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Hadron Couplings in Broken $SU(6)\times O(3)$. II. Meson Decays

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The phenomenological prescriptions outlined in the preceding paper (I) for meson-baryon couplings in a quark model of broken $SU(6)\times O(3)$ are employed for the construction of meson couplings in an almost identical spirit in respect of relativistic generalization as well as the inclusion of the recoil contribution. The multiplying form factor for a supermultiplet transition from an initial $Q\bar{Q}$ state (mass M) to a final $Q\bar{Q}$ state (mass m) is assumed to have the analogous structure $f_L(k^2)=\bar{g}_L\mu^{-L-1}(\mu/m_\pi)^{1/2}(\mu/\omega_k)^{L\pm1}(2M)$, where $\bar{g}_L$ is a free parameter different from g_L for all cases except $L=0$ (when both are equal to the $\bar{Q}QP$ coupling constant f_q). The agreement of this meson coupling scheme with experiment is found to be extremely good for a large number and variety of decays from $L^P=0^-$, 1^+, and 2^- supermultiplets. These results not only provide a sensitive test of the (rapidly varying) structure of the form factor, but also serve to emphasize the essential unity of description of both meson and baryon couplings through a common set of assumptions. A more sensitive test of this model is provided by its polarization predictions for $B\to\omega\pi$ and $A_1\to\rho\pi$ decays which are appreciably different from, and in somewhat better experimental accord than, those of the simple quark model.

I. INTRODUCTION

IN the preceding paper,[1] a new phenomenological scheme has been proposed for baryon couplings with pseudoscalar mesons, which depends on a single "coupling constant" governing an entire supermultiplet transition. From the point of view of parametrization, the scheme clearly represents a great improvement over earlier phenomenological studies of baryon couplings beyond the simple **56** states,[2] since the parameters now appearing in the form factor are the very masses of the involved mesons and baryons. Relativistic kinematics have been so incorporated as to give a Lorentz-invariant structure to a significant fraction of the coupling, while the multiplying form factor has merely the effect of replacing the momenta by the velocities in the total interaction. In terms of their "looks," these couplings thus have a good deal of similarity to those derivable from formal theories such as $U(6)\times U(6)$ or $SU(6)_W$,[3] but with the added advantage of including such important features as the experimentally observed enhancements in heavy-meson modes for s-wave decays.

The purpose of this paper is to give a corresponding model for meson couplings so as to bring out the essential similarity between the two cases in terms of the algebraic structure of the couplings as well as agreement with experiment.

From the point of view of their utility with respect to quantum number [$SU(3)$, J^P, etc.] assignments to the higher meson resonances, the studies of meson couplings are just as important as those of baryon couplings. Such studies have ranged from purely phenomenological analyses (based on the quark model)[4–6] to formal group-theoretical calculations based on compact and noncompact groups.[3,7,8] Indeed, meson couplings seem to lend themselves to more detailed investigations in terms of formal groups like $\tilde{U}(12)$, $SL(6,C)$, $SU(6)_W$, and $O(4,2)$ than have their baryonic counterparts to similar groups. On the other hand, phenomenological investigations on meson couplings have been much more qualitative than corresponding ones on baryon couplings essentially because of lack of reliable data, even for the positive-parity states in the mass range 1000–1300 MeV where a good deal of confusion still exists in several 0^+ and 1^+ mesons. For a semiquantitative comparison it is therefore even more imperative than for the baryon case, that such models be dependent on as few parameters as possible. The model to be presented in this paper is designed mainly to serve this objective and thus to narrow the gap between quark phenomenology and formal group theory for meson couplings, in much the same way as has been described in the preceding paper[1] for baryon couplings.

From the point of view of quark phenomenology mesons are somewhat simpler objects than baryons, though this advantage is partly compensated by the fact that in the nonrelativistic framework, quarks (Q) and antiquarks ($\bar{Q}$) are entirely different objects, so that the symmetry requirements on the baryon (QQQ) states are no longer valid for the meson ($Q\bar{Q}$) states. Thus, unlike the baryon states, the $SU(3)$ degrees of freedom for mesons can be completely factored out from the spin-*cum*-spatial coordinates. And with the possibility of strong mixing between the $I=0$ components of **8** and **1** $Q\bar{Q}$ states, the appropriate $SU(3)$ description is in terms of nonets. In this respect an

[1] D. L. Katyal and A. N. Mitra, preceding paper, Phys. Rev. D **1**, 338 (1970); hereafter referred to as I.
[2] A. N. Mitra and M. H. Ross, Phys. Rev. **158**, 1630 (1967); D. L. Katyal and A. N. Mitra, *ibid.* **169**, 1322 (1968).
[3] R. J. Rivers, Phys. Rev. **161**, 1687 (1967); P. N. Dobson, Jr., *ibid.* **160**, 1501 (1967).
[4] J. Uretsky, in *High Energy Theoretical Physics* (University of Beirut, Beirut, Lebanon, 1967).
[5] M. Elitzur, H. R. Rubinstein, H. Stern, and H. J. Lipkin, Phys. Rev. Letters **17**, 420 (1966).
[6] A. N. Mitra and P. P. Srivastava, Phys. Rev. **164**, 1803 (1967).
[7] K. C. Tripathy and R. N. White, Phys. Rev. **159**, 1374 (1967).
[8] A. O. Barut and K. C. Tripathy, Phys. Rev. Letters **19**, 918 (1967).

especially interesting possibility is the "ideal mixing" angle, first suggested by Okubo,[9] which finds a natural expression in the $Q\bar{Q}$ model through the assumptions of $I=0$ meson structures in terms of "strange" and "nonstrange" quarks only. This possibility will be explored in this paper for several $I=0$ meson pairs beyond the familiar (ω,ϕ) states.

As to the coupling scheme for mesons in terms of the quark model, it is beset with some formal difficulties. Since the idea of a $\bar{Q}QP$ coupling is based on the assumption that the meson is an elementary object (radiation quantum), while the baryon is a QQQ composite, a literal application of the same idea for meson couplings would be faced with the dilemma as to which meson in the final state should be regarded as "elementary" and which one as a composite. While the "problem" may be resolved for cases like MVP couplings where only one pseudoscalar (P) meson is involved, by regarding the P meson as elementary and the vector (V) meson as a $Q\bar{Q}$ composite, it assumes a more embarrassing form for a coupling like MPP, where both the final mesons are pseudoscalar. Such ambiguities, which do not have any counterparts for baryon couplings ($\bar{B}BP$), must be resolved through some further conventions which may or may not have a formal theoretical basis. It has been shown[10] that from the point of view of obtaining "correct" formal *results* for a coupling like $\rho\pi\pi$, it is immaterial as to which π meson is regarded as elementary, but that care should be taken *not* to *add* the amplitudes for these separate possibilities. This prescription does not, however, cover cases like $M\eta\pi$ or $MK\pi$ where two unequal P mesons are involved. A convention is clearly necessary for such cases, since the $\bar{Q}QP$ interaction involves the mass (μ) of the P meson through a derivative coupling, and this mass would be very sensitive to the choice of the "elementary quantum." We adopt the rather unorthodox convention that the *heavier* of the two P mesons be regarded as elementary, since the opposite choice would make the decay rates for heavy-meson modes enormously larger than those for pionic modes, a result which is completely contradictory to observation.[11]

In Sec. II we present our new coupling scheme for higher meson supermultiplets in an $SU(6)\times O(3)$ classification, preceded by a short résumé of more familiar **36** couplings in order to clarify the role of relativistic normalization and symmetry-breaking effects in this new coupling scheme. In Sec. III a detailed comparison of this scheme with experiment is made with respect to decay patterns of mesons of $L=1$ and 2, while the polarization predictions are discussed in Sec. IV. Section V summarizes the main conclusions together with a comparative assessment of its significance. Preliminary results of this model were reported some time ago.[12]

II. NEW COUPLING SCHEME FOR MESONS

In this section we outline the coupling scheme of higher mesons (M_L) with the pseudoscalar (P) and vector (V) mesons. The basic couplings of the pseudoscalar meson, which is emitted with a momentum $\mathbf{k}$, at the quark (Q) and the antiquark ($\bar{Q}$) are, respectively,

$$H(Q)=\sum_{\alpha=0}^{8}\mu_\alpha^{-1}f_q\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}-\omega_kM_Q^{-1}\mathbf{p})\lambda_\alpha\Pi_\alpha\,, \tag{2.1}$$

$$H(\bar{Q})=-\sum_{\alpha=0}^{8}\mu_\alpha^{-1}f_q\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{k}+\omega_kM_Q^{-1}\mathbf{p})\lambda_\alpha\Pi_\alpha\,, \tag{2.2}$$

where $\boldsymbol{\sigma}^{(1)}$, $\boldsymbol{\sigma}^{(2)}$ and $\mathbf{p}$, $-\mathbf{p}$ are the spin operators and recoil momenta of quark and the antiquark, respectively; ω_k is the energy of the emitted meson (mass μ) and M_Q is the mass of the quark. From (2.1) and (2.2), it is clear that the recoil effect vanishes for couplings among the **36** mesons.

As in I, the general procedure for the evaluation of the couplings is first to evaluate the interactions in the nonrelativistic limit and then to supplement the same by relativistic prescriptions which include the normalization of states. Thus, for $\rho\pi\pi$ coupling, which is a special case of coupling among **36** mesons, the relativistic coupling constant defined by $H_{\rho\pi\pi}=if_{\rho\pi\pi}\rho_\mu{}^a\pi^b\partial_\mu\pi^c\epsilon_{abc}$ is given by the VW[10] prescription as

$$f_{\rho\pi\pi}(2m_\rho)^{-1/2}(2\omega_\pi)^{-1/2}=\mu^{-1}f_qF_M(k^2)\,, \tag{2.3}$$

where the energy ω_π of the final $Q\bar{Q}$ pionic state must be taken as equal to m_ρ, the energy of the initial $Q\bar{Q}$ ρ-meson state at rest, in the PCAC (partial conservation of axial-vector current) limit. The multiplying form factor $F_M(k^2)$, which is normalized by $F_M(0)=1$, may be parametrized by the Weisskopf factor $(\mu/m_\pi)^{1/2}$ (which equals unity in this case). The other VPP couplings among **36** mesons, which can be written down with the same prescription, lead to the following decay predictions, where the bracketed quantities denote the experimental values of the decay widths in MeV:

$$\begin{aligned}\Gamma(\rho\to\pi\pi)&=185(110\text{–}150)\,,\\ \Gamma(K^*\to K\pi)&=75(49.5\pm1.1)\,,\\ \Gamma(\phi\to K\bar{K})&=2.4(2.9\pm0.8)\,.\end{aligned} \tag{2.4}$$

As for the VVP couplings, the ideal mixing angle forbids $\phi\rho\pi$ coupling, while $\omega\to\rho\pi$ and $\phi\to K^*\bar{K}$ are not allowed by phase space.

For the couplings M_LPP and M_LPV of the higher meson nonets (M_L) of $SU(6)\times O(3)$ to P and V mesons of **36**, the procedure is very close to I, except that the

[9] S. Okubo, Phys. Letters **5**, 165 (1963).
[10] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967). Hereafter referred to as VW.
[11] Opposite choice was taken in Ref. 6; but then there was an *ad hoc* reduction factor. Also the Gaussian form obscured the dependence on μ.
[12] A. N. Mitra, Nuovo Cimento **61**, 344 (1969).

$SU(3)$ and spin structures are separately factored out (since there is no symmetry requirement for $Q\bar{Q}$ states). As in I, the spatial overlap integral is expressible in the forms

$$\bar{f}_L(k^2)B_{\alpha_1\cdots\alpha_L}{}^{LM}k_{\alpha_1}\cdots k_{\alpha_L}, \tag{2.5}$$

where $\bar{f}_L$ is a scalar form factor and $B_{\alpha_1\cdots\alpha_L}{}^{LM}$ is a normalized spherical tensor of rank L, symmetrical and traceless in all the three-dimensional indices $(\alpha_1\cdots\alpha_L)\equiv\alpha$. The spin-matrix elements for the ${}^3S_1\rightarrow{}^1S_0$ and ${}^3S_1\rightarrow{}^3S_1$ transition are, respectively, of the form

$$V_\alpha k_\alpha,\quad V_\alpha k_\beta\omega_\gamma\epsilon_{\alpha\beta\gamma}, \tag{2.6}$$

where $\mathbf{V}$ and $\boldsymbol{\omega}$ are unit three-vectors representing the polarization states of the relevant vector mesons. The Clebsch-Gordan expansion of the direct product of (2.5) and (2.6) leads eventually to the coupling structures of the forms[13]

$$M_LPP:\ A_{\alpha\alpha_1\cdots\alpha_L}{}^{L+1}\,k_\alpha k_{\alpha_1}\cdots k_{\alpha_L} +\left[\frac{L(L+1)}{2(2L+1)}\right]^{1/2}\mathbf{k}^2A_{\alpha_2\cdots\alpha_L}{}^{L-1}\,k_{\alpha_2}\cdots k_{\alpha_L}, \tag{2.7}$$

$$M_LPV:\ A_{\alpha\alpha_1\cdots\alpha_L}{}^{L+1}\,k_{\alpha_1}\cdots k_{\alpha_L}\epsilon_{\alpha\beta\gamma}k_\beta\omega_\gamma +\left(\frac{L}{L+1}\right)^{1/2}[(\mathbf{k}\cdot\boldsymbol{\omega})A_{\alpha_1\cdots\alpha_L}{}^Lk_{\alpha_1}-\mathbf{k}^2A_{\alpha_1\cdots\alpha_L}{}^L\omega_{\alpha_1}]\times k_{\alpha_2}\cdots k_{\alpha_L}, \tag{2.8}$$

$$B_LPV:\ k_\alpha V_\alpha B_{\alpha_1\cdots\alpha_L}{}^L\,k_{\alpha_1}\cdots k_{\alpha_L}. \tag{2.9}$$

Here $A_{(\alpha)}{}^J$ is a normalized tensor of the type ${}^3L_J$ $(J=L\pm1,\ L)$ and $B_{(\alpha)}{}^L$ is a similar tensor of the type ${}^1L_J$ $(J=L)$. Using the same prescription as outlined in I for incorporation of the recoil effect, the relativistic extensions of these couplings are, respectively,

$$A_{\mu\mu_1\cdots\mu_L}{}^{L+1}\,k_\mu k_{\mu_1}\cdots k_{\mu_L} +(-\mu^2)\left[\frac{L(L+1)}{2(2L+1)}\right]^{1/2}A_{\mu_2\cdots\mu_L}{}^{L-1}k_{\mu_2}\cdots k_{\mu_L}, \tag{2.10}$$

$$iM^{-1}\partial_\rho A_{\mu\mu_1\cdots\mu_L}{}^{L+1}(k_{\mu_1}\cdots k_{\mu_L})\epsilon_{\mu\nu\lambda\rho}k_\nu\omega_\lambda +\left(\frac{L}{L+1}\right)^{1/2}[k_\mu\omega_\mu k_{\mu_1}+\mu^2\omega_{\mu_1}] \times k_{\mu_2}\cdots k_{\mu_L}A_{\mu_1\cdots\mu_L}{}^L, \tag{2.11}$$

and

$$B_{\mu_1\cdots\mu_L}{}^Lk_\mu\omega_\mu k_{\mu_1}\cdots k_{\mu_L}. \tag{2.12}$$

The multiplying form factor is taken as

$$\bar{f}_L(k^2)=\bar{g}_L\mu^{-L-1}(\mu/\omega_k)^{L\pm1}(\mu/m_\pi)^{1/2}(2M), \tag{2.13}$$

where all the factors have the same significance as (2.10) of I, except for the replacement $(M/m)^{1/2}\rightarrow(2M)$ reflecting the difference in the normalization of baryon and meson states. Again, there is a single over-all constant $\bar{g}_L$ governing the entire supermultiplet transition $L^P\rightarrow0^-$. While there is no obvious connection between g_L and $\bar{g}_L$ for $L\neq0$, the quark model predicts[14] $g_0=\bar{g}_0=f_q$, a universal principle like Sakurai's[15] which goes beyond group theory.

Finally, for couplings within higher meson supermultiplets $(L>0)$, the spatial overlap is again essentially a normalization effect as in the baryon case, so that the form factor simplifies to

$$\bar{f}_L'(k^2)=\mu^{-1}\bar{g}_0(\mu/m_\pi)^{1/2}(2M), \tag{2.14}$$

which covers the cases of couplings among **36** mesons as well.

TABLE I. $L=1$ nonet scheme.

Configuration	$(J,P)^C$	$I=I$	$I=\frac{1}{2}$	$I=0$	$I=0$
${}^3P_2$	$(2,+)^+$	$A_2(1300)$	$K_V(1419)$	$f(1260)$	$f'(1515)$
${}^3P_1$	$(1,+)^+$	$A_1(1070)$	$K_A(1320)$	$\cdots$	$D(1285)$
${}^3P_0$	$(0,+)^+$	either of $\delta(965)$, $\pi_V(1010)$, $\pi_N(980)$	$K\pi(\approx1100)$	$\cdots$	$\eta_V(1070)$
${}^1P_1$	$(1,+)^-$	$B(1220)$	$K_C{}^*(1230)$		$\cdots$

III. COMPARISON OF M_LPP AND M_LPV COUPLINGS WITH EXPERIMENT

The data on higher meson decays are limited mostly to $L^P=1^+$ states which are listed in Table I. The information on the kaon-type states is still very incomplete except for $K_V(1420)$ of $J^P=2^+$. For the nonstrange mesons, the $SU(3)$ mixing possibilities are confined mainly to the $I=0$ members. For example, the ideal mixing angle between f and f' is indicated from the absence of $\pi\pi$ modes from f' and that of $K\bar{K}$ from f. As recently discussed by Harari,[16] this result mainly comes from $SU(3)$ plus phase-space considerations. As for (η,X^0) mixing, it is difficult to believe that η is made up of $\lambda\bar{\lambda}$, though this was suggested by some authors.[5,16] We think, on the other hand, that if it were so certain η modes of N resonances [especially $N(1500)\rightarrow N\eta$] would not be allowed (see I).

A. $L^P=1^+$ Meson Decays

Table II is a representative sample of the better-known $L^P=1^+$ meson decays to PP and PV systems on the basis of the model proposed in Sec. II. While the

[13] For details of derivations, see A. N. Mitra, in *Lectures in High Energy Theoretical Physics*, edited by H. H. Aly (Gordon and Breach, Science Publishers, Inc., New York, to be published).

[14] C. Becchi and G. Morpurgo, Phys. Rev. **149**, 1284 (1966).

[15] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[16] H. Harari, rapporteur talk, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 195.

Table II. Decays from $L=1$ to $L=0$ states for mesons. Ideal mixing angle is used for $f(1260)$ and $f'(1514)$. For (η,X^0), ideal mixing angle as well as a pure octet assignment to η is used. $\eta_V(1070)$ is assumed to be in the octet.

Particle	Mode	Γ_{calc} (MeV)	Γ_{expt} (MeV)	Γ (MeV) in $O(4,2)$
$\pi_V(1016)$	$K\bar{K}$	41.1	only mode seen $\Gamma_{\text{tot}}=25\pm5$	...
	$\eta\pi$; $\eta_8\pi$	0; 24.3	$\lesssim 24$	...
$\delta(965)$	$\eta\pi$; $\eta_8\pi$	0; 22.3	$\lesssim 5$	...
$\pi_N(980)$	$\eta\pi$; $\eta_8\pi$	0; 22.7	total width $=80\pm30$	...
$\eta_V(1070)$	$\pi\pi$	55.0	$\lesssim 56$	...
	$K\bar{K}$	24.8	>24.0	...
$A_1(1070)$	$\rho\pi$	65.0	total $\Gamma(A_1\rightarrow\rho\pi)$ $=80\pm35$	...
$B(1220)$	$\omega\pi$	129.0	123 ± 16	...
$f(1260)$	$\pi\pi$	75.0	large	100
	$K\bar{K}$	8.1	$\lesssim 3.6$	2.4
	$\eta\eta$; $\eta_8\eta_8$	0; 0.6	...	...
$f'(1516)$	$K\bar{K}$	76.2	52.5 ± 16.5	31.0
	$K^*\bar{K}+\bar{K}^*K$	17.6	7.3 ± 2.3	18.0
	$\pi\pi$	0	$\lesssim 10$	1.7
	$\eta\eta$; $\eta_8\eta_8$	91; 35	$\lesssim 38$	...
$A_2(1300)$	$\rho\pi$	82	$77\ \pm8.5$	75
	$K\bar{K}$	11.2	2.2 ± 1.0	6
	$\eta\pi$; $\eta_8\pi$	0; 8.8	10.9 ± 1.2	11.0
	$X^0\pi$; $\eta_0\pi$	1; 9.0	≈ 1	...
$K_V(1420)$	$K\pi$	42.2	$45\ \pm3$	42.0
	$K^*\pi$	30.4	29.3 ± 2.0	28.5
	$K\rho$	9.0	$10\ \pm1$	8.2
	$K\omega$	2.7	3.0 ± 1.0	3.0
	$K\eta$; $K\eta_8$	17.2; 2.8	1.8 ± 0.1	1.4

list is by no means exhaustive, it includes a sufficient number and variety of decay modes so as to bring out the rather sensitive mass and momentum dependence of the form factor (2.13), as well as the relativistic structure of the couplings (2.10)–(2.12). For purposes of comparison with experiment,[17] we have taken the first and second members of the pairs (f,f'), and (ω,ϕ) as made up of nonstrange and strange quarks, respectively, while for (η,X^0) we have considered both the possibilities of a pure $SU(3)$ assignment as well as the ideal mixing angle in a comparative spirit. The only adjustable parameter is $\bar{g}_1$ which has been estimated from the rates of the rather well-defined mode $A_2\longrightarrow\rho\pi$ as

$$\bar{g}_1{}^2/4\pi\approx0.08\pm0.01. \tag{3.1}$$

The over-all agreement with experiment seems to be quite impressive, considering the fact that there are no other free parameters. Indeed the agreement seems to be somewhat better than for (**70**, 1^-) baryon decays discussed in I. The results of the $O(4,2)$ theory[8] (which mainly deal with 2^+ meson decays) have also been listed for comparison. We see that both the theories predict almost identical branching ratios for 2^+ meson decays.

[17] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).

An interesting feature of Table II is that the $SU(3)$ assignments for (η,X^0) seem to work somewhat better than the ideal mixing angle except for $\delta\longrightarrow\eta\pi$. In particular, if the recently discovered[18] $\pi_N(980)$ is considered as the genuine $(I=1)$ scalar meson, then its principal experimental mode $\pi_N\longrightarrow\eta\pi$ can be explained only on the assumption that η is either a pure octet or has at most a slight admixture of the singlet component. While the results given in Table II for the $\eta\pi$ modes are in apparent discord with those of Harari[16] on the basis of $SU(3)$ analysis alone, the discrepancy may be due to the manner in which the momentum dependence has been considered in the present model versus pure $SU(3)$ calculations. Thus in an $SU(3)$ investigation[19] it is usual to take the phase space as $\sim k^{2l+1}$, while in the present model, this factor is multiplied by an additional factor $[(\mu/\omega_k)^{2l}]$, which considerably tones down the effect of large k. Since we have already seen the efficacy of this last factor for baryon decays in I, and since it is reasonable to assume that the couplings of both types of hadron states should show quantitatively similar features, it would be hard to give up the factor $(\mu/\omega_k)^{2l}$ for the meson case. Also, in the context of this factor, the $SU(3)$ breaking for meson decays is given by the $\mu^{1/2}$ and $\mu^{-3/2}$ laws for s- and d-wave decays, respectively, exactly as for baryon couplings. One should perhaps interpret the good agreement of this model with the data as an "experimental evidence" for the need for such extra phase-space factors, which are otherwise indicated on general, theoretical grounds as well.

The s-wave decays, such as the $K\bar{K}$ modes of threshold resonances like η_V or π_V, are a direct manifestation of the recoil effect inserted phenomenologically through the replacements of $\mathbf{k}^2\longrightarrow(-\mu^2)$ in the $(L-1)$-wave couplings. The only "bad" case is $\delta\longrightarrow\eta\pi$ which is still predicted to be "large." A possible way out might lie in the recognition that the proximity of $\delta(960)$ to $\pi_V(1003)$ near the threshold could strongly affect the rate of $\delta\longrightarrow\eta\pi$. A similar mechanism might occur for η_V versus S^*.

B. Higher Meson Decays

The experimental situation on the higher meson states $(L\geqslant 2)$, which have been recently summarized by Dalitz[20] seems to be much more confused than the corresponding situation for higher baryon states. Though the famous R-S-T-U series has been known to lie on a linear plot in (mass),[2] only the R region has so far been explored in some detail. The following nonstrange mesons have been fairly well identified:

[18] R. Ammar *et al.*, Phys. Rev. Letters **21**, 1832 (1968).
[19] S. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).
[20] R. H. Dalitz, in Proceedings of the Conference on Meson Spectroscopy, University of Philadelphia, 1968 (unpublished).

(i) $\rho_V(1650)$, or the g meson, which decays into $\pi\pi$ with a total width of $\sim$120 MeV.

(iii) $\rho(1700)$, which is seen in 4π final states and has a well-defined $\pi\omega$ mode with a width of 27 ± 6 MeV.

(iii) $\pi_A(1640)$, which decays mainly into three pions.

Among the strange mesons, the best-established one is the $K^*(1780)$, or the L meson, which has a number of well-defined decay modes, mainly to PV pairs, but hardly to PP pairs.

Dalitz[20] has given a discussion of the possible spin-parity and $SU(3)$ assignments for these mesons in terms of various evidences bearing on their decay modes, including angular correlation for some cases [especially $\rho(1700)$]. The most natural L value to be considered for these states is $L=2$, which gives rise to the possible nonet structures ${}^1D_2$ and ${}^3D_{1,2,3}$. We have considered some of these possibilities, partly on the basis of general selection rules (*à la* Dalitz) for the allowed two-body modes of decay, and partly on the basis of various quantitative predictions on the various decay rates provided by our model (without any reference to radial excitations). The best pattern of the various decay modes is obtained by putting $\rho(1700)$ in ${}^3D_3$, $\rho_V(1650)$ in ${}^3D_1$, and $K^*(1780)$ in ${}^3D_3$.[21] For $\pi_A(1640)$, an obvious possibility is to regard it as the Regge recurrence of π, thus suggesting the choice ${}^1D_2$ for this state. Table III gives the predicted versus experimental decay rates for all these mesons under the assignments stated above, with the input value

$$\bar{g}_2{}^2/4\pi\approx0.08 \tag{3.2}$$

adjusted to the $\rho_V\to2\pi$ decay rate. It also lists the decay rates of $A_{2L}(1270)$ under the ${}^1D_2$ assignment. The poor agreement for $\pi_A(1640)\to\rho\pi$ under the ${}^1D_2$ assignment suggests that π_A is more likely to be a radially excited ($n=2$) meson of $L=0$ than a Regge recurrence of π. On the other hand, the assignment ${}^1D_2$ for A_{2L} appears quite reasonable. A similar difficulty appears for the $K\bar{K}$ mode of $\rho_V(1650)$ under the ${}^3D_1$ assignment. Note however that this state has a strong possibility for mixing with the ${}^3S_1$ radial ($n=2$) excitation of the (more familiar) ρ meson.

It is remarkable that the modes for these higher meson decays which involve large variations in quantities appearing in the form factor (2.13), show a rather good pattern of agreement with experiment, with the help of a single input parameter $\bar{g}_2$. From analogy with the corresponding baryon results (discussed in I), one would have expected $\bar{g}_2$ to be nearly equal to $\bar{g}_0$ ($\equiv f_q$) on the basis of the general idea of universal coupling of the Regge trajectories of the $L=0$ states. A part of the discrepancy should perhaps be ascribed to the role of radial exciation which could well be more important for meson states like ρ_V and π_A than for some of the baryon states whose decay modes were considered in I. Such a mechanism however requires more detailed assumptions (e.g., harmonic-oscillator wave functions)[22] which are beyond the scope of the present model.

TABLE III. Decays from $L=2$ to $L=0$ states for mesons.

Particle	LSJ	Mode	$\Gamma_{\rm calc}$ (MeV)	$\Gamma_{\rm expt}$ (MeV)
$\rho(1700)$	${}^3D_3$	$\omega\pi$	21.2	27.5 ± 6.0
		$\pi\pi$	26.6	11 [if $\rho(1700)\neq\rho(1650)$]
$\rho_V(1650)$	${}^3D_1$	$\pi\pi$	102	appears dominant (Dalitz puts as 120 MeV)
		$\omega\pi$	0	...
		$K\bar{K}$	96	$\lesssim12.0$
$K^*(1780)$	${}^3D_3$	$K\pi$	3.1	$\lesssim1.6$
		$K^*\pi$	17.7	17.2 ± 6.0
		$K\rho$	14.4	7.2 ± 2.6
		$K\omega$	4.5	3.5 ± 1.1
		$K\eta$	1.3	$\lesssim1$
		$K\eta_8$	1.9	$\lesssim1$
$\pi_A(1640)$	${}^1D_2$	$K\bar{K}$	0	$\lesssim37.2$
		$\rho\pi$	422	$\lesssim19.0$
		$\pi\eta$; $\pi\eta_8$	0; 0	$\lesssim$ 8.3
$A_{2L}(1270)$	${}^1D_2$	$\rho\pi$	47	dominant } 27 ± 7
		$\eta\pi$; $\eta_8\pi$	0	indication seen } 27 ± 7
		$K\bar{K}$	0	not seen } 27 ± 7

IV. POLARIZATION PREDICTIONS

A more sensitive test of the coupling scheme would be provided by polarization measurements, such as those now available in $B\to\omega\pi$ decay. This process which may be visualized as a ${}^1P_1\to{}^3S_1$ transition in the $Q\bar{Q}$ state, with the emission of a pion, is described (see Sec. II) by a nonrelativistic coupling of the form

$$F_D(k^2)(B_\alpha{}^a k_\alpha)(\omega_\beta k_\beta)\Pi_a\,, \tag{4.1}$$

which arises from the direct term in the $\bar{Q}QP$ coupling. [Here a is the isospin index, (α,β) represent the three-dimensional Cartesian indices, and $F_D(k^2)$ is the associated form factor.] It was pointed out by Uretsky[4] and Lipkin[23] that the interaction (4.1) leads to a $\cos^2\chi$ distribution in the angle χ between the normal $\hat{n}$ to the plane of $\omega\to3\pi$ decay and the direction of the outgoing pion in $B\to\omega\pi$. This follows directly from the structure of (4.1) which explicitly exhibits the factor $(\boldsymbol{\omega}\cdot\mathbf{k})$, since the polarization vector $\boldsymbol{\omega}$ is parallel to $\hat{n}$ and the momentum vector $\mathbf{k}$ is the pion direction in $B\to\omega\pi$ decay. However, the recent experimental distribution[24] in this angle seems to suggest an almost pure $\sin^2\chi$ law. A

[21] For $K^*(1780)$ or L meson, a ${}^1D_2$ assignment is ruled out by charge conjugation. A ${}^3D_1$ assignment will give zero coupling to PV states in total disagreement with experiment, while a ${}^3D_2$ assignment gives too large widths by several orders of magnitude.

[22] We note in this connection a theory of radially excited mesons recently suggested by P. G. O. Freund [ICTP Report, 1968 (unpublished)], according to which the decay rates $\rho_V\to2\pi$ are in the ratio $1:18/35:\frac{2}{5}$ under the respective assumptions of (i) radial excitation of ${}^3S_1$, (ii) ${}^3D_3$, and (iii) ${}^3D_1$. However, this theory still does not say anything about the effects of mixtures.

[23] H. J. Lipkin, Phys. Rev. **159**, 1303 (1967).

[24] G. Ascoli *et al.*, Phys. Rev. Letters **20**, 1411 (1968).

possible way out of this dilemma lies in the role of the recoil term which by itself leads to a $B\omega\pi$ coupling term of the form

$$F_R(k^2)B_\alpha{}^a\omega_\alpha\pi_a\,, \qquad (4.2)$$

where $F_R(k^2)$ is the corresponding form factor. It may be seen from (4.2) that the polarization vectors $\mathbf{B}$ and $\boldsymbol{\omega}$ are directly coupled to each other by index contraction, unlike the structure of (4.1) where these vectors are each coupled to $\mathbf{k}$, but not to each other. It is thus clear that a suitable combination of (4.1) and (4.2), through a special choice of form factors, viz.,

$$F_R(k^2)\approx -k^2F_D(k^2)\,, \qquad (4.3)$$

would lead to a resultant $B\omega\pi$ coupling of the form

$$F_D(k^2)[k_\alpha k_\beta-\delta_{\alpha\beta}\mathbf{k}^2]B_\alpha{}^a\omega_\beta\pi_a\,, \qquad (4.4)$$

which in turn would give rise to a $\sin^2\chi$ law in the angle χ between $\mathbf{k}$ and $\boldsymbol{\omega}$. In a similar way the nonrelativistic $A_1\rho\pi$ interaction according to the direct and recoil terms in $\bar{Q}QP$ coupling are of the respective forms

$$F_D(k^2)(k_\alpha k_\beta-\delta_{\alpha\beta}\mathbf{k}^2)A_\alpha{}^a\rho_\beta{}^b\pi_c\epsilon_{abc} \qquad (4.5)$$

and

$$F_R(k^2)A_\alpha{}^a\rho_\alpha{}^b\pi_c\epsilon_{abc}\,, \qquad (4.6)$$

where the relative normalizations between (4.1) and (4.5) and between (4.2) and (4.6) have been taken into account according to the predictions of the quark model. In this case the direct term gives the angular distribution $\sin^2\theta$ in the angle θ between $\mathbf{k}$ and $\boldsymbol{\rho}$, while the recoil term predicts isotropy in this angle. The special combination (4.3) of F_D and F_R used for $B\rightarrow\omega\pi$, now gives for $A_1\rho\pi$ coupling the structure

$$F_D(k^2)[k_\alpha k_\beta-2_{k\alpha\beta}\mathbf{k}^2]A_\alpha{}^a\rho_\beta{}^b\pi_c\epsilon_{abc}\,, \qquad (4.7)$$

which predicts the distribution $(\cos^2\theta+4\sin^2\theta)$ in the angle θ.

According to the relativistic prescription outlined in Sec. II, the $B\omega\pi$ and $A_1\rho\pi$ couplings would be of the respective forms

$$B_\nu k_\mu\omega_\mu k_\nu\pi\,, \qquad (4.8)$$

$$(1/\sqrt{2})(k_\mu\rho_\mu k_\nu+\mu^2\rho_\nu)A_\nu\pi\,, \qquad (4.9)$$

each multiplied by the form factor (2.13). Equation (4.8) still predicts essentially a $\cos^2\chi$ distribution in the angle χ between $\boldsymbol{\omega}$ and $\mathbf{k}$, and (4.9) a $\sin^2\theta$ distribution in the angle θ between $\mathbf{k}$ and $\boldsymbol{\rho}$, just as the direct-term couplings (4.1) and (4.5) for $B\omega\pi$ and $A_1\rho\pi$, respectively. This seems to suggest that our relativistic prescription for a partial inclusion of the recoil term is not enough to produce the desired angular distribution in $B\rightarrow\omega\pi$ decay. It may, however, be noted that the couplings (4.8) and (4.9) are not really the strict consequences of our prescription since both the $B\omega\pi$ and $A_1\rho\pi$ couplings are mixtures of s and d waves. Thus, for $B\omega\pi$ coupling, viz.,

$$B_\beta k_\alpha\omega_\alpha k_\beta\,,$$

we first separate these two parts explicitly in the form

$$T_{\alpha\beta}k_\alpha k_\beta+\tfrac{1}{3}\mathbf{k}^2\omega_\alpha B_\alpha\,, \qquad (4.10)$$

where

$$T_{\alpha\beta}=\tfrac{1}{2}(B_\alpha\omega_\beta+B_\beta\omega_\alpha-\tfrac{2}{3}\delta_{\alpha\beta}\mathbf{B}\cdot\boldsymbol{\omega})\,, \qquad (4.11)$$

before the relativistic prescription is applied. Our relativistic prescription now not only consists in making the replacements

$$T_{\alpha\beta}\rightarrow T_{\mu\nu}\,,\quad k_\alpha\rightarrow k_\mu\,,\quad B_\alpha\rightarrow B_\mu\,,\quad \omega_\alpha\rightarrow\omega_\mu\,,$$

but in addition the special replacement $\mathbf{k}^2\rightarrow-\mu^2$ in the second (s-wave) term of (4.10). It may then be seen that with these replacements, the nonrelativistic form of the resultant $B\omega\pi$ coupling becomes

$$(\mathbf{B}\cdot\mathbf{k})(\boldsymbol{\omega}\cdot\mathbf{k})-\tfrac{1}{3}(\mu^2+\mathbf{k}^2)(\mathbf{B}\cdot\boldsymbol{\omega})\,, \qquad (4.12)$$

which predicts an angular distribution

$$W(\chi)=[(k^2-\tfrac{1}{3}\omega_k{}^2)^2/3\mu^4]\cos^2\chi+(\omega_k{}^4/27\mu^4)\sin^2\chi\,, \quad (4.13)$$

where the factor μ^4 has been introduced to make the coefficients of $\cos^2\chi$ and $\sin^2\chi$ dimensionless. Substitution of the magnitude of the actual momentum in $B\rightarrow\omega\pi$ decay gives

$$W(\chi)=4.25\cos^2\chi+1.74\sin^2\chi\,. \qquad (4.14)$$

While this result is an appreciable improvement over a pure $\cos^2\chi$ distribution obtained from the simple quark model, the contribution from the s-wave interaction does not seem to be large enough to make the $\sin^2\chi$ term predominant.[25] Thus, while our relativistic prescription goes a part of the way towards resolving the anomaly in the B-meson polarization, it still falls short of the experimental requirements. In a similar manner, we separate the nonrelativistic $A_1\rho\pi$ coupling,

$$(k_\alpha\rho_\alpha k_\beta-\mathbf{k}^2\rho_\beta)A_\beta\,, \qquad (4.15)$$

into s- and d-wave parts explicitly in the form

$$T_{\alpha\beta}'k_\alpha k_\beta+[\tfrac{1}{3}\delta_{\alpha\beta}\mathbf{k}^2(\boldsymbol{\rho}\cdot\mathbf{A})-\mathbf{k}^2\rho_\beta A_\beta]\,, \qquad (4.16)$$

where

$$T_{\alpha\beta}'=\tfrac{1}{2}(\rho_\alpha A_\beta+\rho_\beta A_\alpha-\tfrac{2}{3}\delta_{\alpha\beta}\boldsymbol{\rho}\cdot\mathbf{A})\,. \qquad (4.17)$$

Use of our relativistic prescription on (4.16) gives the structure

$$T_{\mu\nu}'k_\mu k_\nu+[\tfrac{1}{3}\delta_{\mu\nu}(-\mu^2)\rho_\lambda A_\lambda+\mu^2\rho_\lambda A_\lambda]\,, \qquad (4.18)$$

whose nonrelativistic form now becomes

$$(\mathbf{A}\cdot\mathbf{k})(\boldsymbol{\rho}\cdot\mathbf{k})+(\mu^2-\tfrac{1}{3}\omega_k{}^2)(\boldsymbol{\rho}\cdot\mathbf{A})\,. \qquad (4.19)$$

This coupling predicts for the distribution in the angle

[25] In a more qualitative way, this result was already noted in the preliminary version of this model (Ref. 12) by one of the authors.

θ between $\mathbf{k}$ and $\boldsymbol{\rho}$ the expression

$$W'(\theta)=(4/g)(\omega k^4/\mu^4)\cos^2\theta+[(\mu^2-\tfrac{1}{3}\omega_k{}^2)^2/\mu^4]\sin^2\theta. \quad (4.20)$$

Substitution of the magnitude of the actual momentum in $A_1\to\rho\pi$ decay yields

$$W'(\theta)=7.33\cos^2\theta+12.54\times10^{-2}\sin^2\theta, \quad (4.21)$$

which predicts almost a pure $\cos^2\theta$ distribution in contrast to the simple $\sin^2\theta$ distribution given by (4.5). Thus our relativistic prescription, viz., replacing $\mathbf{k}^2$ by $-\mu^2$ in the s-wave part of the interaction, seems to play an important role in A_1 decay, in that it changes the angular distribution almost all the way from $\sin^2\theta$ to $\cos^2\theta$. Such a situation has indeed been reported recently[26] from the observation that the ρ meson in $A_1\to\rho\pi$ is purely longitudinally polarized giving rise to a pure $\cos^2\theta$ distribution. Thus our relativistic prescription predicts an angular distribution for $A_1\to\rho\pi$ in excellent agreement with experiment, which was beyond the scope of the simple quark model.

V. CONCLUSION

We have presented a phenomenological model of meson couplings on almost identical lines to one proposed for baryon couplings in the preceding paper, except for (i) some trivial modifications necessitated by the requirement of relativistic normalizations for the meson fields and (ii) some additional prescriptions bearing on the question of which meson in a final PP state should be regarded as the radiation quantum. The agreement of the meson-decay predictions with experiment is no less impressive (perhaps even more in some respects) than that obtained for baryons in the preceding paper. The model makes some interesting predictions on the angular distributions in some decays such as $B\to\omega\pi$ and $A_1\to\rho\pi$. The prediction for the former mode in our model gives a substantial improvement over the prediction of the simple quark model, although it is still short of the experimental distribution. On the other hand, a dominant $\cos^2\theta$ distribution (longitudinal polarization of the ρ meson) for $A_1\to\rho\pi$ predicted in our model seems to be indicated by some recent evidence[26]; a simple quark model without the inclusion of recoil terms would predict only a pure $\sin^2\theta$ distribution. Our relativistic prescription for the inclusion of the recoil contribution in the present model thus seems to yield much better predictions for certain angular distributions than earlier calculations with a simple quark model. The experimental success of a common form of parametrization for both meson and baryon couplings seems to be a very encouraging result by itself (despite the empirical structure of the coupling scheme), and can perhaps be relied upon to give some indications of the type of symmetry breaking due to masses which a more complete theory is expected to predict in the limit of low momenta.

[26] As quoted by Leith after the review talk by G. Morpurgo, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 249.

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