Production Mechanism for Reactions Forbidden in Single-Meson Exchange

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Observed angular distributions for the processes $\pi^+ + n \to \pi^- + \Delta^{++}(1236)$ and $K^- + p \to Y_1^{*-}(1385) + \pi^+$ have been explained both in shape and in magnitude using the rescattering model.

The Lawrence Radiation group[1,2] has made an extensive study of reactions forbidden in single-meson exchange which exhibit forward peaks. We have shown[3–5] that in such cases the rescattering box diagram with the lowest t-channel singularity can be evaluated to yield a convergent contribution which reproduces not only the shape of the angular distribution but also the magnitudes at various energies. The assumptions made are that (i) the two-particle-exchange diagram in the t channel can be approximated by a box diagram, and (ii) the imaginary part of the amplitude gives the dominant contribution. The purpose of this paper is to test the validity of these assumptions by evaluating the angular distributions for the processes

$$\pi^+ + n \to \pi^- + \Delta^{++}(1236), \quad 1.9\text{–}2.4 \text{ GeV}/c \tag{1}$$

$$K^- + p \to \pi^+ + Y_1^{*-}(1385), \quad 2.1\text{–}2.6 \text{ GeV}/c \tag{2}$$

for which data are now available.[1]

In Fig. 1, a stands for π^+ (K^-), b for $p(\Lambda)$, c for π^+ (π^-), and d for ρ^0 (ρ^0) for processes (1) and (2). If we put s-channel intermediate states, i.e., b and d, on the mass shell,[6,7] the absorptive part of the amplitude for the rescattering diagram shown in Fig. 1 can be written[3] as

$$T_4{}^i = \int d\cos\theta' d\phi' \frac{|\mathbf{q}'|m_b}{16\pi^2 W}\bar{U}_\sigma(p_2) \times \frac{p_\sigma'}{m_c}\frac{g_{Dbc}g_{Ccd}}{Q_2{}^2+m_c{}^2}i(q_2-Q_2)_\nu\left(\delta_{\mu\nu}+\frac{q_\mu' q_\nu'}{m_d{}^2}\right)i(q_1+Q_1)_\mu \times \frac{g_{Bba}g_{Aad}}{Q_1{}^2+m_a{}^2}\frac{-i\gamma\cdot p'+m_b}{2m_b}\gamma_5 u(p_1), \tag{3}$$

where $\cos\theta' = \hat{q}_1\cdot\hat{q}'$, ϕ' is the azimuth angle of $\mathbf{q}'$, $|\mathbf{q}'|$ is the center-of-mass momentum of the intermediate particles, W is the center-of-mass total energy, $U_\sigma(p_2)$ is the Rarita-Schwinger wave function, and $u(p_1)$ is the Dirac wave function. For the two-step process

$$A+B \to d+b \to C+D, \tag{4}$$

the pion mediator for the process $b+d\to C+D$ goes on the mass shell inside the physical region. This pole occurs for $Q_2{}^2 = -t'' = -m_c{}^2$. To evaluate the integral in (1), we can do the ϕ' integration first and get[8]

$$T_4{}^i = 2\int_{-1}^{+1} d\cos\theta'' \int_{x_-}^{x_+} d\cos\theta' \times[\Gamma(\cos\theta,\cos\theta',\cos\theta'')]^{-1/2} \times\frac{|\mathbf{q}'|m_b}{16\pi^2 W}\bar{U}_\sigma(p_2)\frac{p_\sigma'}{m_c}\frac{g_{Dbc}g_{Ccd}}{Q_2{}^2+m_c{}^2}i(q_2-Q_2)_\nu \times\left(\delta_{\mu\nu}+\frac{q_\mu' q_\nu'}{m_d{}^2}\right)i(q_1+Q_1)_\mu\frac{g_{Bba}g_{Aad}}{Q_1{}^2+m_a{}^2} \times\frac{-i\gamma\cdot p'+m_b}{2m_b}\gamma_5 u(p_1), \tag{5}$$

where

$$x_\pm = \cos\theta\cos\theta'' \pm \sin\theta\sin\theta'',$$

$$\cos\theta'' = \hat{q}_2\cdot\hat{q}', \quad \cos\theta = \hat{q}_2\cdot\hat{q}_1,$$

$$\Gamma(\cos\theta,\cos\theta',\cos\theta'') = 1-\cos^2\theta-\cos^2\theta'-\cos^2\theta''+2\cos\theta\cos\theta'\cos\theta''.$$

The $\cos\theta''$ integration can be performed by the method of residues. The remaining integration, over $\cos\theta'$, can be done by noting that

$$F = \{[\Gamma(\cos\theta,\cos\theta',\cos\theta'')]^{1/2}(Q_1{}^2+m_a{}^2)\}^{-1} \tag{6}$$

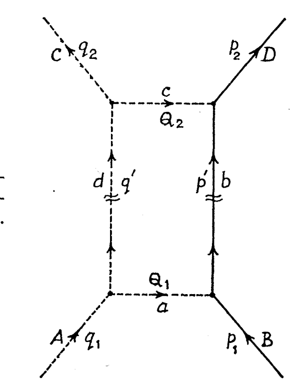

Fig. 1. Rescattering square diagram for the resonance D production in the reaction $A+B\to C+D$.

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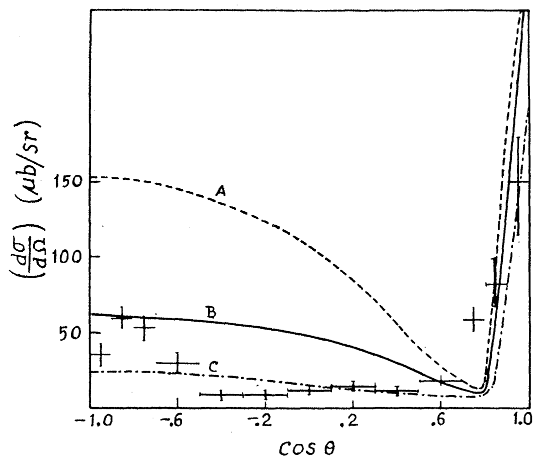

FIG. 2. Production angular distributions from Eq. (8) for process (1) ($\cos\theta = \hat{q}_2\cdot\hat{q}_1$). Curve A represents the calculation at 1.9 GeV/c, B yields the calculation at 2.1 GeV/c, and C gives the theoretical calculation at 2.4 GeV/c. The experimental data have been taken from Ref. 1.

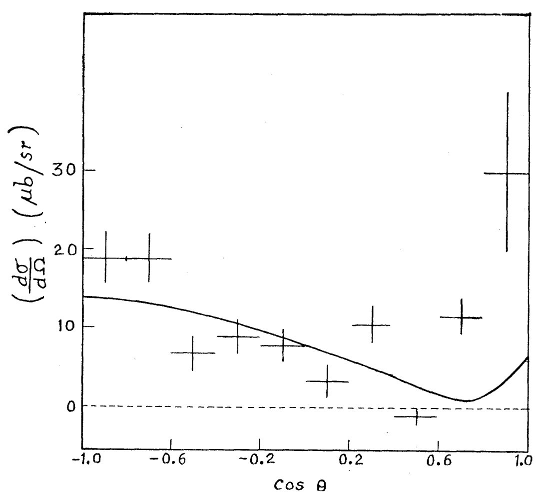

FIG. 3. Production angular distribution from Eq. (8) for process (2) at 2.1 GeV/c ($\cos\theta = \hat{q}_2\cdot\hat{q}_1$). The experimental data have been taken from Ref. 1.

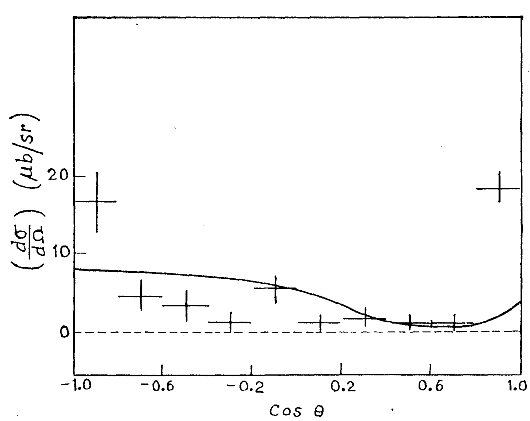

FIG. 4. Production angular distribution from Eq. (8) for process (2) at 2.6 GeV/c compared with the experimental data of Dauber *et al.* (Ref. 1) ($\cos\theta = \hat{q}_2\cdot\hat{q}_1$).

is a rapidly varying function of $\cos\theta'$ and hence essentially determines the angular integration. We substitute[9] in the rest of the integrand that value of $\cos\theta'$ for which the function F has a maximum (i.e., $\cos\theta' = 1$). Equation (3) then becomes

$$T_4^i = \frac{4\pi^2 i|\mathbf{q}'|m_b}{4|\mathbf{q}_1|\,|\mathbf{q}'|^2|\mathbf{q}_2|W}\frac{1}{16\pi^2\sqrt{(-\beta)}}$$

$$\times\bar{U}_\sigma(p_2)\frac{p_\sigma'}{m_c}g_{Dbc}g_{Ccd}i(q_2-Q_2)_\nu\left(\delta_{\mu\nu}+\frac{q_\mu' q_\nu'}{m_d^2}\right)$$

$$\times i(q_1+Q_1)_\mu g_{Bba}g_{Aad}\frac{-i\gamma\cdot p'+m_b}{2m_b}\gamma_5 u(p_1)\,, \quad (7)$$

where

$$\beta = 1-\cos^2\theta-\alpha_1^2-\alpha_2^2+2\alpha_1\alpha_2\cos\theta\,,$$

$$\alpha_1 = (2q_{10}q_0'-m_d^2)/2|\mathbf{q}_1|\,|\mathbf{q}'|\,,$$

$$\alpha_2 = (2q_{20}q_0'-m_d^2)/2|\mathbf{q}_2|\,|\mathbf{q}'|\,.$$

The usual sum over the polarization and spin states gives the differential cross section as follows:

$$\frac{d\sigma}{d\Omega} = \frac{0.38935m_B m_D}{2048(2\pi W)^4}$$

$$\times\frac{\pi^2 g_{Dbc}^2 g_{Bba}^2 g_{Aad}^2 g_{Ccd}^2}{(-\beta)|\mathbf{q}_2|\,|\mathbf{q}'|^2|\mathbf{q}_1|^3}XYZ \quad \text{mb/sr}, \quad (8)$$

where

$$X = 16\left(q_2\cdot q_1+\frac{q_2\cdot q' q_1\cdot q'}{m_d^2}\right)^2,$$

$$Y = \frac{2}{3m_c^2}\left(p'^2+\frac{1}{m_D^2}p_2\cdot p'^2\right),$$

$$Z = -\frac{2}{m_B m_D}\left[(m_D m_b-p_2\cdot p')(m_B m_b+p_1\cdot p')\right].$$

We have used the following values of the coupling constants[10]:

$$g_{\pi NN}^2/4\pi = 15, \qquad g_{\rho\pi\pi}^2/4\pi = 2.4\,,$$

$$g_{\pi NN^*}^2/4\pi = 0.38, \qquad g_{\bar{K}p\Lambda}^2/4\pi = 7.4\,,$$

$$g_{\bar{K}K\rho}^2/4\pi = 1.2, \qquad g_{Y_1^*\Lambda\pi}^2/4\pi = 0.14\,.$$

There are thus no free parameters left in our calculation.

Angular distributions according to Eq. (8) for processes (1) and (2) are given in Fig. 2 and Figs. 3 and 4, respectively. At 2.4 GeV/c, our value in the forward direction for reaction (1) is 200 μb/sr, which agrees well with the experimental value. For process (1), we find

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a strong energy dependence of the cross section. The experimental data are not resolved highly enough with respect to energy to show this trend. However, the effect is clearly evident both in experiment and in theory for reaction (2). More data on these reactions would be useful to decide this point. In conclusion, we can say that the two-meson-exchange contributions merit further study for processes where one-meson exchange is forbidden.

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Regge Classification of the Cascade Resonances

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By using a previously developed dispersion technique, we investigate the Chew-Frautschi and the Imα-versus-s plots for the cascade resonances. The eight resonances are found to lie on three nearly linear plots on both graphs as was found to be the case previously for the baryon N_γ and Δ_δ sequences. Of the two types of resonances, probable and doubtful, all of the former are fitted (with no gaps on the trajectories) and none of the latter is needed. Severe self-consistency requirements are well satisfied by the two plots.

I. INTRODUCTION

Because of the improving experimental situations, more interest has developed recently in the Imα-versus-s plot for resonances.[1–6] The Chew-Frautschi (Reα-versus-s) plot has, of course, received attention for much longer. While most work has centered on the meson trajectories, one paper in particular[6] has done a study of the two best-established baryon sequences: the N_γ and Δ_δ.

This work reached a conclusion that the Imα-versus-s curves for the resonances were consistent with linearity and had a slope of 0.135 BeV^{-2}. The Chew-Frautschi plots were nearly linear, being only very slightly distorted by the dispersion integral used to calculate the Reα-versus-s curve from Imα. Some stringent self-consistency requirements are satisfied by these plots, and this lends confidence to the results.

There are a number of reasons for wishing to extend this technique to other baryon resonances. The most immediate is, of course, to enable one to classify the particle resonances themselves and offer suggestions for their appropriate spins and parities, most of which are still undetermined.

However, there are two other reasons for such investigations. While the baryon widths (and hence the imaginary part of α) definitely rise in essentially linear fashion, the meson widths may well rise and then fall sharply, thus forming a peaked Imα-versus-s curve.[2–4] In fact, the latest experimental evidence on high-spin meson resonances indicates they are very narrow.[7] While this does not preclude linear plots (see Ref. 1), it is more compatible with a peaked plot of Imα versus s. If the meson widths do give rise to such peaked curves, it is of great interest to gather more evidence on bayon curves and see if they peak (like the mesons) or are linear (like the N_γ and Δ_δ). A difference in baryon and meson behavior would be very puzzling.

Recently, Ball and Zachariasen[8] have suggested the interesting possibility that the trajectory function $\alpha(t)$ is complex for $t<0$. One of the experimental consequences of this is that the Imα-versus-s plot of resonances should be a linear plot near the Imα threshold. Any plot such as we do below is therefore a partial substantiation of the Ball-Zachariasen conjecture.

In the next section we briefly review the technique as developed in Ref. 1 and apply it to a recent compilation of data on the cascade resonances. In the last section we discuss the results of the plots.

II. TECHNIQUE AND CALCULATION

In Ref. 1 it was shown that for linearly rising Imα curves we could write

$$\alpha(s)=\tau_1+\tau_2+\frac{s(s-z)}{\pi}\int_{s_0}^{\infty}\frac{\mathrm{Im}\alpha(s')}{s'(s'-z)(s'-s)}ds', \quad (1)$$

with τ_1 and τ_2 real. (We assume that the trajectories are functions of s, not $\sqrt{s}$.) A very important threshold

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