Noncausal Behavior of Classical Tachyons*

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It is shown by example that causality violation can take place through the transmission of classical tachyons between three or more observers in such a way that each observer receives and emits only tachyons whose energies he measures to be positive. Such examples are not open to the ambiguities of interpretation proposed by Feinberg; they imply that a classical-particle description of tachyons is not physically viable.

THE possibility of faster-than-light particles—tachyons—has been resurrected in recent years by Sudarshan[1] and Feinberg,[2] among others.[2a] The latter has taken some pains to show that because of ambiguities of interpretation, no manifest violation of causality can take place through the exchange of tachyons between two uniformly moving observers. Attempts to exhibit noncausality in such cases lead to difficulties of interpreting the behavior of tachyon receivers which may become spontaneous emitters. Such difficulties may be avoided by adoption of the following convention: Each observer describes the motion of those tachyons which interact with his apparatus in such a way that their energies are always measured to be positive.[3] With this convention, the experiments discussed by Feinberg cannot lead to violations of causality. However, these experiments involve motion in only one space dimension; by suitable arrangements of observers involving at least *two* space dimensions, one can construct thought experiments satisfying the convention but leading to manifest causality violations. In what follows, one such experiment is described in detail.

The experimental arrangement is represented in Fig. 1. Four observers A, B, C, and D move in the xy plane. The solid arrows denote their velocity three-vectors in a given system of reference S (all four-vector components and space directions are specified in S). For simplicity, the velocity three-vectors are chosen to be of equal magnitude and to bisect the angles between the coordinate directions. The wiggly lines, which run parallel to the coordinate directions, denote the paths of the tachyons, which are numbered 1, 2, 3, and 4. In order for their energies, measured in S, to be positive, the tachyons would have to be described as traveling in senses *opposite* to those shown by the arrows. However, the velocities of the observers are chosen so that the arrows show correctly the sense of motion assigned by each observer to the tachyons emitted or received by his apparatus. Thus a tachyon signal initiated by A travels counterclockwise around the figure and returns to A, supposed moving uniformly, *before* the moment of initiation.

More precisely, the following sequence of events takes place:

A emits tachyon 1;
B receives tachyon 1, and at once emits tachyon 2;
C receives tachyon 2, and at once emits tachyon 3;
D receives tachyon 3, and at once emits tachyon 4;
A receives tachyon 4, before his emission of tachyon 1 (and tachyon 4 triggers the destruction of his emitter).

Of course, the observers are given suitable instructions in advance. Except for A, each observer is involved in the sequence at only one point of his world line, so that the rest of his motion is irrelevant; it is assumed for simplicity that A and all the tachyons move uniformly.

The details of the emission and reception events are given in Table I, where for the moment six *positive* parameters are left unassigned. They are v, which characterizes the observer velocities, α, which characterizes the tachyon velocities, and l_k ($k=1$, 2, 3, and 4) which are the lengths in S of the tachyon paths.

If again for simplicity the tachyon four-momenta are chosen equal to their four-velocities [an additional (positive) scalar factor would make no difference to the argument], it will be seen from the Table I that each observer measures the energies[4] of the tachyons he emits and receives to be $v(1+\alpha^2)^{1/2}-\alpha(1+2v^2)^{1/2}$. In order that the energies all be positive, it is therefore necessary and sufficient that

$$\alpha^2<v^2/(1+v^2).$$

Positive values of α satisfying this inequality may be found for any given v. Conversely, a v satisfying the inequality may be found provided that $\alpha<1$. This limitation on α arises from the geometry chosen for the experiment.

Further conditions have to be imposed on the parameters to ensure that the final event in the sequence—reception of tachyon 4 by observer A—actually takes place. Let τ measure A's proper time, positive towards the future, from the event of emission of tachyon 1.

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[2] G. Feinberg, Phys. Rev. **159**, 1089 (1967).
[2a] O. M. P. Bilaniuk and E. C. G. Sudarshan, Phys. Today, **22**, 43 (1969); O. M. P. Bilaniuk, S. L. Brown, B. S. DeWitt, W. A. Newcomb, M. Sachs, E. C. G. Sudarshan, and S. Yoshikawa, *ibid.* **22**, 47 (1969).
[3] Senses of motion assigned in this way will agree with those which the same observer fixes by counter measurements.
[4] Velocity of light = 1.

TABLE I. Details of four-tachyon experiment. The $[x,y,t]$ components of events and four-velocities in the frame S are given. The z component, which is zero in all cases, is omitted. Some information is duplicated so that verification of arithmetic becomes trivial.

Tachyon No.	Emission event	Emitter and his four-velocity	Tachyon four-velocity	Tachyon path vector	Reception event	Receiver and his four-velocity
1	$[0,0,0]$	A $[-v,\ v,\ (1+2v^2)^{1/2}]$	$[(1+\alpha^2)^{1/2},\ 0,\ -\alpha]$	il_1	$[l_1,\ 0,\ -\alpha(1+\alpha^2)^{-1/2}l_1]$	B $[-v,\ -v,\ (1+2v^2)^{1/2}]$
2	$[l_1,\ 0,\ -\alpha(1+\alpha^2)^{-1/2}l_1]$	B $[-v,\ -v,\ (1+2v^2)^{1/2}]$	$[0,\ (1+\alpha^2)^{1/2},\ -\alpha]$	jl_2	$[l_1,\ l_2,\ -\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{2} l_k]$	C $[v,\ -v,\ (1+2v^2)^{1/2}]$
3	$[l_1,\ l_2,\ -\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{2} l_k]$	C $[v,\ -v,\ (1+2v^2)^{1/2}]$	$[-(1+\alpha^2)^{1/2},\ 0,\ -\alpha]$	$-il_3$	$[l_1-l_3,\ l_2,\ -\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{3} l_k]$	D $[v,\ v,\ (1+2v^2)^{1/2}]$
4	$[l_1-l_3,\ l_2,\ -\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{3} l_k]$	D $[v,\ v,\ (1+2v^2)^{1/2}]$	$[0,\ -(1+\alpha^2)^{1/2},\ -\alpha]$	$-jl_4$	$[l_1-l_3,\ l_2-l_4,\ -\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{4} l_k]$	A $[-v,\ v,\ (1+2v^2)^{1/2}]$

Then the parametric equations of A's world line are

$$x=-v\tau\,,$$
$$y=v\tau\,,$$
$$t=(1+2v^2)^{1/2}\tau\,,$$

and therefore (from the last line of Table I), he will receive tachyon 4 at proper time τ if

$$l_1-l_3=-v\tau\,,$$
$$l_2-l_4=v\tau\,,$$
$$(1+2v^2)^{1/2}\tau=-\alpha(1+\alpha^2)^{-1/2}\sum_{k=1}^{4} l_k\,.$$

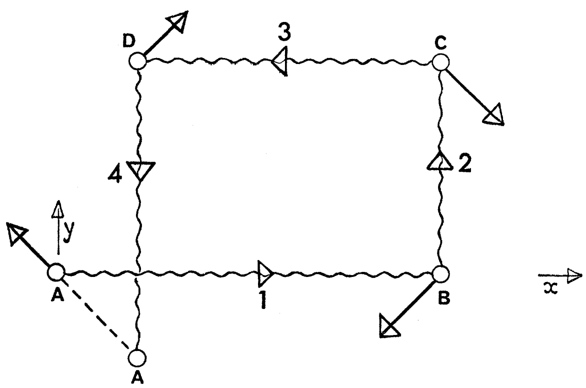

FIG. 1. Experimental arrangement for causality violation with tachyons. Solid arrows denote observer three-velocities, wiggly lines, tachyon paths.

Since α, v, and all the l's are positive, it follows from the last of these conditions that τ is negative, which yields the causality violation. If l_2 and l_3 are chosen arbitrarily, then the other two conditions fix l_4 and l_1:

$$l_1=l_3+v|\tau|\,,\quad l_4=l_2+v|\tau|\,,$$

and therefore

$$|\tau|>2\alpha(1+\alpha^2)^{-1/2}(1+2v^2)^{-1/2}(l_2+l_3)\,,$$

so that $|\tau|$ may be made as large as you please by conducting the experiment on a sufficiently large scale.

It is not difficult to convince oneself that a similar experiment involving only *two* observers cannot succeed in producing the same violation, but essentially the same results are obtained with *three* observers whose velocity three-vectors point along the sides of an equilateral triangle. A variety of generalizations to larger numbers of observers and other geometries is evident.

It is difficult to see how in the face of this example a classical-particle description of tachyons can be sustained. All the emission and absorption processes admit conventional descriptions, and only positive-energy particles are involved in measurements, yet a sequence of such processes leads to the return of a tachyon signal to its originator at a proper time before the moment of initiation.

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