will vary with the incident energy. All this implies that the goodness of the fit is determined by the model itself and is independent of the choice of the parameter sets. Our choice of parameters is conditioned by the demand that we include the inelastic process in the backward direction. The fitted parameter x decreases as the incident momentum increases. This dependence may reveal the importance of the inelastic process at higher energies. The ratio of the parameter a_1 to the incident momentum k shows a slight decrease as the incident energy increases. This may indicate the transparency of the optical medium at higher energies. The π^+p backward scattering data exhibit more complicated structure than the π^-p data. It is apparent that our simple model cannot accommodate the π^+p backward data. We hope that with some additional mechanism, our simple model may be made to include the π^+p results. The success of the π^-p fits indicates that our model may provide valuable clues as to the nature of high-energy scattering, and that the exchange mechanism is not essential for a description of the backward scattering.

ACKNOWLEDGMENTS

We have enjoyed useful conversations on this subject with our colleagues in the physics department of Virginia Polytechnic Institute. We especially thank Professor R. A. Arndt and Professor S. P. Almeida for helping in the computational programming, and Professor D. Kaplan for reading the manuscript.

PHYSICAL REVIEW D VOLUME 1, NUMBER 1 1 JANUARY 1970

Hadron Couplings in Broken $SU(6)\times O(3)$. I. Baryon Decays

D. L. Katyal* and A. N. Mitra

Department of Physics and Astrophysics, University of Delhi, Delhi-7, India

(Received 1 July 1969)

A new form of parametrization is proposed for meson-baryon couplings in a phenomenological quark (Q) model of broken $SU(6)\times O(3)$ for the decays of baryons (mass M) belonging to the representations $(\mathbf{56},2l^+)$ or $[\mathbf{70},(2l+1)^-]$ to baryons (mass m) belonging to **56**, together with the emission of pseudoscalar (P) mesons (mass μ). The starting point is the use of the direct term in $\bar{Q}QP$ coupling for the evaluation of the meson-baryon coupling structures which are reexpressed in terms of nonrelativistic Rarita-Schwinger fields together with multiple derivative structures in the meson field. A relativistic generalization of the latter is then proposed through a simple extension of the index structures in the $(L+1)$ partial-wave coupling terms. For the $(L-1)$-wave coupling terms, which appear with an extra multiplying factor $\mathbf{k}^2$ in the meson three-momentum $\mathbf{k}$, an additional ansatz $\mathbf{k}^2\rightarrow k_\mu k_\mu$ $(\equiv -\mu^2)$ is used in order to include the contributions of the recoil terms for $(L-1)$-wave transitions in a certain special combination, so as to incorporate the experimental feature of enhanced heavy-meson decays in the s wave. Finally, all these coupling terms are assumed to be multiplied by the following form factor, the plausibility of whose structure is defended on physical grounds: $f_L(k^2)=g_L\mu^{-L-1}(\mu/m_\pi)^{1/2}(\mu/w_k)^{L\pm1}(M/m)^{1/2}$, where the exponents $(L\pm1)$ are used for emissions in the corresponding waves, and g_L is a single free parameter governing the entire supermultiplet transition. The scheme, which is subjected to a detailed experimental test in respect to a large number of baryonic transitions from $(\mathbf{70},1^-)$ and $(\mathbf{56},2^+)$ states involving a wide range of masses and momenta, is found to provide an impressive number and quality of agreements with experiment. It is also shown to yield almost equal values of the coupling constants g_L in respect to the couplings of several Regge recurrences of the Δ resonances, in conformity with the general expectation of a universal coupling for the Regge trajectory of a given particle.

1. INTRODUCTION

ONE of the most fruitful studies of higher resonances has been through their couplings with the **56** baryons and **36** mesons. The usefulness of these couplings lies partly in their mathematical simplicity (being merely three-point functions) and partly in their direct physical manifestations through the two-body decays of various resonances into lighter objects. Moreover, the decay rates of these resonances are generally very sensitive to their spin-parity and $SU(3)$ assignments, so that they provide fairly unambiguous means of testing these assignments without going too much into the details of a theory. In this respect decay properties are a better guide to the identifications of $SU(6)$ quantum numbers for resonances than, e.g., the studies of mass formulas or mass splittings which not only are less sensitive to the input potentials but also are much more model-dependent.

Studies of hadron couplings can be classified under two broad heads, (i) those which are based on fairly elaborate relativistic groups, which leave little scope for parametrization, and (ii) those which use the quark model as a pedagogical device for the evaluation of coupling coefficients in terms of certain phenomenological form factors which are used as free parameters. In the

* Permanent address: Basic Physics Division, National Physical Laboratory, Hillside Road, Delhi-12.

former category belong the formal theories based on noncompact groups like $\tilde{U}(12)$, $\tilde{SL}(6,C)$, and $O(4,2)$,[1–3] each of which allows, in principle, the evaluation of various decay properties in terms of a single input parameter. The physical features of such a theory are so rigidly connected with the basic group assumption itself that any possible adjustment (to meet the experimental situation) can be made only at the cost of the theory itself. The second type of approach makes use of less elaborate group assumptions but makes up for the deficiency through a more liberal form of parametrization in the couplings. While this flexibility limits the second approach to a level far lower than that of full-fledged theory, such phenomenology has, nevertheless, the advantage of staying closer to experiment, and hence of exhibiting the desirable features to be possessed by a more complete theory of the future. The size of the group in these phenomenological approaches does not usually exceed[4] $SU(6)$ or at most $SU(6)\times O(3)$, which is considerably smaller than the (noncompact) structures characterizing the more formal theories. An intermediate position is occupied by certain collinear, relativistic, yet compact groups, such as $SU(6)_W$,[5] $U(6)_W\times O(2)W$,[6] or $U(6)\times U(6)$ (chiral[7] and nonchiral[8]), which, while depending on fewer parameters than do phenomenological theories based on quarks, have a much more limited range and scope of predictions than possessed by the more formal noncompact theories.

An important problem common to all these approaches is concerned with how to take account of breaking of symmetry in the spaces of spin and unitary spin, due to the masses of the particles involved. One common prescription which is almost always followed is to take account of the actual masses and energies involved in the phase space. However, this is frequently not adequate and must be supplemented by additional prescriptions for symmetry breaking in the couplings themselves. In the more formal theories, which facilitate the explicit evaluation of the form factors in terms of the dynamical variables of the particles involved, these additional prescriptions are simply to take account of the actual masses in the structures of the form factors as well. In the more phenomenological quark theories, which do not provide for explicit recipes for the evaluation of the form factors, no such theoretical prescription is possible for breaking the symmetry in the couplings, so that the only guidance available in this regard is from experiment itself.

All these models have much the same predictions for couplings among the **56** baryons (B) and the **36** mesons (M); viz., the geometrical factors associated with the couplings[9] $\bar{B}BP$, $\bar{B}BV$, $\bar{B}B^*P$, and $\bar{B}B^*V$ are those of $SU(6)$ in the limit of small momentum. As for electromagnetic and weak interactions, their physical interest is confined mainly to the corresponding couplings among some of these lowest hadron states, where such interactions have a fair chance of explicit realization under the energetically unfavorable conditions preventing competition from possible strong interactions (e.g., $\Lambda\rightarrow p+\pi^-$, $\Sigma\rightarrow\Lambda+\gamma$, etc.). For higher resonances among baryons and mesons, the role of electromagnetic and weak interactions is considerably eclipsed by the background of strong interactions which almost exclusively govern the decay characteristics of these resonances. Therefore, physical interest in the couplings of higher resonances to the **56** baryons and **36** mesons is almost entirely confined to the strong interaction.[10] Electromagnetic and weak interactions will not concern us any further in this article; our scope is limited mainly to the couplings of higher resonances with the **56** baryons and **36** mesons, and not so much to the couplings of the latter among themselves.

A large number of phenomenological investigations[11–16] on the couplings of higher resonances have been performed during the last few years. The essential idea behind all these calculations has been to suppose the transition between two hadron states (looked upon as quark composites) to proceed through the emission of a meson (looked upon as a radiation quantum) by one quark (Q) or antiquark ($\bar{Q}$) constituent at a time. The total amplitude for the transition (and hence the coupling scheme) is then obtained by adding

[1] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); A. Salam, J. Strathdee, P. T. Mathews, and J. Charap, Phys. Letters **15**, 184 (1965); F. J. Dyson, *Symmetry Groups* (W. A. Benjamin, Inc., New York, 1966).

[2] P. Dudini and R. White, Phys. Rev. **151**, 1287 (1966); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965); W. Rühl, Phys. Letters **14**, 346 (1965).

[3] A. O. Barut and H. Kleinert, Phys. Rev. Letters **18**, 754 (1967); A. O. Barut and K. C. Tripathy, *ibid.* **19**, 1081 (1967); A. O. Barut and H. Kleinert, Phys. Rev. **157**, 1180 (1967).

[4] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Paris, *ibid.* **13**, 175 (1964).

[5] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965); D. Horn, M. Kugler, H. J. Lipkin, S. Meshkov, J. C. Carter, and J. Coyne, *ibid.* **14**, 717 (1965); P. N. Dobson, Jr., Phys. Rev. **160**, 1501 (1967); R. H. Capps, *ibid.* **153**, 1503 (1967); **158**, 1433 (1967).

[6] P. G. O. Freund, A. N. Maheshwari, and E. Schonberg, Phys. Rev. **159**, 1232 (1967).

[7] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964); M. Gell-Mann and R. F. Dashen, Phys. Letters **17**, 142 (1965); **17**, 145 (1965); R. Delbourgo, A. Salam, and J. Strathdee, Phys. Rev. **138**, 420 (1965); K. Baradacki, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964); **14**, 48 (1965); **14**, 264 (1965).

[8] S. Okubo and R. E. Marshak, Phys. Rev. Letters **13**, 818 (1964). See also Baradacki *et al.*, Ref. 7.

[9] Here B and B^* are the **8** and **10** baryons, and P and V are the nonets of pseudoscalar and vector mesons, respectively.

[10] Electromagnetic processes can, however, manifest themselves through the *inverse* process of photoproduction of certain resonances. On the other hand, such production processes cannot be studied in isolation from the sequential processes involving the decays (strong) of the same resonances.

[11] C. Becchi and G. Morpurgo, Phys. Rev. **149**, 1284 (1966).

[12] A. N. Mitra and M. Ross, Phys. Rev. **158**, 1630 (1967); hereafter referred to as MR.

[13] H. J. Lipkin, H. R. Rubinstein, and H. Stern, Phys. Rev. **161**, 1502 (1967).

[14] D. L. Katyal and A. N. Mitra, Phys. Rev. **169**, 1322 (1968).

[15] J. Utretsky, Argonne Report, 1967 (unpublished); A. N. Mitra and P. P. Srivastava, Phys. Rev. **164**, 1803 (1967).

[16] R. Van Royen and V. F. Weisskopf, Nuovo Cimento **50A**, 617 (1967).

these individual amplitudes and calculating the matrix element of this sum between the (quark) wave functions for the initial and final states. This involves the evaluation of (i) certain purely geometrical coefficients depending on the symmetries or group structures assumed for the quark wave functions, and (ii) certain form factors representing the overlap of the initial and final wave functions. While the group structure is generally assumed to be given by $SU(6)\times O(3)$, a lack of precise knowledge of the quark wave functions necessitates a direct parametrization of the overlap integrals with the help of some broad guiding principles, such as correct threshold behavior near zero momentum and a reasonably rapid falloff at large momentum. Indeed, many of the differences in various contemporary investigations can be traced to differences in the assumed structures of those form factors, for which enough freedom is available. Therefore, unlike the quantitative predictions of the more formal theories, the predictions of such phenomenological theories can be given at most a qualitative significance. Some important qualitative features that have emerged from such studies concern the roles of (i) representation mixing and (ii) the recoil term for decays in low partial waves. Thus the study of s-wave decays from $(\mathbf{70},1^-)$ states by MR suggested that these should more properly be described by the quark-recoil term in the $\bar{Q}QP$, rather than the direct term, in order to account for the (experimentally observed) enhanced rates for heavy-meson emission. This important feature does not seem to be present in most of the formal theories on hadron couplings proposed to date. On the other hand, separate parametrizations of decay widths for $(L\pm1)$-wave emissions on the assumptions of direct and recoil terms, respectively, for these two modes introduces so many free parameters in the model that it gets too far removed from a formal theory to warrant any meaningful comparison with the latter.

This article represents an attempt to bridge the long gap between formal theories and quark phenomenology through certain explicit assumptions on the structure of the form factors designed to reduce the number of free parameters to a minimum. The obvious advantage of the recoil terms for providing enhanced heavy-meson modes for $(L-1)$-wave emission is also retained without the introduction of additional parameters, by using certain additional prescriptions to obtain a special form of parametrization for such amplitudes. These devices will be shown not only to give a semirelativistic look to the coupling scheme so as to bring its structure considerably nearer to more formal theories like $SU(6)_W$, but also to have the additional advantage of better accord with experiment. The model, which will amount to an explicit ansatz on the $SU(3)$-symmetry breaking, partly in the form factors due to the various masses, and partly through a relativistic prescription for the couplings, will be shown to depend on a single free parameter, governing an entire supermultiplet transition, almost like the predictions of more formal theories like $SL(6,C)$ or $O(4,2)$. The agreement with experiment will also be shown to be surprisingly good, in spite of the absence of free parameters other than the masses of the particles involved. It turns out that both baryon and meson couplings admit of very similar treatment with almost identical structures for the form factors. This paper and the following one[17] are devoted to the cases of baryon and meson couplings, respectively. It is hoped that this type of treatment will be of help in the eventual formulation of a more complete theory which stays explicitly closer to experiment in respect of several observed features. The preliminary results of this model have been reported earlier.[18]

In Sec. 2, we summarize the new baryon coupling scheme in terms of a relativistic Rarita-Schwinger (RS) type of field representing the baryons[19] together with a certain ansatz on the multiplying (scalar) form factor, in terms of the masses and momenta, so that a single parameter describes the entire supermultiplet transitions. Section 3 is devoted to a detailed comparison with experiment for several types of available supermultiplet transitions, with special reference to (i) $SU(3)$ breaking due to masses, (ii) heavy-meson modes of emission in lower partial waves, and (iii) the strong momentum dependence of the decay widths predicted by the model. To the best of our knowledge, the comparison is much more comprehensive (from the point of view of both the number of cases for each type of transition and the number of different types of two-body transitions) than any available so far, either in the quark model or using more formal group theories. The possibility of mixing between certain $SU(3)$ states of given J^p values is discussed empirically in relation to their observed decay widths, as well as the prescriptions of other authors. An interesting feature of the model is the prediction of almost equal magnitudes for all the coupling constants governing the $(\mathbf{56},2l^+)\rightarrow(\mathbf{56},0^+)$ transitions up to values of $L(=2l)$ as high as $L=8$. This result is interpreted in terms of a universal coupling to the **56** baryons of all the particles lying on their Regge trajectories. Section 4 summarizes the main conclusions as to the theoretical and experimental status of the model.

2. NEW COUPLING SCHEME

In order to describe the couplings of baryons with a pseudoscalar-meson (P) nonet in terms of the quark model, we shall make free use of the methods and notations in MR, without detailed explanation. The basic $\bar{Q}QP$ interaction is of the form[11,12,16]

$$f_q\sum_{\alpha=0}^{8}\sum_{i=1}^{3}\mu_\alpha^{-1}\boldsymbol{\sigma}^{(1)}\cdot\left(\mathbf{k}-\frac{W_k}{M_Q}\mathbf{P}_i\right)\lambda_\alpha^{(i)}\Pi_\alpha, \qquad (2.1)$$

[17] D. K. Choudhury and A. N. Mitra, following paper, Phys. Rev. D **1**, 351 (1970).

[18] A. N. Mitra, Nuovo Cimento **61A**, 344 (1969); also in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 248.

[19] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

where $\boldsymbol{\sigma}^{(i)}$ is the spin matrix for the quark number i, $\mathbf{P}_i$ is its momentum, $\lambda_\alpha^{(i)}$ are the Gell-Mann matrices associated with it ($\alpha=0$ corresponds to the 3×3 unit matrix), and Π_α is the $SU(3)$ label for the emitted P meson of mass μ_α and three-momentum $\mathbf{k}$; f_q is a dimensionless coupling constant. The momentum structure of the second term (coming from Galilean invariance[12]) is very different from that of the first term (the direct term). Thus, while the direct term is proportional to $\mathbf{k}$ and hence vanishes at the threshold, the recoil term can be quite appreciable at threshold. The quark mass M_Q which appears in the recoil term is a sort of effective mass[20] of the bound quark, and may even be small, in contrast to the mass of the free quark, which may not be so. We see that (2.1) has a built-in mechanism of symmetry breaking at the $SU(3)$ level if the mass μ_α is made to vary with the $SU(3)$ label α.

For the evaluation of the baryon couplings, we take the matrix elements of the operator (2.1) between appropriate QQQ states representing the initial and final baryons. As in MR, we shall use symmetric wave functions for quarks in this article, whose construction has already been given.[12,21]

For the evaluation of Yukawa couplings of baryon states, the first requirement is to express the matrix elements in the composite QQQ space in terms of those between baryon states regarded as "elementary" particles. Thus, e.g., the spin matrix elements between **56** states are related by

$$\langle\chi''|\boldsymbol{\sigma}^{(1)}|\chi''\rangle=-\tfrac{1}{3}\langle\chi'|\boldsymbol{\sigma}^{(1)}|\chi'\rangle \equiv-\tfrac{1}{3}\langle\chi|\boldsymbol{\sigma}|\chi\rangle, \tag{2.2}$$

where in the last form, χ represents the spin function of an "elementary" baryon of spin $\frac{1}{2}$, and $\boldsymbol{\sigma}$ represents its spin operator. Similarly for $\Delta\to N\pi$ transitions, we have the correspondence

$$\langle\chi^S|\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}|\chi''\rangle\Rightarrow\chi_\alpha^\dagger k_\alpha\chi,$$

where χ is a spin-$\frac{1}{2}$ Pauli field and χ_α is a nonrelativistic RS field of spin $\frac{3}{2}$.[19] A similar correspondence between matrix elements in composite and elementary baryon spaces applies to transitions between **70** and **56** states. These identifications lead rapidly to the usual $SU(6)$ relations for the couplings within the **56** baryons and their necessary extensions for other supermultiplet transitions. These spin and $SU(3)$ factors must be multiplied by a spatial integral of the form

$$\int\psi_{LM}^*\left(\mathbf{k}-\frac{W_k}{M_Q}\mathbf{P}_i\right)\psi_0 \tag{2.3}$$

for an $L^P\to0^+$ transition. For easy reference we give in Tables I and II the multiplying $SU(6)$ coefficients as well as the isotopic structures for the couplings of important states.

TABLE I. Isotopic structures of the various couplings between two **56**-type states. The $SU(6)$ coefficients for the different cases must be read as square roots, together with the appropriate sign given in front of each. The asterisk indicates radial excitation without change in L value.

$SU(2)$ structure	$(\mathbf{8})_d$	$(\mathbf{10})_q$
$N^+\tau_a\pi_aN$	25/3	
$N^+K\Lambda$	−3	
$N^+\eta\Lambda$	4/3	
$N^+\pi_a\Delta_a$	16	
$\Lambda^+\bar{K}N$	−6	
$\Lambda^+\pi_a\Sigma_a$	1	
$\Lambda^+\eta\Lambda$	−4/3	
$\Lambda^+\pi_a\Sigma_a{}^*$	12	
$\Sigma_a{}^+\bar{K}\tau_aN$	−2/9	−8/3
$\Sigma_a{}^+\pi_a\Lambda$	1/3	−4
$\Sigma_a{}^+\pi_b\tau_b\Sigma_a$	32/9	8/3
$\Sigma_a{}^+\eta\Sigma_a$	1/3	4
$\Sigma_a{}^+\tau_aK\Xi$		8/3
$\Sigma_a{}^+\pi_b\tau_b\Sigma_a{}^*$	8/3	
$\Xi^+\tau_a\pi_a\Xi$	1/3	4
$\Xi^+\bar{K}\Lambda$	4/3	−4
$\Xi^+\bar{K}\tau_a\Sigma_a$	−25/3	4
$\Xi^+\eta\Lambda$	−3	12
$\Delta_a{}^+\pi_aN$		−8
$\Delta_a{}^+\tau_b\pi_b\Delta_a$		15
$\Delta_a{}^+K\Sigma_a$		8

TABLE II. Isotopic structures of the various couplings between **70**- and **56**-type states. The $SU(6)$ coefficients for the different cases must be read as square roots, together with the appropriate sign given in front of each.

$SU(2)$ structure	$\mathbf{8}_q$	$\mathbf{8}_d$	$\mathbf{10}_d$	$\mathbf{1}_d$
$N^+\tau_a\pi_aN$	−1	8/3		
$N^+K\Lambda$	0	−3/2		
$N^+\eta N$	1	1/6		
$N^+\pi_a\Delta_a$	−12	8		
$\Lambda^+\bar{K}N$	0	3		−3
$\Lambda^+\pi_a\Sigma_a$	−3	1/2		9/2
$\Lambda^+\eta\Lambda$	1	1/6		3/2
$\Lambda^+\pi_a\Sigma_a{}^*$	−9	6		
$\Sigma_a{}^+\tau_aKN$	−8/3	−1/9	1/72	
$\Sigma_a{}^+\eta\Lambda$	1	−1/6	1/18	
$\Sigma_a{}^+\eta\Sigma_a$	1		1/18	
$\Sigma_a{}^+\pi_b\tau_b\Sigma_a$	2/3	25/9	1/72	
$\Sigma_a{}^+\pi_b\tau_b\Sigma_a{}^*$	2	−4/3	16/3	
$\Sigma_a{}^+\bar{K}\Delta_a$	−8	16/3		
$\Xi^+\tau_a\pi_a\Xi$	4	1/6	1/48	
$\Xi^+\bar{K}\Lambda$	−1	−2/3	1/48	
$\Xi^+\bar{K}\tau_a\Sigma_a$	−1	8/3	1/48	
$\Xi^+\eta\Xi$	0	3/2	1/48	
$\Xi^+\pi_a\tau_a\Xi^*$	3	−2	2	
$\Xi^+\bar{K}\tau_a\Sigma_a{}^*$	−3	2		
$\Delta_a{}^+\pi_aN$			1/3	
$\Delta_a{}^+\tau_b\pi_b\Delta_a$			10	

[20] H. J. Lipkin, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967* (North-Holland Publishing Co., Amsterdam, 1968).

[21] A. N. Mitra, Ann. Phys. (N. Y.) 43, 126 (1967).

TABLE III. Some of the better-known baryon octet states. For the cases marked (?), the assignments are tentative.

L	Spin S	J^P				
0	$\frac{1}{2}$	$\frac{1}{2}^+$	$N(938)$	$\Lambda(1115)$	$\Sigma(1190)$	$\Xi(1318)$
1	$\frac{1}{2}$	$\frac{1}{2}^-$	$N(1710)$	$\Lambda(1670)$?	$\Sigma(1670)$?	
1	$\frac{3}{2}$	$\frac{1}{2}^-$	$N(1550)$	$\Lambda(1670)$?	$\Sigma(1670)$?	
1	$\frac{1}{2}$	$\frac{3}{2}^-$	$N(1518)$	$\Lambda(1690)$?	$\Sigma(1660)$?	$\Xi(1816)$?
1	$\frac{3}{2}$	$\frac{3}{2}^-$	$N(1675)$	$\Lambda(1690)$?	$\Sigma(1660)$	$\Xi(1816)$?
1	$\frac{3}{2}$	$\frac{5}{2}^-$	$N(1690)$	$\Lambda(1830)$	$\Sigma(1767)$	$\Xi(1930)$
2	$\frac{1}{2}$	$\frac{5}{2}^+$	$N(1688)$	$\Lambda(1816)$	$\Sigma(1910)$	$\Xi(2020)$?
2	$\frac{1}{2}$	$\frac{3}{2}^+$	$N(1863)$			
2	$\frac{1}{2}$	$\frac{7}{2}^-$	$N(2190)$?		$\Sigma(2260)$?	$\Xi(2460)$?
2	$\frac{3}{2}$	$\frac{7}{2}^+$	$N(2190)$?		$\Sigma(2260)$?	$\Xi(2460)$?

We reexpress (2.3) in a more transparent form in order to process the couplings further. From general invariance considerations, the "direct" term of (2.3) is expressible as

$$f_L(k^2)B_{\alpha_1\ldots\alpha_2}{}^{LM}k_{\alpha_1}\cdots k_{\alpha_L}\times 1\,, \tag{2.4}$$

where $f_L(k^2)$ is a scalar function of k^2, and $B_{\alpha_1\ldots\alpha_L}{}^{LM}$ is an irreducible tensor field of rank L, symmetric in all the three-dimensional indices $\alpha_1\cdots\alpha_L$, and normalized by

$$B_{\alpha_1\ldots\alpha_L}{}^{LM}B_{\alpha_1\ldots\alpha_L}{}^{LM'}=\delta_{MM'}. \tag{2.5}$$

The couplings between different baryon states of given J are now obtained by expressing the direct product of factors like $\chi^\dagger\boldsymbol{\sigma}\cdot\mathbf{k}\chi$ or $\chi_\alpha{}^\dagger k_\alpha\chi$, with (2.4) in terms of Clebsch-Gordan (CG) series separately for initial and final states. For example, the CG series for $\mathbf{J}=\mathbf{L}+\frac{1}{2}$ is

$$\chi\otimes B_{\alpha1\ldots\alpha L}{}^{L}=\chi_{\alpha1\ldots\alpha L}{}^{L+1/2}+\left(\frac{L}{2L+1}\right)^{1/2}S_L\sigma_{\alpha1}\chi_{\alpha2\ldots\alpha L}{}^{L-1/2}, \tag{2.6}$$

where $\chi_{\alpha_1\ldots\alpha_{J-\frac{1}{2}}}{}^{J}$ is a normalized RS spinor of spin J and S_L is a symmetrizer of indices.[22]

The structures and relative strengths of couplings of nonrelativistic RS fields χ^J are illustrated for $\mathbf{J}=\mathbf{L}+\frac{1}{2}$ by

$$\begin{aligned}[B_{\alpha1\ldots\alpha L}{}^{L}\otimes\chi]^\dagger(\boldsymbol{\sigma}\cdot\mathbf{k})(k_{\alpha1}\cdots k_{\alpha L})\chi &=\chi_{\alpha1\ldots\alpha L}{}^{L+1/2\dagger}(\boldsymbol{\sigma}\cdot\mathbf{k})(k_{\alpha1}\cdots k_{\alpha L})\chi\\ &\quad+\left(\frac{L}{2L+1}\right)\mathbf{k}^2\chi_{\alpha2\ldots\alpha L}{}^{L-1/2\dagger}(k_{\alpha2}\cdots k_{\alpha L})\chi,\end{aligned} \tag{2.7}$$

where the extra threshold factor $\mathbf{k}^2$ is visible with the $(L-1)$-wave term, characteristic of direct term coupling. Similarly, expressions for the couplings of $\mathbf{J}=\mathbf{L}+\frac{3}{2}$ to a RS field χ of $J=\frac{1}{2}$ and of $\mathbf{J}=\mathbf{L}+\frac{1}{2}$ and $\mathbf{J}=\mathbf{L}+\frac{3}{2}$ fields to a RS field χ_α of $J=\frac{3}{2}$ can be written down. These couplings must be considered in association with the content of Tables I and II, which give the necessary $SU(6)$ factors and isotopic structures for the various cases. Similar structures can be written down with the recoil term, with the difference that the $(L\pm1)$-wave couplings now appear with different form factors.

To make relativistic extension of these couplings, we follow very similar lines to the boosting of **56** and **36** states.[23] Thus the nonrelativistic spinors χ^J should be replaced by relativistic RS spinors ψ^J with four-dimensional indices and the three-momentum indices by four-momentum indices.[22] We also have the replacement

$$\chi^\dagger(\boldsymbol{\sigma}\cdot\mathbf{k})\chi\rightarrow\bar{\psi}\gamma_5\boldsymbol{\gamma}\cdot\mathbf{k}\psi\rightarrow(M+m)i\bar{\psi}\gamma_5\psi. \tag{2.8}$$

These prescriptions are adequate for $(L+1)$ couplings. For $(L-1)$-wave couplings, we use an additional prescription $\mathbf{k}^2\rightarrow k_\mu k_\mu$. This must be interpreted as incorporating the effect of the recoil term in the special ratio $1:(-\omega_k{}^2)$ to the direct term. This simple device has the effect of incorporating the heavy-meson feature discussed in MR (since $k_\mu k_\mu=-\mu^2$ on the energy shell) for the lower $(L-1)$ waves, without the price of extra parametrization. Since this prescription is *ad hoc*, it can be judged only by its experimental performance. Thus Eq. (2.7) now becomes

$$(M+m)\bar{\psi}_{\mu1\ldots\mu L}{}^{L+1/2}i\gamma_5k_{\mu1}\cdots k_{\mu L}\psi+(-\mu^2)\left(\frac{L}{2L+1}\right)^{1/2}\bar{\psi}_{\mu2\ldots\mu L}{}^{L-1/2}k_{\mu2}\cdots k_{\mu L}\psi. \tag{2.9}$$

For the multiplying form factor $f_L(k^2)$ of Eq. (2.4), we choose the structure

$$f_L(k^2)=g_L\mu^{-L-1}\left(\frac{\mu}{m_\pi}\right)^{1/2}\left(\frac{\mu}{\omega_k}\right)^{L\pm1}\left(\frac{M}{m}\right)^{1/2}, \tag{2.10}$$

where the indices $(L\pm1)$ are associated with the corresponding waves of emission of the radiation quanta if a single partial wave is involved. However, the *higher* of the two exponents must be taken if the transition amplitudes include contributions from both $(L+1)$ and $(L-1)$ waves.

The detailed arguments of the structure have been described elsewhere.[22] In short, $(\mu/m_\pi)^{1/2}$ is the Von Royen-Weisskopf factor[16] and $(M/m)^{1/2}$ is a sort of "energy compensation factor" for baryons in the PCAC (partial conservation of axial-vector current) limit.[18] Similar considerations apply for mesons. The other two factors come from considerations of dimension and of toning down the effect of large momenta in the coupling structure. While this structure is still empirical, it depends on far fewer parameters (essentially one for each supermultiplet transition) than in the earlier inves-

[22] C. Fronsdal, Nuovo Cimento Suppl. **9**, 416 (1958); H. Umezawa, *Theory of Quantized Fields* (North-Holland Publishing Co., Amsterdam, 1956). For details of construction, see A. N. Mitra, in *Lectures in High-Energy Theoretical Physics*, edited by H. H. Aly (Gordon and Breach, Science Publishers, Inc., New York, to be published).

[23] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965).

tigations and thus has some formal similarity to other theories like $SU(6)$ or $O_W(2)\times U_W(6)$.

It may be noted that for couplings within **56** there are no free parameters over and above f_q. The same is true for couplings *within* the same supermultiplet, for example, $(\mathbf{70},1^-)$ or $(\mathbf{56},2^+)$, since the spatial overlap integral is a normalization effect. Indeed, for such coupling the form factor is given by

$$f_L(k^2)=g_0\mu^{-1}\left(\frac{\mu}{m_\pi}\right)^{1/2}\left(\frac{M}{m}\right)^{1/2} \tag{2.11}$$

as for **56** couplings, where g_0 is related to $G_{NN\pi}$ by

$$\frac{g_0^2}{4\pi}=\frac{f_q^2}{4\pi}=\frac{9}{25}\left(\frac{m_\pi}{2m_N}\right)^2\frac{G^2}{4\pi}\approx 0.03. \tag{2.12}$$

3. COMPARISON WITH EXPERIMENT

Before comparing this model with experiment, we note some of its general features. It breaks $SU(3)$ not only in phase space, but also in the couplings themselves, mostly through the factor $\mu^{-L-1/2}$ in the limit $\mathbf{k}\to 0$. For $(L+1)$-wave couplings, this entire factor is operative, so that these break $SU(3)$ by a large amount. For $(L-1)$-wave couplings, on the other hand, the presence of the extra factor $(-\mu^2)$ in the corresponding terms makes the proportionality factor in $SU(3)$ breaking mainly $\mu^{-L+3/2}$. Thus, the model predicts entirely different mechanisms of $SU(3)$ breaking in the $(L\pm1)$-wave modes of emission, the higher wave being more strongly affected by this breaking than the lower wave. For example, for $L=1$, the d-wave amplitudes are proportional to $\mu^{-3/2}$, but the s-wave amplitudes vary as $\mu^{1/2}$. This leads to heavy-meson (K,η) enhancements relative to π modes for s-wave decays and to heavy-meson suppression (compared to π modes) for d-wave decays.

The techniques for calculating various decay widths are similar to those of Behrends and Fronsdal.[24] For a general coupling of the form

$$G_l(\psi_{\mu_1\ldots\mu_l}{}^{l+1/2\dagger}\rho k_{\mu_1}\cdots k_{\mu_l})\psi, \tag{3.1}$$

where $\psi_\mu{}^{l+1/2}$ is a RS field of $J=l+\frac{1}{2}$, ψ is a spinor field,

TABLE IV. Decuplet states up to $J^P=\frac{7}{2}^+$.

L	Spin S	J^P				
0	$\frac{3}{2}$	$\frac{3}{2}^+$	$\Delta(1238)$	$\Sigma^*(1385)$	$\Xi^*(1530)$	$\Omega(1675)$
1	$\frac{1}{2}$	$\frac{1}{2}^-$	$\Delta(1640)$	$\Sigma^*(1770)$?	$\cdots$	$\cdots$
1	$\frac{1}{2}$	$\frac{3}{2}^-$	$\Delta(1691)$	$\cdots$	$\Xi^*(1816)$	$\cdots$
2	$\frac{3}{2}$	$\frac{1}{2}^+$	$\Delta(1934)$	$\cdots$	$\cdots$	$\cdots$
2	$\frac{3}{2}$	$\frac{5}{2}^+$	$\Delta(1913)$	$\cdots$	$\cdots$	$\cdots$
2	$\frac{3}{2}$	$\frac{7}{2}^+$	$\Delta(1950)$	$\Sigma^*(2030)$	$\cdots$	$\cdots$

[24] R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1958).

TABLE V. Decays within the same supermultiplets.

$SU(3)$ decay	Mode	Γ_{theor} (MeV)	Γ_{expt} (MeV)	$\Gamma_{SU(6)}$ (MeV)
$(\mathbf{10})_{3/2+}\to(\mathbf{8})_{1/2+}$	$\Delta(1238)\to N\pi$	101.0	120.0	77
	$\Sigma^*(1385)\to\Lambda\pi$	36.4	34 ± 3.0	24.0
	$\Sigma\pi$	4.4	3.5 ± 1.0	3.3
	$\Xi^*(1530)\to\Xi\pi$	12.4	7.3 ± 1.7	8.9
$(\mathbf{8})_{5/2-}\to(\mathbf{1})_{3/2-}$	$\Sigma(1767)\to\Lambda(1520)\pi$	12.4	13.3	
$(\mathbf{8})_{3/2-}\to(\mathbf{1})_{1/2-}$	$\Sigma_d(1660)\to\Lambda(1520)\pi$	3.1		
	$\Sigma_q(1660)\to\Lambda(1405)\pi$	5.4	$\leqslant 16.0$	
$(\mathbf{10})_{7/2+}\to(\mathbf{8})_{5/2+}$	$\Delta(1950)\to N(1688)\pi$	61.0		

ρ is 1 or $i\gamma_5$, the decay width is given by

$$\Gamma_l=G_l^2(4\pi)^{-1}l![(2l+1)!!]^{-1}k^{2l+1}(m/M)(E_k/m\pm1), \tag{3.2}$$

where for G_l one must use the form factor (2.10) for the relevant l value, together with the appropriate isospin factors given by Tables I and II, and E_k is the total energy of the product baryon.

The current experimental status of supermultiplets, which has been summarized by Dalitz[25] as well as by Harari,[26] seems to indicate the occurrence of only the following types of supermultiplets:

$$(\mathbf{56},0^+),(\mathbf{56},2^+),(\mathbf{70},1^-),(\mathbf{56},0^+)_R,(\mathbf{70},3^-),\ldots. \tag{3.3}$$

The asterisk in Tables I and II indicates radial excitation without change of the l value. This list, which is considerably smaller than one based on the harmonic-oscillator potential for the QQQ system,[27] can be summarized in terms of only the following types of states:

$$(\mathbf{56},2l^+),\quad(\mathbf{70},(2l+1)^-), \tag{3.4}$$

together with their radial excitations. We note in passing that it is precisely such states that are predicted to be the lowest-lying ones, under the assumption of only s-wave Q-Q forces.[28] However, for the purpose of this paper we shall not discuss the dynamical significance of the content of (3.4) any further, and merely take the list (3.4) for granted, referring to the appropriate reviews for experimental details. For convenience, Tables III and IV summarize the $[SU(3),J^P]$ assignments for the various baryon resonances.[25,26]

A. Decays within Same Supermultiplets

For simplicity we first discuss the cases of decays within the same supermultiplets, for which the predictions are free from the effects of recoil. Moreover, the coupling structures for such cases are relativistically invariant [since the associated form factors (2.11) are independent of any three-momentum]. Finally, the

[25] R. H. Dalitz, in Proceedings of the Conference on π-N Scattering, Irvine, Calif., 1967 (unpublished).

[26] H. Harari, rapporteur talk, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 195.

[27] D. Faiman and A. W. Hendry, Phys. Rev. **173**, 1720 (1968).

[28] A. N. Mitra and D. L. Katyal, Nucl. Phys. **B5**, 308 (1968); A. N. Mitra, Nuovo Cimento **56A**, 1164 (1968).

coupling constant, being directly related to $G_{NN\pi}$, is not a free parameter, unlike the case of transitions between different supermultiplets.

Table V gives the results for decay within **56**, as well as for certain cases such as $\Sigma(1767) \to \Lambda(1530)\pi$ and $\Sigma(1660) \to \Lambda(1405)$ within the $(\mathbf{70},1^-)$ in relation to experiment. The role of the factor $(M/m)^{1/2}$ is exhibited through a comparison of results with and without this factor [the latter being identical with the predictions of relativistic $SU(6)$, except for $(\mu/m_\pi)^{1/2}$]. It is clearly seen that the factor $(M/m)^{1/2}$ is very helpful in bridging the large gap between the $SU(6)$ prediction of 77 MeV and the experimental value (120 MeV) for the $\Delta \to N\pi$ width. Its role is equally useful for the other **56** cases except for $\Xi^* \to \Xi\pi$. While we are unable to suggest a precise dynamical significance for this factor (except for its occurrence through relativistic normalization), a possible mechanism may be provided by the role of rescattering of the radiation quantum by the quarks in the baryon, before it finally escapes from the system. Indeed, some recent estimates of the latter effect,[29,30] which bring about similar effects, would seem to encourage such a belief. For decays within $(\mathbf{70},1^-)$, while the case of $\Sigma(1767) \to \Lambda(1520)\pi$ is in good accord with experiment, the decay rate of $\Sigma(1660) \to \Lambda(1405)\pi$ under the assumption of an $\mathbf{8}_d$ assignment is rather small. This discrepancy illustrates the necessity for mixing, which is especially important for Σ-type states as discussed later in this section. Indeed, the mixture suggested in Table VII is seen to yield an improved estimate for the same rate. The case of $\Delta(1950) \to N(1688)\pi$ as an example for decays within $(\mathbf{56},2^+)$ is listed as a prediction at this stage, but the fairly large magnitude of this decay warrants an early experimental search for this mode.

B. $(\mathbf{70},1^-) \to (\mathbf{56},0^+)$ Decays

Next we consider decays for $(\mathbf{70},1^-) \to (\mathbf{56},0^+)$, which are listed in Table VI together with the experimental data. We refrain from giving the comparison figures for the other phenomenological models, since these involve several more parameters. Even the predictions of a more formal theory like $SU(6)_W$ lose a considerable amount of theoretical significance because of the appearance of as many as three free parameters for each supermultiplet transition, and have therefore not been included in the table. These decays are now governed by the single free parameter g_1, whose value is taken as

$$g_1^2/4\pi = 0.14 \pm 0.02 \qquad (3.5)$$

in order to reproduce a couple of well-known measured modes such as $\Sigma(1767) \to N\bar{K}$ and $\Lambda(1830) \to \Sigma\pi$ fairly well. Considering that there is no other adjustable parameter, the general pattern of agreement is seen to be rather good on the whole, though there are some important cases of disagreement as well. However, it is remarkable that the data seem to support the very sensitive dependence of the form factor (2.10) on both the masses and momenta, illustrated by the factors μ^{-L-1} and $(\mu/\omega_k)^{L\pm1}$, respectively, without which the nonpionic and the energetic modes, respectively, would be nowhere near agreement. We now discuss some specific features of these decays.

For s-wave decays, the relative enhancements in the heavy-meson modes are particularly noticeable in the case of the threshold resonances $N(1550) \to N\eta, \Lambda(1670) \to N\bar{K}$ or $\Lambda\eta$, and $\Sigma(1770) \to \Sigma\eta$, whose decay widths would have been almost negligible if only the direct-term contribution had been considered. These results also suggest that $N(1550)$ is mostly in $\mathbf{8}_q$ and $N(1710)$ in $\mathbf{8}_d$, in agreement with the earlier analysis of MR, but in disagreement with the conclusions of Dalitz[25] on the basis of proximity of split mass levels due to a spin-orbit force. In this respect we feel that decay widths should provide a more sensitive test of the $[SU(3),J^P]$ assignments than would mass patterns due to simple symmetry-breaking potentials. A further test of the assignment would be furnished by the observation of the (energetically allowed) ΛK mode for $N(1710)$. Thus, the quark model predicts, as a general selection rule, that an N state gives appreciable and zero ΛK widths according as this state belongs to $\mathbf{8}_d$ or $\mathbf{8}_q$, respectively. As yet, experiments seem inconclusive on this point, but it is significant that the $N\pi$ model of $N(1710)$ falls considerably short of the total width ($\sim$300) of the latter.

On the whole, s-wave decays agree very well with experiment, thus providing good support for the $SU(3)$-breaking structure of the form factor for such cases. Even the $\Lambda(1405) \to \Sigma\pi$ decay is very well reproduced, considering the fact that the model predicts large widths for most other $\Sigma\pi$ modes. Effects of mixing among states which undergo s-wave decays do not seem to be as pronounced as would be expected on general grounds. This is at least partly due to the fact that the role of mixing has been taken over by the effect of the recoil term which (in this model) is present only in such couplings (through the *ad hoc* replacement $\mathbf{k}^2 \to -\mu^2$). It should perhaps be emphasized that no amount of mixing could ever lead to the observed enhancements in heavy-meson modes, without at least a partial reliance on the recoil term. However, the case of states such as $\Lambda(1670)$ shows that even with the recoil term, one still requires some octet-singlet mixing among certain $I=0$ states, which have a common J^P value.[31]

For the d-wave decays, the model predicts a stronger $SU(3)$-breaking effect ($\sim\mu^{-3/2}$). This mass dependence

[29] A. H. Rosenfeld *et al.*, Wallet Cards, 1969; R. D. Tripp, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 173.

[30] J. D. Anand, V. S. Bhasin, and A. N. Mitra, Phys. Rev. **176**, 1891 (1968).

[31] We note in passing that the ideal mixing angle between $\Lambda(1670)$ and $\Lambda(1405)$ fails to reproduce the data.

TABLE VI. Decays from $(\mathbf{70},1^-)$ states to octets of $\mathbf{56}$.

$SU(3)$ decay	Mode	Γ_{theor} (MeV) q	d	Γ_{expt} (MeV)	Γ_{mixing} (MeV) (see Table VII)
$(\mathbf{8})_{5/2^-}\to(\mathbf{8})_{1/2^+}$	$N(1690)\to N\pi$	20.8		68.0	
	$N\eta$	6.1		4.0	
	ΛK	0		2.7	
	$\Lambda(1830)\to\Sigma\pi$	56.7		33±6	
	$N\bar{K}$	0		6±1	
	$\Sigma(1767)\to N\bar{K}$	48.3		44±8	
	$\Lambda\pi$	17.8		14±6	
	$\Sigma\eta$	0.1		0.5	
	$\Sigma\pi$	10.0		2.0	
	$\Xi(1930)\to\Xi\pi$	67.9		Seen	
	$\Lambda\bar{K}$	18.0		Seen	
	$\Sigma\bar{K}$	11.6			
	$\Xi\eta$	0			
$(\mathbf{8})_{3/2^-}\to(\mathbf{8})_{1/2^+}$	$N(1518)\to N\pi$		164.1	62.0	92.8
	$N\eta$		0.12	0.6	0.3
	$N(1675)\to N\pi$	36.0			
	$N\eta$	11.2			138.0
	ΛK	0			5.3
	$\Lambda(1690)\to\Sigma\pi$	52.7	21.9	35±15	36.9
	$\Lambda\eta$	0.07	0.12		
	$N\bar{K}$	0	153.8	11±4	31.4
	$\Sigma(1660)\to N\bar{K}$	42.0	4.4	5.0	39.0
	$\Sigma\pi$	10.7	111.3	25.0	44.3
	$\Lambda\pi$	20.9	8.7	14±2	13.8
	$\Xi(1816)\to\Xi\pi$	68.1	7.1	~1.6	4.0
	$\Lambda\bar{K}$	13.6	22.6	~10.4	11.5
	$\Sigma\bar{K}$	4.6	30.4	~0.5	0.01
$(\mathbf{10})_{3/2^-}\to(\mathbf{8})_{1/2^+}$	$\Delta(1691)\to N\pi$		31.6	37.0	
	$\Sigma^*(1660)\to N\bar{K}$		1.75	5.0	
	$\Sigma\pi$		1.70	25.0	
	$\Lambda\pi$		8.8	14±2	
	$\Xi^*(1816)\to\Xi\pi$		7.1	1.6	
	$\Lambda\bar{K}$		5.7	10.4	
	$\Sigma\bar{K}$		1.9	0.3	
$(\mathbf{1})_{3/2^-}\to(\mathbf{8})_{1/2^+}$	$\Lambda(1520)\to N\bar{K}$		10.0	7.2	17.1
	$\Sigma\pi$		85.4	7.2	24.7
	$N(1550)\to N\pi$	47.3		39.0	31.0
	$N\eta$	80.6		91.0	89.0
	ΛK	0			
	$N(1710)\to N\pi$		159.3	240.0	192.0
	$N\eta$		26.7	12±8	9.2
	ΛK		137.4	6.4±4.0	137.0
	$\Lambda(1670)\to N\bar{K}$	0	434.3	3±0.5	3.6
	$\Lambda\eta$	9.6	4.1		13.1
	$\Sigma\pi$	45.2	18.8	10±2	3.7
$(\mathbf{10})_{1/2^-}\to(\mathbf{8})_{1/2^+}$	$\Sigma(1670)\to N\bar{K}$	375.0	15.8		
	$\Sigma\pi$	25.1	103.3		
	$\Lambda\pi$	43.1	7.2		
	$\Delta(1640)\to N\pi$		18.4	54.0	
	$\Sigma^*(1770)\to N\bar{K}$		7.3		
	$\Sigma\pi$		1.8		
	$\Lambda\pi$		8.4		
	$\Sigma\eta$		10.4		
	$\Xi^*(1816)\to\Lambda\bar{K}$		8.3		
	$\Xi\pi$		2.5		
	$\Sigma\bar{K}$		6.2		
$(\mathbf{1})_{1/2^-}\to(\mathbf{8})_{1/2^+}$	$\Lambda(1405)\to\Sigma\pi$		59.1	50.0	64.0

TABLE VII. Empirical mixtures for some (70,1⁻) states, in the present model, versus those of Ref. 34.

State (mass)	(J)	Amplitudes Present model			Ref. 34		
		$\mathbf{8}_d$	$\mathbf{8}_q$	**10**	$\mathbf{8}_d$	$\mathbf{8}_q$	**10**
$N(1518)$	$(\frac{3}{2})$	$(0.83)^{1/2}$	$(0.17)^{1/2}$	0	1	0	0
$N(1675)$	$(\frac{3}{2})$	$-(0.17)^{1/2}$	$(0.83)^{1/2}$	0	0	1	0
$\Sigma(1660)$	$(\frac{3}{2})$	$(0.65)^{1/2}$	$-(0.25)^{1/2}$	$(0.10)^{1/2}$	0.79	−0.58	−0.23
$\Xi(1816)$	$(\frac{3}{2})$	$-(0.05)^{1/2}$	0	$(0.95)^{1/2}$	0.86	0.42	−0.29
$N(1550)$	$(\frac{1}{2})$	$(0.25)^{1/2}$	$(0.975)^{1/2}$	0	1	0	0
$N(1710)$	$(\frac{1}{2})$	$(0.975)^{1/2}$	$-(0.025)^{1/2}$	0	0	1	0
		$\mathbf{8}_d$	$\mathbf{8}_q$	**1**	$\mathbf{8}_d$	$\mathbf{8}_q$	**1**
$\Lambda(1690)$	$(\frac{3}{2})$	$(0.70)^{1/2}$	$(0.15)^{1/2}$	$(0.15)^{1/2}$	0.94	−0.17	0.29
$\Lambda(1520)$	$(\frac{3}{2})$	$-(0.25)^{1/2}$	$(0.06)^{1/2}$	$(0.69)^{1/2}$	−0.30	−0.03	0.95
$\Lambda(1670)$	$(\frac{1}{2})$	$(0.10)^{1/2}$	$(0.85)^{1/2}$	$(0.05)^{1/2}$	0.86	0.14	0.49
$\Lambda(1405)$	$(\frac{1}{2})$	$(0.035)^{1/2}$	$(0.03)^{1/2}$	$(0.935)^{1/2}$	−0.49	0.01	0.87

seems to be in good accord with experiment for the $J^P=\frac{5}{2}^-$ cases which, being stretched states, are least affected by mixing. However, the d-wave decays of $J^P=\frac{3}{2}^-$ states are not in such good accord, the widths (especially for $\Sigma\pi$ modes) being considerably larger than experiment, though the relative decay widths within the supermultiplet are quite reasonable. This discrepancy is due partly to the structure of the couplings for unstretched states like $J^P=\frac{3}{2}^-$ and partly to a lack of proper knowledge of configuration mixing for such states. Regarding the first point, we observe that the couplings of unstretched states involve the factor $(M+m)$, while those of stretched states $(J^P=\frac{5}{2}^-)$ do not involve this factor. According to (2.8), the factor $\boldsymbol{\sigma}\cdot\mathbf{k}$ goes over in the relativistic case to the form $(M+m)(i\gamma_5)$ on the mass shell, thus bringing with it the unusually large quantity $(M+m)$, in contrast to a mere (k_μ) for the couplings of stretched states. Thus, the relative discrepancy between $J^P=\frac{5}{2}^-$ and $J^P=\frac{3}{2}^-$ is partly traceable to the conventional prescriptions for the relativistic generalization of the coupling terms $\chi^+\boldsymbol{\sigma}\cdot\mathbf{k}\chi$ versus $\chi^+k_\alpha\chi_\alpha$. One could expect to overcome this difficulty through the possibility of configuration-mixing effects for unstretched states. Such mixings can occur at the levels of $SU(3)$ or spin or both. For N-type states, $SU(3)$ mixing is not available, so that one must depend only on spin mixings (e.g., between $\mathbf{8}_d$ and $\mathbf{8}_q$ states). A good example of spin mixing is provided by the width of $N(1518)$, which is too large or too small on the assignments $\mathbf{8}_d$ or $\mathbf{8}_q$, respectively. The discovery of its more recent counterpart $N(1675)$ should help remove this discrepancy when the decay properties of the latter are better known.

For the strange members Λ, Σ, and Ξ, mixing can occur at both the $SU(3)$ and spin levels. That $SU(3)$ mixings are important for such states is exemplified by the general failure of the following predictions for unmixed states [$\Lambda(1690)$, $\Sigma(1660)$, $\Lambda(1520)$]; viz., apart from phase-space corrections, any unmixed Σ state has equal decay rates to $\Sigma\eta$ and $\Lambda\pi$ and an unmixed Λ state should have a 3:1 ratio between its $\Sigma\pi$ and $\Lambda\eta$ rates.[26] Indeed, a literal interpretation of this result would imply that Σ and Λ states should strongly prefer π to η modes of emission. This is precisely what we see from Table VI, for the predictions of unmixed states; but there is disagreement with experiment. Several recent phenomenological analyses have suggested mixings between $\Lambda(1520)$ and $\Lambda(1690)$ states[32,33] by angles varying between 16° and 21°. Moreover, the decay rates are generally rather sensitive functions of mixing angles, as had been observed in the very earliest analysis of MR.

C. Mixing Effects

Unfortunately, the present model does not give any prescription for mixing effects. However, if the latter are regarded as small, the model is capable of making fairly unambiguous assignments to most resonances to well-defined multiplets. Further, on the basis of the general agreement of the model with experiment, we can use these couplings structures to determine certain plausible sets of mixing angles for several mixed states so as to make the pattern of their decay rates conform to experiment. One such set of assignments for the mixed states, which has been worked out in a purely empirical manner, is listed in Table VII together with the suggestion of Divgi and Greenberg.[34] On the basis of these assignments, the widths of well-defined decay modes for such states are also shown in Table VI.

It may be seen from Table VII that though some of our suggested mixtures are appreciably different from those of Ref. 34, the agreement of the former with experiment is in general good. As for $N(1518)$, it is still mainly in $\mathbf{8}_d$, in agreement with other authors, though an appreciable (~15%) mixture of $\mathbf{8}_q$ is indicated. Even for $\Sigma(1600)$ ($\mathbf{8}_d$), the decay modes seem to be in fairly good agreement with experiment. A case of strik-

[32] R. D. Tripp *et al.*, Phys. Rev. Letters **21**, 1721 (1968).
[33] R. H. Capps, Phys. Rev. Letters **22**, 215 (1969).
[34] D. R. Divgi and O. W. Greenberg, Phys. Rev. **175**, 2024 (1968).

ing difference from other authors[34,35] is provided by $\Xi(1816)$, whose assignment mainly to **10** seems to work much better than if it is put in an **8** (d or q), in agreement with the earlier analysis of MR.

Certain ratios of the decay modes are of inherent importance in discussing the significance of the mixings. Thus, for $\Lambda(1520)$, the ratio

$$\Gamma[\Lambda(1520)\to N\bar{K}]/\Gamma[\Lambda(1520)\to\Sigma\pi]=0.69\,,$$

in comparison to the ratio 0.11 without mixing and 1.0 experimentally. Divgi[35] gets this ratio as 0.43 without mixing and 1.0 after 10% mixing with $\mathbf{8}_d$ and $\mathbf{8}_q$, as shown in Table VII. Similarly, for the $\Lambda(1690)$ of $J^P=\frac{3}{2}^-$, the ratio

$$\Gamma[\Lambda(1690)\to N\bar{K}]/\Gamma[\Lambda(1690)\to\Sigma\pi]=0.85$$

after mixing (7.0 without mixing), while experimentally it is 0.35. This may be compared to the values 1.1 and 7.4 given in Ref. 35 with and without mixing, respectively. Thirdly, if we treat $\Lambda(1670)$ as belonging mainly to $\mathbf{8}_q$, the ratio

$$\Gamma[\Lambda(1670)\to N\bar{K}]/\Gamma[\Lambda(1670)\to\Sigma\pi]\simeq 1.0$$

after mixing (exactly zero in the absence of any mixing), while the experimental value is ≈ 0.30. Thus even a small mixing plays an important role in changing these decay ratios.

We conclude this subsection with the remark that while our model in general predicts large $\Sigma\pi$ modes, the same modes are considerably suppressed through mixing effects.

D. $(\mathbf{56},2^+)\to(\mathbf{56},0^+)$ Decays

The next higher resonances, which presumably belong to the $(\mathbf{56},2^+)$ representation, as the first Regge recurrence of the $(\mathbf{56},0^+)$ states, have a good number of decay modes to the latter. Note that for such states there is much less possibility of $SU(3)$ mixing than for the $(\mathbf{70},1^-)$, since this mixing is now confined only to the Σ- and Ξ-type states. These states do not also admit of spin mixing, since the **56** representations have only spin-doublet states for **8** and spin-quartet ones for **10**. Therefore, one would expect the predictions for $(\mathbf{56},2^+)$ decays to be much cleaner than those of $(\mathbf{70},1^-)$ decays, so that a comparison with the experimental data would provide a more direct test of the model. This comparison is shown in Table VIII for several types of the decay modes of these resonances, to bring out the mass and momentum dependence involved in the form factor (2.10). The input value of the coupling constant g_2 is taken as

$$g_2^2/4\pi\simeq 0.04\pm 0.01\,, \tag{3.6}$$

designed to give a fit to some of the principal $\frac{5}{2}^+$ modes, especially $N(1688)\to N\pi$, to within 10%. The general agreement is again surprisingly good, and indeed it is

[35] D. R. Divgi, Phys. Rev. **175**, 2027 (1968).

TABLE VIII. (**56**, 2^+)decays to octets of **56**.

$SU(3)$ decay	Mode	Γ_{theor} (MeV)	Γ_{expt} (MeV)
$(\mathbf{10})_{7/2+}\to(\mathbf{8})_{1/2+}$	$\Delta(1950)\to N\pi$	28.8	88.0
	ΣK	5.9	Seen
	$\Sigma^*(2030)\to N\bar{K}$	10.5	14.2
	$\Lambda\pi$	12.6	43.2
	$\Sigma\pi$	7.5	10.8
	ΞK	1.3	2.4
	$\Xi^*(2020)\to\Sigma\bar{K}$	5.3	
	$\Lambda\bar{K}$	7.8	
	$\Xi\pi$	9.1	
	$\Xi\eta$	1.6	
$(\mathbf{8})_{5/2+}\to(\mathbf{8})_{1/2+}$	$N(1688)\to N\pi$	76.3	84.5
	ΛK	0.25	0.17
	$N\eta$	1.6	1.95
	$\Lambda(1816)\to N\bar{K}$	38.4	46.6 ± 3.2
	$\Sigma\pi$	6.5	8.1 ± 0.07
	$\Lambda\eta$	0.6	0.74 ± 0.07
	$\Sigma(1910)\to N\bar{K}$	2.4	4.8
	$\Lambda\pi$	3.7	6.0
	$\Sigma\pi$	28.8	1.8
	$\Sigma\eta$	0.85	
	$\Xi(2020)\to\Sigma\bar{K}$	42.5	
	$\Lambda\bar{K}$	10.5	
	$\Xi\pi$	2.6	
	$\Xi\eta$	1.1	
$(\mathbf{10})_{5/2+}\to(\mathbf{8})_{1/2+}$	$\Delta(1913)\to N\pi$	46.3	56.0
$(\mathbf{10})_{1/2+}\to(\mathbf{8})_{1/2+}$	$\Delta(1934)\to N\pi$	159.4	102
$(\mathbf{10})_{3/2+}\to(\mathbf{8})_{1/2+}$	$N(1863)\to N\pi$	64.7	63

better than the $SU(6)_W$ predictions.[5,36] Since several types of meson masses are involved in Table VIII, this agreement can safely be interpreted as bringing out the validity of the mass effect on $SU(3)$ breaking, which is now proportional to $\mu^{-1/2}$ and $\mu^{-5/2}$ for p- and f-wave decays, respectively. For example, independent of the value of g_2, the ratios of the modes $\Lambda(1816)\to N\bar{K}$ and $\Delta(1913)\to N\pi$ to $N(1688)\to N\pi$ are determined as 0.50 and 0.61, respectively, in almost exact agreement with the corresponding experimental ratios.[29] There are, however, certain important points of disagreement similar to the negative-parity cases, viz., that the predicted widths for the stretched states ($J^P=\frac{7}{2}^+$) are rather depressed in relation to those for the unstretched states ($J^P=\frac{5}{2}^+$). The reason for such a discrepancy is very similar to what was mentioned already in connection with $(\mathbf{70},1^-)$ decays, viz., that the relativistically generalized couplings for the unstretched states bring in the unreasonably large factor $(M+m)$, which does not find a counterpart in the corresponding generalization for stretched states. Also, since the input value of g_2 has been determined from the decays of certain unstretched states, the decay widths for stretched ($J^P=\frac{7}{2}^+$) states like $\Delta(1950)$ and $\Sigma^*(2030)$ are considerably below their experimental values. However, the relative widths within the category of stretched states are seen to be very well reproduced. Another interesting feature of Table VIII is that, unlike the case of many modes for $(\mathbf{70},1^-)$, most such modes of $(\mathbf{56},2^+)$ states are in quite

[36] It would have been interesting also to compare our results with the $O(4,2)$ predictions of Barut and collaborators (Ref. 3), but such data are confined mainly to the pionic modes of Δ-type states.

TABLE IX. Decuplet modes of decay from some (**70**,1⁻) and (**56**,2⁺) states. $\Gamma_{\text{theor}}^{(1)}$ and $\Gamma_{\text{theor}}^{(2)}$ are, respectively, the results of the incomplete prescriptions given by type B and the fuller prescriptions (3.9). For other notations see text.

$SU(3)$ decay	Mode	$\Gamma_{\text{theor}}^{(1)}$ (MeV) q	d	$\Gamma_{\text{theor}}^{(2)}$ (MeV) q	d	Γ_{expt} (MeV)
$(\mathbf{8})_{5/2^-}\to(\mathbf{10})_{3/2^+}$	$\Sigma(1767)\to\Sigma^*\pi$	25.2		25.2		13.3
	$N(1690)\to\Delta\pi$	212		212		
	$\Lambda(1830)\to\Sigma^*\pi$	155		155		
	$\Xi(1930)\to\Xi^*\pi$	55		55		
$(\mathbf{8})_{3/2^-}\to(\mathbf{10})_{3/2^+}$	$N(1675)\to\Delta\pi$	54.7		28.3		
	$N(1518)\to\Delta\pi$		91.0		60.0	50.0
	$\Lambda(1690)\to\Sigma^*\pi$	52.6	87.7	15.9	47.4	
	$\Sigma(1660)\to\Sigma^*\pi$	8.8	14.7	4.7	4.6	$\lesssim$8.0
	$\Xi(1816)\to\Xi^*\pi$	12.4	20.8	6.7	6.3	3.2+1.6
	Σ^*K	3.2	5.3	0.9	$\approx$0.0	
$(\mathbf{10})_{3/2^-}\to(\mathbf{10})_{3/2^+}$	$\Delta(1691)\to\Delta\pi$	347			220	
$(\mathbf{10})_{1/2^-}\to(\mathbf{10})_{3/2^+}$	$\Delta(1640)\to\Delta\pi$		341		341	
$(\mathbf{8})_{5/2^+}\to(\mathbf{10})_{3/2^+}$	$N(1688)\to\Delta\pi$				24	
	$\Lambda(1816)\to\Sigma^*\pi$				18	
	$\Sigma(1910)\to\Sigma^*\pi$				6.7	
	$\Delta\bar{K}$				2.5	
$(\mathbf{10})_{7/2^+}\to(\mathbf{10})_{3/2^+}$	$\Delta(1950)\to\Delta\pi$	150	150			
	$\Sigma^*(2030)\to\Sigma^*\pi$	71	71			

good agreement with experiment, without the need for mixing. This is rather encouraging for the model, since the possibility of mixing is now very much restricted.

E. Decays to 10 States

There are several examples of decays from (**70**,1⁻) and (**56**,2⁺) states into the **10** states of (**56**,0⁺) which need some special attention because each of these is governed by more than a single partial wave. Thus, for decays from (**70**,1⁻) states of $J^P=\frac{3}{2}^-$, both s and d partial waves are allowed. One must also take proper account of the intrinsic spin structure of the decaying state. Thus for a transition from an initial spin-quartet (q) state to a final **10** state of **56**, the appropriate matrix element is of type $g\bar{\psi}_\mu\rho k^2\psi_\mu\pi$ (type A), where the operator ρ is 1 or $i\gamma_5$, according as the initial state is a (**70**,1⁻) or (**56**,2⁺), respectively. On the other hand, for a transition from an initial spin-doublet (d) state to a final **10** state of **56**, the matrix element is of type $g\bar{\psi}_\mu\rho k_\mu k_\nu\psi_\nu\pi$ (type B). Both these matrix elements are mixtures of s and d, or p and f, according as the transition is from (**70**,1⁻) or (**56**,2⁺), respectively. However, the type A does not admit of any further modification in its structure on the basis of our prescriptions outlined in Sec. 2. On the other hand, type B, whose nonrelativistic form is

$$\chi_\beta{}^\dagger\rho k_\beta k_\alpha\chi_\alpha\pi\,, \qquad (3.7)$$

needs to be further modified in keeping with the spirit of the above prescriptions. For this purpose, we rewrite (3.7) as

$$\chi^\dagger\rho k_\beta k_\alpha\chi_{\alpha\beta}\pi+[L/(2L+1)]^{1/2}\chi^\dagger\rho\mathbf{k}^2\chi\pi \qquad (3.8)$$

to bring out the s- and d-wave structures explicitly. In this reduction we have, so to say, "transferred" the tensor index β from the final state χ_β to the initial state χ_α, so that these now become χ and $\chi_{\alpha\beta}$, respectively, where χ is merely a Pauli spinor and $\chi_{\alpha\beta}$ is a nonrelativistic RS spinor of spin $\frac{5}{2}$. While these new states do not correspond to any *physical* states as such, they are nevertheless very convenient for bringing out the types of relevant partial waves that are involved in the transitions. It is now a matter of straightforward application of our relativistic prescriptions given in Sec. 2, according to which the three-vector indices (α,β) become four-vector indices (μ,ν) and the factor $\mathbf{k}^2$ in the second term of (3.8) goes over to $k_\mu k_\mu$ ($=-\mu^2$ on the mass shell), so that the resulting matrix element is now

$$\chi^\dagger\rho k_\nu k_\mu\chi_{\mu\nu}\pi+[L/(2L+1)]^{1/2}\chi^\dagger\rho(-\mu^2)\chi\pi\,. \qquad (3.9)$$

The entire expression must now be multiplied by the form factor (2.10) together with the exponent ($L+1$) associated with the higher partial wave. The structure (3.9) brings out the full effect of our prescriptions for decays into decuplets (of spin $\frac{3}{2}$), while the analogous type B represents only a partial implementation of the same prescription. For comparison with experiment, we exhibit separately the results obtained with both (3.9) and the matrix element of type B, on the basis of pure spin assignments for some of the (**70**,1⁻) states,[37] together with the experimental values[29] for the measured cases in Table IX. It may be seen that the agreement with experiment is greatly improved by the substitution of (3.9) for type B. We consider this result a rather neat test of our detailed prescription when the complications

[37] There is, of course, no problem of spin mixing for the (**56**,2⁺) states.

due to the presence of more than one partial wave are taken into account. Finally, if one takes account of the empirical mixtures listed in Table VII for some of the $(\mathbf{70},1^-)$ states, one obtains the following results:

$$\Gamma[N(1518)\to\Delta\pi]=46\text{ MeV},$$
$$\Gamma[\Lambda(1690)\to\Sigma^*\pi]=13\text{ MeV},$$
$$\Gamma[\Sigma(1660)\to\Sigma^*\pi]=0.2\text{ MeV},$$
$$\Gamma[N(1675)\to\Delta\pi]=75\text{ MeV},$$
$$\Gamma[\Xi(1816)\to\Xi^*\pi]=3.5\text{ MeV}.$$

These figures seem to improve the agreement with the experimental values even further (see Table IX).

F. $(\mathbf{70},3^-)\to(\mathbf{56},0^+)$ Decays

This case is much more tentative than the two previous ones since so far only a few higher-lying negative-parity states have been identified, and that too with inadequate measurement of their spins. However, if we assume these states to belong to $(\mathbf{70},3^-)$, or the Regge recurrences of $(\mathbf{70},1^-)$, we can use this model as a guide to their detailed quantum-number assignments until more data become available. The input value of g_3 used is

$$g_3^2/4\pi=0.11. \qquad (3.10)$$

With this value of g_3, one can obtain reasonable agreement if $N(2190)$ is taken as mostly in $\mathbf{8}_d$ and $\Lambda(2100)$ is mostly in the singlet. The features for the other cases at this stage are mainly predictions (so they have not been tabulated) which must await further experimental investigations.

The cases of radially excited states, especially $N(1470)$, $\Delta(1688)$, and possibly $N(1570)$ of $\frac{1}{2}^+$, cannot be discussed without the use of a formal coupling scheme such as the harmonic-oscillator model of baryons.[27]

G. Universal Coupling of Regge Recurrences

A very interesting consequence of this model relates to the magnitudes of the coupling constants for the various supermultiplet transitions. A comparison of the magnitudes of (2.12) and (3.6) for g_0 and g_2, respectively, shows such a close proximity that it is most unlikely to be a mere chance coincidence. Since according to our traditional ideas, the $(\mathbf{56},2^+)$ states represent the first Regge recurrences of $(\mathbf{56},0^+)$ states, it is quite reasonable to interpret the near equality of g_0 and g_2 in terms of a universal coupling for the entire Regge trajectory, which contains the **56** baryons of positive parity. Consequently, it is interesting to compare the predictions of this model for the still higher resonances of positive parity. In this regard, the only available data are on the Δ-type states $\Delta(1950)$, $\Delta(2420)$, $\Delta(2850)$, and $\Delta(3230)$ of successive J values of $\frac{7}{2}^+$, $\frac{11}{2}^+$, $\frac{15}{2}^+$, and $19/2^+$, respectively, each of which has a well-defined $N\pi$ mode. If these states are regarded as successive Regge recurrences of $\Delta(1238)$, with $L^P=2^+$, 4^+, 6^+, and 8^+, respectively, the model predicts a very simple result for their $N\pi$ decay widths. For this purpose we use the following general formula for decay[24,22]:

TABLE X. $\Delta\to N\pi$ decays from successive Regge recurrences of Δ-type states in the present model and in $O(4,2)$ theory.

L	J^P	Mode	Γ_{theor} (MeV)	Γ_{expt} (MeV)	$\Gamma_{O(4,2)}$ (MeV)
2	$7/2^+$	$\Delta(1920)\to N\pi$	28.8	88.0	76.7
4	$11/2^+$	$\Delta(2420)\to N\pi$	9.2	27.5	45.0
6	$15/2^+$	$\Delta(2850)\to N\pi$	2.8	10.0	13.9
8	$19/2^+$	$\Delta(3230)\to N\pi$	0.75	2.2	3.9

$$\Gamma(\Delta_{L+3/2}\to N\pi)\approx\frac{g_L^2}{4\pi}\frac{(L+1)!}{(2L+3)!!}\frac{M+m-\omega_k}{m}(8k). \qquad (3.11)$$

Here the stretched value $J=L+\frac{3}{2}$ has been used, and certain simplifications, based on the relativistic approximation for the pion energy, have been performed.

It is tempting to use the same value for all the g's corresponding to $(L=4,6,8)$ as for the parameter g_2 given by (3.6). With this extra assumption the predictions of the model are given in Table X, together with the experimental data as well as the predictions of $O(4,2)$.[3] These figures show that, apart from an over-all reduction factor of 0.3 ± 0.04 (caused by our input assumption on g_2 through a fit to the widths of some stretched states), the agreement is excellent. This strengthens our belief that these particles, which are supposed to lie on the same Regge trajectory, have a universal coupling to the $N\pi$ system. The fact that our empirical model has led to such a neat result suggests that its physical features are in accord with some general principles which a more formal theory is expected to obey while claiming to fit experimental data.

H. Polarization Effects

The more interesting consequences of the model concern the predictions of polarizations of the resonances as should manifest themselves through specified angular distributions in the decay products. Such a possibility was suggested by Lipkin *et al.*[3] on the basis of the direct term in the $\bar{Q}QP$ coupling which predicts geometrical ratios for the $(L\pm1)$-wave amplitudes. However, as was argued in Ref. 14, this possibility would no longer exist if both the direct and recoil contributions to the couplings are taken into account, since separate parametrization would be necessary for both. In the present model, we have in a way *restored* the above possibility through the prescription $\mathbf{k}^2\to k_\mu k_\mu$, which yields a definite ratio of the $(L+1)$- to the $(L-1)$-wave amplitudes, in spite of the inclusion of both the direct and recoil contributions. However, the present state of experimental knowledge about the polarization states of resonances does not as yet warrant any detailed predictions on the latter. For certain meson states, on the other hand, the availability of polarization data (e.g., in $B \to \omega\pi$ decay) would lend greater physical interest in the predictions of this model and these are discussed in the following paper.[17]

4. SUMMARY AND CONCLUSIONS

We have tried to present an improved form of parametrization for baryon couplings which, though within the general phenomenological framework of the quark model, nevertheless has several desirable features in common with a more formal theory. The main features of the new coupling are (i) their relativistically invariant structures, (ii) a unified treatment of direct and recoil terms through a simple and plausible ansatz, and (iii) an efficient form of parametrization of the multiplying form factor, so that a single over-all constant now describes an entire supermultiplet transition. The model, which predicts a definite pattern of $SU(3)$ breaking in the coupling constants arising for the various masses, has been put to a detailed experimental test for the entire structure of the form factor, with respect to a large variety of decays involving a wide range of masses and momenta. As to the $SU(3)$-breaking effects, the data seem to bring out the feature of a strong suppression of heavy-meson modes for $(L+1)$ couplings $(\sim u^{-L-1/2})$ in relation to the pionic modes, and a much weaker effect for the corresponding $(L-1)$ couplings $(\sim \mu^{-L+3/2})$, to the extent that the s-wave decays actually show a heavy-meson enhancement $(\sim \mu)$ in the decay rates. This last feature is fairly well substantiated by the observation of the $(\mathbf{70},1^-)$ decays in the s wave. While the predictions of the model for d-wave decays for $(\mathbf{70},1^-)$ states are not as good, the discrepancies are generally confined to cases involving the decays from unstretched states $(J^P=\frac{3}{2}^-)$, for which the model unfortunately overestimated the coupling strengths. However, such cases are also ones in which one expects significant effects of configuration mixing [$SU(3)$ and/or internal spin]. Since the model, by itself, does not provide any mechanism for the latter, one could perhaps employ its general structure in making some empirical determination of mixing parameters for certain states with well-defined decay modes. For the $(\mathbf{56},2^+)$ decays, the predictions of the model are extremely good, and these are also cases which are much less affected by configuration mixing than are some $(\mathbf{70},1^-)$ states. A particularly interesting (though empirical) result is the prediction of near equality in the reduced coupling constants (g_L) governing the couplings of the Regge recurrences of Δ-type states belonging to the **56** representation up to L values as high as $L=8$. Since such a result is in accord with the general expectation of the universal couplings of Regge trajectories, its explicit realization within an empirical model of rapidly varying form factors should perhaps be interpreted as a good numerical support for the broad validity of the model.

The model has its unsatisfactory features, especially the prediction of (i) large $\Sigma\pi$ modes in d-wave decays from $(\mathbf{70},1^-)$ states, and (ii) large decuplet modes for both $(\mathbf{70},1^-)$ and $(\mathbf{56},2^+)$ states. While a part of these discrepancies could be ascribed to configuration-mixing effects, a good part of the blame must be taken by the very nature of relativistic generalization from a nonrelativistic $(\boldsymbol{\sigma}\cdot\mathbf{k})$-type coupling between two baryon states which brings in an unusually large factor $(M+m)$. We have seen, however, that a more faithful implementation of the prescriptions of the model for the $(L+1)$-wave couplings to Δ-type states in **56** tends to bring about a substantial improvement in the decay widths to $\Delta\pi$-type modes.

Being empirical in nature, the model cannot be compared with the more formal theories of couplings, yet it has several desirable features such as relativistic invariance and economy of parametrization and, above all, fairly good agreement with experiment on the energy shell (i.e., for decay widths). It should thus be of considerable interest to apply the model to processes involving the *off-shell* manifestations of the coupling constants such as in scattering and production processes. A generalization of the model has been made to include couplings of vector mesons with baryons.[38] Calculations of certain elastic and inelastic processes with this model $(PB \to PB,\ PB \to VB)$ are currently in progress.

ACKNOWLEDGMENTS

Most of this work was done when one of the authors (A. N. M.) was a visitor at the International Centre for Theoretical Physics, Trieste, during the summer of 1968. He wishes to thank Professor Abdus Salam and P. Budini as well as the IAEA for the warm hospitality of the Centre. Discussions with Professor Fronsdal have greatly helped in the understanding of generalized Rarita-Schwinger fields. The interest shown by Professor R. C. Majumdar and Professor F. C. Auluck in this investigation is also gratefully acknowledged.

[38] A. N. Mitra, ICTP Report No. IC/69/49, 1969 (unpublished).