phase-shift analyses for the S_{31} partial wave using Eq. (1) with the potential[7]

$$B=\begin{bmatrix} \dfrac{0.72}{W-6.7}-\dfrac{10.0}{W-2} & \dfrac{5.48}{W-6.7} \\ \dfrac{5.48}{W-6.7} & \dfrac{140}{W-6.7} \end{bmatrix}. \quad (2)$$

$\alpha_a=m_N+1$ and ρ_a is taken[8] to be simply the momentum k_a, whereas for the quasi-two-body D-wave $\pi\Delta$ channel, we have $\alpha_b=m_N+2$ and

$$\rho_b=\int_{m_N+1}^{W-1}dm\left(\frac{[W^2-(m+1)^2][W^2-(m-1)]}{4W^2}\right)^{5/2}\times\frac{1}{W^4}\frac{\Gamma_\Delta{}^2/4}{(m-m_\Delta)^2+\Gamma_\Delta{}^2/4}, \quad (3)$$

where Γ_Δ and m_Δ are the width and mass of the $\Delta(1236)$. With the choice of B as a sum of poles, the integral equations for N_{ij} in (1) are easily reduced to quadrature.

[7] We use units $m_\pi=1$ and denote the nucleon mass by m_N.

[8] The nature of the fit is not sensitive to the detailed choice of the asymptotic behavior of ρ.

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Theorem on K_{l3} Form Factors

FAYYAZUDDIN AND RIAZUDDIN

Institute of Physics, University of Islamabad, Rawalpindi, Pakistan and International Centre for Theoretical Physics, Trieste, Italy

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A simple proof of a theorem derived recently by Dashen and Weinstein on K_{l3} form factors is presented. With the further assumption that $D(t)\equiv\langle\pi|\partial_\lambda V_\lambda{}^1|K\rangle$ is dominated by the 0^+ κ meson, a relation between κ-meson mass and $f_K/f_\pi F_+(0)$ is obtained.

RECENTLY Dashen and Weinstein[1] have proved a theorem on the form factors in K_{l3} decay which leads to the result

$$\xi(0)\equiv\frac{F_-(0)}{F_+(0)}=\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)-\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}\lambda_+ +O(\epsilon^2), \quad (1)$$

where $F_+(t)$ and $F_-(t)$ are the two form factors which determine the hadronic part of the matrix elements of K_{l3},

$$\lambda_+=m_\pi{}^2[d\ln F_+(t)/dt]_{t=0}, \quad (2)$$

and ϵ denotes a parameter which characterizes the simultaneous breaking of $SU(3)$ and $SU(3)\times SU(3)$. The purpose of this paper is to present a simple proof of relation (1). With the further assumption that

$$D(t)\equiv\langle\pi|\partial_\lambda V_\lambda{}^1|K\rangle=(m_K{}^2-m_\pi{}^2)F_+(t)+tF_-(t) \quad (3)$$

is dominated by a $\kappa(0^+)$ meson, a relation between the experimentally known number $f_K/f_\pi F_+(0)$ and the κ-meson mass is obtained.

By using the current-algebra techniques, we have

$$-k_\nu\Gamma_{\nu\lambda}=f_\pi\Gamma_\lambda+i\int d^4x\, \times e^{-ik\cdot x}\langle 0|\delta(x_0)[A_0{}^{1-i2},V_\lambda{}^{4+i5}(0)]|\bar{K}^0(p)\rangle, \quad (4)$$

where $\Gamma_{\nu\lambda}$ and Γ_λ are defined by

$$i\int d^4x\, e^{-ik\cdot x}\langle 0|T(A_\nu{}^{1-i2}(x)V_\lambda{}^{4+i5}(0))\bar{K}^0(p)\rangle =i\Gamma_{\nu\lambda}+i\frac{f_\pi k_\nu}{k^2+m_\pi{}^2}\Gamma_\lambda, \quad (5a)$$

$$i\int d^4x\, e^{-ik\cdot x}\langle 0|T(\partial_\nu A_\nu{}^{1-i2}(x)V_\lambda{}^{4+i5}(0))|\bar{K}^0(p)\rangle =\frac{f_\pi m_\pi{}^2}{k^2+m_\pi{}^2}\Gamma_\lambda, \quad (5b)$$

$$\Gamma_\lambda=\langle\pi^+(k)|V_\lambda{}^{4+i5}|\bar{K}^0(p)\rangle =[F_+(t)(p+k)_\lambda+F_-(t)(p-k)_\lambda]. \quad (5c)$$

Using the current commutation relation

$$\delta(x_0)[A_0{}^{1-i2}(x),V_\lambda{}^{4+i5}(0)]=V_\lambda{}^{6+i7}(0)\delta^4(x) \quad (6a)$$

and the definition

$$\langle 0|V_\lambda{}^{6+i7}|\bar{K}^0(p)\rangle=if_Kp_\lambda, \quad (6b)$$

we obtain from (4)

$$-k_\nu\Gamma_{\nu\lambda}=f_\pi[F_+(t)(p+k)_\lambda+F_-(t)(p-k)_\lambda]-f_Kp_\lambda. \quad (7)$$

[1] R. Dashen and M. Weinstein, Phys. Rev. Letters 22, 1337 (1969).

From Lorentz covariance, the most general form for $\Gamma_{\nu\lambda}$ is

$$\Gamma_{\nu\lambda}=F_1(t)\delta_{\nu\lambda}+F_2(t)p_\nu p_\lambda+F_3(t)k_\nu k_\lambda+F_4(t)p_\nu k_\lambda+F_5(t)k_\nu p_\lambda. \quad (8)$$

Substituting (8) in (7), one obtains expressions for $F_+(t)$ and $F_-(t)$ which give

$$F_+(t)+F_-(t)=f_K/f_\pi-(1/f_\pi)[F_2(t)\tfrac{1}{2}(t-m_K^2-m_\pi^2)-m_\pi^2F_5(t)] \quad (9)$$

or, for $t=m_K^2-m_\pi^2$,

$$F_+(m_K^2-m_\pi^2)+F_-(m_K^2-m_\pi^2)=f_K/f_\pi+(m_\pi^2/f_\pi)F, \quad (10a)$$

where $F=F_2(m_K^2-m_\pi^2)+F_5(m_K^2-m_\pi^2)$. We rewrite (10a) as

$$[F_+(m_K^2-m_\pi^2)-F_+(0)]+F_-(m_K^2-m_\pi^2)+[F_+(0)-f_K/f_\pi]=(m_\pi^2/f_\pi)F. \quad (10b)$$

Now $F_-(t)$ and $F_+(0)-f_K/f_\pi$ are obviously of order λ [where λ denotes the strength of $SU(3)$-symmetry breaking], while

$$F_+(m_K^2-m_\pi^2)=F_+(0)[1+\lambda_+(m_K^2-m_\pi^2)/m_\pi^2+O(\lambda^2)]=F_+(0)[1+O(\lambda)].$$

Thus the left-hand side of (10a) is of order λ, so that F must be of order λ. Hence from (10a)

$$F_+(m_K^2-m_\pi^2)+F_-(m_K^2-m_\pi^2)=f_K/f_\pi+m_\pi^2O(\lambda). \quad (11)$$

Let ϵ' denote the strength of $SU(2)\times SU(2)$ breaking, so that $m_\pi^2=O(\epsilon')$ and $m_\pi^2O(\lambda)=O(\epsilon'\lambda)$. Thus from (11) we obtain the result of Dashen and Weinstein[1]:

$$F_+(m_K^2-m_\pi^2)+F_-(m_K^2-m_\pi^2)=f_K/f_\pi+O(\epsilon'\lambda). \quad (12)$$

To derive relation (1), we make the expansions

$$F_+(t)/F_+(0)=1+\lambda_+/m_\pi^2t+\cdots, \quad (13a)$$

$$F_-(t)/F_+(0)=\xi(0)+\Lambda_+/m_\pi^2t+\cdots, \quad (13b)$$

where $\xi(0)$ and Λ_+ are of order λ each. Taking $t=m_K^2-m_\pi^2$, we put expansions (13) in (12) to obtain

$$F_+(0)[1+\lambda_+(m_K^2-m_\pi^2)/m_\pi^2+O(\lambda^2)+\xi(0)+O(\lambda^2)]=f_K/f_\pi+O(\epsilon'\lambda). \quad (14)$$

This gives [note that[2] $F_+(0)=1+O(\lambda^2)$]

$$[(m_K^2-m_\pi^2)\lambda_+/m_\pi^2+\xi(0)]=f_K/f_\pi F_+(0)-1+O(\epsilon'\lambda)+O(\lambda^2) \quad (15a)$$

or

$$\xi(0)=-\frac{m_K^2-m_\pi^2}{m_\pi^2}\lambda_++\left(\frac{f_K}{f_\pi F_+(0)}-1\right)+O(\epsilon'\lambda)+O(\lambda^2), \quad (15b)$$

which is equivalent to (1) if one notes the fact that the difference between

$$\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right) \quad \text{and} \quad \left(\frac{f_K}{f_\pi F_+(0)}-1\right)$$

is of order λ^2.

If we assume that $D(t)$ is dominated by the (0^+) κ meson so that

$$D(t)\equiv(m_K^2-m_\pi^2)F_+(t)+tF_-(t)=m_\kappa^2D(0)/(m_\kappa^2-t), \quad (16)$$

where $D(0)=(m_K^2-m_\pi^2)F_+(0)$, we obtain from (16), by taking $t=m_K^2-m_\pi^2$ and relation (12),

$$\frac{f_K}{f_\pi}+O(\epsilon'\lambda)=\frac{m_\kappa^2F_+(0)}{m_\kappa^2-(m_K^2-m_\pi^2)}$$

or

$$\frac{f_K}{f_\pi F_+(0)}=\frac{m_\kappa^2}{m_\kappa^2-(m_K^2-m_\pi^2)}+O(\epsilon'\lambda). \quad (17)$$

Taking $f_K/f_\pi F_+(0)=1.28$ from experiments,[3] one obtains, on neglecting terms of order $(\epsilon'\lambda)$ (this is better than neglecting terms of order λ^2, since ϵ' is believed to be smaller than λ),

$$m_\kappa=1021 \text{ MeV}. \quad (18)$$

On the other hand, if one assumes κ to be the daughter of K^* as in the Veneziano model,[4] so that

$$m_\kappa=m_{K^*},$$

one predicts from (17), neglecting small corrections of order $\epsilon'\lambda$, that

$$f_K/f_\pi F_+(0)=m_{K^*}^2/m_\rho^2\approx1.36, \quad (19)$$

where we have used the relation $m_{K^*}^2-m_\rho^2=m_K^2-m_\pi^2$. The value (19) is in fair agreement with its experimental value[3] 1.28 ± 0.06.

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