a strong energy dependence of the cross section. The experimental data are not resolved highly enough with respect to energy to show this trend. However, the effect is clearly evident both in experiment and in theory for reaction (2). More data on these reactions would be useful to decide this point. In conclusion, we can say that the two-meson-exchange contributions merit further study for processes where one-meson exchange is forbidden.

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Regge Classification of the Cascade Resonances

Richard M. Spector
Physics Department, Wayne State University, Detroit, Michigan 48202
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By using a previously developed dispersion technique, we investigate the Chew-Frautschi and the Imα-versus-s plots for the cascade resonances. The eight resonances are found to lie on three nearly linear plots on both graphs as was found to be the case previously for the baryon N_γ and Δ_δ sequences. Of the two types of resonances, probable and doubtful, all of the former are fitted (with no gaps on the trajectories) and none of the latter is needed. Severe self-consistency requirements are well satisfied by the two plots.

I. INTRODUCTION

Because of the improving experimental situations, more interest has developed recently in the Imα-versus-s plot for resonances.[1–6] The Chew-Frautschi (Reα-versus-s) plot has, of course, received attention for much longer. While most work has centered on the meson trajectories, one paper in particular[6] has done a study of the two best-established baryon sequences: the N_γ and Δ_δ.

This work reached a conclusion that the Imα-versus-s curves for the resonances were consistent with linearity and had a slope of 0.135 BeV^{-2}. The Chew-Frautschi plots were nearly linear, being only very slightly distorted by the dispersion integral used to calculate the Reα-versus-s curve from Imα. Some stringent self-consistency requirements are satisfied by these plots, and this lends confidence to the results.

There are a number of reasons for wishing to extend this technique to other baryon resonances. The most immediate is, of course, to enable one to classify the particle resonances themselves and offer suggestions for their appropriate spins and parities, most of which are still undetermined.

However, there are two other reasons for such investigations. While the baryon widths (and hence the imaginary part of α) definitely rise in essentially linear fashion, the meson widths may well rise and then fall sharply, thus forming a peaked Imα-versus-s curve.[2–4] In fact, the latest experimental evidence on high-spin meson resonances indicates they are very narrow.[7] While this does not preclude linear plots (see Ref. 1), it is more compatible with a peaked plot of Imα versus s. If the meson widths do give rise to such peaked curves, it is of great interest to gather more evidence on bayon curves and see if they peak (like the mesons) or are linear (like the N_γ and Δ_δ). A difference in baryon and meson behavior would be very puzzling.

Recently, Ball and Zachariasen[8] have suggested the interesting possibility that the trajectory function $\alpha(t)$ is complex for $t<0$. One of the experimental consequences of this is that the Imα-versus-s plot of resonances should be a linear plot near the Imα threshold. Any plot such as we do below is therefore a partial substantiation of the Ball-Zachariasen conjecture.

In the next section we briefly review the technique as developed in Ref. 1 and apply it to a recent compilation of data on the cascade resonances. In the last section we discuss the results of the plots.

II. TECHNIQUE AND CALCULATION

In Ref. 1 it was shown that for linearly rising Imα curves we could write

$$\alpha(s)=\tau_1+\tau_2+\frac{s(s-z)}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im}\alpha(s')}{s'(s'-z)(s'-s)}ds', \quad (1)$$

with τ_1 and τ_2 real. (We assume that the trajectories are functions of s, not $\sqrt{s}$.) A very important threshold

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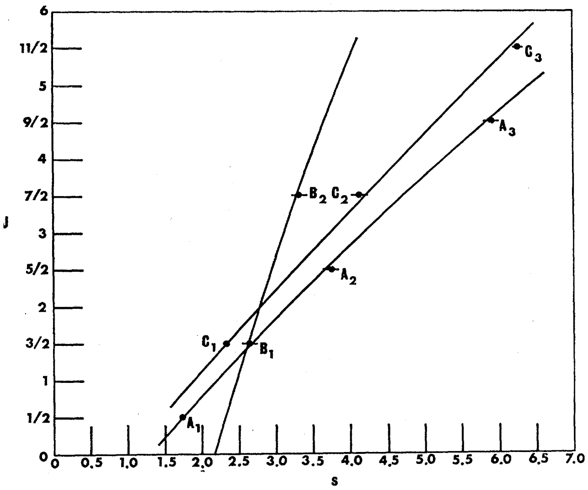

FIG. 1. Chew-Frautschi plots (s in units of BeV^2) of the series A, B, and C of Table I as plotted from Eq. (5). The lines are not straight, as may be seen by holding a straight edge against them.

condition exists which requires

$$\mathrm{Im}\alpha(s) \approx (s-s_0)^{\mathrm{Re}\alpha(s_0)} \tag{2a}$$

or

$$\mathrm{Im}\alpha(s) \approx (s-s_0)^{\mathrm{Re}\alpha(s_0)+1}. \tag{2b}$$

Equation (2a) holds for positive-parity resonances of a spinless pseudoscalar particle and a spin-$\frac{1}{2}$ particle, while Eq. (2b) holds for negative-parity resonances. If the $\mathrm{Im}\alpha$ plots are to be linear, then at the energy for which $\mathrm{Im}\alpha$ is zero the $\mathrm{Re}\alpha(s)$ curve must be unity or zero for (2a) or (2b), respectively. This is a severe constraint which was very well satisfied by the N_γ and Δ_δ resonances.

In addition, we need the formula relating the experimental width to $\mathrm{Im}\alpha(s)$. This is

$$\Gamma = (2\,\mathrm{Im}\alpha\,\mathrm{Re}\alpha')/|\alpha'|^2, \tag{3}$$

$$\mathrm{Im}\alpha \approx \tfrac{1}{2}\Gamma\,\mathrm{Re}\alpha'. \tag{4}$$

The prime means derivative with respect to s and the Γ is the width in s, not in the mass. Since the worst case $(\mathrm{Im}\alpha'/\mathrm{Re}\alpha')^2 \approx 0.02$, the approximation in Eq. (4) is very good.

TABLE I. Cascade data.

Label	Mass (BeV)	Width (MeV)	J^P Expt	J^P Theor
A_1	1.31	stable	$\frac{1}{2}+$	$\frac{1}{2}+$
A_2	1.937 ± 0.008	130 ± 16	?	$\frac{5}{2}+$
A_3	$2.43\ \pm0.02$	150_{-40}^{+60}	?	$\frac{9}{2}+$
B_1	1.635 ± 0.010	20 ± 10	?	$\frac{3}{2}-$
B_2	1.818 ± 0.006	45 ± 13	?	$\frac{7}{2}-$
C_1	1.530 ± 0.001	7.3 ± 1.7	$\frac{3}{2}+$	$\frac{3}{2}+$
C_2	2.030 ± 0.010	45_{-20}^{+40}	?	$\frac{7}{2}+$
C_3	2.500 ± 0.010	59 ± 27	?	$\frac{11}{2}+$

TABLE II. Parameters of the fits.

Series	λ (BeV^{-2})	z (BeV^2)	s_0 (BeV^2)	σ_1	σ_2 (BeV^{-2})
A	0.135	5	2.2	-1.16	0.98
B	0.22	3	2.18	-6.91	3.17
C	0.03	2	1.88	-1.05	1.09

Putting $\mathrm{Im}\alpha(s) = \lambda(s-s_0)$, we may write Eq. (1) as

$$\mathrm{Re}\alpha(s) = \sigma_1 + \sigma_2 s - \frac{\lambda}{\pi}(s-s_0)\ln\left|\frac{s-s_0}{z-s_0}\right|, \tag{5}$$

where some terms from the integral have been absorbed into the first two terms. We can now see the self-consistency circle. The value of $\mathrm{Im}\alpha(s)$ for a resonance depends, from Eq. (4), on its value of $\mathrm{Re}\alpha'(s)$. But this depends on λ, from Eq. (5). But λ is the slope of the $\mathrm{Im}\alpha(s)$ curve, which cannot be determined until $\mathrm{Im}\alpha(s)$ is known for the resonances. In addition, once a curve is plotted for $\mathrm{Im}\alpha$, the value of s_0 can be determined. This then requires $\mathrm{Re}\alpha(s_0)$ to be 0 or 1. If we must alter τ_2 in Eq. (5) to achieve this (the third term vanishes at $s=s_0$), then this affects $\mathrm{Re}\alpha'(s)$, which affects $\mathrm{Im}\alpha(s)$, which changes s_0.

Our technique is to fit a straight line through the resonances and determine σ_1 and σ_2 in Eq. (5). We then use $\mathrm{Re}\alpha'(s) = \sigma_2$ and plot the curve from Eq. (4). Picking out λ and s_0, we adjust σ_1 and σ_2 to make $\mathrm{Re}\alpha(s_0)$ equal 0 or 1 and calculate the third term in Eq. (5) using the best of a series of choices for z. We then go back with the adjusted $\mathrm{Re}\alpha(s)$ and recycle. Since λ is small, the procedure converges in one, two, or three cycles. Because of the general state of the experimental data, we have not tried to do a best fit in any statistical sense. Any small improvement that such a fit would yield is not worth the considerable effort in light of the present state of the data.

As can be seen from Fig. 1, the contribution of the integral in Eq. (5) is quite small. The additional free

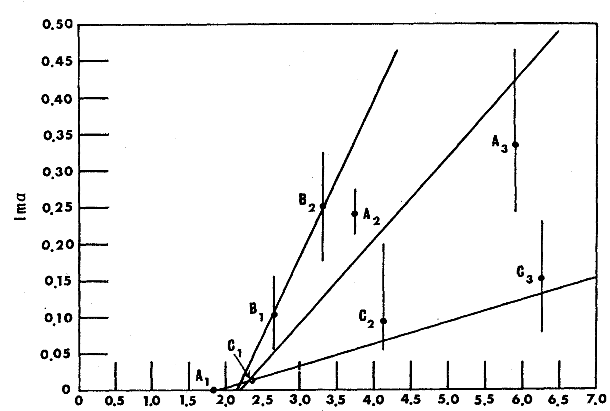

FIG. 2. Straight-line plots of $\mathrm{Im}\alpha$ versus s (in BeV^2) which determine s_0 from $\mathrm{Im}\alpha(s_0) = 0$.

parameter z is ineffective for other than very minor alterations in the curves. Thus, effectively there are only two free parameters σ_1 and σ_2 which must be varied to fit four points on the A and C curves and three on the B curve. Note that the requirement on $\mathrm{Re}\alpha(s_0)$ is, in effect, another point on the curve.

III. RESULTS AND DISCUSSION

In Table I we list all the cascade resonances used along with the known experimental information, as well as the assigned spin and parity based on Fig. 1. These experimental data are preliminary and not to be taken as final. However, for our purposes we do not need to be concerned with any latter small changes in the data, with the possible exception of the A_2 or A_3 width (see Ref. 9). Of course, most of these resonances are only poorly determined and the errors, especially on the widths, are not really hard figures. These seven resonances (and the stable cascade) are the only ones considered to be reasonably well established. There are a number of other possible resonances which are probably not resonances. We note that we have used *none* of the questionable resonances and *all* of the probable resonances and have no gaps in our trajectories.

In Fig. 1, the Chew-Frautschi plot, we see that the lines are nearly straight and two of them have slopes of nearly unity. In Fig. 2 we find that straight lines give a consistent fit. The parameters as determined by self-consistency are given in Table II.

We have not used exchange degeneracy and have three sequences beginning with $\frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{3}{2}^-$.

There is really little free play in adjusting these curves. We give four examples of this, since the difficulty of making these plots self-consistent is not immediately obvious.

(i) For the A curve, we have $\mathrm{Re}\alpha(s_0)=1$, since it has positive parity. $\mathrm{Re}\alpha(s_0)=0$ is wildly inconsistent because the value of s_0 for this to be true would be about 1.3 BeV. But this is *below* the stable cascade rest mass and $\mathrm{Im}\alpha(s)$ must go to zero at no lower than this mass.

(ii) If we supposed the C resonances to have negative parity, we would need $\mathrm{Re}\alpha(s_0)=0$ and s_0 would be about 1.1 BeV. But $\mathrm{Im}\alpha(1.1)=0$ would require an $\mathrm{Im}\alpha$ line which, passing through C_1, would lie well below the lowest end of the error bars of the C_2 and C_3 particles.

(iii) The $\mathrm{Im}\alpha$ curve for the A's misses A_2. If we alter the line only slightly to pass through the bottom of the A_2 error bar and the top of the A_3 error bar, we would have $s_0\approx 1.9$ and $\mathrm{Re}\alpha(s_0)\approx 0.7$. But if we assume, consistent with this, that $\mathrm{Im}\alpha(s)\approx\lambda(s-s_0)^{0.7}$, we find that the $\mathrm{Im}\alpha(s)$ curve comes in at a shallower angle and $s_0<0.7$. The situation rapidly deteriorates from the self-consistent point of view.[10]

(iv) At first glance, it might appear reasonable to put A_1, B_2, and C_3 on the same trajectory. This would form a reasonably smooth Chew-Frautschi plot, though with a much more pronounced curvature than our lines have. However, a look at Fig. 2 indicates that such a grouping would cause a radical shift in the shape of the $\mathrm{Im}\alpha$ curve. One would have to abandon entirely the idea of straight-line fits.

We conclude that the $\mathrm{Im}\alpha$-versus-s curves for the probable cascade resonances are consistent with being linear and that spin-parity assignments as given in Table I are likely.

ACKNOWLEDGMENT

I would like to thank Professor Yodh for making information available on the experimental situation before publication.

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[10] Up well away from s_0, the $\mathrm{Im}\alpha(s_0)$ curve may well develop a little curvature, which may explain why the A curve misses A_2. Or possibly the experimental data are not sufficiently accurate.