lines connected to both the inside and the outside of the loop. We have developed techniques for computing the infinite products required in these examples, techniques which obviously can be extended to cases with more external lines. The situation for more than one loop, that is, additional points in the interior of the dual surface, is certainly more complicated, and additional tricks are undoubtedly required.

In each case we found that there was a corner of the integration hypercube where all of the infinite number of factors simultaneously went to 1, and that divergences in the infinite products caused the integral to be undefined. We can only speculate as to how this difficulty might be eventually resolved. One possibility is that factorization constraints may lead to the inclusion of additional factors[8] in the integrands to assure convergence. Another is that these diagrams, redefined with the additional pole, may combine with other diagrams also containing poles (possibly of higher order) arising in the same manner. In any case, the essential point of this paper is to show that the infinite products arising in at least some of the diagrams can be computed.

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Particles with Integral Spin and $B \neq 0$

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Constructing particles out of quarks (via infinite-component fields), we show that if an "elementary" particle has integral spin, baryon number $B\neq 0$, and a symmetric spin-isospin wave function, then its electromagnetic charge form factor vanishes.

INTRODUCTION

In this paper we present a simple argument, based on quarks, showing that if an "elementary" particle has integral spin, baryon number $B\neq 0$, and a symmetric spin-isospin wave function, then it has a vanishing electromagnetic charge form factor. The particle, therefore, would not be detectable via its charge vertex. Thus we obtain insight into the question of why integral spin implies $B=0$ for any given elementary particle.

We show explicitly that if an elementary particle is made up of an even number of symmetrized quarks (i.e., if it has integral spin, $B=2/3$, $4/3$, or $6/3$, etc., and a symmetric spin-isospin wave function), then the electromagnetic charge form factor of the particle vanishes. The model used is the infinite-component field model of Cocho *et al.*[1]

In Sec. I we set up the problem and review the infinite-component field formalism. The representations are discussed, and the symbols defined. In Sec. II we define and discuss the functions entering into a calculation of the electromagnetic charge form factor of a particle. In Sec. III we show that the electromagnetic charge form factor vanishes identically whenever the strongly interacting particle is made up of an even number of quarks.

I. ELECTROMAGNETIC FORM FACTORS AND INFINITE-COMPONENT FIELDS

In this section, we discuss the framework for calculating electromagnetic form factors via infinite-component fields.[2] The vertex we wish to consider is shown in Fig. 1: a strongly interacting particle interacts with a photon. The strongly interacting particle is represented by a wave function in momentum space, and we wish to compute the electromagnetic charge form factor associated with the process of Fig. 1.

Ordinarily, one would represent a strongly interacting particle with spin J by a wave function that transformed according to spin J. In the infinite-component field

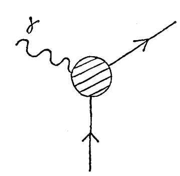

FIG. 1. Electromagnetic vertex. The solid lines represent the strongly interacting particle.

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[1] G. Cocho, C. Fronsdal, Harun Ar-Rashid, and R. White, Phys. Rev. Letters **17**, 275 (1966).

[2] A summary of work using infinite-component fields in connection with form factors is listed in C. Fronsdal, Phys. Rev. **182**, 1564 (1969).

formalism, which is the formalism used in the present paper, the wave function is more general and contains many different spins instead of just one spin. To represent a specific particle with a given spin J, one then just takes the appropriate projection of the more general wave function.

The concept of spin J is next generalized to a particle transforming according to an irreducible representation of $SU(n)$. The general wave function will now contain many representations of $SU(n)$. To represent a given particle, one takes the appropriate projection of this general wave function.

One choice for the more general wave function is a wave function with N dotted (upper) spinor indices and $N+k$ undotted (lower) spinor indices[1]:

$$\psi_{\alpha_1,\cdots,\alpha_{N+k}}{}^{\dot\beta_1,\cdots,\dot\beta_N}(p)\,. \tag{1}$$

This wave function represents a particle of momentum p; it contains many representations of $SU(n)$, where $n=2k$. The **56** representation of $SU(6)$, for example, would correspond to a certain projection of this wave function that is completely *symmetric* in both upper and lower indices, with k set equal to $6/2=3$.

For a complete discussion of the representations one can use, the reader is referred to a report by Fronsdal.[3] A different representation one might try is, for example,[3]

$$\psi_{\alpha_1,\cdots,\alpha_M,\dot\beta_1,\cdots,\dot\beta_M}{}^{\kappa_1,\cdots,\kappa_M,\dot\lambda_1,\cdots,\dot\lambda_1,\cdots,\dot\lambda_M}(p)\,. \tag{2}$$

Still another approach[4] [for integral spin and without $SU(n)$] is to use a wave function with four-vector indices, instead of spinor indices as in (1) and (2):

$$\psi_{\mu_1,\cdots,\mu_N}(p)\,. \tag{3}$$

In this paper we shall use the choice (1). The actual representation used is that projection of (1) which corresponds to a completely symmetrized (in spin and isospin) state of k quarks. Thus for $k=3$, we would be dealing with the **56** representation of $SU(6)$, which contains the $N^*(1236)$ decuplet and the nucleon octet. If there were a spin-1 particle made up of two *quarks* (no antiquarks; i.e., the particle would have $B=\frac{2}{3}$), this particle would be included in our considerations. In other words, we are including all "elementary" particles made of k quarks, with the quarks symmetrized in spin-isospin variables. We shall proceed to show that if the number k of quarks is *even*, then the charge vertex of Fig. 1 vanishes. The vanishing of this vertex of Fig. 1 is consistent with the experimental fact that there are no observed elementary particles made up of an even number of quarks. That is, there are no experimental elementary particles with baryon number $B=2/3$, $4/3$, $6/3$, etc. Thus our result is consistent with the fact that for elementary particles with integral spin, $B=0$.]

[3] C. Fronsdal, Trieste Report No. IC/66/51 (unpublished).
[4] C. Fronsdal, Phys. Rev. **156**, 1653 (1967).

The projection of (1) corresponding to our completely symmetrized state of k quarks turns out to be[1,3]

$$\psi_{\alpha_1,\cdots,\alpha_{N+k}}{}^{\dot\beta_1,\cdots,\dot\beta_N}(p)\rightarrow \times m^{-N} S p_{\alpha_1}{}^{\dot\beta_1}\cdots p_{\alpha_N}{}^{\dot\beta_N}\psi_{\alpha_{N+1},\cdots,\alpha_{N+k}}(p)\,, \tag{4}$$

where m is the mass of the strongly interacting particle, p is its momentum, and $\psi_{\alpha_{N+1},\cdots,\alpha_{N+k}}(p)$ is a wave function with k indices and corresponding to momentum p. $p_\alpha{}^{\dot\beta}$ is a $2k\times2k$ matrix defined as

$$p_\alpha{}^{\dot\beta}=(p_0-\mathbf{p}\cdot\boldsymbol{\sigma})_A{}^B\delta_a{}^b\,,$$

where $(p_0,\mathbf{p})$ is the four-momentum of the strongly interacting particle, $\boldsymbol{\sigma}$ is the set of 2×2 Pauli spin matrices, and a and b run from 1 to k. The symbol S stands for symmetrization in the indices $\alpha_1,\cdots,\alpha_{N+k}$.

Now the charge part of the electromagnetic vertex of Fig. 1 is given, in Born approximation, by[1]

$$\text{vertex}=\frac{(p+p')_\mu}{2m}\bar\psi_{\dot\beta_1,\cdots,\dot\beta_N}{}^{\alpha_1,\cdots,\alpha_{N+k}}(p') \times Q\psi_{\alpha_1,\cdots,\alpha_{N+k}}{}^{\dot\beta_1,\cdots,\dot\beta_N}(p)\,,$$

where $p=(p_0,\mathbf{p})$ and $p'=(p'_0,\mathbf{p}')$ are the four-momenta of the incoming and outgoing strongly interacting particle, respectively, and Q is the charge operator. Using Eq. (4), we can reduce this expression:

$$\psi_{\alpha_1,\cdots,\alpha_{N+k}}{}^{\dot\beta_1,\cdots,\dot\beta_N}(p)\rightarrow \times m^{-N} S p_{\alpha_1}{}^{\dot\beta_1}\cdots p_{\alpha_N}{}^{\dot\beta_N}\psi_{\alpha_{N+1},\cdots,\alpha_{N+k}}(p)\,,$$

$$\bar\psi_{\dot\beta_1,\cdots,\dot\beta_N}{}^{\alpha_1,\cdots,\alpha_{N+k}}(p')\rightarrow m^{-N} S\bar\psi^{\alpha_1,\cdots,\alpha_k}(p')p'_{\dot\beta_1}{}^{\alpha_{k+1}}\cdots p'_{\dot\beta_N}{}^{\alpha_{N+k}}\,,$$

$$\text{vertex}\rightarrow\frac{(p+p')_\mu}{2m}m^{-2N} \times(S\bar\psi^{\alpha_1,\cdots,\alpha_k}(p')p'_{\dot\beta_1}{}^{\alpha_{k+1}}\cdots p'_{\dot\beta_N}{}^{\alpha_{k+N}}) \times Q(Sp_{\alpha_1}{}^{\dot\beta_1}\cdots p_{\alpha_N}{}^{\dot\beta_N}\psi_{\alpha_{N+1},\cdots,\alpha_{N+k}}(p))\,. \tag{5}$$

Here

$$p_\alpha{}^{\dot\beta}=(p_0-\mathbf{p}\cdot\boldsymbol{\sigma})_A{}^B\delta_a{}^b \quad\text{and}\quad p'_{\dot\alpha}{}^{\beta}=(p'_0+\mathbf{p}'\cdot\boldsymbol{\sigma})_A{}^B\delta_a{}^b\,.$$

This vertex has, in fact, been calculated for the case of a proton [$SU(6)$],[1] and the result is fairly simple. In this paper we shall evaluate the vertex (5) for the case of $SU(n)$, where $n=2k$, and where k *is even* (i.e., the number of quarks is even). We shall, in fact, show that when k is even, the vertex (5) vanishes.

II. DEFINITIONS

The charge part of the electromagnetic vertex of Fig. 1 has been written in Eq. (5). This expression is fairly involved, but it can be written in a simpler form,[1] namely,

$$\text{vertex}=\frac{(p+p')_\mu}{2m}\bar{\psi}^{\alpha_1,\cdots,\alpha_k}(p')$$
$$\times[f_0\delta_{\alpha_1}{}^{\alpha'_1}\cdots\delta_{\alpha_k}{}^{\alpha'_k}+f_1\delta_{\alpha_1}{}^{\alpha'_1}\cdots\delta_{\alpha_{k-1}}{}^{\alpha'_{k-1}}T_{\alpha_k}{}^{\alpha'_k}+\cdots$$
$$+f_kT_{\alpha_1}{}^{\alpha'_1}\cdots T_{\alpha_k}{}^{\alpha'_k}]Q\psi_{\alpha'_1,\cdots,\alpha'_k}(p)\,,\quad(6)$$

where $T_\alpha{}^{\alpha'}=m^{-2}p_\alpha{}^k p'_k{}^{\alpha'}$, and the f_j's are functions of $p\cdot p'$. Cocho *et al.*[1] then derive the result (the proof is straightforward but very tedious) that

$$f_j=\binom{N+k}{k}^{-1}\sum_{i=j}^{k}\binom{k}{i}\binom{i}{j}(-1)^{i-j}[{}^kQ_{N-i+j}{}^{(i)}],\quad(7)$$

where the Q functions are defined by

$${}^kQ_N{}^{(i)}=(y-y^{-1})^{-1}\sum_{j=0}^{N-1}(y^{N-j}-y^{-N+j})\,{}^kQ_j{}^{(i-1)}\,,$$
$${}^kQ_N{}^{(0)}\equiv{}^kQ_N=m^{-2N}[S(pp')_{\alpha_1}{}^{\beta_1}\cdots(pp')_{\alpha_N}{}^{\beta_N}]$$
$$\times\delta_{\beta_1}{}^{\alpha_1}\cdots\delta_{\beta_N}{}^{\alpha_N}.\quad(8)$$

Here

$$y=m^{-2}\{p\cdot p'+[(p\cdot p')^2-m^4]^{1/2}\}\,.$$

Thus we see that if we knew each function ${}^kQ_{N-i+j}{}^{(i)}$, then we could straightforwardly find each f_j by means of Eq. (7). Equation (6) could then be used to compute the complete charge vertex. In particular, if each ${}^kQ_{N-i+j}{}^{(i)}$ were to vanish, this would immediately imply that the charge vertex of Fig. 1 vanishes. In the following section we show that for k even, each ${}^kQ_{N-i+j}{}^{(i)}$ does indeed vanish identically. Thus the electromagnetic charge form factor of Fig. 1 vanishes when the strongly interacting elementary particle is composed of an even number of symmetrized quarks.

III. EVEN NUMBER OF QUARKS IMPLIES VERTEX VANISHES

In this section we show that the charge part of the electromagnetic vertex of Fig. 1 vanishes for k (the number of quarks) even. As discussed in the preceding section, it is sufficient to show that

$${}^kQ_{N-i+j}{}^{(i)}\equiv 0\,,$$

where the function ${}^kQ_{N-i+j}{}^{(i)}$ is defined by Eq. (8).

Now, starting with the definition of Eq. (8), it is possible to derive a more simple expression for ${}^kQ_{N-i+j}{}^{(i)}$:

$${}^kQ_{N'}{}^{(i)}=[(k+i-1)!]^{-1}\left(\frac{d}{d\omega}\right)^{k+i-1}\left(\frac{y^{N'+k}-y^{-N'-k}}{y-y^{-1}}\right),\quad(9)$$

where $\omega=y+y^{-1}+2$, and

$$y=m^{-2}\{p\cdot p'+[(p\cdot p')^2-m^4]^{1/2}\}\,.$$

This expression [Eq. (9)] has been derived for general k [5]; it has also been previously derived for $k=1$, 2, and 3.[1,6] The proof of Eq. (9) is rather messy and will not be discussed here; see, instead, Refs. 1, 5, and 6.

We shall instead simply use Eq. (9) to show that ${}^kQ_{N-i+j}{}^{(i)}=0$ when k is even. Now unitarity and the existence of a parity operator require $N=-\frac{3}{2}k$.[3] But then Eq. (9) says that

$${}^kQ_{N-i+j}{}^{(i)}=[(k+i-1)!]^{-1}\left(\frac{d}{d\omega}\right)^{k+i-1}$$
$$\times\left\{\frac{y^{-k/2-i+j}-y^{k/2+i-j}}{y-y^{-1}}\right\}.\quad(10)$$

We restrict ourselves to k even. Then the quantity in curly brackets in Eq. (10) can easily be shown to be a polynomial in ω ($\omega=y+y^{-1}+2$), of order $\frac{1}{2}k+i-j-1$ (note that $\frac{1}{2}k$ is an integer). But then $(d/d\omega)^{k+i-1}$ acting on this quantity must vanish, since $k+i-1>\frac{1}{2}k+i-j-1$. Thus, by Eq. (10),

$${}^kQ_{N-i+j}{}^{(i)}\equiv 0\quad(k\text{ even}).\quad(11)$$

This result then trivially [via Eqs. (6) and (7)] implies that the electromagnetic charge vertex of Fig. 1 vanishes, for k even.

Thus we have shown that in this model an "elementary" particle made of an even number of quarks does not have any electromagnetic charge coupling; an elementary particle with integral spin, $B=2/3$, $4/3$, $6/3$, or $8/3$, etc., and a symmetric spin-isospin wave function in this model does not have any charge coupling to a photon. In this fashion we obtain insight into the question of why integral spin implies $B=0$ for any given elementary particle.

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