Multiperipheral Mechanism for a Schizophrenic Pomeranchon*

GEOFFREY F. CHEW AND DALE R. SNIDER
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
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It is demonstrated through an explicit model that the weak high-subenergy tail of the multiperipheral kernel, acting in conjunction with the strong low-subenergy component, is capable of producing a high-ranking output Regge doublet with vacuum quantum numbers. We show that association of the upper doublet member with the P (Pomeranchon) and the lower with the P' is consistent with experimental total, elastic, and diffractive dissociation cross sections, as well as with multiplicity of produced pions, and predicts a Pomeranchon slope near $t=0$ that is roughly half normal. As t becomes negative, the Pomeranchon slope decreases to a small value, while for t positive the slope increases to a normal value, the P trajectory containing the particles usually assigned to the P'. The latter trajectory has a converse behavior, with small slope for positive t and normal slope at negative t. The P and P' trajectories thus exchange "normal" and "abnormal" roles near $t=0$.

I. INTRODUCTION

THE observed large multiplicity of particles produced in high-energy hadron collisions precludes a dominant role for the Pomeranchon in the multiperipheral kernel. Large multiplicity implies a low average subenergy for adjacent particle pairs in the multiperipheral "chain" and thus a controlling influence for the low-energy resonance component of the kernel. The high-energy Pomeranchuk component can be no more than a perturbation if the kernel is to generate the requisite number of final-state pions. Even in the role of perturbation, however, it has been noted that the weak Pomeranchon "tail" of the kernel may be responsible for qualitatively interesting effects in the complex J plane.[1–3] This paper draws attention to the possibility that the input Pomeranchon perturbation may split the leading output pole into two poles, whose combined strength corresponds to the single output pole that would be generated by the kernel's low-energy resonance component acting alone. We show that the magnitude of this splitting may be sufficiently large to permit identification of the resulting Regge-pole doublet with the P and P' trajectories. The P trajectory is then relatively flat for $t<0$ but has a normal slope for $t\gtrsim 1$ GeV^2. The converse is true for the P'. Each has a slope roughly half normal near $t=0$.

To achieve a tractable framework, we employ a factorizable kernel.[4,5] Support for this simplification is given by recent numerical work on the "ABFST" model,[6] where the leading output Regge pole turns out to be adequately described by the trace approximation to the Fredholm determinant, even when the low-energy resonance component of the kernel is dominant. Validity of the trace approximation implies the existence of an approximately equivalent factorizable kernel. Our model, then, is of the CP type,[1,4] but we shall alter the detailed structure of the kernel so as to make it identifiable with the more explicit and realistic ABFST model.

II. MODEL

The ABFST model is based on pion exchange, the kernel being proportional to the elastic $\pi\pi$ cross section, as shown in Fig. 1. Our model further approximates the elastic $\pi\pi$ cross section by the sum of a "Pomeranchuk component"—the high-energy tail, and a low-energy resonance component. Opening up the $\pi\pi$ vertex, we thus may think of a "three-channel" system, π, R, and P, as shown in Fig. 2, with no diagonal couplings and no R-P coupling. (We lean on the duality concept to justify speaking of the resonance link as a crossed "channel.") In the notation of Ref. 1, the

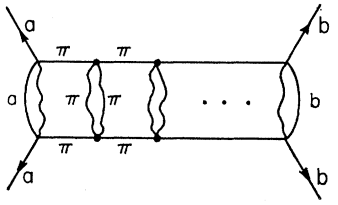

FIG. 1. ABFST model, each vertex being proportional to the elastic $\pi\pi$ amplitude.

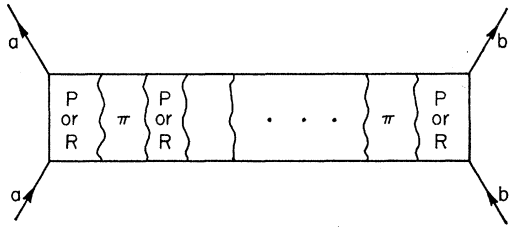

FIG. 2. CP version of the ABFST model.

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[1] G. F. Chew and W. R. Frazer, Phys. Rev. **181**, 1914 (1969).
[2] L. Caneschi and A. Pignotti, Phys. Rev. **184**, 1915 (1969).
[3] J. S. Ball and G. Marchesini, Phys. Rev. **188**, 2508 (1969).
[4] G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968).
[5] G. F. Chew, M. L. Goldberger, and F. E. Low, Phys. Rev. Letters **22**, 208 (1969).
[6] L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento **25**, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, *ibid.* **26**, 6 (1962).

internal coupling matrix can be written in the form

$$\lambda^{\gamma\gamma'}=\begin{pmatrix}0 & g_R^2 & g_P^2\\ g_R^2 & 0 & 0\\ g_P^2 & 0 & 0\end{pmatrix}, \tag{2.1}$$

while the external coupling is

$$\lambda_a{}^{\gamma'}=\begin{pmatrix}0\\ g_{aR}^2\\ g_{aP}^2\end{pmatrix}. \tag{2.2}$$

It is understood that the crossed-channel quantum numbers are those of the vacuum, so the relevant direct-channel total cross section is an appropriate average over internal quantum numbers. Still using the notation of Ref. 1, the solution to the multiperipheral equation for the forward direction imaginary part, projected onto the Lorentz quantum numbers $\lambda=J+1$ and $M=0$, is

$$A_{ab}(J)=g_{aR}{}^2\rho_R(J)g_{Rb}{}^2+g_{aP}{}^2\rho_P(J)g_{Pb}{}^2 + \frac{[g_{aR}{}^2\rho_R(J)g_R{}^2+g_{aP}{}^2\rho_P(J)g_P{}^2]\rho_\pi(J)[g_R{}^2\rho_R(J)g_{Rb}{}^2+g_P{}^2\rho_P(J)g_{Pb}{}^2]}{1-\rho_\pi(J)[g_R{}^4\rho_R(J)+g_P{}^4\rho_P(J)]}, \tag{2.3}$$

the first two terms corresponding to the elastic cross section and the third to the inelastic. The "propagator" functions $\rho_{\pi,R,P}(J)$ are the Lorentz projections of the subenergy dependence of the π, R, and P links, respectively. For $t\neq 0$ these propagators become functions of t.[7] Otherwise (2.3) remains unchanged.

Regge poles arise when the denominator of the second term of Eq. (2.3) vanishes, and a qualitative understanding of the main point of this paper can be based on the following observations: The "propagators" $\rho_{\pi,R,P}(J)$ are real and analytic functions in a J-plane cut along the real axis from $J=-\infty$ to $J=\beta_{\pi,R,P}$, where for $t=0$, $\beta_P=2\alpha_P(0)-1\approx 1$, $\beta_\pi=2\alpha_\pi(0)-1\approx -1$, and (by duality) $\beta_R=2\alpha_\rho(0)-1\approx 0$. In each case, the branch point at $J=\beta$ is logarithmic and, at least for $\rho_P(J)$, is infinite. To the right of the branch point at β, each propagator is positive and monotonically decreasing as J increases. The region of concern here is the interval $0.5\lesssim J\lesssim 1$, where $\rho_\pi(J)$ and $\rho_R(J)$ are positive, real, and smoothly decreasing, but toward the upper end of this interval there occurs the infinite branch point in $\rho_P(J)$.

As already discussed, the Pomeranchuk coupling $g_P{}^4$ is much smaller than the resonance coupling $g_R{}^4$, and to achieve the experimentally indicated multiplicity it is necessary that the magnitude of $g_R{}^4$ lead to a zero of the denominator close to $J=1$ when the input Pomeranchon is neglected completely. At the same time, if we set $g_R{}^4=0$, even a small value of $g_P{}^4$ unavoidably generates a zero for J just above β_P and therefore near the point $J=1$. This pole has a very small residue and a correspondingly small physical effect. With $g_R{}^4$ and $g_P{}^4$ *both* nonzero, nevertheless, *two* poles near $J=1$ are expected in the absence of R-P interaction, one pole of large residue generated by the "resonance channel" and one pole of small residue generated by the "Pomeranchon channel." Because of the indirect coupling between R and P channels via the pion channel, there is a "mixing" of the two poles together with a mutual repulsion. If, before mixing, the two poles are sufficiently close together, the final pair of poles may have comparable residues.

III. REGGE PROPAGATORS

To illustrate the above mechanism we now proceed to make specific choices for the propagators $\rho_{\pi,R,P}(J)$. Treating the exchanged pions as particles of fixed, zero spin suggests the approximation $\rho_\pi(J)\approx(J+1)^{-1}$, replacing the cut by a pole at $J=-1$. A study of the ABFST model confirms the presence of such a pole but also reveals an effective pole near $J=0$ when the trace approximation is employed, owing to the very small pion mass.[8] Thus we take

$$\rho_\pi(J)=1/(J-\tilde{\beta}_\pi), \tag{3.1}$$

where $-1<\tilde{\beta}_\pi<0$.

The simplest reasonable choice for the "resonance propagator" is $\rho_R(J)=1$. Although a branch point occurs near $J=0$, as noted above, numerical estimates show that the corresponding high-energy tail of the resonance component of $\sigma_{\pi\pi}{}^{\text{el}}$ is small compared with the contribution of the individual ρ and f_0 resonances. Cutting off the tail of the resonance spectrum removes all singularities from $\rho_R(J)$. (We ignore the exponential decrease at large positive J associated with multiperipheral phase space.[9])

We can afford to be crude in our treatment of $\rho_\pi(J)$ and $\rho_R(J)$ because the singularities of these functions do not fall into the region of concern: $0.5\lesssim J\lesssim 1$. It is necessary to be more careful with $\rho_P(J)$. If the Pomeranchuk trajectory were flat we could take $\rho_P(J)=(J-\beta_P)^{-1}$. The most recent evidence on the energy dependence of the forward diffraction peak in pp scatter-

[7] W. R. Frazer and C. H. Mehta, Phys. Rev. Letters **23**, 258 (1969).

[8] G. F. Chew, T. W. Rogers, and D. R. Snider, UCRL Report No. UCRL-19457, 1970 (unpublished).

[9] G. F. Chew and D. R. Snider, Phys. Letters **31B**, 75 (1970).

ing suggests that $\alpha_P'(0)\approx 0.47$ GeV^{-2}.[10] Now, if

$$d\sigma_{\pi\pi}{}^{\rm el}/dt \propto e^{2at} s^{2[\alpha_P(t)-1]}, \tag{3.2}$$

the effective length of the cut in $\rho_P(J)$ is given by

$$\Delta J \approx \alpha_P'(0)/a. \tag{3.3}$$

Assuming the value of a to be the same as in pp scattering,[10] we have $a\approx 3.4$ GeV^{-2} and thus $\Delta J\approx 0.14$. The smallness of this interval justifies the replacement of the cut by an equivalent pole at

$$\bar{\beta}_P \approx \beta_P - \Delta J, \tag{3.4}$$

if we are concerned with structure in the J plane on a scale large compared with ΔJ. Since the P-P' splitting is of the order 0.4, it is not unreasonable to take

$$\rho_P(J) = 1/(J-\bar{\beta}_P). \tag{3.5}$$

IV. REGGE POLES AND RESIDUES

Our model solution (2.3) then becomes

$$A_{ab}(J) = g_{aR}{}^2 g_{Rb}{}^2 + \frac{g_{aP}{}^2 g_{Pb}{}^2}{J-\bar{\beta}_P} + \frac{[g_{aR}{}^2 g_R{}^2 + g_{aP}{}^2 g_P{}^2/(J-\beta_P)][g_R{}^2 g_{Rb}{}^2 + g_P{}^2 g_{Pb}{}^2/(J-\beta_P)]}{J-\bar{\beta}_\pi - g_R{}^4 - g_P{}^4/(J-\bar{\beta}_P)}. \tag{4.1}$$

Observe that the "pole" in the elastic cross section at $J=\bar{\beta}_P$, which represents the AFS[6] cut, is canceled by a corresponding "pole" in the inelastic cross section, so that the only poles in the total cross section arise from the two zeros of the denominator of the third term. These are the true Regge poles.

The two Regge poles are located at

$$\alpha_\pm = \tfrac{1}{2}\{\bar{\beta}_P + \beta' \pm [(\bar{\beta}_P - \beta')^2 + 4g_P{}^4]^{1/2}\}, \tag{4.2}$$

where

$$\beta' \equiv \bar{\beta}_\pi + g_R{}^4 \tag{4.3}$$

is the position of the single pole that would be generated by the resonance component of the kernel, acting alone. The residues of the two poles are easily calculated, allowing us to write

$$A_{ab}(J) = \frac{r_+}{J-\alpha_+} + \frac{r_-}{J-\alpha_-} + g_{aR}{}^2 g_{Rb}{}^2, \tag{4.4}$$

with

$$r_\pm = \frac{1}{2}\left(1 \mp \frac{\Delta\beta}{\Delta\alpha}\right) \times\left[\left(g_{aR}{}^2 g_R{}^2 + \frac{2g_P{}^2 g_{aP}{}^2}{\pm\Delta\alpha - \Delta\beta}\right)\times(a\leftrightarrow b)\right], \tag{4.5}$$

where

$$\Delta\beta \equiv \bar{\beta}_P - \beta' \tag{4.6}$$

and

$$\Delta\alpha \equiv +[(\Delta\beta)^2 + 4g_P{}^4]^{1/2}. \tag{4.7}$$

Note that as $g_P{}^2 \to 0$ with $\Delta\beta$ fixed, $\Delta\alpha \to \Delta\beta$ and the residue of the pole at $J=\alpha_+$ approaches $g_{aP}{}^2 g_{Pb}{}^2$, corresponding to purely elastic scattering. The residue of the pole at $J=\alpha_-$ in this limit is much larger, since it carries the entire inelastic cross section. On the other hand, if $\Delta\beta \to 0$ with $g_P{}^2$ fixed, $\Delta\alpha \to 2g_P{}^2$ and the ratio of the pole residue factors becomes

$$(g_{aR}{}^2 g_R{}^2 + g_{aP}{}^2)/(g_{aR}{}^2 g_R{}^2 - g_{aP}{}^2), \tag{4.8}$$

favoring the pole at α_+. Thus, for a sufficiently small ratio of $\Delta\beta$ to $2g_P{}^2$, even though the Pomeranchon coupling $g_P{}^2$ is small, the two poles will have comparable residues. It is easy to show that the *sum* of the two residues is independent of $g_P{}^2$, being given by

$$r_+ + r_- = g_{aP}{}^2 g_{Pb}{}^2 + g_{aR}{}^2 g_R{}^4 g_{Rb}{}^2, \tag{4.9}$$

so the resonance component of the kernel alone determines the sum of the two residues, even when these residues are comparable in magnitude.

To the extent that the splitting $\Delta\alpha$ between the two poles can be considered small, they behave in effect as a single pole with residue given by (4.9). In this sense the input Pomeranchon effect is weak. Experiments, however, have been sufficiently precise to resolve two separate high-ranking poles with the vacuum quantum numbers, the P and the P'. Let us now see whether the observed P-P' splitting and residue ratio can be reasonably accommodated.

V. NUMERICAL ESTIMATE OF PARAMETERS

The observed P-P' splitting is roughly 0.4, requiring us to set

$$\Delta\alpha \approx 0.4. \tag{5.1}$$

From Eq. (3.4),

$$\bar{\beta}_P \approx 2\alpha_+ - 1 - \Delta J, \tag{5.2}$$

and since

$$\alpha_+ = \tfrac{1}{2}(\bar{\beta}_P + \beta' + \Delta\alpha), \tag{5.3}$$

it follows that

$$\beta' \approx 1 - \Delta\alpha + \Delta J. \tag{5.4}$$

Thus we choose

$$\beta' \approx 0.7. \tag{5.5}$$

Next, if we set

$$\alpha_+ = \beta' + \tfrac{1}{2}(\Delta\beta + \Delta\alpha) \tag{5.6}$$

[10] G. G. Beznogikh *et al.*, Phys. Letters **30B**, 274 (1969). Note that for the purposes of this paper it is the "effective slope" of the Pomeranchon, as measured by the actual rate of forward peak shrinkage, that is important.

equal to 1, we have

$$\Delta\beta = 2(1-\beta') - \Delta\alpha \approx 0.2. \tag{5.7}$$

The parameter g_P^2 is determined now from (4.7):

$$2g_P^2 = [(\Delta\alpha)^2 - (\Delta\beta)^2]^{1/2} \approx (0.12)^{1/2} \tag{5.8}$$

or

$$g_P^4 \approx 0.03. \tag{5.9}$$

From (4.3),

$$g_R^4 = \beta' - \tilde{\beta}_\pi, \tag{5.10}$$

and since $\tilde{\beta}_\pi$ is supposed to lie between 0 and -1, we choose for simplicity

$$g_R^4 \approx 1. \tag{5.11}$$

To discuss residues, we must select a particular pair of incident particles. We choose these to be pions and assume that at moderate energies, say $s \approx 20$ GeV^2, the ratio of total to elastic cross sections is about 6; this estimate is based on the corresponding ratios for πN and NN scattering, together with factorization. In our model the elastic cross section has an energy dependence characterized by $\tilde{\beta}_P \approx 0.9$, while the total cross section contains the two powers $\alpha_+ \approx 1.0$ and $\alpha_- \approx 0.6$ with comparable weight: At not too high an energy the ratio of total to elastic cross section is thus slowly varying. Formula (4.9) then leads us to take

$$g_{\pi R}^4 / g_{\pi P}^4 \approx 5. \tag{5.12}$$

We are now in a position to calculate the residue ratio. From (4.5) we have

$$\frac{r_+}{r_-} = \frac{1-\Delta\beta/\Delta\alpha}{1+\Delta\beta/\Delta\alpha}\left(\frac{g_{\pi R}^2}{g_{\pi P}^2}g_R^2 + \frac{2g_P^2}{\Delta\alpha-\Delta\beta}\right)^2 \Bigg/ \left(\frac{g_{\pi R}^2}{g_{\pi P}^2}g_R^2 + \frac{2g_P^2}{-\Delta\alpha-\Delta\beta}\right)^2$$

$$\approx \frac{0.5}{1.5}\left((\sqrt{5}) + \frac{2(0.173)}{0.2}\right)^2 \Bigg/ \left((\sqrt{5}) - \frac{2(0.173)}{0.6}\right)^2$$

$$= 1.9.$$

Such a number is in reasonable accord with the phenomenologically determined ratio of P and P' residues for πN and NN scattering,[11] translated by factorization to $\pi\pi$.

VI. MULTIPLICITY

At moderate energies the average multiplicity of produced pions in our model is roughly the same as in a model with only the resonance component in the kernel, that is, with $g_P^2 = 0$. This average multiplicity is easily calculated to be

$$\bar{n} = 2g_R^4 \ln s. \tag{6.1}$$

As the energy increases the burden shifts gradually to the upper pole at α_+, which carries a fraction $\approx \frac{1}{2}$ of the residue sum. The coefficient of $\ln s$ has a corresponding gradual decrease to a limiting value which a short calculation shows to be

$$\bar{n} \sim 2\left[\tfrac{1}{2}g_R^4\left(1 - \frac{\Delta\beta}{\Delta\alpha}\right) + \frac{g_P^4}{\Delta\alpha}\right]\ln s. \tag{6.2}$$

With the numbers of Sec. V, we thus have a gradual shift from $\bar{n} \approx 2 \ln s$ at moderate energies to $\bar{n} \approx 0.65 \ln s$ at extremely high energies.[12] Cosmic-ray emulsion data, interpreted superficially, suggest multiplicity at very high energy that is larger by a factor of 3 than what we predict, but complex nuclei are involved and, in addition, low multiplicity events may be missed. Accelerator data at moderate energies are consistent with (6.1). We therefore regard the multiplicity prediction of the model to be at least marginally satisfactory.

VII. DIFFRACTIVE DISSOCIATION AT INTERMEDIATE ENERGIES

At moderate energies the effect of the Pomeranchon in the kernel is a relatively small perturbation, and the cross section can meaningfully be expanded in powers of g_P^2. The linear term in such an expansion corresponds to a single Pomeranchon link occurring at one end of the multiperipheral chain and may approximately be identified with the cross section for "diffractive dissociation." The experimental magnitude of the latter thus provides a check on the magnitude of g_P^2.

The full cross-section formula corresponding to (4.4) is[13]

$$\begin{aligned}\sigma_{ab}(s) &= r_+ s^{\alpha+} + r_- s^{\alpha-} \\ &= \tfrac{1}{2}(r_+ + r_-)(s^{\alpha+} + s^{\alpha-}) + \tfrac{1}{2}(r_+ - r_-)(s^{\alpha+} - s^{\alpha-}) \\ &= (r_+ + r_-)s^{\bar{\alpha}} \cosh(\tfrac{1}{2}\Delta\alpha \ln s) \\ &\quad + (r_+ - r_-)s^{\bar{\alpha}} \sinh(\tfrac{1}{2}\Delta\alpha \ln s),\end{aligned} \tag{7.1}$$

where

$$\bar{\alpha} \equiv \tfrac{1}{2}(\tilde{\beta}_P + \beta') \tag{7.2}$$

and where we have omitted the resonance δ function corresponding to the third term of (4.4). If $\ln s$ is only moderate, the cosh and sinh functions in (7.1) may be expanded to give the simpler form

$$\sigma_{ab}(s) \approx (r_+ + r_-)s^{\bar{\alpha}} + (r_+ - r_-)s^{\bar{\alpha}}(\tfrac{1}{2}\Delta\alpha) \ln s. \tag{7.1'}$$

The first term of (7.1') is independent of g_P^2, and

[11] R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

[12] This predicted shift in the rate of multiplicity growth is a characteristic qualitative feature of any multiperipheral doublet model.

[13] Our unit of energy squared is 1 GeV^2.

keeping only the linear component then gives

$$\sigma_{ab}^{\text{diff. diss.}} \approx g_P^2 g_R^2 (g_{aR}^2 g_{Pb}^2 + g_{aP}^2 g_{Rb}^2) s^{\bar{\alpha}} \ln s, \quad (7.3)$$

to be compared with the first term of (7.1′), which approximates the total cross section at intermediate energies. Using (4.9) to evaluate the latter, we have, for comparison purposes,

$$\sigma_{ab}^{\text{tot}} \approx (g_{aP}^2 g_{Pb}^2 + g_{aR}^2 g_R^4 g_{Rb}^2) s^{\bar{\alpha}}, \quad (7.4)$$

or

$$\frac{\sigma_{ab}^{\text{diff. diss.}}}{\sigma_{ab}^{\text{tot}}} \approx g_P^2 g_R^2 \left(\frac{g_{aR}^2 g_{Pb}^2 + g_{aP}^2 g_{Rb}^2}{g_{aP}^2 g_{Pb}^2 + g_{aR}^2 g_R^4 g_{Rb}^2} \right) \ln s. \quad (7.5)$$

The numbers of Sec. V, when inserted into Eq. (7.5), lead to

$$\sigma_{\pi\pi}^{\text{diff. diss.}} / \sigma_{\pi\pi}^{\text{tot}} \approx 0.13 \ln s, \quad (7.6)$$

a result which evidently is meaningful only for moderate values of $\ln s$.

Current experimental evidence is too meager to yield a reliable integrated diffractive dissociation cross section. However, an ABFST-model extrapolation from the A_1 region, where such a model is successful, suggests a diffractive-dissociation cross section that at moderate energies is of the same order of magnitude as the corresponding elastic cross section,[8] in agreement with (7.6). Thus the value of g_P^2 needed to produce the observed P-P' splitting is consistent with that implied by diffractive dissociation.

VIII. DISCUSSION

The model employed here oversimplifies a complex multiperipheral mechanism, but relatively few aspects of the mechanism are essential to the point in question. The essential features are:

(a) The multiperipheral kernel chiefly generates low subenergies for adjacent pion pairs in the chain, while at the same time the kernel possesses a weak high-subenergy "tail."

(b) In the absence of this "tail" the kernel strength must be capable of generating an output "vacuum" Regge pole with position fairly close to $J=1$ and with a residue whose magnitude corresponds roughly to the high-energy inelastic cross section.

(c) The kernel "tail," acting alone, generates a weak-output vacuum pole, also near $J=1$.

(d) The complete kernel then generates *two* vacuum poles which split apart by the usual repulsive mechanism. If the uncoupled spacing is sufficiently small, the two final residues are of comparable magnitude.

The function of our model is to see whether the strength of the kernel "tail" can

(i) be large enough to produce the observed P-P' splitting,

(ii) be large enough with respect to the uncoupled-pole separation so that the P and P' residues are comparable,[14]

(iii) be small enough to avoid pushing the P above $J=1$, in violation of the Froissart limit,

(iv) be small enough to accord with the observed probability of diffractive dissociation.[15]

All the foregoing requirements can be satisfied within our crude model, leading us to suggest that the actual multiperipheral kernel may generate a P-P' doublet. The most questionable aspect of our model is its replacement of the input Pomeranchon cut by a pole. This replacement depends on the effective Pomeranchon slope's being sufficiently small that the effective length of the input cut is smaller than other significant J-plane intervals. If the input cut is not concentrated, the splitting of the two output poles has a substantial imaginary component in the J plane (one of the poles residing on the unphysical sheet) and the observed P-P' configuration may not be achieved.

A number of questions are raised by the suggestion of a doublet P-P' relationship:

A. Phenomenological schemes often have assigned to the P' a role closely related to that of other meson trajectories, while the P is considered a unique phenomenon. Is our proposal incompatible with such a picture?

B. With the doublet mechanism, are particles expected on both the P and P' trajectories?

C. Ting[16] has suggested a baryon-exchange multiperipheral mechanism for the P' and ω. Does our proposal exclude his, and vice versa?

We do not have a definite answer to these questions but suggest a partial answer to A and B in order to illustrate the subtlety of the situation. The model employed here can be extended to nonzero momentum transfer t, and one may continue to decompose the kernel into a resonance component plus a high-energy tail. If the effective Pomeranchon slope is substantially smaller than "normal," the uncoupled output trajectories will have different slopes, crossing at some small positive value of t (see Fig. 3). At either large positive t or large negative t, therefore, the mixing of the two poles will be weak. The trajectory controlled by the resonance component of the kernel is expected to be "normal" and to contain particles, but the trajectory controlled by the high-energy tail of the kernel will be "abnormal." Note, as shown in Fig. 3, that an exchange of roles for the two final trajectories occurs in the crossover region. The slope of each near $t=0$ is expected to be

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[14] Satisfaction of this condition ensures a substantial multiplicity at very high energy.

[15] The strength of the kernel tail inferred here is much larger than that inferred earlier from diffractive dissociation data by Chew and Pignotti in Ref. 4. The earlier determination incorrectly omitted the contribution to the diffractive dissociation cross section from produced masses above 2 GeV.

[16] P. Ting, Phys. Rev. **181**, 1942 (1969).

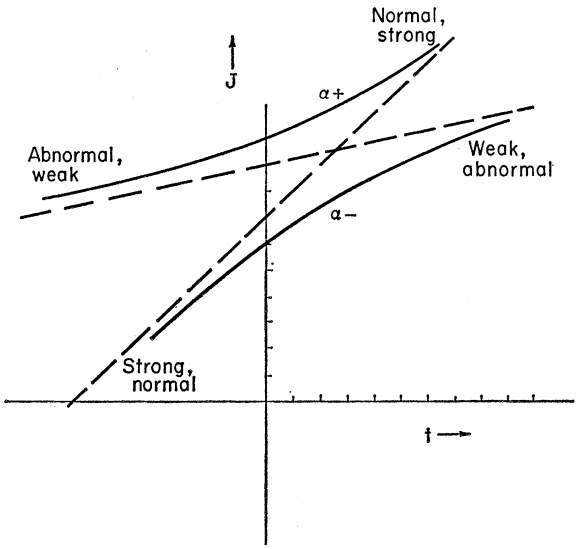

FIG. 3. Uncoupled (dashed) and coupled (solid) P and P' trajectories.

about half the "normal" slope. For the Pomeranchon, at least, there is experimental evidence for such an "in-between" slope.[10] One of the further striking predictions of our model is that the Pomeranchon slope will decrease as $|-t|$ grows.[17]

The picture represented by Fig. 3, for negative t away from the crossover region, allows revival of the suggestion in Ref. 5 that the tendency of the Pomeranchon to remain near 1 is related to the smallness of $g_P{}^4$. The uncoupled trajectory [$\alpha_+(t)$, that is] occurs at $\tilde{\beta}_P(t)+O(g_P{}^4)$, where $\tilde{\beta}_P(t)\approx 2\alpha_+(t/4)-1-\Delta J(t)$. The natural way to accomplish such a condition, with $\Delta J(t)$ small, is to have $\alpha_+(t)$ always close to 1.

[17] The Frazer-Mehta generalization of the CP model in Ref. 7 has $\rho_P(J,t)$ decreasing strongly with $|-t|$. If $\rho_P(J,t)\to 0$ as $t\to-\infty$, the model predicts that $\alpha_P(t)\to 1$ as $t\to-\infty$, so there would be a region in which the Pomeranchon slope is actually negative.

The mechanism described here is expected to double other trajectories as well, such as the ρ, but because the weak input cut is now less concentrated, the output-pole splitting may be less visible experimentally. The reason to expect larger ΔJ with nonvacuum quantum numbers is that trajectories with slopes near 1 GeV^{-2} will occur in the combinations that give rise to the weak-input cut.

In conclusion, we refer to three other recent multiperipheral calculations that are related to the doublet phenomenon. Ball and Marchesini[3] studied the ABFST Bethe-Salpeter equation with a kernel qualitatively similar to ours, but the resonance component of their kernel was not sufficiently strong to produce the needed approximate coincidence of uncoupled poles. The ABFST model kernel, if based on the on-shell elastic $\pi\pi$ cross section, contains a resonance component that is too weak by a factor ≈ 2. Ball and Marchesini thus did not achieve adequate pole mixing. In this paper we have assumed that some correction to the ABFST model will suitably augment the low-subenergy component of the kernel.

The model of Ref. 4, in particular Eq. (4.12), contains an explicit doublet but, as remarked in our Ref. 15, the splitting was taken in Ref. 4 to be so much smaller than in this paper that there was no temptation to associate the doublet with P-P'.

In a completely different but still essentially multiperipheral calculation, using the on-shell strip model, Collins and Johnson[18] found only a single high-lying output vacuum pole, of large residue. In their equations, however, they employed a cutoff which effectively removed the high-energy tail of the kernel—leaving only the resonance component. We conjecture that if the strip-model equation can be solved without a cutoff, two vacuum poles will emerge, each with residue about half that found by Collins and Johnson.

[18] P. D. B. Collins and R. C. Johnson, Phys. Rev. **185**, 2020 (1969); **177**, 2472 (1969).