

Reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c*

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The differential cross section and ρ density-matrix elements for the reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c are presented. The data are interpreted in terms of an absorption model with conventional pion exchange. The transverse ρ^0 production is compared with the predictions of vector dominance.

THE highly peripheral reaction $\pi p \rightarrow \rho\Delta$ is of considerable importance in distinguishing between various exchange models. The results of this experiment and similar experiments at 1 8 GeV/c or less suggest that the above reaction is dominated by pion exchange. According to LeBellac,² if the above reaction is described by a model of conspiring Regge poles the pion contribution has a dip in the forward direction. This is unlike the np charge-exchange reaction where conspiracy implies a forward peak. In contrast, most models with absorption effects have a forward peak structure. However, since the finite widths of the ρ and Δ tend to smear out dips in the differential cross section, an energy of 8 GeV/c or greater is required to resolve the forward structure, as suggested by Donohue.³

We have analyzed the differential cross section and ρ density matrix for the reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c. The data were obtained from 230 000 pictures taken in the SLAC 82-in. bubble chamber. The four-prong events were measured on SMP's (scanning measuring projectors) and analyzed by the local version of the TVGP-SQUAW data reduction and fitting programs. Some 10 000 events in the $\pi^+\pi^+\pi^-p$ final state were fitted; after correction for background, the $\rho^0\Delta^{++}$ cross section was found to be 0.17 ± 0.02 mb, normalized to the total $\pi^+\pi^+\pi^-p$ cross section of 1.23 ± 0.04 mb. The ρ and Δ widths are observed to be $\Gamma_\rho = \Gamma_\Delta = 0.12$ GeV. For the subsequent analysis, the $\rho^0\Delta^{++}$ region is defined by $M_{\pi^+\pi^-} = 0.765 \pm 0.1$ GeV and $M_{\pi^+p} = 1.236 \pm 0.1$ GeV, yielding a sample of 1037 events. The distribution of events in $|t'|$ is presented in Fig. 1, in which the two dashed lines correspond to a coarse representation of the data by an exponential, $N(t') \sim e^{B_1 t'}$, where B_1 and B_2 are given by 20 and 3.8 (GeV/c)⁻², respectively [very close to the values 19.1 and 3.2 (GeV/c)⁻² quoted in the 8-GeV/c experiment]⁴; $t' = t - t_{\min}$, and t_{\min} is the minimum momentum transfer for a given event. A rise in the forward direction, $|t'| \lesssim \mu^2$, is clearly exhibited in the

inset to Fig. 1. This distribution is described quite nicely in terms of a pion-exchange model with absorption, and clearly is not consistent with an $M=1$ pion conspiracy.

The detailed comparisons with models, given below, use only those events with $|t'| < 0.2$ (GeV/c)². Beam momentum spread and resonance widths cause $|t_{\min}|$ to vary from 0.008 to 0.028 (GeV/c)²; but 72% of the events have $|t_{\min}|$ between 0.012 and 0.02 (GeV/c)², with an average of 0.016 (GeV/c)². By comparison, the average $|t_{\min}|$ in the 8-GeV/c experiment was 0.028 (GeV/c)².

The two prominent features of the data are a strong forward peak of width approximately μ^2 in the differential cross section as a function of $|t'|$ with a secondary break at $|t'| \simeq 0.15$ GeV/c² (Fig. 1), and a large value of

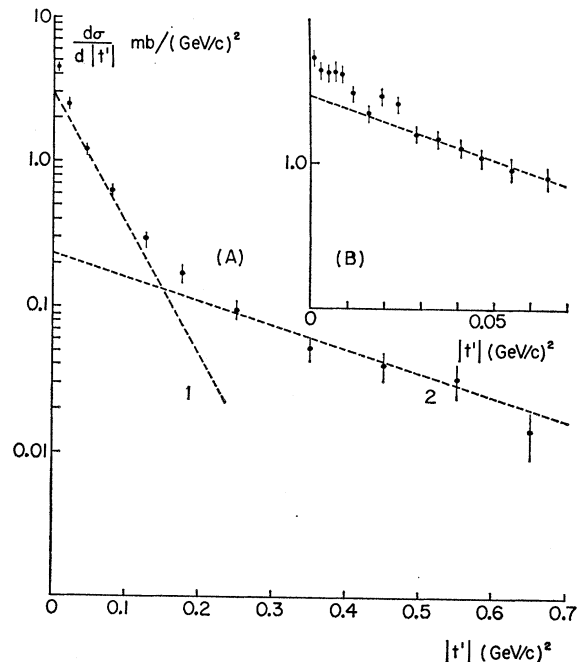


FIG. 1. (a) Differential cross section as a function of $|t'|$ for $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c. The lines correspond to slopes of 20 and 3.8 (GeV/c)⁻². (b) Same as (a), but for small $|t'|$; the line has a slope of 20 (GeV/c)⁻².

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¹ Aachen-Berlin-CERN collaboration, Phys. Letters **27B**, 174 (1968).

² M. LeBellac, Phys. Letters **25B**, 524 (1967).

³ J. T. Donohue, Nuovo Cimento **55A**, 527 (1968).

⁴ M. Aderholz *et al.*, Nucl. Phys. **B8**, 45 (1968).

($\rho_{00} - \rho_{11}$) in the forward direction (Fig. 2). The values of $\text{Re}\rho_{10}$ and ρ_{1-1} are consistent with zero for $|t'| < 0.2$ (GeV/c)². The observed fore-aft asymmetry in the Jackson angle distribution is 0.36 ± 0.04 , indicating some s -wave background. If the value of $\rho_{11}(t')$ is estimated from the absorption model given below and this contribution subtracted from the constant part of $d\sigma(t')/d\cos\theta_J$, the s -wave background is less than 10% for $|t'| < 0.04$ and less than 5% thereafter. Neglecting the s -wave contribution, the fitted values of ρ_{00} are shown in Fig. 3, along with the data from the 8-GeV/c experiment,⁵ where the dashed lines represent calculations based on the absorption model, at 13 and 8 GeV/c.

An absorption model is presented which fits all of the above data; the sharp forward peak is given by a pole term, the secondary break by amplitudes which vanish in the forward direction but rise smoothly to maxima at $|t'| \sim 0.1$ (GeV/c)². The absorption terms are relatively small in the forward direction but tend to give large contributions for larger $|t'|$. The nonhelicity-flip cross section is consistent with a very different model containing interference between pion and A_1 -meson Regge poles without absorption.

Standard Regge analysis begins with the t -channel amplitudes for the reaction $\bar{\Delta}(\lambda')N(\lambda) \rightarrow \rho(\mu)\pi$ which are denoted by $f_{\lambda'\lambda\mu}$, where λ' , λ , and μ are the helicities of the Δ , N , and ρ , respectively. These amplitudes satisfy the kinematic constraints for values of t corresponding to pseudothresholds, the one of interest here being $(M_\Delta - M_N)^2 \sim 4\mu^2$. The Regge-pole contribution is given by

$$f_{\lambda'\lambda\mu} \sim \frac{1}{2}\pi\alpha' (\gamma_{\lambda'\lambda\mu} / \sin\frac{1}{2}\pi\alpha) e^{-\frac{1}{2}i\pi\alpha} (s/s_0)^\alpha,$$

where $\alpha(t)$ is the Regge trajectory and $\rho(t)$ the residue. The existence of a forward peak and the large value of ρ_{00} prove in a model-independent way that $f_{\frac{3}{2}\frac{3}{2}0}$ dominates in the forward direction. We start with $\gamma_{\frac{3}{2}\frac{3}{2}0} = 4g_\rho g_\Delta (\hat{p}_\pi \cdot \epsilon_\rho) (\bar{U}_\nu U P_N')$ suggested by the Feynman diagram for pion exchange, where U_ν , U , and ϵ are the

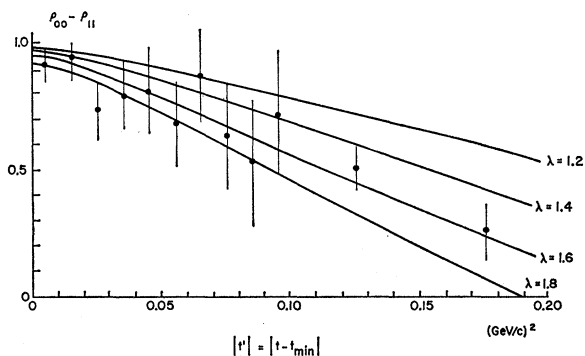


FIG. 2. ρ density-matrix element as a function of $|t'|$ in the Jackson frame for $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c; the curves are based on the model of Henyey *et al.* (Ref. 8); λ is a parameter which modifies the absorptive contribution.

⁵ K. Paler (private communication).

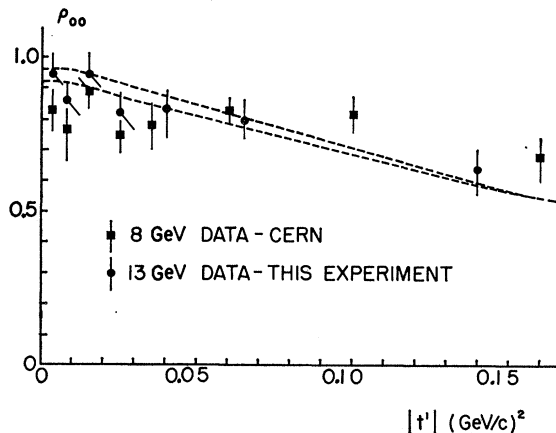


FIG. 3. $\rho_{00}(|t'|)$ in the Jackson frame from data at 13.1 and 8 GeV/c. The dashed lines are obtained from the absorption model at these energies, the upper line corresponding to the higher energy.

spin- $\frac{3}{2}$, $-\frac{1}{2}$ and -1 wave functions, respectively, and the couplings g_ρ , g_Δ are related to the widths in the usual way.⁶ This is the only coupling for a physical pion, and

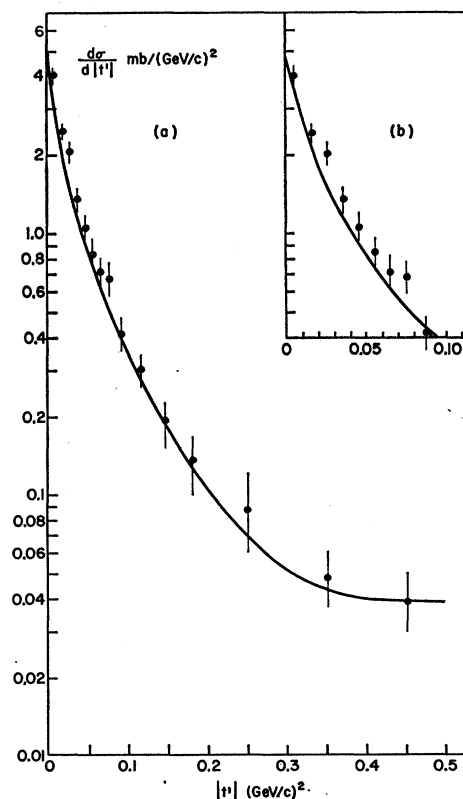


FIG. 4. (a) Differential cross section as a function of $|t'| = |t - t_{\min}|$ for the reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c. The curve is a calculation based on the model of Henyey *et al.* (Ref. 8) (not normalized to the data). (b) Same as (a), but enlarged for the region $0 < |t'| < 0.1$.

⁶ P. G. Federbush, M. T. Grisaru, and M. Tausner, *Ann. Phys.* (N. Y.) **18**, 23 (1962).

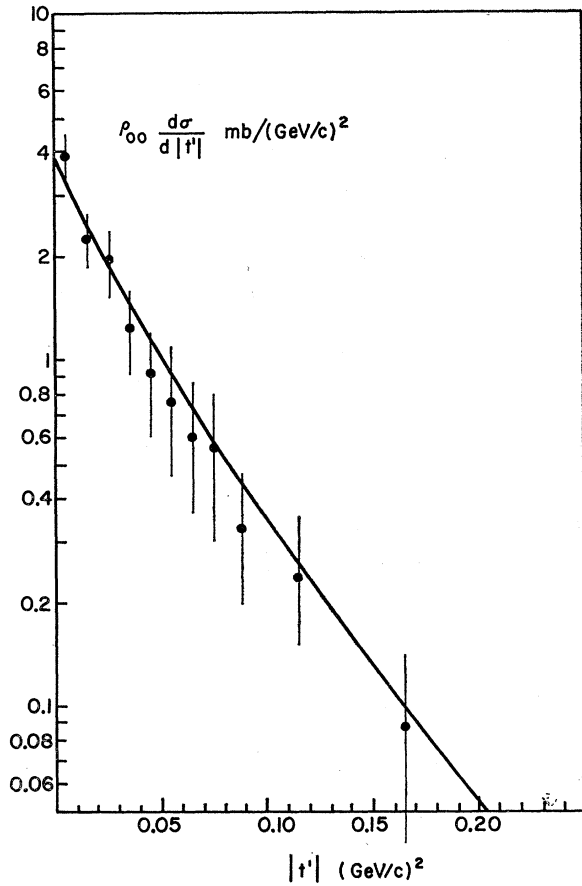


FIG. 5. $\rho_{00} d\sigma/d|t'|$ for the reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ at 13.1 GeV/c. The curve is a calculation from the model of Arbab and Brower (Ref. 11) (not normalized to the data).

it automatically satisfies the pseudethreshold constraints. This term alone is essentially the model used by Haber *et al.*,⁷ and it fits the forward part of the cross section reasonably well, but not the density-matrix elements. Absorption is introduced following Henyey, Kane, Pumplin, and Ross,⁸ whose procedure is similar to that of Gottfried and Jackson,⁹ but simpler to use. The reaction amplitudes with absorption are calculated in the s channel, where $F_{\lambda'\lambda\mu}$ is the amplitude for $\pi N(\lambda) \rightarrow \rho(\mu)\Delta(\lambda')$. The model gives

$$F(t') = F^\pi(t') - \frac{\lambda\sigma}{8\pi} \int_{-\infty}^0 dy' e^{\lambda A(t'+y')} I_n(A(t'y')^{1/2}) F^\pi(y'),$$

where the helicity indices have been suppressed; F^π is the s -channel amplitude corresponding to $f_{\frac{3}{2}\frac{3}{2}0}$ in the t channel, σ and A are parameters from elastic πN

⁷ B. Haber, U. Maor, G. Yekutieli, and E. Gotsman, Phys. Rev. **168**, 1773 (1968).

⁸ F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, Phys. Rev. **182**, 1579 (1969).

⁹ K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1967).

scattering, I_n is the modified Bessel function, and n is the net helicity flip. The factor λ is introduced to represent contributions from processes other than elastic scattering or as a measure of elastic $\rho\Delta$ scattering. Henyey *et al.* find $1.5 < \lambda < 2.0$. The falloff with $|t'|$ of the density-matrix element $\rho_{00} - \rho_{11}$ requires $\lambda = 1.6 \pm 0.2$ as shown in Fig. 2, which is in good agreement with the result of Henyey *et al.* With $\lambda = 1.6$, the values $A = 4$ (GeV/c)⁻², and $\sigma = 24.5$ mb from elastic scattering and the accepted values of $s_0 = 1.0$ (GeV/c)² and $\alpha_{\pi'} = 1.0$ (GeV/c)⁻², there are no free parameters. The curve in Fig. 4 corresponds to these values and yields an 85% confidence level for the fit.¹⁰ The fit is not very sensitive to variations of the parameters within the range

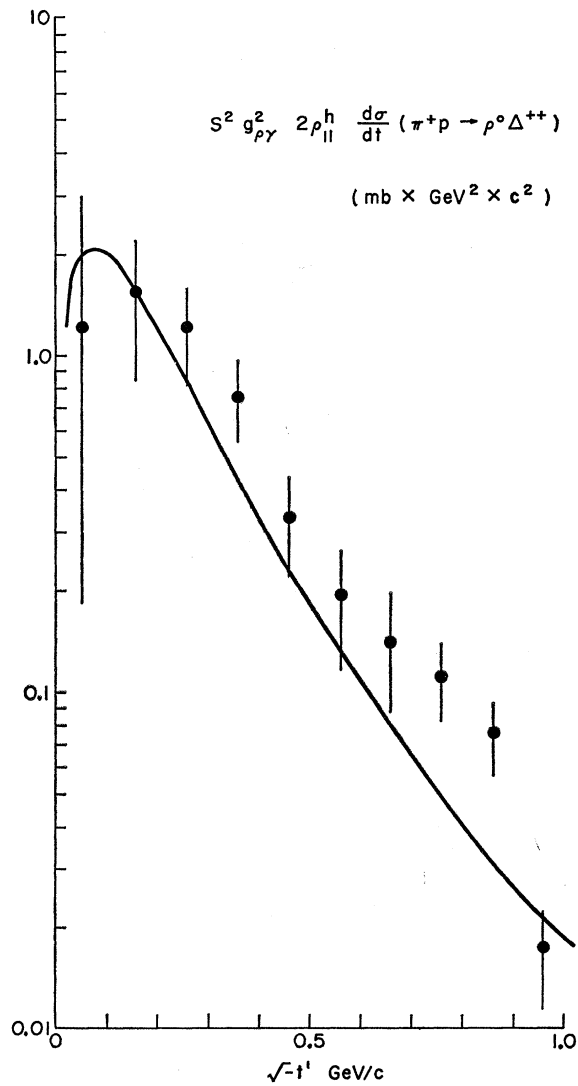


FIG. 6. $s^2 g_{\rho\gamma}^2 2\rho_{11}^h (d\sigma/dt') (\pi^+p \rightarrow \rho^0\Delta^{++})$ at 13.1 GeV/c. The smooth curve is discussed in the text.

¹⁰ A detailed analysis of the charged four-prong final state will be published elsewhere. The errors in the data presented are statistical only, and do not reflect possible over-all systematic errors.

$1.4 < \lambda < 1.8$, $0.6 < s_0 < 1.0$ (GeV/c)², and $0.8 < \alpha' < 1.2$ (GeV/c)⁻².

An alternative consideration is the model of Arbab and Brower,¹¹ with strong interference between the π and A_1 . Although the pion contribution vanishes at $t=0$, $f_{\frac{1}{2} \frac{1}{2} 0}$ is nonzero because of strong A_1 contributions. This model requires a strong decrease of all residues with t in addition to the $(s/s_0)^\alpha$ term. Also, since $f_{\frac{1}{2} \frac{1}{2} 0}$ does not satisfy the pseudethreshold constraint by itself, the two "nonsense" amplitudes $f_{\frac{3}{2} \frac{1}{2} 0}$ and $f_{\frac{1}{2} -\frac{1}{2} 0}$ are needed. The results are interpreted in terms of $\rho_{00} d\sigma/d|t'|$, where a smooth interpolation of the data was used for ρ_{00} . With $s_0=1$ (GeV/c)² and $\alpha_{\pi'}=1.2$ (GeV/c)⁻², this model fits the data reasonably well (Fig. 5).

Both calculations agree with the 8-GeV/c data when evaluated at that energy.¹²

A sum rule derived from vector dominance arguments, which relates the cross section for $\gamma p \rightarrow \pi\Delta$ to the cross section of $\pi p \rightarrow \rho\Delta$, has been proposed recently by Dar¹³; specifically,

$$\frac{d\sigma}{dt}(\gamma p \rightarrow \pi^-\Delta) + \frac{d\sigma}{dt}(\gamma n \rightarrow \pi^+\Delta) \approx g_\gamma^2 \frac{d\sigma}{dt}(\pi^+p \rightarrow \rho_{tr}^0\Delta),$$

where ω and Φ contributions have been ignored. With

$$\frac{d\sigma}{dt}(\pi^+p \rightarrow \rho_{tr}^0\Delta) = 2\rho_{11}^h \frac{d\sigma}{dt}(\pi^+p \rightarrow \rho^0\Delta),$$

where ρ_{11}^h is the density-matrix element in the heli-

¹¹ F. Arbab and R. C. Brower, Phys. Rev. **175**, 1991 (1968).

¹² The coupling constant of Ref. 11 has been reduced by 25% to make their fit agree with the final experimental results at 8 GeV/c. The same value is used in this paper.

¹³ A. Dar, Nucl. Phys. **B11**, 634 (1969); A. Dar and V. F. Weisskopf, CERN Report No. CERN-Th 934, 1968 (unpublished).

city frame and $g_\rho\gamma^2=4.6 \times 10^{-3}$, the distribution of $s^2 g_\rho\gamma^2 2\rho_{11}^h d\sigma/d|t'|$ is shown in Fig. 6.

From lower energy $\pi^+p \rightarrow \rho\Delta, \omega\Delta$ experiments and the data for the process $\gamma p \rightarrow \pi^-\Delta$, comparison with vector-dominance relations is made and the ratio

$$R = \frac{d\sigma}{dt}(\gamma n \rightarrow \pi^+\Delta) \bigg/ \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^-\Delta)$$

is predicted.¹³ The curve shown in Fig. 6 is a drawn interpolation of the cross section $(1+R)s^2(d\sigma/dt)(\gamma p \rightarrow \pi^-\Delta)$ from Dar.¹³ The rather good agreement of the data with the curve may be somewhat fortuitous because of the imprecise definition of the $\rho^0\Delta^{++}$ cross section. However, the slopes agree quite well. The 13.1-GeV/c data are in excellent agreement with the data at 8 GeV/c.

In conclusion, the $|t'|$ distribution of the reaction $\pi^+p \rightarrow \rho^0\Delta^{++}$ exhibits a peak for $|t'| \leq \mu^2$ superimposed upon an exponential decrease, where the exponent is ~ 20 (GeV/c)⁻² for $|t'| \lesssim 0.15$ and ~ 3 (GeV/c)⁻² for $0.15 < |t'| < 1$, the value of ρ_{00} is large ($\gtrsim 0.9$) in the forward direction and decreases to ≈ 0.5 at $|t'| = 0.17$ (GeV/c)². These facts are described well by the model of Henyey *et al.* with conventional pion exchange plus absorption. The pion pole term does not vanish at $t=0$ and is evasive in the sense that no conspiring relation between Regge trajectories is needed. The alternative model of Arbab and Brower with conspiring π and A_1 exchange terms gives an adequate description of $\rho_{00} d\sigma/d|t'|$.

There is good agreement between the relation of transverse ρ production to $\pi\Delta$ photoproduction rates given by vector dominance.

One of us (FTM) would like to acknowledge the contribution of Professor Gordon Kane concerning the interpretation of the absorption model.