4. CONCLUSIONS

From the above discussion, one can draw the following conclusions:

(i) The second-order corrections in $SU(3)$ breaking to $f_+(0)$ are small (3.7 to 2.5%) in contrast to what people previously thought.[5] $f_K/f_\pi \approx 1.24$ to 1.31 and $|f_\kappa/f_\pi|$ comes out to be quite small (0.42 to 0.35).

(ii) The value of ξ is small, in general, and is nearly zero:

$$\lambda_- = m_\pi^2/m_{K^*}^2 \approx 0.024 \approx \lambda_+ \quad \text{if} \quad m_\kappa = m_{K^*}$$

but

$$\lambda_- \approx 0.041 \quad \text{if} \quad m_\kappa = 1021 \text{ MeV}.$$

(iii) $\kappa \to K\pi$ width is large. The formula for this width is nearly the same as in the Veneziano model if $m_\kappa = m_{K^*}$. For the case $m_\kappa = m_{K^*}$, this is exactly the same as in the Veneziano model if, in addition, one assumes that the matrix elements of the divergence of the $|\Delta Y| = 1$ vector current between K and π goes to 0 as $t \to \infty$. In this case $a[\equiv f_K/f_\pi f_+(0)]$ is predicted to be $m_{K^*}/m_\rho^2 \approx 1.36$, in fair agreement with its experimental value of 1.28 ± 0.06.

Note added in proof. After submission of this paper, it came to our notice that some of the results similar to ours have also been obtained by P. R. Auvil and N. G. Deshpande, Phys. Rev. **183**, 1463 (1969); see also L. K. Pande, Phys. Rev. Letters **23**, 353 (1969).

ACKNOWLEDGMENTS

The authors thank Dr. David Morgan and Dr. P. K. Kabir for hospitality at the Rutherford High Energy Laboratory.

PHYSICAL REVIEW D VOLUME 1, NUMBER 1 1 JANUARY 1970

Derivation of Relations of the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin Type*

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(Received 22 April 1968; revised manuscript received 3 October 1969)

An alternative method for deriving Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin- (KSRF-) type relations, which uses a low-energy theorem in quantum electrodynamics, is presented. The method is applied to processes of the kind $V \to P + P' + \gamma$. The amplitude for this process is exactly calculable to order zero in the photon momentum k in terms of the strong vertex $f_{VPP'}(m_V^2, m_P^2, m_{P'}^2)$, with all particles on the mass shell. KSRF-type relations are obtained when the momentum-independent term from the series expansion (in powers of the photon momentum) of the radiative amplitude is equated with the parallel term obtained from an explicit calculation of the amplitude. The latter calculation involves the use of current-algebra, partial conservation of axial-vector current, and hard-pion techniques. The derivation is worked out in detail for the $\rho \to \pi^+\pi^-\gamma$ process, and the method is further considered in connection with the appropriate radiative decays of ϕ^0 and K^{*0} mesons.

I. INTRODUCTION

ONE of the most widely discussed applications of the current-algebra methods and the partial conservation of axial-vector current is the relation obtained[1,2] by Kawarabayashi, Suzuki, Riazuddin, and Fayyazudin (KSRF), $f_{\rho\pi\pi} = m_\rho^2/2F_\pi^2 f_\rho$. Through this expression, the mass of the ρ meson, the $\rho\pi\pi$ coupling ($f_{\rho\pi\pi}$), and the ρ coupling to the vector current (f_ρ) are related to F_π, the constant of the partial conservation of the axial-vector current (PCAC). This relation was subsequently rederived by the use of other approaches as well.[3] However, in all these derivations,[1–3] the KSRF relation is obtained only at the expense of various undesirable approximations. For example, in the original derivation, the relation obtained is in fact $f_{\rho\pi\pi}(0,0,0) = m_\rho^2/2F_\pi^2 f_\rho$, i.e., for zero-mass ρ and π mesons. The difficulty stems here mainly from the unwanted "softening" of the ρ, which occurs when the soft-pion limit is taken for the fairly energetic pions in the $\rho \to \pi\pi$ process. As a result, one is led to inquire whether the KSRF relation will retain its original form when the approximations are removed.

Recently, several articles have appeared[4–6] in which

* Work supported at Northwestern University by the National Science Foundation.

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the deficiencies of the existing derivations[1–3] are critically analyzed. It becomes obvious, especially from the more complete analysis of Ref. 6, that in fact current algebra and PCAC alone are not sufficient to deduce the KSRF formula. On the other hand, by using hard-pion methods and vector-meson dominance, Arnowitt *et al.*[6] derive a KSRF-type relation, which supplements the original formula with additional constants which relate to the A_1 meson. Their expression[7–10] reads

$$f_{\rho\pi\pi}(m_\rho^2,m_\pi^2,m_\pi^2)=\frac{m_\rho^2}{2F_\pi^2 f_\rho}\left[1-\frac{f_\rho^2}{f_A^2}(1+\delta)\right], \quad (1)$$

where f_A is the coupling of the A_1 meson to the axial-vector current, and δ is related to the anomalous magnetic moment of A_1.

In this article we describe an alternative method by which KSRF-type relations can be derived. In our approach, we consider radiative decay amplitudes, which have both an inner bremsstrahlung and a direct decay part, and we use a theorem on the series expansion of these amplitudes in powers of the photon four-momentum. KSRF-type relations result when one equates the momentum-independent term from the expansion, with the parallel term obtained from an explicit calculation of the decay amplitude. The latter calculation involves the use of current algebra, PCAC, and hard-pion techniques.

In this work we consider decays of the type $V\to P+P'+\gamma$, for which the strong decay $V\to P+P'$ also occurs. The procedure is carried out in detail in Sec. II for the radiative decay $\rho^0\to\pi^+\pi^-\gamma$. In Sec. III the application to the decays $\phi\to K^+K^-\gamma$ and $K^{*0}\to K\pi\gamma$ is outlined, without, however, our going into the details of the hard-meson technique for K mesons. In Sec. IV, we discuss several features of our derivation as well as the limited experimental information available for checking the various KSRF-type relations.

II. KSRF-TYPE RELATION FROM ρ-RADIATIVE DECAY

Consider the radiative process $\rho^0\to\pi^+\pi^-\gamma$. This decay has been discussed previously in the literature, both without[11] and with[12,13] the current-algebra approach.[14] We follow here the method developed by Weinberg[15,16] and we write for the decay transition off the mass shell

$$(2\pi)^3(4p_0q_0)^{1/2}\langle\pi^+(q)\pi^-(p)|J_\mu(0)|\rho^0\rangle$$
$$=(2m_\pi^4F_\pi^2)^{-1}(q^2-m_\pi^2)(p^2-m_\pi^2)\int d^4x d^4y e^{iq\cdot x}e^{ip\cdot y}$$
$$\times\langle 0|T\{\partial_\nu A_+^\nu(x),\,\partial_\sigma A_-^\sigma(y),\,J_\mu(0)\}|\rho^0\rangle. \quad (2)$$

In Eq. (2), J_μ is the electromagnetic current taken as having the $SU(3)$ transformation properties

$$J_\mu(x)=e[V_\mu^3(x)+(1/\sqrt{3})V_\mu^8(x)], \quad (3)$$

and F_π is defined through the pion decay amplitude

$$\langle 0|\partial_\mu A_+^\mu(0)|\pi^\pm(p)\rangle=\sqrt{2}F_\pi m_\pi^2(2\pi)^{-3/2}(2p_0)^{-1/2}, \quad (4)$$

where

$$A_\pm^\mu(x)=A_1^\mu(x)\pm iA_2^\mu(x). \quad (5)$$

One now expands the time-ordered product $T\{\partial_\nu A_+^\nu(x),\partial_\sigma A_-^\sigma(y),J_\mu(0)\}$ in analogy with Eq. (4) of Ref. 16, obtaining

$$T\{\partial_\nu A_+^\nu(x),\,\partial_\sigma A_-^\sigma(y),\,J^\mu(0)\}$$
$$\equiv\frac{\partial}{\partial x^\nu}\frac{\partial}{\partial y^\sigma}T\{A_+^\nu(x),\,A_-^\sigma(y),\,J^\mu(0)\}-\delta(x^0-y^0)T\{[A_-^0(y),\,\partial_\nu A_+^\nu(x)],\,J^\mu(0)\}$$
$$-\tfrac{1}{2}\delta(x^0-y^0)T\left\{\left(\frac{\partial}{\partial y^\sigma}+\frac{\partial}{\partial x^\sigma}\right)[A_+^0(x),\,A_-^\sigma(y)],\,J^\mu(0)\right\}-\frac{1}{2}\left(\frac{\partial}{\partial y^\sigma}\quad\frac{\partial}{\partial x^\sigma}\right)\delta(x^0-y^0)T\{[A_+^0(x),\,A_-^\sigma(y)],\,J^\mu(0)\}$$
$$-\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_-^0(y),\,[A_+^0(x),\,J^\mu(0)]]-\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_+^0(x),\,[A_-^0(y),\,J^\mu(0)]]$$
$$-\delta(y^0)T\{[A_-^0(y),\,J^\mu(0)],\,\partial_\nu A_+^\nu(x)\}-\delta(x^0)T\{[A_+^0(x),\,J^\mu(0)],\,\partial_\sigma A_-^\sigma(y)\}. \quad (6)$$

[7] This formula is implicitly contained also in several other works dealing with hard-pion techniques (see Refs. 5 and 8–10).
[8] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
[9] H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).
[10] S. G. Brown and G. B. West, Phys. Rev. **168**, 1605 (1968).
[11] P. Singer, Phys. Rev. **130**, 2441 (1963); **161**, 1694 (1967).
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[13] R. N. Chaudhuri and R. Dutt, Phys. Rev. **177**, 2337 (1969).
[14] In Ref. 12 the rate of $\rho^0\to\pi^+\pi^-\gamma$ decay is calculated using the soft-pion limit of the amplitude obtained with the current-algebra technique. This is inadequate, since by taking the soft-pion limit one loses also the terms of order $1/k$ in photon momentum, which give essentially the major contribution to the rate. This deficiency is remedied by the authors of Ref. 13, who use the hard-pion method in their analysis. The results of the latter work are in remarkable agreement with the original pole-model calculation (Ref. 11).
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[16] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966); **18**, 1178 (1967).

In order to evaluate these terms, we use the following current commutation relations[17]:

$$[A_0^a(x), A_\nu^b(y)]\delta(x_0-y_0)=if_{abc}V_\nu^c(x)\delta^4(x-y)\,, \tag{7}$$

$$[A_0^a(x), V_\nu^b(y)]\delta(x_0-y_0)=if_{abc}A_\nu^c(x)\delta^4(x-y)\,, \tag{8}$$

as well as the additional commutation relation of the σ model,

$$[A_0^a(x), \partial_\mu A_b^\mu(y)]\delta(x_0-y_0)=i\delta_{ab}\sigma(x)\delta^4(x-y)\,. \tag{9}$$

Then one easily sees that the third term in the expansion of the right-hand side of Eq. (6) vanishes because of $V_\mu^3(x)$ conservation, and the fourth term vanishes by the requirement of charge-conjugation invariance. The nonvanishing contributions are calculated by double application of the commutation relations and by using the identity

$$\partial_\mu T\{A_b^\lambda(0), A_a^\mu(x)\}\equiv\delta(x^0)[A_a^0(x), A_b^\lambda(0)]+T\{A_b^\lambda(0), \partial_\mu A_a^\mu(x)\}\,. \tag{10}$$

Finally, using partial integration, the invariant amplitude for the decay is

$$M(\rho^0\to\pi^+\pi^-\gamma)=(2F_\pi^2)^{-1}\epsilon_\mu^{(\gamma)}\bigg\{q_\sigma p_\tau\int d^4xd^4y e^{i(q\cdot x+p\cdot y)}\langle 0|T\{A_+^\sigma(x), A_-^\tau(y), J^\mu(0)\}|\rho^0\rangle$$
$$-2\int d^4x e^{i(q+p)\cdot x}\langle 0|T\{\sigma(x)J^\mu(0)\}|\rho^0\rangle+iep_\nu\int d^4y e^{ip\cdot y}\langle 0|T\{A_+^\mu(0), A_-^\nu(y)\}|\rho^0\rangle$$
$$-ieq_\nu\int d^4x e^{iq\cdot x}\langle 0|T\{A_-^\mu(0), A_+^\nu(x)\}|\rho^0\rangle+2e(m_\rho^2/f_\rho)\epsilon^\mu{}_{(\rho)}\bigg\}\,. \tag{11}$$

$\epsilon_\mu^{(\gamma)}$ and $\epsilon^\mu{}_{(\rho)}$ are the polarization four-vectors of the photon and ρ meson. In writing the last term in (11), we used the definition of the ρ coupling to the vector current,

$$(2\pi)^{3/2}(2p_0)^{1/2}\langle 0|V_3^\mu(0)|\rho^0(p)\rangle=(m_\rho^2/f_\rho)\epsilon^\mu{}_{(\rho)}\,. \tag{12}$$

In general, the amplitude for a radiative decay can be written[11,18,19] as the sum of an inner bremsstrahlung term M_{ib} and a direct term M_d, which are separately gauge-invariant.[20] In an expansion of the amplitude in powers of the photon four-momentum, one has $M_{ib}=O_{ib}(k^{-1})+O_{ib}(0)+O_{ib}(k)+\cdots$, while $M_d=O_d(k)+\cdots$. M_d has no terms of order k^{-1} which come from charged-particle propagators, since the photon vertex is never on a free line in M_d. In addition, by using the technique developed by Low[21] for scattering, Chew[18] and Pestieau[19] have shown that in the radiative decay of a hadron to two hadrons, terms of order zero in k can originate only from the M_{ib} part of the amplitude. As a result, terms up to zero order in the photon momentum in the radiative amplitude $V\to P+P'+\gamma$ are *exactly calculable* in terms of the strong vertex $f_{VPP'}(m_V^2,m_P^2,m_{P'}^2)$, *with all particles on the mass shell.*

[17] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[18] H. Chew, Phys. Rev. **123**, 377 (1961).
[19] J. Pestieau, Phys. Rev. **160**, 1555 (1967).
[20] We define M_{ib} so as to include already the "direct" part which is related to the soft-photon terms by gauge invariance. The M_d part on the other side is model-dependent. For instance, in the vector-dominance model, the M_d part of $\rho^0\to\pi^+\pi^-\gamma$ vanishes (Ref. 11).
[21] F. Low, Phys. Rev. **110**, 974 (1958).

By using this theorem, we obtain

$$M(\rho^0\to\pi^+\pi^-\gamma)$$
$$=2ef_{\rho\pi\pi}(m_\rho^2,m_\pi^2,m_\pi^2)\bigg[\epsilon^{(\rho)}\cdot\epsilon^{(\gamma)}+\frac{(\epsilon^{(\rho)}\cdot p)(\epsilon^{(\gamma)}\cdot q)}{q\cdot k}$$
$$+\frac{(\epsilon^{(\rho)}\cdot q)(\epsilon^{(\gamma)}\cdot p)}{p\cdot k}\bigg]+O_{ib}(k)+\cdots+M_d\,. \tag{13}$$

From Eq. (13), it is evident that the term of zero order in k, which depends only on the constants e and $f_{\rho\pi\pi}$, has no pion momentum dependence either. One can also easily check that the sum of the first three terms on the right-hand side of (13) is gauge-invariant.

Now we have to identify the momentum-independent term from (11), and by equating it to the parallel term of (13) a KSRF-type relation obtains, with $f_{\rho\pi\pi}$ defined for all particles on the mass shell. As a first step, let us consider the soft-pion limit q_μ, $p_\mu\to 0$. In this limit, the first, third, and fourth expressions on the right-hand side of (11) vanish, being of first or second order in pion momenta. Considering $\sigma(x)$ to represent a scalar particle, the second term is related to the amplitude for $\rho\to\sigma+\gamma$, which belongs to M_d. Hence, the only term of order zero in k surviving in the soft-pion limit in (11) is the last one, and equating now the momentum-independent terms in (11) and (13), one obtains

$$f_{\rho\pi\pi}(0,0,0)=m_\rho^2/2F_\pi^2f_\rho\,. \tag{14}$$

This is the original expression of KSRF,[1,2] and since we deal here with soft pions and real photon, the coupling constant $f_{\rho\pi\pi}$ appearing in (14) is obtained at the point $m_\rho^2=m_\pi^2=0$. One should also remark that in the limit taken above, the M_{ib} part of the radiative amplitude is not gauge-invariant any more.

In order to obtain $f_{\rho\pi\pi}(m_\rho^2,m_\pi^2,m_\pi^2)$, one has to calculate explicitly the expressions in (11) which vanish in the soft-pion limit. To this end, we use the so-called hard-meson current-algebra techniques which allow us to calculate the expressions in Eq. (11) for physical pions. As developed by the authors of Refs. 9 and 22–24, these techniques give explicit expressions for the three- and four-point functions of axial-vector and vector currents, which appear in our Eq. (11) after the ρ is brought into the bracket by the standard reduction procedure. These T products of current operators are evaluated on the assumptions (e.g., Refs. 24 and 25) (1) that one may saturate the sums over the intermediate states with the lowest-lying single-meson states, i.e., π, ρ, and A_1 particles; (2) that the resulting particle vertex functions have the smoothest dependence on the momenta of the single particles involved; and (3) that the currents appearing in the T products satisfy conservation of vector current (CVC), PCAC, and the $SU(2)\times SU(2)$ chiral algebra [the latter being sufficient for the derivation of (1)]. The expressions we used for our calculation are summarized in Chap. IV of Ref. 22. A straightforward calculation then gives for the momentum-independent (m.i.) part of (11)

$$M(\rho^0\to\pi^+\pi^-\gamma)_{\text{m.i.}}=\epsilon^{(\rho)}\cdot\epsilon^{(\gamma)}(2F_\pi^2)^{-1}\times\frac{2em_\rho^2}{f_\rho}\left[1-\frac{f_\rho^2}{f_A^2}(1+\delta)\right]. \quad (15)$$

Equating this to the appropriate term in (11), one obtains

$$f_{\rho\pi\pi}(m_\rho^2,m_\pi^2,m_\pi^2)=\frac{m_\rho^2}{2F_\pi^2f_\rho}\left[1-\frac{f_\rho^2}{f_A^2}(1+\delta)\right]. \quad (1)$$

δ is the parameter entering in the definition of the $A_1A_1\rho$, $\rho\pi\pi$, and $A_1\rho\pi$ vertices as defined, e.g., in Ref. 9, and is related to the magnetic moment of the A_1 meson. f_A is the A_1 coupling to the axial-vector current A^μ, defined by

$$(2\pi)^{3/2}(2p_0)^{1/2}\langle 0|A_3^\mu(0)|A_1^0(p)\rangle=(m_A^2/f_A)\epsilon^\mu{}_{(A_1)}. \quad (16)$$

III. KSRF-TYPE RELATIONS FROM ϕ AND K^* RADIATIVE DECAYS

A similar approach is used to deal with the ϕ and K^* appropriate radiative decays. For the decay amplitude $\phi\to K^+K^-$ one has an expression analogous to (2), except that we replace $\rho^0\to\phi^0$, $\pi^\pm\to K^\pm$, and $A_\pm{}^\mu(x)\to A_{K^\pm}(x)$, where

$$A_{K^{\pm\mu}}(x)=A_4{}^\mu(x)\pm iA_5{}^\mu(x). \quad (17)$$

Partial conservation of the axial-vector strangeness-changing current is now used:

$$\langle 0|\partial_\mu A_{K^{\pm\mu}}(0)|K^\pm(q)\rangle=\sqrt{2}F_KM_K^2(2\pi)^{-3/2}(2q_0)^{-1/2}. \quad (18)$$

Using the commutation relations (7) and (8) and an SU_3 generalization of (9), as well as the definition

$$(2\pi)^{3/2}(2q_0)^{1/2}\langle 0|V_\mu{}^8(0)|\phi(q)\rangle=(m_\phi^2/f_\phi)\epsilon_\mu{}^{(\phi)}, \quad (19)$$

we obtain the relation[25]

$$f_{\phi K^+K^-}(m_\phi^2,m_K^2,m_K^2)=\sqrt{3}m_\phi^2/4F_K^2f_\phi+H_\phi. \quad (20)$$

In arriving at (20), the procedure followed is identical with the one described in the previous section. In the expansion of the product $T\{\partial_\nu A_{K^{+\nu}}(x),\ \partial_\sigma A_{K^{-\sigma}}(y),\ J_\mu(0)\}$, one has now the third term vanishing because of $V_\mu{}^3(x)$ and $V_\mu{}^8(x)$ conservation, and the fourth term because of charge-conjugation invariance. In Eq. (20), H_ϕ is the addition to a "soft" KSRF-type relation, which can be obtained by calculating terms similar to the first, third, and fourth on the right-hand-side of (11), with the replacements $\rho^0\to\phi^0$, $\pi^\pm\to K^\pm$, and $A_\pm{}^\mu(x)\to A_{K^{\pm\mu}}(x)$.

We further apply the same method to $K^{*0}\to K^+\pi^-\gamma$ decay.[26] Now, in the expansion of the T-ordered product, one obtains from the third term a "κ term," if we assume the divergence of the strangeness-changing vector current to be proportional to a scalar κ field. A similar κ term arises also from the second term. These terms do not affect the KSRF-type relation to be obtained, since they describe the direct decay $K^*\to\kappa+\gamma$, which is at least of first order in the photon momentum k. The fourth term vanishes because of the assumed form of $J^\mu(x)$, which gives zero magnetic moment for K^{*0}. Defining

$$(2\pi)^{3/2}(2q_0)^{1/2}\langle 0|V_\mu{}^6(0)+iV_\mu{}^7(0)|K^{*0}(q)\rangle=(\sqrt{2}m_{K^*}^2/f_{K^*})\epsilon_\mu{}^{(K^{*0})}, \quad (21)$$

we now obtain the relation

$$f_{K^{*0}K^+\pi^-}(m_{K^*}^2,m_K^2,m_\pi^2)=m_{K^*}^2/2\sqrt{2}F_KF_\pi f_{K^*}+H_{K^*}. \quad (22)$$

Here again H_{K^*} is the correction to the formula which is obtained when K^* and π are on the mass shell. Discussions on the extension of the hard-meson technique to $SU(3)\times SU(3)$ current algebra which is needed to evaluate H_ϕ and H_{K^*} have appeared recently (e.g., Refs. 27 and 28). We feel, however, that at present the "single-particle approximation" to the needed three-

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[26] M. Sapir and P. Singer, Phys. Rev. **163**, 1756 (1967).

[27] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968).

[28] R. Arnowitt and P. Nath, Lectures at the Summer Institute of Theoretical Physics, University of Colorado, 1968 (unpublished).

and four-point functions is not as meaningful (from the experimental point of view) as in the $SU(2)\times SU(2)$ case, and we shall therefore refrain for the time being from a detailed calculation of H_ϕ and H_{K^*}.

IV. DISCUSSION

By considering the radiative decays of vector mesons to S-wave pseudoscalar mesons and a low-energy theorem in quantum electrodynamics, we arrived here at a new technique for deriving KSRF-type relations. In our approach, we can obviously avoid the soft limit for p-wave mesons of the original derivations, as well as the zero-mass limit for the decaying particle. In addition, our method provides a compact procedure for treating both the soft case as well as the on-the-mass-shell one. The method can be extended, in principle, to obtain additional relations by using radiative decays such as $\omega\to 3\pi+\gamma$, etc.

For deriving a KSRF-type relation, we consider the (convenient) zero-order term in the expansion of the radiative amplitude. If the vector particle has no magnetic moment, the same relation results by equating the appropriate $1/k$ terms, and we checked this explicitly.

In deriving Eq. (1), we found the following interesting result: After explicitly calculating the expressions of Eq. (9) by the hard-pion technique, it turns out that in collecting the various momentum-independent terms, the pion mass disappears from the final term. Hence, somewhat paradoxically, the hard-pion correction does not depend explicitly on the pion mass. Therefore, the same result will be obtained,[29] for instance, for zero-mass pions; i.e.,

$$f_{\rho\pi\pi}(m_\rho^2,0,0)=\frac{m_\rho^2}{2F_\pi^2 f_\rho}\left[1-\frac{f_\rho^2}{f_A^2}(1+\delta)\right],$$

with, however, the ρ being on its mass shell.

The experimental validity of (1) has already been discussed in the literature. If one uses ρ dominance of the pion form factor ($f_{\rho\pi\pi}=f_\rho$), then, neglecting the correction term $(f_\rho^2/f_A^2)(1+\delta)$, one would predict from (1) a ρ width of approximately 140 MeV. The experimental value obtained from colliding-beam experiments is 110–130 MeV.[30] Hence, δ is expected to lie between -0.5 and -0.2, the correction to the "soft" KSRF formula being indeed fairly small (with our notation, $f_A=2f_\rho$ from the second Weinberg sum rule).

In order to check (20) and (22), we proceed by assuming H_ϕ and H_{K^*} to be also small compared to the main terms. Neglecting the term $(f_\rho^2/f_A^2)(1+\delta)$ in (1), we rewrite (1), (20), and (22) as

$$\frac{\sqrt{2}f_{\phi K^+K^-}}{f_{\rho\pi\pi}}=\left(\frac{m_\phi}{m_\rho}\right)^2\left(\frac{F_\pi}{F_K}\right)^2\frac{f_\rho}{(\sqrt{2}/\sqrt{3})f_\phi},\tag{23}$$

$$\frac{2f_{K^{*0}K^+\pi^-}}{f_{\rho\pi\pi}}=\left(\frac{m_{K^*}}{m_\rho}\right)^2\frac{F_\pi}{F_K}\frac{f_\rho}{f_{K^*}}.\tag{24}$$

Various approximations can be made for the right-hand sides of (23) and (24). Using vector-meson dominance for appropriate processes, one has $f_\rho=f_{\rho\pi\pi}$, $f_{K^*}=\sqrt{2}f_{K^{*0}K^+\pi^-}$, and $f_\phi=\sqrt{3}f_{\phi K^+K^-}$, and the SU_3 ratios of the $\phi\to KK$, $\rho\to\pi\pi$, and $K^*\to K\pi$ decay widths are modified by factors proportional to the squares of the masses, if $F_\pi/F_K=1$. Then Eqs. (23) and (24) give, for a ρ width of 120 ± 10 MeV, $\Gamma_{K^*}=47\pm4$ MeV (versus $\Gamma_{K^*}{}^{\rm expt}=49.2\pm1.0$ MeV) and $\Gamma_{\phi\to K\bar{K}}=4.4\pm0.4$ MeV (versus $\Gamma_{\phi\to K\bar{K}}{}^{\rm expt}=3.5\pm0.4$ MeV). However, it is quite possible that $F_K/F_\pi=1.16$, and these results should be then appropriately corrected. Alternatively, if we assume that the f_V's obey exact nonet symmetry, the SU_3 ratios for the decays are now modified by mass ratios to the fourth power. Equally good agreement with experiment is obtained now if one takes $F_K/F_\pi=1.16$, giving $\Gamma_{K^*}=46\pm4$ MeV and $\Gamma_{\phi\to K\bar{K}}=4.1\pm0.3$ MeV. Sakurai and Oakes[31] have recently obtained somewhat similar relations from an analysis of sum rules.

Our analysis hence shows that H_ϕ and H_{K^*} are also small compared to the appropriate main terms, since this assumption leads to consistent values for the quantities discussed above.

Finally, we wish to remark on the meaning of KSRF-type relations. This becomes more explicit if we consider for simplicity the soft-pion case. In deriving the last term in (11), we used the commutator (7). This is of the type $[A,A]=V$, i.e., the commutator which was "scaled" by Gell-Mann in proposing a closed A,V algebra.[17] In demanding that the expression obtained from the commutator (which is essentially also the main term of the correct KSRF relation) conform with the exact quantum-electrodynamics calculation, one obtains (1) as a necessary consistency relation.

ACKNOWLEDGMENTS

I would like to acknowledge helpful discussions and correspondence with R. Arnowitt, P. Auvil, L. M. Brown, N. G. Deshpande, S. Gasiorowicz, H. Munczek, and H. J. Schnitzer. I am also indebted to Mrs. Naomi Levy for checking the calculations leading to Eq. (15).

[29] This result was also obtained using a different method by the authors of Ref. 6.

[30] J. E. Augustin *et al.*, Nuovo Cimento Letters **2**, 214 (1969).

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