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Propagation of Electromagnetic Waves in a Degenerate Neutrino Sea

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The propagation of electromagnetic waves in a Fermi sea of neutrinos is investigated on the basis of the photon-neutrino weak-coupling theory and is compared with the result obtained by Royer on the basis of the $(e\nu)(e\nu)$ coupling. It is shown that a photon propagating through the neutrino sea satisfies a dispersion relation of the form $\omega^2=\omega_0^2+k^2c^2$, where $(\hbar\omega_0)^2\approx 6.2\times10^{-23}\, E_F^2$, E_F being the Fermi energy of the sea. Finally, some remarks are made as to the possibility of the destruction of a photon when propagating through the sea due to the process $\gamma+\nu\rightarrow\nu+\nu+\bar{\nu}$. It is found that the mean free path corresponding to this process is $\approx 10^{69}$ cm, much larger than the radius of the universe.

It is generally believed that there is a Fermi sea of neutrinos in the universe[1] and the magnitude of the Fermi energy E_F is related to various cosmological theories. To establish a meaningful limit of the Fermi energy, Royer[2] has investigated the propagation of electromagnetic waves in a degenerate neutrino sea, considering the polarization effect arising out of the $(e\nu)(e\nu)$ coupling. However, because of the small magnitude of the weak-coupling constant, the term introduced in the Maxwell's equation due to this polarization is of unobservable effect. But Royer made the interesting observation that the magnetic field from a localized static current should have a term which drops off as $1/r$ instead of the usual $1/r^2$, though the experimental observation of this effect is impossible at present. In view of this anomalous result, we shall consider here the propagation of electromagnetic waves in a Fermi sea of neutrinos on the basis of the photon-neutrino weak-coupling theory as proposed by Bandyopadhyay,[3] and compare the result with that obtained by Royer.

According to the photon-neutrino coupling theory, the polarization effect in a neutrino sea arises mainly from the process shown in Fig. 1. The effect is similar to the propagation of an electromagnetic wave in a plasma as investigated by Tsytovich.[4] Following Tsytovich[4] and Adams *et al.*,[5] we write the modified Maxwell's equation as follows:

$$(k_\lambda k^\lambda \delta_{\mu\nu}-k_\mu k_\nu-4\pi\Pi_{\mu\nu})A_\nu=4\pi j_\mu\,, \qquad (1)$$

where A_ν is the electromagnetic four-potential and j_μ is the external four-current. $\Pi_{\mu\nu}$ is the polarization tensor, and gauge invariance implies that

$$k_\mu\Pi_{\mu\nu}=0\,. \qquad (2)$$

$\Pi_{\mu\nu}$ is given by the expression

$$\Pi_{\mu\nu}=\frac{ig^2}{(2\pi)^4}\int \mathrm{Tr}\gamma_\mu(1+\gamma_5) \times G(\mathbf{p}+\mathbf{k},\,\omega+\lambda)\gamma_\nu(1+\gamma_5)G(\mathbf{p},\lambda)d\mathbf{p}d\lambda\,. \qquad (3)$$

Here g is the photon-neutrino weak-coupling constant. The Green's function for the propagation of a neutrino is modified by the neutrino sea through which it moves.

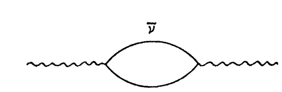

FIG. 1. Feynman diagram that contributes to the dielectric constant in a neutrino sea.

[1] S. Weinberg, Phys. Rev. **128**, 1457 (1962).
[2] J. Royer, Phys. Rev. **174**, 1719 (1968).
[3] P. Bandyopadhyay, Phys. Rev. **173**, 1481 (1968); Nuovo Cimento **55A**, 367 (1968).
[4] V. N. Tsytovich, Zh. Eksperim. i Teor. Fiz. **40**, 1775 (1961) [English transl.: Soviet Phys.—JETP **13**, 1249 (1961)].
[5] J. B. Adams, M. A. Ruderman, and C. H. Woo, Phys. Rev. **129**, 1382 (1963).

We have

$$G(x)=\frac{i}{(2\pi)^4}\int_{(\alpha)} d^4p\,\exp(-ip_\lambda x_\lambda+i\mu t)\,\frac{1}{2E_\nu}$$
$$\times(E_\nu\gamma_4-\boldsymbol{\gamma}\cdot\mathbf{p})\frac{1-n^-(\mathbf{p})}{p_4+\mu-E_\nu+i\delta}+\frac{n^-(\mathbf{p})}{p_4+\mu-E_\nu-i\delta}$$
$$+(E_\nu\gamma_4+\boldsymbol{\gamma}\cdot\mathbf{p})\frac{1-n^+(\mathbf{p})}{p_4+\mu+E_\nu-i\delta}+\frac{n^+(\mathbf{p})}{p_4+\mu+E_\nu+i\delta}. \quad (4)$$

Here E_ν is the neutrino energy, δ is a positive infinitesimal, μ is the chemical potential, and $n^-(\mathbf{p})$ [$n^+(\mathbf{p})$] is the distribution of neutrinos [antineutrinos] in momentum space. Noting that the factor $(1+\gamma_5)$ in the trace (3) contributes a multiplicative factor 2 in the expression for $\Pi_{\mu\nu}$, we write

$$\Pi_{\mu\nu}=\frac{i2g^2}{(2\pi)^4}\int \mathrm{Tr}\gamma_\mu G(\mathbf{p}+\mathbf{k},\,\omega+\lambda)\gamma_\nu G(\mathbf{p},\lambda)d\mathbf{p}d\lambda. \quad (5)$$

Since the medium is isotropic and $\Pi_{\mu\nu}=\Pi_{\nu\mu}$, we can now write the polarization term as follows[4]:

$$\Pi_{\mu\nu}(\mathbf{k},\omega)=(1/4\pi)\omega^2(\epsilon^t-1)(P_T)_{\mu\nu}+(1/4\pi)k_\lambda k^\lambda(\epsilon^l-1)(P_L)_{\mu\nu}, \quad (6)$$

where

$$(P_T)_{\mu\nu}=\delta_{\mu\nu}-k_\mu k_\nu/|k|^2, \quad (7a)$$

$$(P_T)_{i4}=(P_T)_{44}=0, \quad (7b)$$

and

$$(P_L)_{\mu\nu}=e_\mu e_\nu,\quad e_\mu=(k_\lambda k^\lambda)^{-1/2}(\omega\hat{k},k). \quad (7c)$$

ϵ^t and ϵ^l are the dielectric constants. Substituting Eq. (6) in Eq. (1), and setting the external current equal to zero, the equation has two independent solutions, a longitudinal part satisfying the relation

$$\epsilon^l(\omega,\mathbf{k})=0, \quad (8)$$

and a transverse part satisfying

$$\omega^2\epsilon^t(\omega,\mathbf{k})=k^2. \quad (9)$$

From the dispersion relation (9), we can derive a relation of the form

$$\epsilon^t=1-\omega_0^2/\omega^2. \quad (10)$$

Here ω_0^2 is given by

$$\omega_0^2=8\pi g^2\int d^3p\frac{f(E_\nu)}{E_\nu}\left(1-\tfrac{1}{3}\frac{\mathbf{p}^2}{E_\nu}\right), \quad (11)$$

where $f(E_\nu)$ is the neutrino distribution function. For a degenerate neutrino gas at $T=0°$, we find

$$\omega_0^2=8g^2p_F^3/3\pi E_F. \quad (12)$$

Taking the photon-neutrino coupling constant

$$g=10^{-10}e,^3$$

we have

$$(\hbar\omega_0)^2\approx 6.2\times10^{-23}E_F^2. \quad (13)$$

So a photon propagating through a degenerate neutrino sea of Fermi energy E_F will appear to have a "mass" given by $\hbar\omega_0/c^2\approx 8\times10^{-12}E_F/c^2$.

The above result is to be compared with that obtained by Royer, where the polarization effect is taken to be due to a $(e\nu)(e\nu)$ coupling. In this case the polarization tensor is given by

$$\Pi_{\mu\nu}=\frac{1}{9\pi^4}e^2\frac{G}{\sqrt{2}}E_F^3\epsilon^{0\mu\nu\alpha}q_\alpha+O\left(\frac{q^2}{m_e^2}\right), \quad (14)$$

where G is the $(e\nu)(e\nu)$ coupling constant. Maxwell's equation is thus modified by the term $K\mathbf{B}$ in the expression for $\nabla\times\mathbf{B}$ ($\nabla\times\mathbf{B}=4\pi\mathbf{J}+K\mathbf{B}+\partial\mathbf{E}/\partial t$), where

$$K=(1/9\pi^4)e^2(G/\sqrt{2})E_F^3\approx10^{-27}E_F^3\ (E_F\ \text{in eV}).$$

One significant implication of Royer's result is that the magnetic field from a localized static current should have a term which drops off as $1/r$ instead of the usual $1/r^2$. But our above analysis shows that when the polarization effect is estimated on the basis of the photon-neutrino weak-coupling theory, no such anomalous term arises.

From the dispersion relation (10), we note that the existence of a degenerate Fermi sea of neutrinos will forbid any photon of energy E to propagate through the sea when $E<\hbar\omega_0\approx 8\times10^{-12}E_F$. If the neutrino energy density of the universe is taken to be equal to the universal matter density ($\approx10^{-29}$ g/cm^3), E_F is found to be 0.01 eV. Taking this value of E_F, we find the lower limit of energy for photon propagation, $E\approx 8\times10^{-14}$ eV.

Finally, we make certain remarks as to the possibility of a photon's being destroyed by the process $\gamma+\nu\to\nu+\nu+\bar{\nu}$ when passing through the Fermi sea of neutrinos. According to the photon-neutrino coupling theory, the process $\gamma+\nu\to\nu+\nu+\bar{\nu}$ is allowed and can be described by a diagram with three weak vertices. For a 1-eV photon, the cross section is $\sigma\approx10^{-76}$ cm^2. The number density of neutrinos is given by

$$n_\nu=(1/3\pi^2)(E_F/c\hbar)^3.$$

For $E_F=0.01$ eV, $n_\nu\approx 5\times10^6$/cm^3. So the mean free path for a 1-eV photon is given by $1/\sigma n\approx 2\times10^{69}$ cm. This is much larger than the radius of the universe ($\approx10^{28}$ cm). Thus the probability of a photon's being lost by the process $\gamma+\nu\to\nu+\nu+\bar{\nu}$ when passing through the neutrino sea is almost nil.