Diffraction Scattering for Inelastic Processes*

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Previous treatments on high-energy scattering in quantum electrodynamics are extended to diffractive inelastic processes. We explore the model in which the diffractive excitation proceeds through the exchange of vector mesons. Six inelastic impact factors are explicitly calculated in the lowest order: (1) scalar to scalar, (2) pseudoscalar to pseudoscalar, (3) vector to vector, (4) axial vector to axial vector, (5) scalar to axial vector, and (6) pseudoscalar to axial vector. For all six of these impact factors, the masses of the incoming and outgoing particles may be different; and for the last two impact factors, the spins and the parities of the incoming and outgoing particles may be different. The nonvanishing of the impact factors demonstrates that mass, spin, and parity can change during a diffractive process. We also conclude that C (charge conjugation quantum number), S (strangeness), I (isotopic spin), B (baryon number), and G parity, etc., must remain the same during a diffractive process. Some general properties of the impact factor are also discussed.

1. INTRODUCTION

RECENTLY, a study[1,2] was made of all two-body elastic scattering processes in quantum electrodynamics at high energies. Out of this study a picture of diffraction scattering for elastic processes has emerged. We shall now extend these considerations to two-body inelastic processes at high energies. To be more specific, we shall give here a model of "diffractive excitation,"[3] which accounts for near-constant cross section and small angular width for certain two-body, high-energy inelastic processes.

Since the constancy of the cross section at high energies implies the proportionality of the amplitude to s, the square of the center-of-mass energy, the relevant diagrams must be the ones in which vector mesons are exchanged. For the inelastic process $a+b \to a'+b'$, the scattering amplitude for such exchanges can be conveniently expressed in terms of the impact factors $\mathcal{I}^{aa'}$ and $\mathcal{I}^{bb'}$. We shall calculate explicitly the impact factor in the lowest order. In order to avoid a discussion on the isotopic spin, we shall concentrate on the class of impact factors $\mathcal{I}^{aa'}$, where a and a' are neutral mesons. The method used here is a direct extension[4] of the one developed in IV, which allows us to calculate the impact factor $\mathcal{I}^{aa'}$ directly, without specifying what the other participating particles b and b' are.

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[1] H. Cheng and T. T. Wu, Phys. Rev. Letters **22**, 666 (1969).
[2] H. Cheng and T. T. Wu, Phys. Rev. **182**, 1852 (1969); **182**, 1868 (1969); **182**, 1873 (1969); **182**, 1899 (1969). These papers are hereafter referred to as I, II, III, and IV.
[3] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).
[4] The investigation can be carried through in a similar way if this fermion field is replaced by a scalar field which couples to the vector meason A_μ through a conserved current. The qualitative behavior of the impact factor is not expected to differ.

2. MODEL OF DIFFRACTIVE EXCITATION

For the scattering process $a+b \to a'+b'$, let us denote

$$r_1=\tfrac{1}{2}(p_{a'}-p_a)=\tfrac{1}{2}(p_b-p_{b'}), \tag{2.1}$$

$$r_2=\tfrac{1}{2}(p_a+p_{a'}), \tag{2.2}$$

and

$$r_3=\tfrac{1}{2}(p_b+p_{b'}), \tag{2.3}$$

where p_a is the momentum of particle a, etc. In this notation the standard energy invariants are given by

$$s=(r_2+r_3)^2, \tag{2.4}$$

$$t=4r_1^2, \tag{2.5}$$

and

$$u=(r_2-r_3)^2. \tag{2.6}$$

The masses of a, a', b, and b' will be denoted by M_a, $M_{a'}$, M_b, and $M_{b'}$, respectively. From (2.1)–(2.3), we get

$$r_1\cdot r_2=\tfrac{1}{4}(M_{a'}^2-M_a^2), \quad r_1\cdot r_3=\tfrac{1}{4}(M_b^2-M_{b'}^2) \tag{2.7}$$

and

$$r_2^2+r_1^2=\tfrac{1}{2}(M_a^2+M_{a'}^2), \quad r_3^2+r_1^2=\tfrac{1}{2}(M_b^2+M_{b'}^2). \tag{2.8}$$

It is sometimes helpful to know the components of r_1, r_2, and r_3 in the center-of-mass system. Let us choose the z axis to be parallel to $\mathbf{r}_2$; then these components are

$$r_2\sim[\omega+(4\omega)^{-1}(M_a^2+M_{a'}^2-\tfrac{1}{2}t),\ \omega,\ 0], \tag{2.9}$$

$$r_3\sim[\omega+(4\omega)^{-1}(M_b^2+M_{b'}^2-\tfrac{1}{2}t),\ -\omega,\ 0], \tag{2.10}$$

$$r_1\sim[(8\omega)^{-1}(-M_a^2+M_b^2+M_{a'}^2-M_{b'}^2),\ (8\omega)^{-1}(M_a^2+M_b^2-M_{a'}^2-M_{b'}^2),\ \mathbf{r}_{1\perp}], \tag{2.11}$$

where $\omega=|\mathbf{r}_2|$. From (2.11), we have

$$r_1^2=\tfrac{1}{4}t\sim-\mathbf{r}_{1\perp}^2. \tag{2.12}$$

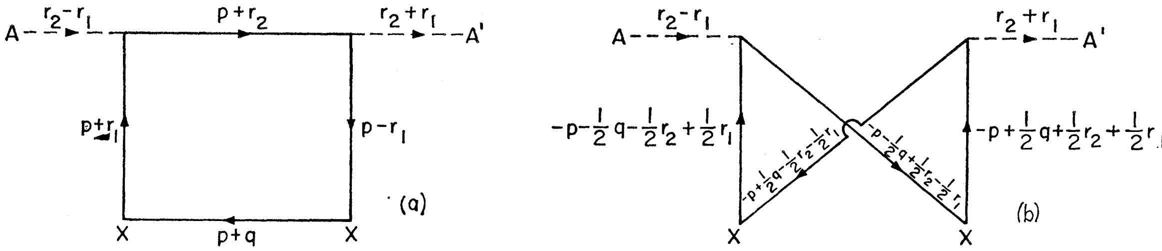

FIG. 1. Lowest-order diagrams for diffraction scattering.

We shall explore the model in which the exchanged vector mesons (A_μ) are coupled to a fermion field[4] (ψ) through the interaction $g\bar{\psi}\gamma_\mu\psi A_\mu$. The external mesons also couple to this fermion field, and the diffractive excitation of a to a' proceeds through the following steps: (1) A fermion and an antifermion are created by the incoming meson a; (2) these two particles are scattered by exchanging vector mesons with the other group of particles in collision; (3) they then annihilate to form the outgoing particle a'. The diffractive excitation of b to b' occurs similarly.

The impact factor $\mathcal{g}^{aa'}$ will be explicitly calculated in this model, where a and a' may be scalar mesons, pseudoscalar mesons, vector mesons, or axial-vector mesons, which will be abbreviated by S, P, V, and A, respectively.

3. IMPACT FACTORS FOR INELASTIC PROCESSES

In this section we evaluate explicitly six of the inelastic impact factors. More precisely, we shall calculate $\mathcal{g}^{SS}$, $\mathcal{g}^{PP}$, $\mathcal{g}^{VV}$, $\mathcal{g}^{AA}$, $\mathcal{g}^{PA}$, and $\mathcal{g}^{SA}$ in the lowest order of perturbation. For this purpose we shall generalize slightly the method[5] outlined in IV to deal with the situation in which r_1 has longitudinal components. Let us decompose the longitudinal components of a four-vector q into $q_+r_2+q_-r_3$; then

$$d^4q\sim(r_2\cdot r_3)dq_+dq_-d\mathbf{q}_\perp. \tag{3.1}$$

Following IV, we write

$$\mathcal{g}^{aa'}=\mathcal{g}_1^{aa'}+\mathcal{g}_2^{aa'}, \tag{3.2}$$

where $\mathcal{g}_1^{aa'}$ is contributed by the diagram of Fig. 1(a), and is explicitly given by

$$\mathcal{g}_1^{aa'}=\lim_{s\to\infty}4s^{-1}g^2f_af_{a'}(2\pi)^{-5}\int d^4p \times\int_{-\infty}^{\infty}dq_-N_1^{aa'}[(r_2+p)^2-m^2]^{-1}[(p-r_1)^2-m^2]^{-1} \times[(p+q)^2-m^2]^{-1}[(p+r_1)^2-m^2]^{-1}. \tag{3.3}$$

[5] An even simpler method is given in H. Cheng and T. T. Wu, Phys. Rev. Letters 23, 670 (1969). See also H. Cheng and T. T. Wu (to be published).

In (3.3), f_a ($f_{a'}$) is the coupling constant of a (a') to the fermion field ψ, m is the mass of the fermion, and the explicit form of $N_1^{aa'}$ will be given later for all of the cases discussed.

Similarly, $\mathcal{g}_2^{aa'}$ is contributed by the diagram of Fig. 1(b) and is equal to

$$\mathcal{g}_2^{aa'}=\lim_{s\to\infty}2s^{-1}g^2f_af_{a'}(2\pi)^{-5}\int d^4p \times\int_{-\infty}^{\infty}dq_-N_2^{aa'}[(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2]^{-1} \times[(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2]^{-1} \times[(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2]^{-1} \times[(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2]^{-1}, \tag{3.4}$$

where $N_2^{aa'}$ will be explicitly given later for all cases.

Carrying out the integration over q_-, (3.3) becomes

$$\mathcal{g}_1^{aa'}=\lim_{s\to\infty}-2is^{-1}g^2f_af_{a'}(2\pi)^{-4} \times\int d^4p\, N_1^{aa'}(-2pr_3)^{-1}[(r_2+p)^2-m^2]^{-1} \times[(p-r_1)^2-m^2]^{-1}[(p+r_1)^2-m^2]^{-1}. \tag{3.5}$$

Next we put $p=p_+r_2+p_-r_3+p_\perp$; then

$$p^2\sim p_+p_-s+p_+^2r_2^2-\mathbf{p}_\perp^2, \tag{3.6}$$

where a term $p_-^2r_3^2$ is neglected, as the longitudinal components of p is dominantly in the direction of r_2. We also have

$$(p\pm r_1)^2-m^2\sim p_+p_-s+p_+^2r_2^2 -(\mathbf{p}_\perp\pm\mathbf{r}_{1\perp})^2-m^2\pm\tfrac{1}{2}p_+(M_{a'}^2-M_a^2) \tag{3.7}$$

and

$$(p+r_2)^2-m^2\sim sp_-(1+p_+) +(1+p_+)^2r_2^2-\mathbf{p}_\perp^2-m^2. \tag{3.8}$$

Carrying out the integration over p_-, (3.5) becomes

$$\mathcal{G}_1^{aa'}=\lim_{s\to\infty}-g^2f_af_{a'}(2\pi)^{-3}s^{-1}\int d\mathbf{p}_\perp \times\int_{-1}^{0}dp_+N_1^{aa'}(-2pr_3)^{-1}(1+p_+)$$
$$\times\{[\mathbf{p}_\perp+(1+p_+)\mathbf{r}_{1\perp}]^2+m^2+p_+(1+p_+)M_a{}^2\}^{-1}$$
$$\times\{[\mathbf{p}_\perp-(1+p_+)\mathbf{r}_{1\perp}]^2+m^2+p_+(1+p_+)M_{a'}{}^2\}^{-1}. \quad (3.9)$$

Putting $p_+=-(1-\beta)$, we obtain

$$\mathcal{G}_1^{aa'}=-g^2f_af_{a'}(2\pi)^{-3}\int d\mathbf{p}_\perp\int_0^1 d\beta\, \mathfrak{N}_1^{aa'}$$
$$\times[(\mathbf{p}_\perp+\beta\mathbf{r}_{1\perp})^2+m^2-\beta(1-\beta)M_a{}^2]^{-1}$$
$$\times[(\mathbf{p}_\perp-\beta\mathbf{r}_{1\perp})^2+m^2-\beta(1-\beta)M_{a'}{}^2]^{-1}, \quad (3.10)$$

where

$$\mathfrak{N}_1^{aa'}=\lim_{s\to\infty}s^{-1}(-2pr_3)^{-1}\beta N_1^{aa'}. \quad (3.11)$$

Next we turn to $\mathcal{G}_2^{aa'}$. To simplify the calculation, we put

$$k=p+\tfrac{1}{2}q; \quad (3.12)$$

then (3.4) becomes

$$\mathcal{G}_2^{aa'}=\lim_{s\to\infty}2s^{-1}g^2f_af_{a'}(2\pi)^{-5}\int d^4k\int_{-\infty}^{\infty}dq_-N_2^{aa'}$$
$$\times[(-k+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2]^{-1}$$
$$\times[(-k+q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2]^{-1}$$
$$\times[(-k+q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2]^{-1}$$
$$\times[(-k-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2]^{-1}. \quad (3.13)$$

Now

$$q\sim q_-r_3+q_\perp;$$

thus

$$(-k+q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2\sim 2q_-(r_2\cdot r_3)(\tfrac{1}{2}-k_+)+[(\tfrac{1}{2}-k_+)r_2-k_-r_3]^2-(-\mathbf{k}_\perp+\mathbf{q}_\perp+\tfrac{1}{2}\mathbf{r}_{1\perp})^2-m^2+\tfrac{1}{4}(\tfrac{1}{2}-k_+)(M_{a'}{}^2-M_a{}^2), \quad (3.14)$$

$$(-k+q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2\sim 2q_-(r_2\cdot r_3)(-\tfrac{1}{2}-k_+)+[(-\tfrac{1}{2}-k_+)r_2-k_-r_3]^2-(-\mathbf{k}_\perp+\mathbf{q}_\perp-\tfrac{1}{2}\mathbf{r}_{1\perp})^2-m^2+\tfrac{1}{4}(\tfrac{1}{2}+k_+)(M_{a'}{}^2-M_a{}^2), \quad (3.15)$$

where

$$k=k_+r_2+k_-r_3+k_\perp.$$

Carrying out the integration over q_-, we obtain from (3.13)

$$\mathcal{G}_2^{aa'}=is^{-1}g^2f_af_{a'}(2\pi)^{-4}\int d\mathbf{k}_\perp\int dk_+dk_-\lim_{s\to\infty}N_2^{aa'}$$
$$\times[(-k+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2]^{-1}[(-k-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2]^{-1}$$
$$\times[(\mathbf{k}_\perp-\mathbf{q}_\perp-k_+\mathbf{r}_{1\perp})^2-(\tfrac{1}{4}-k_+{}^2)M_{a'}{}^2+m^2]^{-1}. \quad (3.16)$$

Now

$$(-k+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1)^2-m^2\sim k_-(k_+-\tfrac{1}{2})s+(\tfrac{1}{2}-k_+)^2r_2{}^2-(\mathbf{k}_\perp+\tfrac{1}{2}\mathbf{r}_{1\perp})^2-m^2-\tfrac{1}{4}(\tfrac{1}{2}-k_+)(M_{a'}{}^2-M_a{}^2), \quad (3.17)$$

$$(-k-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1)^2-m^2\sim k_-(k_++\tfrac{1}{2})s+(\tfrac{1}{2}+k_+)^2r_2{}^2-(\mathbf{k}_\perp-\tfrac{1}{2}\mathbf{r}_{1\perp})^2-m^2-\tfrac{1}{4}(\tfrac{1}{2}+k_+)(M_{a'}{}^2-M_a{}^2). \quad (3.18)$$

Carrying out the integration over k_-, we get from (3.16)

$$\mathcal{G}_2^{aa'}=-g^2f_af_{a'}(2\pi)^{-3}\int d\mathbf{k}_\perp\int_{-1/2}^{1/2}dk_+\lim_{s\to\infty}N_2^{aa'}s^{-2}$$
$$\times[(\mathbf{k}_\perp+k_+\mathbf{r}_{1\perp})^2+m^2-(\tfrac{1}{4}-\mathbf{k}_+{}^2)M_a{}^2]^{-1}$$
$$\times[(\mathbf{k}_\perp-\mathbf{q}_\perp-k_+\mathbf{r}_{1\perp})^2+m^2-(\tfrac{1}{4}-k_+{}^2)M_{a'}{}^2]^{-1}. \quad (3.19)$$

Putting $k_+=\frac{1}{2}-\beta$, and $\mathbf{p}_\perp=\mathbf{k}_\perp-\frac{1}{2}\mathbf{q}_\perp$, we have

$$\mathcal{G}_2^{aa'}=g^2f_af_{a'}(2\pi)^{-3}\int d\mathbf{p}_\perp\int_0^1 d\beta\,\mathfrak{N}_2^{aa'}$$
$$\times\{[\mathbf{p}_\perp+\tfrac{1}{2}\mathbf{q}_\perp+(\tfrac{1}{2}-\beta)\mathbf{r}_{1\perp}]^2+m^2-\beta(1-\beta)M_a{}^2\}^{-1}$$
$$\times\{[\mathbf{p}_\perp-\tfrac{1}{2}\mathbf{q}_\perp-(\tfrac{1}{2}-\beta)\mathbf{r}_{1\perp}]^2+m^2-\beta(1-\beta)M_{a'}{}^2\}^{-1}, \quad (3.20)$$

where

$$\mathfrak{N}_2^{aa'}=\lim_{s\to\infty}(-s^{-2}N_2^{aa'}). \quad (3.21)$$

We notice that the denominator of (3.20) is equal to that in (3.10) with $\beta\mathbf{r}_1$ replaced by

$$\mathbf{Q}=\tfrac{1}{2}\mathbf{q}_\perp+(\tfrac{1}{2}-\beta)\mathbf{r}_{1\perp}. \quad (3.22)$$

Thus (3.2), (3.10), and (3.20) give

$$\mathcal{G}^{aa'}(\mathbf{r}_1,\mathbf{q}_\perp)=-g^2f_af_{a'}(2\pi)^{-3}\int d\mathbf{p}_\perp\int_0^1 d\beta$$
$$\times\{\mathfrak{N}_1^{aa'}[(\mathbf{p}_\perp+\beta\mathbf{r}_{1\perp})^2+m^2-\beta(1-\beta)M_a{}^2]^{-1}$$
$$\times[(\mathbf{p}_\perp-\beta\mathbf{r}_{1\perp})^2+m^2-\beta(1-\beta)M_{a'}{}^2]^{-1}$$
$$-\mathfrak{N}_2^{aa'}[(\mathbf{p}_\perp+\mathbf{Q})^2+m^2-\beta(1-\beta)M_a{}^2]^{-1}$$
$$\times[(\mathbf{p}_\perp-\mathbf{Q})^2+m^2-\beta(1-\beta)M_{a'}{}^2]^{-1}\}. \quad (3.23)$$

We shall next calculate $\mathfrak{N}_1^{aa'}$ and $\mathfrak{N}_2^{aa'}$ explicitly for six specific cases.

A. Scalar to Scalar

When a and a' are both scalar particles with masses which may or may not be equal, and couple to the fermion field ψ by the interactions $\bar{\psi}\psi\phi_a$ and $\bar{\psi}\psi\phi_{a'}$, respectively, we have

$$N_1{}^{SS}=\mathrm{Tr}[(\not{p}+\not{r}_1+m)\not{r}_3(\not{p}+\not{q}+m)\not{r}_3(\not{p}-\not{r}_1+m)\times(\not{p}+\not{r}_2+m)]\sim 8(p\cdot r_3)^2(4m^2-r_2{}^2+r_1{}^2)-8(p\cdot r_3)(r_2\cdot r_3)(p^2-r_1{}^2-m^2) \quad (3.24)$$

and

$$N_2^{SS}=\mathrm{Tr}[(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)\times r_3(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)\times r_3(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)]\sim 8(p\cdot r_3)^2(r_1^2-r_2^2+4m^2)-8(r_2\cdot r_3)^2(p^2-\tfrac{1}{4}q^2)+16(p\cdot r_3)(r_2\cdot r_3)(r_2\cdot p+\tfrac{1}{2}r_1\cdot q). \quad (3.25)$$

There are cancellations of terms in (3.24) with terms in (3.25) in the same way as in III. Specifically, a term

$$-8(p\cdot r_3)(r_2\cdot r_3)(p^2+r_1^2-m^2) \quad (3.26)$$

in N_1^{SS} exactly cancels a term

$$-2(r_2\cdot r_3)^2[4(p^2-m^2)+r_2^2+r_1^2+q^2]+8(r_2\cdot r_3)(p\cdot r_3)(2r_2\cdot p-r_1\cdot q) \quad (3.27)$$

in N_2^{SS}. Thus N_1^{SS} and N_2^{SS} can be replaced by

$$8(p\cdot r_3)^2(4m^2-r_2^2+r_1^2)+16(p\cdot r_3)(r_2\cdot r_3)r_1^2 \quad (3.28)$$

and

$$8(p\cdot r_3)^2(r_1^2-r_2^2+4m^2)-2(r_2\cdot r_3)^2(4m^2-r_2^2-r_1^2-2q^2)+16(r_2\cdot r_3)(p\cdot r_3)(q\cdot r_1), \quad (3.29)$$

respectively. From (3.11) and (3.28), and remembering that $(p\cdot r_3)\sim-(1-\beta)(r_2\cdot r_3)$, we get

$$\mathfrak{N}_1^{SS}=2(1-\beta)\beta(4m^2-r_2^2+r_1^2)-4\beta r_1^2=\beta(1-\beta)(8m^2-M_a^2-M_{a'}^2)+4\beta^2\mathbf{r}_1^2. \quad (3.30)$$

From (3.21) and (3.29), and remembering that $(p\cdot r_3)\sim(k\cdot r_3)\sim(\tfrac{1}{2}-\beta)(r_2 r_3)$, we have

$$\mathfrak{N}_2^{SS}=-2(\tfrac{1}{2}-\beta)^2(r_1^2-r_2^2+4m^2)+\tfrac{1}{2}(4m^2-r_2^2-r_1^2+2q_\perp^2)+4(\tfrac{1}{2}-\beta)(\mathbf{q}_\perp\cdot\mathbf{r}_1)=\beta(1-\beta)(8m^2-M_a^2-M_{a'}^2)+4\mathbf{Q}^2, \quad (3.31)$$

where $\mathbf{Q}$ is given by (3.22). By (3.30) and (3.31), $\mathfrak{N}_2^{SS}$ is equal to $\mathfrak{N}_1^{SS}$ after $\beta\mathbf{r}_1$ is replaced by $\mathbf{Q}$.

If we carry out the integration over $\mathbf{p}_\perp$ in (3.23) by Feynman parametrization, we get from (3.23), (3.30), and (3.31) that

$$\mathcal{G}^{SS}(\mathbf{r}_1,\mathbf{q}_\perp)=\tfrac{1}{2}g^2f_af_{a'}(2\pi)^{-2}\int_0^1 d\alpha\int_0^1 d\beta\times\{[\beta(1-\beta)(M_a^2+M_{a'}^2-8m^2)-4\beta^2\mathbf{r}_1^2]\times(4\beta^2\alpha(1-\alpha)\mathbf{r}_1^2+m^2-\beta(1-\beta)\times[M_a^2\alpha+M_{a'}^2(1-\alpha)])^{-1}-\text{preceding term with }\beta\mathbf{r}_1\to\mathbf{Q}\}. \quad (3.32)$$

B. Pseudoscalar to Pseudoscalar

When a and a' are both pseudoscalar with masses which may or may not be equal, and couple to the fermion field ψ by the interactions $\bar\psi\gamma_5\psi\phi_a$ and $\bar\psi\gamma_5\psi\phi_{a'}$, respectively, where $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$, we have

$$N_1^{PP}=\mathrm{Tr}[\gamma_5(p+r_1+m)r_3(p+q+m)r_3(p-r_1+m)\times\gamma_5(p+r_2+m)]\sim 8(p\cdot r_3)^2(r_1^2-r_2^2)-8(p\cdot r_3)(r_2\cdot r_3)(p^2-r_1^2-m^2) \quad (3.33)$$

and

$$N_2^{PP}=\mathrm{Tr}[\gamma_5(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)\times r_3(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)\gamma_5\times(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)\times r_3(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)]\sim 8(p\cdot r_3)^2(r_1^2-r_2^2)-8(r_2\cdot r_3)^2(p^2-\tfrac{1}{4}q^2-2m^2)+16(p\cdot r_3)(r_2\cdot r_3)(r_2\cdot p+\tfrac{1}{2}r_1\cdot q). \quad (3.34)$$

Because of the cancellation of (3.26) with (3.27), N_1^{PP} and N_2^{PP} can be replaced by

$$8(p\cdot r_3)^2(r_1^2-r_2^2)+16(p\cdot r_3)(r_2\cdot r_3)r_1^2 \quad (3.35)$$

and

$$8(p\cdot r_3)^2(r_1^2-r_2^2)+2(r_2\cdot r_3)^2(2q^2+4m^2+r_1^2+r_2^2)+16(p\cdot r_3)(r_2\cdot r_3)(r_1\cdot q), \quad (3.36)$$

respectively. From (3.11) and (3.35), we get

$$\mathfrak{N}_1^{PP}=2(1-\beta)\beta(r_1^2-r_2^2)-4\beta r_1^2=4\beta^2\mathbf{r}_1^2-(1-\beta)\beta(M_a^2+M_{a'}^2). \quad (3.37)$$

From (3.21) and (3.36), we get

$$\mathfrak{N}_2^{PP}=-2(\beta-\tfrac{1}{2})^2(r_1^2-r_2^2)-\tfrac{1}{2}[-2q_\perp^2+4m^2+\tfrac{1}{2}(M_a^2+M_{a'}^2)]+4(\tfrac{1}{2}-\beta)(\mathbf{r}_1\cdot\mathbf{q}_\perp)=4\mathbf{Q}^2-(1-\beta)\beta(M_a^2+M_{a'}^2), \quad (3.38)$$

which is indeed equal to the right-hand side of (3.37) with βr_1 replaced by $\mathbf{Q}$.

Substituting (3.37) and (3.38) into (3.23), and carrying out the integration over $\mathbf{p}_\perp$, we get

$$\mathcal{G}^{PP}(\mathbf{r}_1,\mathbf{q}_\perp)=\tfrac{1}{2}g^2f_af_{a'}(2\pi)^{-2}\int_0^1 d\alpha\int_0^1 d\beta\times\{[\beta(1-\beta)(M_a^2+M_{a'}^2)-4\beta^2\mathbf{r}_1^2]\times(4\beta^2\alpha(1-\alpha)\mathbf{r}_1^2+m^2-\beta(1-\beta)\times[M_a^2\alpha+M_{a'}^2(1-\alpha)])^{-1}-\text{preceding term with }\beta\mathbf{r}_1\to\mathbf{Q}\}. \quad (3.39)$$

C. Vector to Vector

There are three polarization directions for a vector particle or an axial-vector particle. Two of them are transverse, and one of them is longitudinal. The components of the longitudinal polarization vectors will be explicitly given.

In the center-of-mass system, p_a and $p_{a'}$ are

$$p_a=r_2-r_1\sim[\omega+(8\omega)^{-1}(3M_a{}^2+M_{a'}{}^2-M_b{}^2+M_{b'}{}^2-t),\ \omega-(8\omega)^{-1}(M_a{}^2+M_b{}^2-M_{a'}{}^2-M_{b'}{}^2),\ -\mathbf{r}_{1\perp}],$$

$$p_{a'}=r_2+r_1\sim[\omega+(8\omega)^{-1}(M_a{}^2+3M_{a'}{}^2+M_b{}^2-M_{b'}{}^2-t),\ \omega+(8\omega)^{-1}(M_a{}^2+M_b{}^2-M_{a'}{}^2-M_{b'}{}^2),\ \mathbf{r}_{1\perp}].$$

The longitudinal polarization vector for particle a in the Feynman gauge is given by

$$M_a{}^{-1}[\omega-(8\omega)^{-1}(M_a{}^2+M_b{}^2-M_{a'}{}^2-M_{b'}{}^2+t),\ \omega+(8\omega)^{-1}(3M_a{}^2+M_{a'}{}^2-M_b{}^2+M_{b'}{}^2),\ -\mathbf{r}_{1\perp}]. \quad (3.40)$$

The components in (3.40) are large when ω is large. In order to avoid complications from these large components we shall invoke gauge invariance and subtract $M_a{}^{-1}p_a$ from (3.40) to obtain the polarization vector

$$\epsilon_{aL}\sim M_a(2\omega)^{-1}[-1,1,0]. \quad (3.41)$$

Similarly, the longitudinal polarization vector for a' will be taken to be

$$\epsilon_{a'L}\sim M_{a'}(2\omega)^{-1}[-1,1,0]. \quad (3.42)$$

When a and a' are both vector particles with masses which may or may not be equal, and couple to the current of the fermion field, we have

$$N_1{}^{VV}=\mathrm{Tr}[\gamma\cdot\epsilon_a(r_1+p+m)r_3(p+q+m)r_3(-r_1+p+m)\gamma\cdot\epsilon_{a'}(r_2+p+m)]\sim 8(p\cdot r_3)^2[4(p\cdot\epsilon_a)(p\cdot\epsilon_{a'}) +2(p\cdot\epsilon_a)(p_a\cdot\epsilon_{a'})+2(p_{a'}\cdot\epsilon_a)(p\cdot\epsilon_{a'})-(\epsilon_a\cdot\epsilon_{a'})(r_1{}^2-r_2{}^2)] +8(p\cdot r_3)(r_2\cdot r_3)[2(r_1\cdot\epsilon_a)(p\cdot\epsilon_{a'})-2(p\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})+(\epsilon_a\cdot\epsilon_{a'})(p^2-r_1{}^2-m^2)] \quad (3.43)$$

and

$$N_2{}^{VV}=\mathrm{Tr}[\gamma\cdot\epsilon_a(-p-\tfrac12q-\tfrac12r_2+\tfrac12r_1+m)r_3(-p+\tfrac12q-\tfrac12r_2-\tfrac12r_1+m)\gamma\cdot\epsilon_{a'}(-p+\tfrac12q+\tfrac12r_2+\tfrac12r_1+m) \times r_3(-p-\tfrac12q+\tfrac12r_2-\tfrac12r_1+m)]\sim 8(p\cdot r_3)^2\{[(2p+q)\cdot\epsilon_a][(2p-q)\cdot\epsilon_{a'}]-(p_{a'}\cdot\epsilon_a)(p_a\cdot\epsilon_{a'})+(\epsilon_a\cdot\epsilon_{a'})(r_2{}^2-r_1{}^2)\} +2(r_2\cdot r_3)^2\{4(\epsilon_a\cdot\epsilon_{a'})(p^2-\tfrac14q^2-m^2)-[(2p-q)\cdot\epsilon_a][(2p+q)\cdot\epsilon_{a'}]+(p_a\cdot\epsilon_a)(p_{a'}\epsilon_{a'})\}-8(p\cdot r_3)(r_2\cdot r_3) \times[-2(r_1\cdot\epsilon_a)(p\cdot\epsilon_{a'})+2(p\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})-(r_2\cdot\epsilon_a)(q\cdot\epsilon_{a'})+(q\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})+(\epsilon_a\epsilon_{a'})(2r_2\cdot p+r_1\cdot q)]. \quad (3.44)$$

Due to the cancellations mentioned in Sec. 3 A, $N_1{}^{VV}$ and $N_2{}^{VV}$ can be replaced by

$$8(p\cdot r_3)^2[4(p\cdot\epsilon_a)(p\cdot\epsilon_{a'})+2(p\cdot\epsilon_a)(p_a\cdot\epsilon_{a'})+2(p_{a'}\cdot\epsilon_a)(p\cdot\epsilon_{a'})-(\epsilon_a\cdot\epsilon_{a'})(r_1{}^2-r_2{}^2)] +16(p\cdot r_3)(r_2\cdot r_3)[(r_1\cdot\epsilon_a)(p\cdot\epsilon_{a'})-(p\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})-(\epsilon_a\cdot\epsilon_{a'})r_1{}^2] \quad (3.45)$$

and

$$8(p\cdot r_3)^2\{[(2p+q)\cdot\epsilon_a][(2p-q)\cdot\epsilon_{a'}]-(p_a{}'\cdot\epsilon_a)(p_a\cdot\epsilon_{a'})+(\epsilon_a\cdot\epsilon_{a'})(r_2{}^2-r_1{}^2)\} +2(r_2\cdot r_3)^2\{-(\epsilon_a\cdot\epsilon_{a'})(r_2{}^2+r_1{}^2+2q^2)-[(2p-q)\cdot\epsilon_a][(2p+q)\cdot\epsilon_{a'}]+(p_a\cdot\epsilon_a)(p_{a'}\epsilon_{a'})\} -8(p\cdot r_3)(r_2\cdot r_3)[-2(r_1\cdot\epsilon_a)(p\cdot\epsilon_{a'})+2(p\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})-(r_2\cdot\epsilon_a)(q\cdot\epsilon_{a'})+(q\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})+2(\epsilon_a\cdot\epsilon_{a'})(r_1\cdot q)], \quad (3.46)$$

respectively.

From (3.11) and (3.45), we get

$$\mathfrak{N}_1{}^{VV}=2\beta(1-\beta)[4(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_a)(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_{a'})-4\beta(1-\beta)(r_2\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})-2\beta(r_2\cdot\epsilon_a)(r_1\cdot\epsilon_{a'}) +2\beta(r_1\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})]+(\epsilon_a\cdot\epsilon_{a'})[\beta(1-\beta)(M_a{}^2+M_{a'}{}^2)-4\beta^2\mathbf{r}_1{}^2], \quad (3.47)$$

and from (3.21) and (3.46), we get

$$\mathfrak{N}_2{}^{VV}=2\beta(1-\beta)[4(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_a)(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_{a'})-4\beta(1-\beta)(r_2\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})-(q\cdot\epsilon_a)(q\cdot\epsilon_{a'}) -(r_2\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})+(r_1\cdot\epsilon_a)(r_2\cdot\epsilon_{a'})+(r_1\cdot\epsilon_a)(r_1\cdot\epsilon_{a'})+(1-2\beta)(q\cdot\epsilon_a)(r_2\cdot\epsilon_{a'}) -(1-2\beta)(r_2\cdot\epsilon_a)(q\cdot\epsilon_{a'})]+(\epsilon_a\cdot\epsilon_{a'})[\beta(1-\beta)(M_a{}^2+M_{a'}{}^2)-4\mathbf{Q}^2]. \quad (3.48)$$

When ϵ_a and $\epsilon_{a'}$ are both transverse, we have $(r_2\cdot\epsilon_a)=-(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_a)$ and $(r_2\cdot\epsilon_{a'})=(\mathbf{r}_1\cdot\epsilon_{a'})$. Thus we get

$$\mathfrak{N}_1{}^{VV}=8\beta(1-\beta)[(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_a)(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_{a'})-\beta^2(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_a)(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_{a'})] -(\boldsymbol{\varepsilon}_a\cdot\boldsymbol{\varepsilon}_{a'})[\beta(1-\beta)(M_a{}^2+M_{a'}{}^2)-4\beta^2\mathbf{r}_1{}^2] \quad (3.49)$$

and

$$\mathfrak{N}_2{}^{VV}=8\beta(1-\beta)[(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_a)(\mathbf{p}_\perp\cdot\boldsymbol{\varepsilon}_{a'})-(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_a)(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_{a'})] -(\boldsymbol{\varepsilon}_a\cdot\boldsymbol{\varepsilon}_{a'})[\beta(1-\beta)(M_a{}^2+M_{a'}{}^2)-4\mathbf{Q}^2]. \quad (3.50)$$

When ϵ_a and $\epsilon_{a'}$ are both longitudinal, we have $(r_2\cdot\epsilon_a)=-M_a$, $(r_2\cdot\epsilon_{a'})=-M_{a'}$, and $(p_\perp\cdot\epsilon_a)=(p_\perp\cdot\epsilon_{a'})=(r_1\cdot\epsilon_a)=(r_1\cdot\epsilon_{a'})=(\epsilon_a\cdot\epsilon_{a'})=0$. Thus we get

$$\mathfrak{N}_1{}^{VV}=\mathfrak{N}_2{}^{VV}=-8\beta^2(1-\beta)^2M_aM_{a'}. \quad (3.51)$$

When ϵ_a is transverse and $\epsilon_{a'}$ is longitudinal, we have

$$\mathfrak{N}_1{}^{VV}=-4\beta^2(1-\beta)(1-2\beta)M_{a'}(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_a) \quad (3.52)$$

and

$$\mathfrak{N}_2{}^{VV}=4\beta(1-\beta)(1-2\beta)M_{a'}(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_a). \quad (3.53)$$

When ϵ_a is longitudinal and $\epsilon_{a'}$ is transverse, we have

$$\mathfrak{N}_1{}^{VV}=4\beta^2(1-\beta)(1-2\beta)M_a(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_{a'}) \quad (3.54)$$

and

$$\mathfrak{N}_2{}^{VV}=-4\beta(1-\beta)(1-2\beta)M_a(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_{a'}). \quad (3.55)$$

Substituting (3.49)–(3.55) into (3.23), introducing a Feynman parameter α, and carrying out the interaction over $\mathbf{p}_\perp$, we get

$$g^{VV}(\mathbf{r}_1,\mathbf{q}_\perp)=-\tfrac{1}{2}g^2f_af_{a'}(2\pi)^{-2}\int_0^1 d\alpha\int_0^1 d\beta \times\{A_1[4\beta^2\alpha(1-\alpha)\mathbf{r}_1^2+m^2-\beta(1-\beta)M_a^2 \times\alpha-\beta(1-\beta)M_{a'}^2(1-\alpha)]^{-1} -A_2[4\alpha(1-\alpha)\mathbf{Q}^2+m^2-\beta(1-\beta)M_a^2 \times\alpha-\beta(1-\beta)M_{a'}^2(1-\alpha)]^{-1}\}. \quad (3.56)$$

In (3.56), A_1 and A_2 are, for transverse to transverse,

$$A_1=-32\alpha(1-\alpha)\beta^3(1-\beta)(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_A)(\mathbf{r}_1\cdot\boldsymbol{\varepsilon}_{a'}) +(\boldsymbol{\varepsilon}_a\cdot\boldsymbol{\varepsilon}_{a'})\{4\beta^2\mathbf{r}_1^2[1-8\beta(1-\beta) \times(\tfrac{1}{2}-\alpha)^2]-\beta(1-\beta)(M_a^2+M_{a'}^2)\} \quad (3.57)$$

and

$$A_2=-32\alpha(1-\alpha)\beta(1-\beta)(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_a)(\mathbf{Q}\cdot\boldsymbol{\varepsilon}_{a'}) +(\boldsymbol{\varepsilon}_a\cdot\boldsymbol{\varepsilon}_{a'})\{4\mathbf{Q}^2[1-8\beta(1-\beta)(\tfrac{1}{2}-\alpha)^2] -\beta(1-\beta)(M_a^2+M_{a'}^2)\}. \quad (3.58)$$

For longitudinal to longitudinal, transverse to longitudinal, and longitudinal to transverse, A_1 and A_2 are equal to $\mathfrak{N}_1^{VV}$ and $\mathfrak{N}_2^{VV}$, respectively, with the explicit expressions given by (3.51)–(3.55).

D. Axial Vector to Axial Vector

When a and a' are both axial-vector particles with masses which may or may not be equal, and couple to the fermion field by $i\bar{\psi}\gamma_5\gamma_\mu\psi\phi_\mu$, we have

$$N_1^{AA}=-\mathrm{Tr}[\gamma_5\gamma\cdot\epsilon_a(r_1+p+m)r_3(p+q+m) \times r_3(-r_1+p+m)\gamma_5\gamma\cdot\epsilon_{a'}(r_2+p+m)] \sim N_1^{VV}-32m^2(p\cdot r_3)^2(\epsilon_a\cdot\epsilon_{a'}) \quad (3.59)$$

and

$$N_2^{AA}=-\mathrm{Tr}[\gamma_5\gamma\cdot\epsilon_a(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m) \times r_3(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m) \times\gamma_5\gamma\cdot\epsilon_{a'}(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m) \times r_3(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)]\sim N_2^{VV} -32m^2(p\cdot r_3)^2(\epsilon_a\cdot\epsilon_{a'})+8m^2(r_2\cdot r_3)^2(\epsilon_a\cdot\epsilon_{a'}), \quad (3.60)$$

where N_1^{VV} and N_2^{VV} are given by (3.43) and (3.44), respectively. From (3.11), (3.21), (3.59), and (3.60), we get

$$\mathfrak{N}_1^{AA}=\mathfrak{N}_1^{VV}-8m^2\beta(1-\beta)(\epsilon_a\cdot\epsilon_{a'}) \quad (3.61)$$

and

$$\mathfrak{N}_2^{AA}=\mathfrak{N}_2^{VV}-8m^2\beta(1-\beta)(\epsilon_a\cdot\epsilon_{a'}). \quad (3.62)$$

Thus we have for longitudinal to longitudinal, logitudinal to transverse, and transverse to longitudinal,

$$\mathfrak{N}_1^{AA}=\mathfrak{N}_1^{VV}, \quad (3.63)$$

$$\mathfrak{N}_2^{AA}=\mathfrak{N}_2^{VV}, \quad (3.64)$$

and

$$g^{AA}(\mathbf{r}_1,\mathbf{q}_\perp)=g^{VV}(\mathbf{r}_1,\mathbf{q}_\perp); \quad (3.65)$$

while for transverse to transverse, we have

$$g^{AA}(\mathbf{r}_1,\mathbf{q}_\perp)-g^{VV}(\mathbf{r}_1,\mathbf{q}_\perp)=-m^2(\boldsymbol{\varepsilon}_a\boldsymbol{\varepsilon}_{a'})g^2f_af_{a'}\pi^{-2} \times\int_0^1 d\alpha\int_0^1 d\beta\beta(1-\beta) \times\{[4\beta^2\alpha(1-\alpha)\mathbf{r}_1^2+m^2-\beta(1-\beta)M_a^2 \times\alpha-\beta(1-\beta)M_{a'}^2(1-\alpha)]^{-1} -[4\alpha(1-\alpha)\mathbf{Q}^2+m^2-\beta(1-\beta)M_a^2 -\beta(1-\beta)M_{a'}^2(1-\alpha)]^{-1}\}. \quad (3.66)$$

E. Pseudoscalar to Axial Vector

Consider the impact factor g^{PA}, where the pseudoscalar particle and the axial-vector particle couple to the fermion field by γ_5 and $i\gamma_5\gamma_j$, respectively. Note that $C=1$ for both the pseudoscalar particle and the axial-vector particle.[6] We shall follow the notations of I. Then we have

$$N_1^{PA}=i\,\mathrm{Tr}[\gamma_5(p+r_1+m)r_3(p+q+m)r_3(p-r_1+m)\gamma_5\gamma_j(p+r_2+m)] \sim-2(p\cdot r_3)i\,\mathrm{Tr}[(p+r_1+m)r_3(p-r_1+m)\gamma_j(p+r_2-m)] =-16(p\cdot r_3)im\{(r_3\cdot p)r_{2j}-[(p+r_3)\cdot r_2]r_{1j}\} \quad (3.67)$$

and

$$N_2^{PA}=i\,\mathrm{Tr}[\gamma_5(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)r_3(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m) \times\gamma_5\gamma_j(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)r_3(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)] \sim i[(-2p-r_2)r_3]\,\mathrm{Tr}[(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1-m)\gamma_j(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)r_3(-p-\tfrac{1}{2}q+\tfrac{1}{2}r_2-\tfrac{1}{2}r_1+m)] -i[(-2p+r_2)r_3]\,\mathrm{Tr}[(-p+\tfrac{1}{2}q-\tfrac{1}{2}r_2-\tfrac{1}{2}r_1-m)\gamma_j(-p+\tfrac{1}{2}q+\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)r_3(-p-\tfrac{1}{2}q-\tfrac{1}{2}r_2+\tfrac{1}{2}r_1+m)] =-4im[(-2p-r_2)\cdot r_3][(-2p+r_2)\cdot r_3]r_{2j}-2im[(-2p+r_2)\cdot r_3]^2(q-r_1)_j+2im[(-2p-r_2)\cdot r_3]^2(q+r_1)_j. \quad (3.68)$$

[6] See, for instance, P. Roman, *Theory of Elementary Particles* (North-Holland Publishing Co., Amsterdam, 1960), p. 285.

From (3.11), we get

$$\mathfrak{N}_1^{PA}=-4i\beta m[\beta r_{1j}+(1-\beta)r_{2j}], \tag{3.69}$$

and from (3.21), we get

$$\mathfrak{N}_2^{PA}=-4im\beta(1-\beta)r_{2j}+2im\beta^2(q-r_1)_j -2im(1-\beta)^2(q+r_1)_j. \tag{3.70}$$

When the polarization of the axial-vector particle is longitudinal, we have

$$\mathfrak{N}_1^{PA}=\mathfrak{N}_2^{PA}=4i\beta(1-\beta)mMa. \tag{3.71}$$

When the polarization of the axial-vector particle is transverse and in the scattering plane, we have

$$\mathfrak{N}_1^{PA}=4i\beta(1-2\beta)m|\mathbf{r}_{1\perp}| \tag{3.72}$$

and

$$\mathfrak{N}_2^{PA}=-4im(1-2\beta)Q_1, \tag{3.73}$$

where Q_1 is the component of $\mathbf{Q}$ in the direction of $\mathbf{r}_1$, with $\mathbf{Q}$ given by (3.22).

When the polarization of the axial-vector particle is transverse and perpendicular to the scattering plane, we have

$$\mathfrak{N}_1^{PA}=0, \tag{3.74}$$

$$\mathfrak{N}_2^{PA}=2im(1-2\beta)q_{\perp2}, \tag{3.75}$$

where $q_{\perp2}$ is the component of $\mathbf{q}_\perp$ in the direction perpendicular to the scattering plane.

The impact factor $\mathfrak{g}^{PA}$ is given by (3.23) together with (3.71)–(3.75).

F. Scalar to Axial Vector

Consider now the impact factor $\mathfrak{g}^{SA}$, where the scalar particle and the axial-vector particle couple to the fermion field by 1 and $i\gamma_5\gamma_j$, respectively. Then we have

$$\begin{aligned}
N_1^{SA}&=i\,\mathrm{Tr}[(\boldsymbol{p}+\boldsymbol{r}_1+m)\boldsymbol{r}_3(\boldsymbol{p}+\boldsymbol{q}+m)\boldsymbol{r}_3(\boldsymbol{p}-\boldsymbol{r}_1+m)\gamma_5\gamma_j(\boldsymbol{p}+\boldsymbol{r}_2+m)]\\
&\sim 2(r_3\cdot p)i\,\mathrm{Tr}[(\boldsymbol{p}+\boldsymbol{r}_1+m)\boldsymbol{r}_3(\boldsymbol{p}-\boldsymbol{r}_1+m)\gamma_5\gamma_j(\boldsymbol{p}+\boldsymbol{r}_2+m)]\\
&=2(r_3\cdot p)im\{\mathrm{Tr}[\boldsymbol{r}_3(\boldsymbol{p}-\boldsymbol{r}_1)\gamma_5\gamma_j(\boldsymbol{p}+\boldsymbol{r}_2)]+\mathrm{Tr}[(\boldsymbol{p}+\boldsymbol{r}_1)\boldsymbol{r}_3\gamma_5\gamma_j(\boldsymbol{p}+\boldsymbol{r}_2)]+\mathrm{Tr}[(\boldsymbol{p}+\boldsymbol{r}_1)\boldsymbol{r}_3(\boldsymbol{p}-\boldsymbol{r}_1)\gamma_5\gamma_j]\}\\
&=4(r_3\cdot p)mi\,\mathrm{Tr}\gamma_5\gamma_j(2\boldsymbol{p}+\boldsymbol{r}_2)\boldsymbol{r}_1\boldsymbol{r}_3\\
&\sim -16(r_3\cdot p)mi[(2p+r_2)\cdot r_3]|\mathbf{r}_{1\perp}|\delta_{j2}
\end{aligned} \tag{3.76}$$

and

$$\begin{aligned}
N_2^{SA}&=i\,\mathrm{Tr}[(-\boldsymbol{p}-\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1+m)\boldsymbol{r}_3(-\boldsymbol{p}+\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}-\tfrac12\boldsymbol{r}_1+m)\\
&\qquad\times\gamma_5\gamma_j(-\boldsymbol{p}+\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1+m)\boldsymbol{r}_3(-\boldsymbol{p}-\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1+m)]\\
&\sim 2i[(-p-\tfrac12r_2)\cdot r_3]\,\mathrm{Tr}[(-\boldsymbol{p}+\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1+m)\gamma_5\gamma_j\\
&\qquad\times(-\boldsymbol{p}+\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1+m)\boldsymbol{r}_3(-\boldsymbol{p}-\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1+m)]\\
&\quad-2i[(-p+\tfrac12r_2)\cdot r_3]\,\mathrm{Tr}[(-\boldsymbol{p}+\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1+m)\gamma_5\gamma_j\\
&\qquad\times(-\boldsymbol{p}+\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1+m)\boldsymbol{r}_3(-\boldsymbol{p}-\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1-m)]\\
&=2im[(-p-\tfrac12r_2)\cdot r_3]\{\mathrm{Tr}[\gamma_5\gamma_j(-\boldsymbol{p}+\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1)\boldsymbol{r}_3(-\boldsymbol{p}-\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1)]\\
&\qquad+\mathrm{Tr}[(-\boldsymbol{p}+\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1)\gamma_5\gamma_j(\boldsymbol{q}+\boldsymbol{r}_1)\boldsymbol{r}_3]\}\\
&\quad-2im[(-p+\tfrac12r_2)\cdot r_3]\{\mathrm{Tr}[\gamma_5\gamma_j(-\boldsymbol{p}+\tfrac12\boldsymbol{q}+\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1)\boldsymbol{r}_3(-\boldsymbol{p}-\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2+\tfrac12\boldsymbol{r}_1)]\\
&\qquad+2\,\mathrm{Tr}[(-\boldsymbol{p}+\tfrac12\boldsymbol{q}-\tfrac12\boldsymbol{r}_2-\tfrac12\boldsymbol{r}_1)\gamma_5\gamma_j(\boldsymbol{p}-\tfrac12\boldsymbol{r}_1)\boldsymbol{r}_3]\}\\
&=-2im[(-p-\tfrac12r_2)\cdot r_3]\,\mathrm{Tr}[\gamma_5\gamma_j(-2\boldsymbol{p}-\boldsymbol{r}_1)\boldsymbol{r}_3(\boldsymbol{q}+\boldsymbol{r}_1)]\\
&\qquad-2im[(-p+\tfrac12r_2)\cdot r_3]\,\mathrm{Tr}[\gamma_5\gamma_j(-2\boldsymbol{p}+\boldsymbol{r}_1)\boldsymbol{r}_3(-\boldsymbol{q}+\boldsymbol{r}_1)]\\
&\sim -4im[(-2p-r_2)\cdot r_3](-2p\cdot r_3)[(q_{\perp1}+|r_{1\perp}|)\delta_{j2}-\delta_{j1}q_{\perp2}]\\
&\qquad-4im[(-2p+r_2)\cdot r_3](-2p\cdot r_3)[(-q_{\perp1}+|r_{1\perp}|)\delta_{j2}+\delta_{j1}q_{\perp2}].
\end{aligned} \tag{3.77}$$

In deriving (3.76) and (3.77), we have made use of the following relations:

$$\mathrm{Tr}\gamma_5\boldsymbol{ABCD}=-\mathrm{Tr}\gamma_5\boldsymbol{BACD},$$
$$\mathrm{Tr}\gamma_5\boldsymbol{AACD}=0,$$

and, when $\boldsymbol{C}$ and $\boldsymbol{D}$ are both transverse vectors,

$$\mathrm{Tr}\gamma_5\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{CD}\sim(r_2\cdot r_3)\,\mathrm{Tr}\gamma_1\gamma_2\boldsymbol{CD}.$$

From (3.76) and (3.11), we get

$$\mathfrak{N}_1^{SA}=-4mi\beta(1-2\beta)|\mathbf{r}_{1\perp}|\delta_{j2}. \tag{3.78}$$

From (3.77) and (3.21), we get

$$\mathfrak{N}_2^{SA}=2im(1-2\beta)(1-\beta)[(q_{\perp1}+|\mathbf{r}_{1\perp}|)\delta_{j2}-q_{\perp2}\delta_{j1}] -2im(1-2\beta)\beta[(-q_{\perp1}+|\mathbf{r}_{1\perp}|)\delta_{j2}+q_{\perp2}\delta_{j1}]. \tag{3.79}$$

Thus, if the polarization of the axial-vector particle is transverse and in the scattering plane, we have

$$\mathfrak{N}_1^{SA}=0, \tag{3.80}$$

$$\mathfrak{N}_2^{SA}=-2im(1-2\beta)q_{\perp2}. \tag{3.81}$$

Note that apart from a minus sign, (3.80) and (3.81) are identical to (3.74) and (3.75).

If the polarization of the axial vector is transverse and perpendicular to the scattering plane, we have

$$\mathfrak{N}_1^{SA} = -4mi\beta(1-2\beta)|\mathbf{r}_{1\perp}|, \quad (3.82)$$

$$\mathfrak{N}_2^{SA} = 4mi(1-2\beta)Q_1. \quad (3.83)$$

Note that, apart from a minus sign, (3.82) and (3.83) are identical to (3.72) and (3.73).

If the polarization of the axial-vector particle is longitudinal, we have

$$\mathfrak{N}_1^{SA} = \mathfrak{N}_2^{SA} = 0. \quad (3.84)$$

The impact factor $\mathcal{G}^{SA}$ is given by (3.23) together with (3.80)–(3.84).

4. DISCUSSION

A. Selection Rules

In the preceding section we have calculated six inelastic impact factors. In all of the cases discussed, the impact factor does not vanish when the masses of the initial and the final particles are unequal. Thus the mass can change during a diffractive process. In case E of the preceding section, the spins of the incoming and the outgoing particles are unequal, while in case F of the preceding section, both the spin and the parity of the incoming and the outgoing particles are unequal. Thus the spin and the parity can both change during a diffractive process.

On the other hand, the charge conjugation quantum number C cannot change in a diffractive process. This is because our diffraction mechanism is through the exchange of two vector mesons, which has $C=1$. This further implies that charge, strangeness, isotopic spin, baryon number, and G parity cannot change in a diffractive process. More generally, $\mathcal{G}^{aa'}$ vanishes only if the system aa' may not have the same quantum numbers as those of a system of two vector mesons.

In all of the six cases considered in the preceding section, the impact factor satisfies

$$\mathcal{G}^{aa'}(\mathbf{r}_1, \pm\mathbf{r}_1) = 0. \quad (4.1)$$

Equation (4.1) had already been established for the photon impact factor in quantum electrodynamics.[2] We can also easily check that all of the impact factors are even functions of $\mathbf{q}_\perp$.

In the forward direction $\mathbf{r}_1=0$, both $\mathcal{G}^{VV}$ and $\mathcal{G}^{AA}$ vanish for transverse to longitudinal and for longitudinal to transverse. This will be shown to hold in all perturbation orders. Denote any of these two impact factors by $\mathcal{G}$ and write

$$\mathcal{G} = \mathcal{G}_{\mu\nu}(\epsilon_a)_\mu(\epsilon_{a'})_\nu, \quad (4.2)$$

where $(\epsilon_a)_\mu$ is the μth component of the polarization vector ϵ_a, etc. Since in the forward direction $\mathbf{r}_{1\perp}=0$ we have $q_\perp \cdot r_2 = q_\perp \cdot r_3 = 0$, the most general form for $\mathcal{G}_{\mu\nu}$ is

$$\mathcal{G}_{\mu\nu} = A\delta_{\mu\nu} + Br_{2\mu}r_{2\nu} + Cq_{\perp\mu}q_{\perp\nu}, \quad (4.3)$$

where A, B, and C can be functions of $\mathbf{q}_\perp^2$ only, and are therefore even functions of $\mathbf{q}_\perp$. There are no terms like $r_{2\mu}q_{\perp\nu}$ in (4.3), as $\mathcal{G}_{\mu\nu}$ must be an even function of $\mathbf{q}_\perp$. All terms in (4.3) vanish after being multiplied by $(\epsilon_A)_\mu(\epsilon_{A'})_\nu$, if one of the polarization vectors is transverse and the other one longitudinal. Similar arguments can be used to show that $\mathcal{G}^{SA}$ and $\mathcal{G}^{PA}$ both vanish in the forward direction $\mathbf{r}_{1\perp}=0$ if the polarization of the axial-vector particle is transverse. It is easy to check that the impact factors in cases E and F of the preceding section indeed satisfy this condition. This means that the helicity cannot change by one unit when $\mathbf{r}_{1\perp}=0$. However, when impact factors higher in the hierarchy are considered and multiparticle intermediate states contribute to the diffractive process,[5,7] this rule may not be valid.

Although $\mathcal{G}^{SA}$ vanishes at $\mathbf{r}_{1\perp}=0$ for all polarizations of the axial-vector particle, this is not true of $\mathcal{G}^{PA}$. More precisely, if the polarization of the axial-vector particle is longitudinal, the impact factor $\mathcal{G}^{PA}$, as given by (3.23) and (3.71), does not vanish at $\mathbf{r}_{1\perp}=0$. We emphasize that this statement holds independent of the mass ratio for the pseudoscalar and the axial-vector particles. This nonvanishing of $\mathcal{G}^{PA}$ in the forward direction is in disagreement with the droplet model.[8] This disagreement must be attributed to the assumption in the droplet model that at high energies the elementary interaction is spin-independent. In our view, at high energies the simplifying features do not come from any change in the elementary interaction, which remains spin-dependent. Accordingly, the considerations of Byers and Frautschi[9] on the effect of mass change is of no relevance here.

It is also interesting to note that $\mathcal{G}^{SA}$ and $\mathcal{G}^{PA}$ both vanish if the fermion mass m is equal to zero.

B. Reversed Processes

The diagrams for the impact factor $\mathcal{G}^{aa'}$ are the same as those for $\mathcal{G}^{a'a}$ after r_2 is replaced by $-r_2$. Therefore

$$\mathcal{G}^{aa'} = \mathcal{G}^{a'a}(r_2 \to -r_2). \quad (4.4)$$

Thus, for example, $\mathfrak{N}_1^{AP}$ and $\mathfrak{N}_2^{AP}$ are, respectively, equal to the right-hand sides of (3.69) and (3.70) with $r_2 \to -r_2$.

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