69) to obtain $V_1(\omega)$ as in Eq. (51). Finally, by Eq. (57) we confirm the expression given for V_2 in Eq. (52).

We now proceed to discuss the associated amplitudes $\mathcal{B}_1$ and $\mathcal{B}_2$. This part of the theory embodies the dynamical content of the third sector. The coupled integral equations satisfied by these amplitudes contain terms, due to the known function P, for which there are no counterparts in the corresponding equations of Ref. 5. Contracting the meson with energy ω_1 in $\mathcal{B}_1$, and the meson with energy ω_2 in $\langle 2N\theta_{k_1}\theta_{k_2}, \text{in}|j|B\theta_{k_3}, \text{in}\rangle$, we obtain

$$\begin{aligned}\mathcal{B}_1(\omega_1,\omega_2) &= \Gamma_0[Y(\omega_2)-\mathcal{B}(\omega_2)]\left(\frac{1}{\omega_2-\omega_1+i\epsilon}+\frac{1}{\omega_1-\Delta}\right)\\ &+\frac{\sqrt{2}}{\pi}\int_\mu^\infty \rho(\omega)P(\omega)R(\omega,\omega_2)\left(\frac{1}{\omega+\omega_2-\omega_0-\omega_1+i\epsilon}+\frac{1}{\omega+\omega_1-\omega_B}\right)d\omega\\ &+\frac{1}{\pi}\int_\mu^\infty \rho(\omega)[Y(\omega)-M(\omega)]\mathcal{B}_1(\omega,\omega_2)\left(\frac{1}{\omega-\omega_1+i\epsilon}+\frac{1}{\omega+\omega_1-\omega_2-\Delta-i\epsilon}\right)d\omega\\ &+\frac{1}{\pi^2}\int_\mu^\infty\int_\mu^\infty \rho(\omega)\rho(\omega')R(\omega,\omega')\mathcal{B}_2(\omega,\omega',\omega_2)\left(\frac{1}{\omega+\omega'-\omega_1-\omega_0+i\epsilon}+\frac{1}{\omega+\omega'+\omega_1-\omega_2-\omega_B-i\epsilon}\right)d\omega d\omega'\\ &+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega)}\sin\eta(\omega)\mathcal{B}_1(\omega,\omega_2)\left(\frac{1}{\omega-\omega_1+i\epsilon}+\frac{1}{\omega+\omega_1-\omega_2-\Delta-i\epsilon}\right)d\omega \end{aligned}\tag{70}$$

and

$$\begin{aligned}\mathcal{B}_2(\omega_1,\omega_2,\omega_3) &= \frac{1}{\sqrt{2}}P(\omega_1)[M(\omega_3)-\mathcal{B}(\omega_3)]\left(\frac{1}{\omega_3-\omega_2+i\epsilon}+\frac{1}{\omega_1+\omega_2-\omega_B}\right)\\ &+\frac{1}{\pi}\int_\mu^\infty \rho(\omega)P(\omega)\,\mathcal{A}_0(\omega_1,\omega,\omega_3)\left(\frac{1}{\omega+\omega_3-\omega_1-\omega_2+i\epsilon}+\frac{1}{\omega+\omega_2-\omega_B}\right)d\omega\\ &+\Omega\sqrt{2}\Gamma_0N_0(\omega_1,\omega_3)\left(\frac{1}{\omega_3+\omega_0-\omega_1-\omega_2+i\epsilon}+\frac{1}{\omega_2-\Delta}\right)+gZ_0\mathcal{B}_1(\omega_1,\omega_3)\left(\frac{1}{\omega_3+\Delta-\omega_1-\omega_2+i\epsilon}+\frac{1}{\omega_2-\omega_0}\right)\\ &+\frac{\Omega\sqrt{2}}{\pi}\int_\mu^\infty \rho(\omega)N_0(\omega_1,\omega)\mathcal{B}_1(\omega,\omega_3)\left(\frac{1}{\omega+\omega_0-\omega_1-\omega_2+i\epsilon}+\frac{1}{\omega+\omega_2-\omega_3-\Delta-i\epsilon}\right)d\omega\\ &+\frac{1}{\pi^2\sqrt{2}}\int_\mu^\infty\int_\mu^\infty \rho(\omega)\rho(\omega')\,\mathcal{A}_0(\omega_1,\omega,\omega')\mathcal{B}_2(\omega,\omega',\omega_3)\left(\frac{1}{\omega+\omega'-\omega_1-\omega_2+i\epsilon}+\frac{1}{\omega+\omega'+\omega_2-\omega_3-\omega_B-i\epsilon}\right)d\omega d\omega'\\ &+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega)}\sin\eta(\omega)\,\mathcal{B}_2(\omega_1,\omega,\omega_3)\left(\frac{1}{\omega-\omega_2+i\epsilon}+\frac{1}{\omega+\omega_2+\omega_1-\omega_3-\omega_B-i\epsilon}\right)d\omega. \end{aligned}\tag{71}$$

As before, we have added and subtracted the last term on the right-hand side of Eq. (70). This term and the last one in Eq. (71) are to be eliminated by treating the remaining terms as in the case of the vertex functions. In choosing to contract mesons from the left as indicated above, and not from the right, we have avoided introducing intermediate states containing five particles. It follows straightforwardly that

$$\begin{aligned}\mathcal{B}_1(\omega_1,\omega_2) &= [\alpha^*(\omega_1)\alpha(\omega_2-\omega_1+\Delta)]^{-1}\\ &\times\left(\frac{\Gamma_0\alpha(\Delta)\alpha(\omega_2)[Y(\omega_2)-\mathcal{B}(\omega_2)](\omega_2-\Delta)}{(\omega_1-\Delta)(\omega_2-\omega_1+i\epsilon)}+\psi(\omega_1-i\epsilon,\,\omega_2)+\psi(\omega_2-\omega_1+\Delta+i\epsilon,\,\omega_2)\right)\end{aligned}\tag{72}$$

and

$$\mathcal{B}_2(\omega_1,\omega_2,\omega_3)=(g/Z_0)[\alpha^*(\omega_1)\alpha(\omega_3+\omega_B-\omega_1-\omega_2)\alpha^*(\omega_2)]^{-1}\Bigg[\frac{(\omega_1+\omega_2-2\omega_0)}{(\omega_1-\omega_0)(\omega_2-\omega_0)}\psi(\omega_1+\omega_2-\omega_0-i\epsilon,\,\omega_3)$$

$$+\frac{(\omega_3+\Delta-\omega_2-\omega_0)}{(\omega_1-\omega_0)(\omega_3+\Delta-\omega_1-\omega_2+i\epsilon)}\psi(\omega_3+\Delta-\omega_2+i\epsilon,\,\omega_3)+\frac{(\omega_3+\Delta-\omega_1-\omega_0)}{(\omega_2-\omega_0)(\omega_3+\Delta-\omega_1-\omega_2+i\epsilon)}\psi(\omega_3+\Delta-\omega_1+i\epsilon,\,\omega_3)$$

$$+\tfrac{1}{2}\Gamma_0\alpha(\Delta)\alpha(\omega_3)\Bigg(\frac{[M(\omega_3)-\mathcal{B}(\omega_3)](\Delta-\omega_0)(\omega_1+\omega_3-\omega_B)}{(\omega_1-\omega_0)(\Delta-\omega_1)(\omega_3-\omega_2+i\epsilon)(\omega_1+\omega_2-\omega_B)}+\frac{[Y(\omega_3)-M(\omega_3)](\omega_3-\omega_0)(\omega_3+\omega_0-\omega_1-\Delta)}{(\omega_1-\omega_0)(\omega_2-\Delta)(\omega_3-\omega_1+i\epsilon)(\omega_3+\omega_0-\omega_1-\omega_2+i\epsilon)}$$

$$+\frac{[Y(\omega_3)-\mathcal{B}(\omega_3)](\omega_3-\Delta)(\omega_3+\Delta-\omega_1-\omega_0)}{(\omega_2-\omega_0)(\omega_1-\Delta)(\omega_3-\omega_1+i\epsilon)(\omega_3+\Delta-\omega_1-\omega_2+i\epsilon)}+(1\rightleftarrows 2)\Bigg)\Bigg].\tag{73}$$

The Bose symmetry of $\mathcal{B}_2(\omega_1,\omega_2,\omega_3)$ under interchange of ω_1 and ω_2 accounts for the factor of $\frac{1}{2}$ and the presence of the terms denoted by $(1\rightleftarrows 2)$ in Eq. (73). The ψ function appearing in these expressions is defined by

$$\psi(\omega_1,\omega_2)=\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega)[Y(\omega)-M(\omega)]\mathcal{B}_1(\omega,\omega_2)\alpha(\omega)\alpha(\omega_2-\omega+\Delta)d\omega}{\omega-\omega_1}$$

$$+\frac{g\sqrt{2}}{\pi Z_0G(\omega_2)}\int_{\mu+\omega_2-\omega_0}^\infty\frac{\rho(\omega+\omega_0-\omega_2)[Y(\omega)-M(\omega)]P(\omega+\omega_0-\omega_2)G(\omega)\alpha(\omega)\alpha(\omega_2-\omega+\Delta)d\omega}{(\omega-\omega_1)G(\omega+\omega_0-\omega_2)}$$

$$+\frac{g}{\pi^2Z_0}\int_{2\mu-\omega_0}^\infty d\omega\frac{[Y(\omega)-M(\omega)]G(\omega)\alpha(\omega)\alpha(\omega_2-\omega+\Delta)}{\omega-\omega_1}\int_\mu^{\omega-\mu+\omega_0}\frac{d\omega'\rho(\omega')\rho(\omega-\omega'+\omega_0)\mathcal{B}_2(\omega-\omega'+\omega_0,\,\omega',\,\omega_2)}{G(\omega')G(\omega-\omega'+\omega_0)}.\tag{74}$$

Here we observe that Eqs. (70)–(74) come into formal alignment with the corresponding set in Ref. 5 when we arbitrarily set $\Delta=\omega_0$. Under this condition, P satisfies the same integral equation as before, but with no inhomogeneous term and, consequently, no homogeneous solutions. Indeed, Eq. (50) shows that P would vanish since $G(\omega_0)=0$. Of course, this situation is not obtained here, but an analogous one does arise for the equation satisfied by the disconnected parts of $A_{24}{}^3$ in Ref. 5.

Clearly, at this point the same procedure as in Ref. 5 yields a singular integral equation for $\psi(\omega_1,\omega_2)$ in its first variable. Inspection of this equation then shows that it may be reformulated as

$$\psi(\omega_1,\omega_2)=\frac{g\sqrt{2}}{\pi Z_0G(\omega_2)}\int_{\mu+\omega_2-\omega_0}^\infty\frac{\rho(\omega+\omega_0-\omega_2)[Y(\omega)-M(\omega)]P(\omega+\omega_0-\omega_2)G^2(\omega)}{(\omega-\omega_0)G(\omega+\omega_0-\omega_2)}\alpha(\omega_2+\Delta-\omega)\Phi(\omega_1,\omega,\omega_2)d\omega$$

$$+\Gamma_0\alpha(\Delta)\alpha(\omega_2)\Bigg([Y(\omega_2)-\mathcal{B}(\omega_2)](\omega_2-\Delta)\frac{1}{\pi}\int_\mu^\infty\frac{\rho(\omega)[Y(\omega)-M(\omega)]e^{-2i\eta(\omega)}\Phi(\omega_1,\omega,\omega_2)d\omega}{(\omega-\Delta)(\omega_2-\omega+i\epsilon)}$$

$$+[Y(\omega_2)-\mathcal{B}(\omega_2)](\omega_2-\Delta)\frac{1}{\pi}\int_{2\mu-\omega_0}^\infty\frac{[Y(\omega)-M(\omega)]G^2(\omega)I_1(\omega,\omega_2)\Phi(\omega_1,\omega,\omega_2)d\omega}{(\omega-\omega_0)(\omega_2+\Delta-\omega-\omega_0+i\epsilon)}$$

$$+[M(\omega_2)-\mathcal{B}(\omega_2)]\frac{(\Delta-\omega_0)}{\pi}\int_{2\mu-\omega_0}^\infty\frac{[Y(\omega)-M(\omega)]G^2(\omega)I_2(\omega,\omega_2)\Phi(\omega_1,\omega,\omega_2)d\omega}{(\omega-\omega_0)(\omega-\Delta)}$$

$$+[Y(\omega_2)-M(\omega_2)]\frac{(\omega_2-\omega_0)}{\pi}\int_{2\mu-\omega_0}^\infty\frac{[Y(\omega)-M(\omega)]G^2(\omega)I_3(\omega,\omega_2)\Phi(\omega_1,\omega,\omega_2)d\omega}{(\omega-\omega_0)(\omega_2-\omega+i\epsilon)}\Bigg),\tag{75}$$

where the integrals $I_n(\omega,\omega_2)$ are defined by

$$I_1(\omega,\omega_2)=\frac{g^2}{\pi Z_0{}^2}\int_\mu^{\omega-\mu+\omega_0}\frac{\rho(\omega')\rho(\omega-\omega'+\omega_0)(\omega_2+\omega'+\Delta-\omega-2\omega_0)(\omega-\omega')d\omega'}{|G(\omega')|^2|G(\omega-\omega'+\omega_0)|^2(\omega_2+\omega'-\omega-\omega_0+i\epsilon)(\omega+\omega_0-\omega'-\Delta-i\epsilon)},$$
$$I_2(\omega,\omega_2)=\frac{g^2}{\pi Z_0{}^2}\int_\mu^{\omega-\mu+\omega_0}\frac{\rho(\omega')\rho(\omega-\omega'+\omega_0)(\omega'+\omega_2-\omega_B)(\omega-\omega')d\omega'}{|G(\omega')|^2|G(\omega-\omega'+\omega_0)|^2(\omega_2+\omega'-\omega-\omega_0+i\epsilon)(\Delta-\omega')},\qquad(76)$$
$$I_3(\omega,\omega_2)=\frac{g^2}{\pi Z_0{}^2}\int_\mu^{\omega-\mu+\omega_0}\frac{\rho(\omega')\rho(\omega-\omega'+\omega_0)(\omega_2+\omega_0-\omega'-\Delta)(\omega-\omega')d\omega'}{|G(\omega')|^2|G(\omega-\omega'+\omega_0)|^2(\omega_2-\omega'+i\epsilon)(\omega+\omega_0-\omega'-\Delta-i\epsilon)},$$

and the function $\Phi(\omega_1,\omega,\omega_2)$ satisfies the integral equation

$$\Phi(\omega_1,\omega,\omega_2)=\frac{1}{\omega-\omega_1}+\frac{1}{\pi}\int_\mu^\infty e^{i[\theta_1(\omega')+\theta_2(\omega')-\eta(\omega')]}\sin[\theta_1(\omega')+\theta_2(\omega')-\eta(\omega')]\frac{\Phi(\omega'-i\epsilon,\,\omega,\,\omega_2)d\omega'}{\omega'-\omega_1}$$
$$+\frac{1}{\pi}\int_\mu^\infty \rho(\omega')[Y(\omega')-M(\omega')]\frac{e^{-2i\eta(\omega')}}{\omega'-\omega_1}\Phi(\omega_2+\Delta-\omega'+i\epsilon,\,\omega,\,\omega_2)d\omega'$$
$$+2\left(\frac{g}{\pi Z_0}\right)^2\int_{2\mu-\omega_0}^\infty\frac{d\omega'[Y(\omega')-M(\omega')]G^2(\omega')}{(\omega'-\omega_1)(\omega'-\omega_0)}\int_\mu^{\omega'-\mu+\omega_0}\frac{d\omega''\rho(\omega'')\rho(\omega'-\omega''+\omega_0)}{|G(\omega'')|^2|G(\omega'-\omega''+\omega_0)|^2}$$
$$\times(\omega'-\omega'')(\omega''-\omega_0)\left(\frac{1}{\omega'-\omega''-i\epsilon}+\frac{1}{\omega_2+\Delta-\omega'-\omega_0+i\epsilon}\right)\Phi(\omega_2+\Delta-\omega''+i\epsilon,\,\omega,\,\omega_2).\qquad(77)$$

Again, if we arbitrarily set $\Delta=\omega_0$, the integrals $I_1(\omega,\omega_2)$ and $I_3(\omega,\omega_2)$ reduce to $E(\omega)$, while the terms in Eq. (75) proportional to $\Delta-\omega_0$, or containing P, vanish. As before, this reduction recreates the mathematical situation found in Ref. 5.

If the singular integral operator in Eq. (77) is eliminated by a now familiar procedure, the ensuing result shows that the dynamics in the channel under consideration reduces to the solution of a Fredholm integral equation for Φ in its first variable. When Φ is known, it then follows from Eqs. (72), (73), and (75) that $\mathcal{B}(\omega)$ is determined algebraically by Eq. (48). As can readily be seen, the expression thus obtained for this amplitude is indeed a very complicated one and no attempt is made here at manipulating it into a final simplified form. If at this point the considerations of Ref. 5 continue to serve as a guide, it may be possible to express $\mathcal{B}(\omega)$ in terms of integrals over the solution of the fundamental equation. However, this may not be practical in the present case because of the mathematical incumberances.

In the remainder of this section we display the S-matrix elements with $B\theta$ on one side. These quantities are formulated in terms of the associated amplitudes $\mathcal{B}_1$ and $\mathcal{B}_2$ which, as we have shown, are determined by the fundamental function Φ. The method of approach follows that used in Ref. 1 in analyzing the production element described by Eq. (16). We begin with

$$S_1\equiv\langle B_0\theta_{k_3}\theta_{k_2},\text{ out}|B\theta_{k_1},\text{ in}\rangle,\qquad(78)$$

and contract a meson from the left. This yields the expression

$$S_1=(2\pi i/\sqrt{2})\delta(\omega_3+\omega_2-\omega_1-\Delta)X(\omega_3)\times X(\omega_2)X(\omega_1)\mathcal{P}_1(\omega_2,\omega_1),\qquad(79)$$

where the factor of $1/\sqrt{2}$ comes from the identity of the mesons, while the amplitude $\mathcal{P}_1$ is defined by

$$\mathcal{P}_1(\omega_2,\omega_1)=X^{-1}(\omega_2)X^{-1}(\omega_1)\langle B_0\theta_{k_2},\text{ out}|j|B\theta_{k_1},\text{ in}\rangle.\qquad(80)$$

Introducing a complete set of intermediate in-states into Eq. (80), and noting that the scalar product $\langle B_0\theta_k,\text{ out}|B\rangle$ vanishes, we find

$$\mathcal{P}_1(\omega_2,\omega_1)=X^{-1}(\omega_2)X^{-1}(\omega_1)\sum_k\langle B_0\theta_{k_2},\text{ out}|B_0\theta_k,\text{ in}\rangle\langle B_0\theta_k,\text{ in}|j|B\theta_{k_1},\text{ in}\rangle$$
$$+X^{-1}(\omega_2)X^{-1}(\omega_1)\sum_k\sum_{k'}\langle B_0\theta_{k_2},\text{ out}|2N\theta_k\theta_{k'},\text{ in}\rangle\langle 2N\theta_k\theta_{k'},\text{ in}|j|B\theta_{k_1},\text{ in}\rangle.\qquad(81)$$

The first and second matrix elements in the single sum, and the second matrix element in the double sum of Eq. (81), are defined by Eqs. (13), (49c), and (49d), respectively. In addition, a simple calculation involving the contraction of the out-state meson in the remaining matrix element of Eq. (81) shows that it can be replaced by $2\pi i\delta(\omega_2+\omega_0-\omega-\omega')X(\omega)X(\omega')R(\omega,\omega')$. Thus, on substituting these expressions into Eq. (81) and making use of

Kronecker and Dirac δ's to evaluate some sums and integrals, we obtain

$$\mathcal{P}_1(\omega_2,\omega_1)=\Gamma_0X^{-1}(\omega_2)X^{-1}(\omega_1)\langle B_0\theta_{k_2},\text{ out}|B_0\theta_{k_1},\text{ in}\rangle+\exp[2i\delta^{B_0\theta}(\omega_2)]\mathcal{B}_1(\omega_2,\omega_1)+2i\sqrt{2}\rho(\omega_2+\omega_0-\omega_1)P(\omega_2+\omega_0-\omega_1)R$$
$$\times(\omega_2+\omega_0-\omega_1,\,\omega_1)+\frac{2i}{\pi}\int_\mu^\infty d\omega\,\rho(\omega)\rho(\omega_2+\omega_0-\omega)R(\omega,\,\omega_2+\omega_0-\omega_1)\mathcal{B}_2(\omega,\,\omega_2+\omega_0-\omega_1,\,\omega_1).\quad(82)$$

The phase shift $\delta^{B_0\theta}(\omega)$ for $B_0\theta$ elastic scattering is defined by

$$\exp[i\delta^{B_0\theta}(\omega)]\,\sin\delta^{B_0\theta}(\omega)=\rho(\omega)Y(\omega).\quad(83)$$

Finally, on combining Eqs. (79) and (82), and keeping in mind the energy conservation imposed by S_1, we have

$$S_1=(2\pi i/\sqrt{2})\delta(\omega_3+\omega_2-\omega_1-\Delta)X(\omega_3)X(\omega_2)X(\omega_1)\Big(\exp[2i\delta^{B_0\theta}(\omega_2)]\mathcal{B}_1(\omega_2,\omega_1)+2i\sqrt{2}\rho(\omega_B-\omega_3)P(\omega_B-\omega_3)R(\omega_B-\omega_3,\,\omega_1)$$
$$+\frac{2i}{\pi}\int_\mu^\infty d\omega\,\rho(\omega)\rho(\omega_2+\omega_0-\omega)R(\omega,\,\omega_2+\omega_0-\omega)\mathcal{B}_2(\omega,\,\omega_2+\omega_0-\omega,\,\omega_1)\Big).\quad(84)$$

The form of the first term on the right-hand side of Eq. (84) is reminiscent of the production amplitudes found in the second sector [see Eq. (21)] and in the $V\theta$ sector [see Eq. (58) of Ref. 6]. In these instances there is a contribution to the amplitude coming from intermediate scattering states involving one meson. As shown in Eq. (81), the present example must also account for intermediate scattering states with two mesons leading to the new terms in Eq. (84).

The production of two mesons in the $B\theta$ channel is indicated by the S-matrix element

$$S_2=\langle 2N\theta_{k_4}\theta_{k_3}\theta_{k_2},\text{ out}|B\theta_{k_1},\text{ in}\rangle.\quad(85)$$

The contraction of an out-state meson, say, θ_{k_4}, leads to the analog of Eq. (79), namely,

$$S_2=(2\pi i/\sqrt{3})\delta(\omega_4+\omega_3+\omega_2-\omega_1-\omega_B)X(\omega_4)X(\omega_3)X(\omega_2)X(\omega_1)\mathcal{P}_2(\omega_3,\omega_2,\omega_1),\quad(86)$$

where the production amplitude $\mathcal{P}_2$ is defined by

$$\mathcal{P}_2(\omega_3,\omega_2,\omega_1)=X^{-1}(\omega_3)X^{-1}(\omega_2)X^{-1}(\omega_1)\langle 2N\theta_{k_3}\theta_{k_2},\text{ out}|j|B\theta_{k_1},\text{ in}\rangle.\quad(87)$$

If a complete set of intermediate states is inserted in Eq. (87), we obtain the expansion

$$\mathcal{P}_2(\omega_3,\omega_2,\omega_1)=X^{-1}(\omega_3)X^{-1}(\omega_2)X^{-1}(\omega_1)\sum_k\langle 2N\theta_{k_3}\theta_{k_2},\text{ out}|B_0\theta_k,\text{ in}\rangle\langle B_0\theta_k,\text{ in}|j|B\theta_{k_1},\text{ in}\rangle$$
$$+X^{-1}(\omega_3)X^{-1}(\omega_2)X^{-1}(\omega_1)\sum_k\sum_{k'}\langle 2N\theta_{k_3}\theta_{k_2},\text{ out}|2N\theta_k\theta_{k'},\text{ in}\rangle\langle 2N\theta_k\theta_{k'},\text{ in}|j|B\theta_{k_1},\text{ in}\rangle.\quad(88)$$

Clearly, the matrix elements with $B\theta$ on one side are common to both Eqs. (88) and (81). The single and double sums in Eq. (88) also contain the two-meson sector S-matrix elements for production and four-particle elastic scattering, respectively. Expressions for all of these matrix elements are provided by Eqs. (15), (16), (49c), and (49d). After introducing these into Eq. (88) and simplifying, we find that $\mathcal{P}_2$ becomes

$$\mathcal{P}_2(\omega_3,\omega_2,\omega_1)=2\sqrt{2}\pi i\Omega\Gamma_0\delta(\omega_1+\omega_0-\omega_2-\omega_3)\mathcal{P}(\omega_2,\omega_1)$$
$$+2\sqrt{2}i\rho(\omega_2+\omega_3-\omega_0)\mathcal{P}(\omega_2,\,\omega_2+\omega_3-\omega_0)\mathcal{B}_1(\omega_2+\omega_3-\omega_0,\,\omega_1)+e^{2i[\eta(\omega_3)+\eta(\omega_2)]}\mathcal{B}_2(\omega_3,\omega_2,\omega_1)$$
$$+(\langle 2N\theta_{k_3},\text{ out}|2N\theta_{k_1},\text{ in}\rangle/\sqrt{2})X^{-1}(\omega_1)X^{-1}(\omega_3)P(\omega_2)e^{2i\eta(\omega_2)}$$
$$+(\langle 2N\theta_{k_2},\text{ out}|2N\theta_{k_1},\text{ in}\rangle/\sqrt{2})X^{-1}(\omega_1)X^{-1}(\omega_2)P(\omega_3)e^{2i\eta(\omega_3)}+2i\rho(\omega_3+\omega_2-\omega_1)P(\omega_3+\omega_2-\omega_1)e^{2i\eta(\omega_2)}\mathcal{A}_0$$
$$\times(\omega_2,\,\omega_1,\,\omega_3+\omega_3-\omega_1)+\frac{i\sqrt{2}}{\pi}e^{2i\eta(\omega_2)}\int_\mu^\infty\rho(\omega)\rho(\omega_3+\omega_2-\omega)\mathcal{A}_0(\omega_2,\,\omega,\,\omega_3+\omega_2-\omega)\mathcal{B}_2(\omega,\,\omega_3+\omega_2-\omega,\,\omega_1)d\omega.\quad(89)$$

It remains to combine this result with Eq. (86). In so doing we note that energy conservation rules out the first, fourth, and fifth terms on the right-hand side of Eq. (89). Finally, we obtain

$$S_2=(2\pi i/\sqrt{3})\delta(\omega_4+\omega_3+\omega_2-\omega_1-\omega_B)X(\omega_4)X(\omega_3)X(\omega_2)X(\omega_1)\Big(e^{2i[\eta(\omega_3)+\eta(\omega_2)]}\mathcal{B}_2(\omega_3,\omega_2,\omega_1)$$
$$+2\sqrt{2}i\rho(\omega_2+\omega_3-\omega_0)\mathcal{P}(\omega_2,\,\omega_2+\omega_3-\omega_0)\mathcal{B}_1(\omega_2+\omega_3-\omega_0,\,\omega_1)$$
$$+2i\rho(\omega_3+\omega_2-\omega_1)P(\omega_3+\omega_2-\omega_1)e^{2i\eta(\omega_2)}\mathcal{A}_0(\omega_2,\,\omega_1,\,\omega_3+\omega_2-\omega_1)$$
$$+\frac{i\sqrt{2}}{\pi}e^{2i\eta(\omega_2)}\int_\mu^\infty\rho(\omega)\rho(\omega_3+\omega_2-\omega)\mathcal{A}_0(\omega_2,\,\omega,\,\omega_3+\omega_2-\omega)\mathcal{B}_2(\omega,\,\omega_3+\omega_2-\omega,\,\omega_1)d\omega\Big).\quad(90)$$

To give a complete discussion of amplitudes in the three-meson sector, one should also study the collisions involving two mesons incident on the VN system, and the scattering of three mesons by two N particles. The analog of each of these processes can also be found in Bronzan's example, and it is expected, as in his case, that the corresponding amplitudes can be determined in terms of the Φ function.

IV. CONCLUDING REMARKS

It is interesting to compare our three-meson sector with the one recently proposed by Bronzan. In both instances there is a factorization of the S matrix in the two-meson sector permitting the dynamical equations of the third sector to reduce to the solution of a Fredholm integral equation in one variable. This factorization property is characteristic of the transition amplitudes for the mesodisintegration of the V or VN particles and for the connected amplitudes describing the scattering of two mesons by one or two N particles. Mathematically speaking, the dynamical equations are more complicated, and thus less manipulatable, in our case than in Ref. 5. For this reason we do not attempt to express $\mathcal{B}(\omega)$ in terms of integrals over the solution of the fundamental equation. However, on pretending that $\Delta=\omega_0$, we have found that our dynamical equations resemble those in Ref. 5. This equality is equivalent to saying that the energy of interaction between two V particles is twice the energy of interaction between an N and a V particle. An inspection of Eq. (31) in Ref. 1 shows that its inhomogeneous term vanishes when $\omega_B=2\omega_0$. As a consequence, the disconnected parts of $\mathcal{B}_2$ also vanish. This behavior describes a valid situation for the associated amplitude $A_{24}{}^3$ of Ref. 5, and explains why our equations simplify under this condition.

In Sec. II we have derived the Low equation for the elastic scattering amplitude Y by following a conventional contraction procedure. Guided by the solution for Y found previously, we have solved this equation through the introduction of an auxiliary function having no elastic cut and a known inelastic cut. In a similar way, we can obtain the Low equation for $\mathcal{B}(\omega)$ and try to solve it directly from a knowledge of the amplitudes gained in Sec. III via Amado's novel form of contraction. The usefulness of this procedure, as expounded in Ref. 5, is that it may suggest how to obtain solutions to crossing symmetric Low equations.

In this paper, we have used dispersion methods in a discussion of reactions involving two composite particles formed by the exchange of mesons between two static sources at zero separation. The lack of various properties such as spin, recoil, and crossing symmetry in these reactions makes it virtually impossible to compare results with physical reality. However, we feel that these calculations are of interest in their own right and may provide insights into more suggestive theories, particularly static models with crossing symmetry. Some exploratory work on the charged-scalar theory of two static nucleons has already been carried out; dispersion methods yield, at least in the one-meson approximation, a system of simultaneous equations for vertex functions and scattering amplitudes reminiscent of the analogous Lee-model problem.[15] As we have stated in Sec. I, the idea is to continue this work into the two- and three-meson dynamical equations. Finally, to refine all of these considerations, we can also think of applying the methods of dispersion theory to the Chew-Low[16] model with two sources. Although this model is also an obvious simplification of strong interactions, it does reproduce the essential features of low-energy mesonic phenomena.

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