setting $g_V^2=1$ and $g_A^2=1.51$, we obtain

$$\frac{d\sigma^{(\nu)}(N_i;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}\simeq\frac{2}{5}\frac{1-Z/N}{1-(Z/2A)\delta}\times\left[2.65Z\left(1-\frac{N}{2A}\delta\right)-5I^{(3)}\right]\times\frac{d\sigma^{(\nu)}(n\to p;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}. \quad (19)$$

Setting $\delta=3.11$ and applying this relation to Al^{27} and Fe^{56}, we find that[13]

$$\frac{d\sigma^{(\nu)}(Al^{27};\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}\simeq1.02\frac{d\sigma^{(\nu)}(n\to p;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0} \quad (20)$$

and

$$\frac{d\sigma^{(\nu)}(Fe^{56};\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}\simeq4.2\frac{d\sigma^{(\nu)}(n\to p;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}. \quad (21)$$

This is in very good agreement with the relation

$$\frac{d\sigma^{(\nu)}(N_i;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}\simeq(N-Z)\frac{d\sigma^{(\nu)}(n\to p;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}, \quad (22)$$

derived by Goulard and Primakoff,[3] but in violent disagreement with the recent experimental value[1]

$$\frac{d\sigma^{(\nu)}(Fe^{56};\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}^{\text{expt}}\simeq30\frac{d\sigma^{(\nu)}(n\to p;\nu,\theta)}{d(\cos\theta)}\bigg|_{\theta=0}. \quad (23)$$

Further experiments are clearly needed to clarify the situation.

Two of the authors (CWK and MR) acknowledge the hospitality of the Summer Institute for Theoretical Physics at the University of Colorado, where some of this work was done.

[13] Direct use of the experimental value of $\Gamma(N_i)$ with $\langle\eta^2\rangle=(0.75)^2$ does not change the value of the cross section for Al^{27} but increases the value for Fe^{56} by 10%.

PHYSICAL REVIEW D VOLUME 1, NUMBER 11 1 JUNE 1970

On-Shell Current Algebra and the Radiative Leptonic Decay of K^+†

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Dispersion relations and the hard-meson method of Schnitzer and Weinberg are used to study the radiative leptonic decay of the K^+. It is shown that in the pole-dominance approximation the relevant form factor in the axial-vector amplitude cannot be unsubtracted. Possible alterations of our results arising from relaxing a smoothness approximation are estimated to be small. We discuss and compare various symmetry-breaking schemes for the evaluation of necessary coupling constants. The branching ratio for $K^+\to\gamma e^+\nu$ is calculated to be $\sim2.5\times10^{-5}$ for interesting structure-dependent decays. This is comparable to that for $K\to e\nu$ and two orders of magnitude larger than one would expect from the usual estimates for electromagnetic decays. The feasibility of experimentally observing the decay is discussed, as are the possible effects of electromagnetic violations of time-reversal invariance. From these results, a soft-pion estimate of F_4, the vector form factor in K_{l4}, yields $|F_4|\approx6.9$.

I. INTRODUCTION

THERE have been several discussions, both experimental[1] and theoretical,[2–6] of the strangeness-conserving decay mode $\pi^+\to\gamma l^+\nu$, where l^+ is a positron or a positive muon. In the present paper we study the decay $K^+\to\gamma l^+\nu$ from the point of view of current-algebra and hard-meson methods. The theoretical situation is much less clear for strangeness-changing decays. There has been some discussion of this K^+ decay mode[7–10] in the literature, with widely different results depending upon the particular symmetry-breaking scheme employed. We shall therefore present a comparison of these results together with those of our own method.

The hard-pion method we use, that of Schnitzer and

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Weinberg,[11] is inherently a low-energy method. As such, it complements the method of dispersion relations which makes specific assumptions as to high-energy behavior according to the number of subtractions assumed. We shall therefore be able to learn some information about the subtractedness of various amplitudes. In an appendix we study a possible source of error in this information: that the smoothness assumption in the Schnitzer-Weinberg method is too restrictive. We make a reasonable estimate of the expected corrections arising from relaxing this assumption and conclude that their effects are small.

The calculated branching ratio for $K \to \gamma e\nu$ comes out $\approx 2.5\times10^{-5}$ for interesting structure-dependent decays. This is comparable to the branching ratio (BR) for the nonradiative decay $K\to e\nu$ ($\text{BR}\approx 1.5\times10^{-5}$) and many times larger than one would expect from the usual estimates for electromagnetic decays. Apparently this fact has not been recognized previously.[12] Although there is an enormous $K^+\to\pi^0 e\nu$ background, we discuss the feasibility of detecting the radiative decay and conclude that it is possible but difficult with existing apparatus.

Since the final state contains a photon and a charged particle with spin, the decay is a candidate for observing the effects of a time-reversal-noninvariant interaction. At present, this point is highly academic because of the small branching ratio. Nevertheless, we briefly discuss final-state correlations that would indicate a time-reversal-violating electromagnetic interaction.

Finally, we add a comment concerning the vector form factor F_4 in K_{l4} decay.

II. GENERAL FORMALISM

The amplitude for the decay $K^+(P)\to\gamma(k)+l^+(p)+\nu(q)$ may be written[2]

$$\mathfrak{M}=\mathfrak{M}_{\text{LB}}+ie(G/\sqrt{2})\sin\theta\,\epsilon^\mu\mathfrak{M}_{\mu\nu}l^\nu, \qquad (2.1)$$

where

$$\mathfrak{M}_{\mu\nu}=\int dx\, e^{ikx}\langle 0|T\{j_\mu(x),\, V_\nu^{4-i5}(0)-A_\nu^{4-i5}(0)\}|K^+(P)\rangle \qquad (2.2)$$

and

$$\mathfrak{M}_{\text{LB}}=ie(G/\sqrt{2})\sin\theta(\sqrt{2}F_K)\times\bar{u}_\nu(1+\gamma_5)\frac{\boldsymbol{P}(2\epsilon\cdot p+\boldsymbol{k}\gamma\cdot\epsilon)}{2k\cdot p}v_l \qquad (2.3)$$

is the amplitude for "lepton bremsstrahlung," i.e., the process in which the photon is emitted from the virtual final charged lepton. In these equations, G is the Fermi constant, θ is the Cabibbo angle, ϵ^μ is the photon polarization vector,

$$l^\lambda=\bar{u}_\nu\gamma^\lambda(1-\gamma_5)v_l \qquad (2.4)$$

is the lepton weak current,

$$j_\mu(x)=V_\mu{}^3(x)+(1/\sqrt{3})V_\mu{}^8(x) \qquad (2.5)$$

is the hadron electromagnetic current, $\sqrt{2}F_K$ is the charged-kaon decay amplitude, and $V_\nu{}^{4-i5}$ and $A_\nu{}^{4-i5}$ are the strangeness-changing weak hadronic vector and axial-vector currents, assumed to obey the usual commutation relations.

The vector form factor v_K and the axial-vector form factor a_K are defined in terms of the vector and axial-vector parts of $\mathfrak{M}_{\mu\nu}$:

$$\mathfrak{M}_{\mu\nu}{}^V\equiv\int dx\, e^{ikx}\langle 0|T\{j_\mu(x),V_\nu{}^{4-i5}(0)\}|K^+(P)\rangle = -iv_K\epsilon_{\mu\nu\lambda\sigma}P^\lambda k^\sigma, \qquad (2.6)$$

$$\mathfrak{M}_{\mu\nu}{}^A\equiv\int dx\, e^{ikx}\langle 0|T\{j_\mu(x),A_\nu{}^{4-i5}(0)\}|K^+(P)\rangle =\sqrt{2}F_K\left[g_{\mu\nu}+\frac{P_\mu(P-k)_\nu}{P\cdot k}\right] +a_K[P\cdot k g_{\mu\nu}-P_\mu k_\nu]. \qquad (2.7)$$

The same equations apply for $\pi\to\gamma l\nu$ with the replacement of $\cos\theta$ for $\sin\theta$, F_π for F_K, and the $1-i2$ component of the currents for $4-i5$. We will denote the corresponding π decay form factors by v_π and a_π. All four form factors are real if time-reversal invariance holds. They are functions of $\nu\equiv P\cdot k$ and k^2 in general.

An alternative parametrization of the axial-vector amplitude which is useful for more general discussions is

$$\mathfrak{M}_{\mu\nu}{}^A=\sqrt{2}F_K\frac{(2P-k)_\mu(P-k)_\nu}{2P\cdot k}+P_\mu P_\nu\tilde{H}_1+P_\mu k_\nu\tilde{H}_2 +k_\mu P_\nu\tilde{H}_3+k_\mu k_\nu\tilde{H}_4-g_{\mu\nu}\tilde{H}_5. \qquad (2.8)$$

The first term exhibits the Born contribution, and the remaining invariants $\tilde{H}_i$ are functions of ν and k^2 and are free of poles at $\nu=0$. For the physical process ($k^2=0$, $\epsilon\cdot k=0$, electromagnetic current conserved), $a_K(\nu)=-\tilde{H}_2(k^2=0,\nu)$.

Decays which take place by lepton bremsstrahlung or by the process in which the photon is emitted from the parent meson [the first term in square brackets in Eq. (2.7)] are called inner bremsstrahlung (IB). Other decays are called structure dependent (SD) and are determined by the form factors v_K and a_K.

We will calculate the vector form factor in Sec. III using dispersion relations, and the axial-vector form factor in Sec. IV using both dispersion relations and the hard-meson method of Schnitzer and Weinberg.[11]

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[11] H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[12] E. Fischbach and J. Smith [Phys. Rev. **184**, 1645 (1969)] remark that "typically $\Gamma(K\to e\nu\gamma)/\Gamma(K\to\text{all})\approx10^{-8}$."

III. VECTOR FORM FACTOR

To calculate the vector form factor we write, for v_K in Eq. (2.6), an unsubtracted dispersion relation (USDR) in the variable Q^2, where $Q=P-k=p+q$, and saturate the intermediate states with the K^* meson. This gives us

$$v_K=\sqrt{2}g_{K^*}G_{K^*K\gamma}/(M_{K^*}^2-Q^2)\,, \qquad (3.1)$$

where g_{K^*} is the coupling to the vector current

$$\langle 0|V_\mu{}^{4-i5}|K^{*+}(\epsilon,Q)\rangle=\sqrt{2}g_{K^*}\epsilon_\mu \qquad (3.2)$$

and the $K^*K\gamma$ vertex is defined by

$$\langle K^{*+}(\epsilon,Q)|V_\mu{}^{\rm em}|K^+(P)\rangle=G_{K^*K\gamma}\epsilon_{\mu\tau\lambda\sigma}\epsilon^\tau P^\lambda Q^\sigma. \qquad (3.3)$$

The use of an USDR for v_K may partly be justified by comparing with the π decay form factor.

An USDR for v_π gives

$$v_\pi=\sqrt{2}g_\rho G_{\rho\pi\gamma}/(M_\rho{}^2-Q^2) \qquad (3.4)$$

when we saturate the intermediate states with the ρ meson. Numerically,

$$v_\pi(\nu=\tfrac{1}{2}m_\pi{}^2)=(0.035\pm0.0025)M_\pi{}^{-1}. \qquad (3.5)$$

We have used $g_\rho=0.105$ GeV^2 from the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF)[13] relation, $g^2/M_\rho{}^2=2F_\pi{}^2$, and $G_{\rho\pi\gamma}=1.0$ GeV^{-1} from $SU(3)$ symmetry and $\omega\to\pi\gamma$ decay.[14]

Another method of determining v_π is from the conserved vector current (CVC) hypothesis,[15] which gives

$$v_\pi(\nu=\tfrac{1}{2}m_\pi{}^2)=4(2\pi W_0)^{1/2}/(e^2m_\pi{}^{3/2})$$
$$=(0.0322\pm0.0016)m_\pi{}^{-1}, \qquad (3.6)$$

using the latest experimental π^0 decay rate[16] of $W_0=(0.56\pm0.06)^{-1}\times10^{+16}$ sec^{-1}. As these two estimates for v_π are close to each other,[14] we suspect that an USDR for v_K is also equally valid.

Returning to Eq. (3.1), we take g_{K^*} from the first Weinberg sum rule,[17] which reads

$$g_{K^*}{}^2/M_{K^*}{}^2=g_\rho{}^2/M_\rho{}^2 \qquad (3.7)$$

if the contribution of the κ meson is ignored. If the κ meson is included, then the first sum rule is

$$g_{K^*}{}^2/M_{K^*}{}^2+F_\kappa{}^2=g_\rho{}^2/M_\rho{}^2. \qquad (3.8)$$

Equation (3.7) yields

$$|g_{K^*}|=0.15M_{K^*}{}^2. \qquad (3.9)$$

g_{K^*} may also be obtained from K^* dominance in K_{l3} decay. If the form factor $f_+(t)$ is unsubtracted, $f_+(t)=g_{K^*}G_{K^*K\pi}/(M_{K^*}{}^2-t)$, then $f_+(0)=1$ implies

$$|g_{K^*}|=0.156M_{K^*}{}^2. \qquad (3.10)$$

If $f_+(t)$ is once subtracted, $f_+(t)=f_+(0)-tg_{K^*}G_{K^*K\pi}/M_{K^*}{}^2(M_{K^*}{}^2-t)$, then the slope parameter $\lambda_+=0.023$ implies

$$|g_{K^*}|=0.143M_{K^*}{}^2. \qquad (3.11)$$

These two values straddle the Weinberg sum-rule value without a κ meson.

The situation on $f_+(0)$ and F_κ has been summarized by Weinberg.[18] Values of $|F_\kappa/F_\pi|^2$ from 0.0 to 0.34 have been suggested. Even the largest of these decreases $g_{K^*}{}^2$ by only 15% in Eq. (3.8), which makes $|g_{K^*}|$ only slightly smaller than the value obtained from K_{l3} decay. There seems to be no compelling reason to choose any particular value of F_κ/F_π. We can say only that a nonzero F_κ will lower g_{K^*}. In view of its small effect, we ignore the κ meson and use the value for g_{K^*} in Eq. (3.9).

$G_{K^*K\gamma}$ in Eq. (3.1) may be obtained from the $SU(3)$-symmetry relation

$$G_{K^*K\gamma}=\tfrac{1}{3}G_{\omega^0\pi^0\gamma}=0.98\text{ GeV}^{-1}\,, \qquad (3.12)$$

the number coming from the experimental $\omega^0\to\pi^0\gamma$ width. If we wish a broken-symmetry value for $G_{K^*K\gamma}$, the only way seems to be to assume the photon leg is dominated by the vector mesons ρ, ω, and φ, i.e.,

$$G_{K^*K\gamma}=\sum_{V=\rho,\omega,\varphi}G_{K^*KV}G_V/M_V{}^2\,, \qquad (3.13)$$

and we try to break the symmetry in the coupling constants in the numerator. In Eq. (3.13), $G_V=G_{\rho,\omega,\varphi}$ is the coupling of the vector meson to the electromagnetic current,

$$\langle 0|V_\mu{}^{\rm em}|\rho,\omega,\varphi(\epsilon,p)\rangle=G_{\rho,\omega,\varphi}\epsilon_\mu. \qquad (3.14)$$

Sometimes the quantities $g_\rho=G_\rho$, $g_\omega=\sqrt{3}G_\omega$, $g_\varphi=\sqrt{3}G_\varphi$ are more convenient.

Even in this way we only have theoretical methods to discuss symmetry breaking in the g_V. We therefore take G_{K^*KV} from nonet symmetry,

$$G_{K^*K\rho}=-(1/\sqrt{2})G_{K^*K\varphi}=G_{K^*K\omega}=\tfrac{1}{2}G_{\rho^0\pi^0\omega^0}\,, \qquad (3.15)$$

and discuss the g_V.

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[14] We note that in the KSRF relation, $g_\rho=\sqrt{2}F_\pi M_\rho$, which employs PCAC in its derivation, there is always the question of whether F_π should be taken from the experimental $\pi\to\mu\nu$ rate or from the Goldberger-Treiman relation, $\sqrt{2}F_\pi=2M_Ng_AG^{-1}$, which is an equally valid equation within the spirit of PCAC (M_N=nucleon mass; $g_AG^{-1}=1.18$). This latter relation predicts $\sqrt{2}F_\pi=0.8M_\pi$ and would make $v_\pi=0.029M_\pi{}^{-1}$, also near the CVC value, Eq. (3.6). This uncertainty, inherent in all PCAC calculations, means that our results cannot claim to be accurate to better than about 15%.

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[16] G. Belletini, C. Bemporad, P. L. Braccini, C. Bradaschia, L. Foa, K. Lubelsmeyer, and D. Schmitz, Nuovo Cimento **66A**, 243 (1970).

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[18] S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 253.

TABLE I. Values of the ratios of the vector-meson coupling constants g_V and their widths Γ_V for several symmetry-breaking models discussed in the text. The resulting value for $G_{K^*K\gamma}$ is given in the last row.

	Expt	DMO[a]	OS[b]	Present $\sin^2\theta=\frac{1}{3}$	Present $\sin^2\theta=0.402$
g_ω^2/g_ρ^2	0.40	0.43	0.24	0.482	0.557
$\Gamma_\omega/\Gamma_\rho$	0.12	0.14	0.078	0.156	0.181
g_φ^2/g_ρ^2	1.33	1.03	1.43	0.96	0.833
$\Gamma_\varphi/\Gamma_\rho$	0.19	0.15	0.208	0.14	0.12
g_ω^2/g_φ^2	0.30	0.42	0.17	0.502	0.668
$\Gamma_\omega/\Gamma_\varphi$	0.65	0.93	0.37	1.1	1.47
$G_{K^*K\gamma}/\frac{1}{3}G_{\omega\pi\gamma}$	1.23	1.35	1.09	1.40	1.35

[a] See Ref. 20.
[b] See Ref. 21. The numerical values of g_ω^2/g_ρ^2 and g_φ^2/g_ρ^2 are taken from Ref. 10.

The g_V may be obtained from the experimental widths Γ_V for $V\to e^+e^-$. Unfortunately, the experiments are not yet very precise. Experimental numbers[19] (without errors) are listed in the first column of Table I.

Now Das, Mathur, and Okubo[20] (DMO) have derived relations among g_ρ, g_ω, and g_φ using the first Weinberg sum rule and a modification of the second sum rule. This modification is to assume that the integral $\int dm^2\rho_\alpha(m^2)$ of the spectral function $\rho_\alpha(m^2)$ [α is the $SU(3)$ index] is not independent of α, but rather obeys the Gell-Mann–Okubo first-order mass-spitting formula. These authors' values are given in column 2 of Table I.

An alternative method of obtaining g_V has been proposed by Oakes and Sakurai[21] (OS). These authors include symmetry breaking in a different way which amounts to assuming that the integral $\int dm^2\, m^{-4}\rho_\alpha(m^2)$ obeys the mass-splitting formula. Together with the first sum rule, this relation predicts the values in the third column of Table I.

A different relation among the g_V may be obtained from the first sum rule and the quark model. ω-φ mixing[22] gives us

$$g_\omega=\sin\varphi\, g_8,\quad g_\varphi=\cos\varphi\, g_8, \tag{3.16}$$

where g_8 is the coupling of the eighth member of the octet to the eighth component of the current,

$$\langle 0|V_\mu{}^8|\omega_8(\epsilon,p)\rangle=g_8\epsilon_\mu, \tag{3.17}$$

and the $SU(3)$ singlet ω_1 does not couple to the octet of currents.

Taking the ratio of Eqs. (3.16) and using the first sum rule, we obtain

$$\frac{g_\rho^2}{g_\omega^2}=\frac{M_\rho^2}{M_\omega^2}+\frac{1}{\tan^2\varphi}\frac{M_\rho^2}{M_\varphi^2}, \tag{3.18}$$

$$\frac{g_\rho^2}{g_\varphi^2}=\tan^2\varphi\frac{M_\rho^2}{M_\omega^2}+\frac{M_\rho^2}{M_\varphi^2}. \tag{3.19}$$

When the quark-model value $\sin^2\varphi=\frac{1}{3}$ is used, we obtain the results in the penultimate column of Table I. If instead the value $\sin^2\varphi=0.402$, obtained from experimental masses and the first-order mass formula, is used, the last column results.

Looking across Table I, we find considerable variation in the values of the g_V's. In the bottom row of Table I we have displayed the corresponding values of $G_{K^*K\gamma}$ using Eqs. (3.15) and (3.13). These numbers indicate the type of effects which could arise from symmetry breaking. Since nonet symmetry has been used for G_{K^*KV}, not all possible symmetry breaking has been included, but it seems reasonable to use as an estimate of $G_{K^*K\gamma}$ the value

$$G_{K^*K\gamma}=1.2\pm0.2\ \text{GeV}. \tag{3.20}$$

Our final values for v_K (with an uncertainty of 20%) are

$$|v_K(Q^2=0)|=0.12M_K^{-1},\quad |v_K(Q^2=M_K^2)|=0.17M_K^{-1}. \tag{3.21}$$

IV. AXIAL-VECTOR FORM FACTOR

A calculation of the axial-vector form factor can be done by dispersion relations or by the hard-meson method of Schnitzer and Weinberg.[11] A generalization of the work of Ref. 11 to the case of strangeness-changing currents has been carried out,[23,24] and we make the following observations.

(i) Since, for the axial-vector amplitude, the vector current involved is the conserved electromagnetic current, we will not be involved with unknown divergences of strangeness-changing vector currents.

(ii) The Ward identities are relations among the proper vertices Γ_λ, $\Gamma_{\nu\lambda}$, and $\Gamma_{\mu\nu\lambda}$. The values of Γ_λ and $\Gamma_{\nu\lambda}$ are determined by these equations in terms of the divergences of $\Gamma_{\nu\lambda}$ and $\Gamma_{\mu\nu\lambda}$, respectively. $\Gamma_{\mu\nu\lambda}$ itself is chosen by a smoothness hypothesis. Now the σ term, arising from the equal-time commutator $[\partial A^a(x), A_0{}^b(y)]\delta(x_0-y_0)$, occurs only in the equation connecting Γ_λ with the divergence $p^\nu\Gamma_{\nu\lambda}$ [Eq. (4) in Ref. 23]. The equation determining $\Gamma_{\nu\lambda}$ in terms of the divergence $q^\mu\Gamma_{\mu\nu\lambda}$ [Eq. (3) in Ref. 23] does not involve the unknown σ term. As our results are drawn only from this latter equation, *the σ term has no effect on our results*.

(iii) We are discussing only real kaons so that we will remain on the K-meson mass shell. Thus no ap-

[19] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[20] T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 470 (1967).
[21] R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters **19**, 1266 (1967).
[22] For these estimates, for which we do not need or claim great accuracy, we choose not to involve ourselves in the complications arising from two mixing angles in the ω-φ system. See R. J. Oakes and J. J. Sakurai, Ref. 21.
[23] K. C. Gupta and J. S. Vaishya, Phys. Rev. **170**, 1530 (1968).
[24] Y. Ueda, Phys. Rev. **174**, 2082 (1968); **184**, 1966(E) (1969).

proximation is made by replacing the interpolating kaon field $\varphi_{K^+}{}^\dagger$ by $(\sqrt{2}F_K M_K{}^2)^{-1}\partial A^{4-i5}$.

The hard-meson method determines $\mathfrak{M}_{\mu\nu}{}^A$ [Eq. (2.7)]. Comparing Eqs. (2.7) and (2.8), we see that $\tilde{H}_5$ and a_K are related by

$$-\tilde{H}_5=\sqrt{2}F_K+P\cdot k a_K\,, \tag{4.1}$$

and we see that to determine a_K we need only the coefficient of $g_{\mu\nu}$ in $\mathfrak{M}_{\mu\nu}{}^A$. Were we considering $\pi\to\gamma l\nu$, only isovector photons would contribute, but for K decay we have both isoscalar and isovector photons. The isovector part of the coefficient of $g_{\mu\nu}$ in $\mathfrak{M}_{\mu\nu}{}^A$ comes out

$$-\frac{i}{\sqrt{2}}f_{ab3}F_K{}^{-1}\left[-\frac{g_\rho{}^2}{M_\rho{}^2}+\frac{g_{K_A}{}^2}{M_{K_A}{}^2}+\nu\delta\frac{g_{K_A}{}^2}{M_{K_A}{}^2(M_{K_A}{}^2-Q^2)}\right]. \tag{4.2}$$

The isoscalar contribution is

$$-\frac{i}{\sqrt{2}}\frac{1}{\sqrt{3}}f_{ab8}F_K{}^{-1}\left[-\frac{g_\omega{}^2}{M_\omega{}^2}-\frac{g_\varphi{}^2}{M_\varphi{}^2}+\frac{g_{K_A}{}^2}{M_{K_A}{}^2}+\nu\delta\frac{g_{K_A}{}^2}{M_{K_A}{}^2(M_{K_A}{}^2-Q^2)}\right]. \tag{4.3}$$

In these equations, $a=4+i5$, $b=4-i5$, and $f_{ab8}=\sqrt{3}f_{ab3}=-i\sqrt{3}$. The parameter δ is defined in Ref. 11. From the first Weinberg sum rule it is clear that the isovector and isoscalar contributions are the same. Thus we have

$$-\tilde{H}_5=-\sqrt{2}F_K{}^{-1}\left[-\frac{g_\rho{}^2}{M_\rho{}^2}+\frac{g_{K_A}{}^2}{M_{K_A}{}^2}+\nu\delta\frac{g_{K_A}{}^2}{M_{K_A}{}^2(M_{K_A}{}^2-Q^2)}\right]. \tag{4.4}$$

This result may be further simplified by using the other first sum rule

$$g_\rho{}^2/M_\rho{}^2=g_{K_A}{}^2/M_{K_A}{}^2+F_K{}^2\,, \tag{4.5}$$

where g_{K_A} is defined by

$$\langle 0|A_\mu{}^{4-i5}|K_A{}^+(\epsilon,p)\rangle=\sqrt{2}g_{K_A}\epsilon_\mu\,.$$

Using this in Eqs. (4.1) and (4.4), we obtain

$$a_K(Q^2)=-\delta(2/\kappa^2-1)\sqrt{2}F_K/(M_{K_A}{}^2-Q^2)\,. \tag{4.6}$$

We have also used the KSRF relation and introduced

$$\kappa=F_K/F_\pi\,. \tag{4.7}$$

The result, Eq. (4.6), is sensitive to κ. Some theoretical models determining κ have been summarized by Weinberg.[18] Numerically, we have

$$a_K(Q^2)=-\begin{Bmatrix}1.00\\0.46\\0.22\end{Bmatrix}\delta\frac{\sqrt{2}F_K}{M_{K_A}{}^2-Q^2}\quad\text{for }\kappa=\begin{Bmatrix}1.00\\1.17\\1.28\end{Bmatrix} \tag{4.8}$$

or

$$a_K(0)=-\delta\begin{Bmatrix}0.042\\0.022\\0.011\end{Bmatrix}M_K{}^{-1} \tag{4.9}$$

for $M_{K_A}=1260$ MeV. Its value varies by about a factor of 4 as κ changes by 28%.

For comparison, the π decay form factor calculated in the same way comes out[5]

$$\begin{aligned}a_\pi(Q^2)&=-\delta\sqrt{2}F_\pi/(M_{A_1}{}^2-Q^2)\,,\\a_\pi(0)&=-\delta(0.053)M_K{}^{-1}.\end{aligned} \tag{4.10}$$

Now if one writes an USDR for $\tilde{H}_5$ and saturates the intermediate states with the K_A meson, one obtains

$$\tilde{H}_5=\sqrt{2}g_{K_A}G_{K_AK\gamma}/(M_{K_A}{}^2-Q^2)\,, \tag{4.11}$$

where $G_{K_AK\gamma}$ is defined by

$$\begin{aligned}&\langle K_A{}^+(\epsilon,q)|j_\mu|K^+(p)\rangle\\&\quad=iG_{K_AK\gamma}[\epsilon_\mu-\epsilon\cdot p(p+q)_\mu/(p^2-q^2)].\end{aligned} \tag{4.12}$$

The current-conservation equation, Eq. (4.1), determines the numerator of (4.11) and one finds

$$\begin{aligned}a_K(Q^2=M_K{}^2)&=-2\sqrt{2}F_K/(M_{K_A}{}^2-M_K{}^2)\\&=-0.115M_K{}^{-1}.\end{aligned} \tag{4.13}$$

If instead we write a once-subtracted dispersion relation (1SDR) for $\tilde{H}_5$,[9] then we find

$$a_K(Q^2)=-2\sqrt{2}g_{K_A}G_{K_AK\gamma}/(M_{K_A}{}^2-M_K{}^2)(M_{K_A}{}^2-Q^2)\,. \tag{4.14}$$

Sarker[9] has calculated the numerator using a ρ-dominance model and $SU(3)$ symmetry to relate $G_{K_AK\gamma}$ to the D-wave amplitude G_D in $K_A\to K^*\pi$. He then used a value for G_D obtained by Srivastava[25] from a dispersion relation. Sarker's result for a_K is

$$a_K(\nu=0)=\frac{\sqrt{2}F_K}{M_{K_A}{}^2-M_K{}^2}\frac{M_{K^*}}{M_\rho}\frac{1-\epsilon}{\kappa}\times\left[1-\frac{4}{\kappa}\frac{\kappa-(1+\epsilon)}{(1-\epsilon)^2}\right], \tag{4.15}$$

where $\epsilon=(M_{K_A}{}^2-2M_{K^*}{}^2)/M_{K_A}{}^2$. ($\epsilon=0.0$ for $M_{K_A}=1260$ MeV, and $\epsilon=0.1$ for $M_{K_A}=1330$ MeV.)

For $\epsilon=0.0$, this is

$$a_K(\nu=0)=\begin{Bmatrix}1.164\\0.424\\0.114\end{Bmatrix}\frac{\sqrt{2}F_K}{M_{K_A}{}^2-M_K{}^2}. \tag{4.16}$$

This result is to be compared with the value from an USDR, Eq. (4.13), and the hard-meson calculation,

[25] P. P. Srivastava, Phys. Letters 26B, 233 (1968).

Eq. (4.9). ($Q^2=0$ and $\nu=0$ arguments cause a difference of about 15%.) δ is known to lie between $-\frac{1}{2}$ and -1,[11] and we shall use the value $-\frac{3}{4}$. The USDR and 1SDR give results differing in sign and, for $\kappa=1.17$, differing in magnitude by a factor of 5. The hard-meson value agrees in sign, and very closely in magnitude, with the 1SDR value.

The conclusion we wish to draw from this is better substantiated in the case of π decay, where there are no complications due to symmetry breaking. The equations analogous to (4.13) and (4.16) show the same result, only more sharply: A 1SDR for $\tilde{H}_5$ gives a result for a_π in excellent agreement in sign and magnitude with the hard-pion method; an USDR gives a result differing in sign and in magnitude by a factor of 8/3.

Now the hard-meson method takes an approach different from that of dispersion relations. In the former the amplitude is taken to have poles corresponding to the nearby resonances, and a numerator which is assumed to be a low-order polynomial *for small arguments*. In particular, no assumption is made about the asymptotic behavior for large momenta; the large-argument behavior is irrelevant for the Schnitzer-Weinberg formulation—it is a low-energy method. This enables one to calculate a_π and a_K without any assumption about their high-energy behavior.

This is in sharp contrast to the dispersion-relation approach in which a specific assumption is made as to the behavior of a_K or $\tilde{H}_5$ at infinity. We have seen that the assumption that $\tilde{H}_5(Q^2)\to 0$ as $Q^2\to\infty$ (i.e., assuming that it satisfies an USDR) gives us an answer that differs by a factor of $-8/3$ for a_π and -5 for a_K from the results of the hard-meson method in which no such assumption was made. We have also seen that the assumption that $\tilde{H}_5\to \text{const}\neq 0$ as $Q^2\to\infty$ (i.e., assuming that it satisfies a 1SDR) leads to a result in agreement with the hard-meson method. We therefore conclude that, within the pole-dominance approximation, $\tilde{H}_5(Q^2)$ cannot satisfy an USDR.

The only way that this conclusion could be false is if the smoothness assumption in the Schnitzer-Weinberg method were too severe, and that even for low momenta a rapid dependence in the numerator $\Gamma_{\nu\lambda}$ is required. It may be that the linearity assumption for $\Gamma_{\mu\nu\lambda}$ (which is simply related to $\Gamma_{\mu\nu}$) is too stringent, and that higher-order terms must be kept. It is then conceivable that this would affect the determination of δ and therefore of a_π and a_K. But in order for this to invalidate our conclusion, it would have to change the sign of δ and change its magnitude considerably. In the Appendix we investigate this possibility and show that such an occurrence is extremely unlikely.

We also point out, again arguing within the vector-meson-dominance approximation, that if $\tilde{H}_5$ is unsubtracted, then the photon coupling to the π and A_1 or to the K_A and K is different in sign and magnitude from that predicted by ρ-meson dominance. This is seen by combining Eqs. (4.1) and (4.11) at $\nu=0$:

$$G_{K_AK\gamma}=(F_K/g_{K_A})(M_{K_A}{}^2-M_K{}^2) \tag{4.17}$$

or its pion counterpart

$$G_{A_1\pi\gamma}=(F_\pi/g_{A_1})(M_{A_1}{}^2-M_\pi{}^2). \tag{4.18}$$

If g_{K_A} and g_{A_1} are taken from the Weinberg sum rule, then these equations predict photon couplings differing by a factor of about -5 and $-8/3$, respectively, from the ρ-dominance calculations of Sarker, and Riazuddin and Fayyazuddin.[5] Thus, if ρ dominance has any merit, this is another simpler reason that $\tilde{H}_5$ cannot be unsubtracted. Of course this result is contained, but not clearly pointed out, in the work of Refs. 9 and 5.

Several authors have attempted to evaluate v_K and a_K.[9,10] The results for v_K agree within 20%, but the values of a_K differ, as can be seen from the values they give for $\gamma_K\equiv a_K/v_K$. Our results, Eqs. (4.9) and (3.21), with $\kappa=1.17$, imply

$$|\gamma_K|=0.14. \tag{4.19}$$

Sarker[9] chose a larger value of κ and finds

$$|\gamma_K|=0.05 \quad \text{(Sarker)}$$

if $M_{K_A}=1260$ MeV and

$$|\gamma_K|=0.11 \quad \text{(Sarker)}$$

if $M_{K_A}=1330$ MeV. Rockmore[10] also finds two values depending upon the symmetry-breaking scheme used for g_ω and g_φ:

$$|\gamma_K|=0.48 \quad \text{(Rockmore)}$$

in the DMO[20] model and

$$|\gamma_K|=0.58 \quad \text{(Rockmore)}$$

in the OS[21] model.

The large discrepancies in the values of $|\gamma_K|$ point to the differences of opinion as to how best to handle symmetry breaking. Had we chosen different values for δ or κ, $|\gamma_K|$ could be as small as 0.05 or as large as 0.36. All authors agree that $|\gamma_K|\lesssim\frac{1}{2}$. Because of this, the decay rate is not very sensitive to γ_K.

V. DECAY RATE

We proceed to discuss the branching ratio for $K^+\to\gamma e^+\nu$. Relevant equations for the differential decay rates are presented in Ref. 2.

Before calculating the rate itself, let us compare qualitatively the decays of π and K. The IB rate is proportional to the nonradiative decays $\pi\to e\nu$ and $K\to e\nu$. Since $\Gamma(K\to e\nu)/\Gamma(\pi\to e\nu)=(M_K/M\pi)\times(\kappa\tan\theta)^2\simeq 0.27$, the absolute bremsstrahlung rate is slightly smaller in K decay. On the other hand, the SD amplitude contains a (mass)^2 of the parent meson. If, for the moment, we drop energy-dependent polynomials in the SD amplitude which are comparable for π and K decay (they would be equal if $\gamma_\pi=\gamma_K$), then the ratio of SD rates is

$$\frac{\Gamma_{\rm SD}(K\to\gamma e\nu)}{\Gamma_{\rm SD}(\pi\to\gamma e\nu)}=\frac{M_K}{M_\pi}\left(\frac{M_K{}^2}{M_\pi{}^2}\tan\theta\frac{M_K v_K}{M_\pi v_\pi}\right)^2\simeq 680. \quad (5.1)$$

Thus SD decays are much more prominent in K decay than π decay. Similarly, the total rate will also be much greater for K decay.

That the factor $M_K{}^2$ must appear in the SD amplitude can be seen from dimensional reasons alone. In our normalization the amplitude is dimensionless. Since it is proportional to the Fermi constant G, which has dimensions M^{-2}, there must appear an additional (mass)^2. As it is an excellent approximation to drop the lepton mass, this factor must be $M_K{}^2$ since there is no other mass.

We can briefly inquire as to the physical origin of the (mass)^2 factor by studying a simple K^* exchange diagram for v_K such as we used to saturate the dispersion relation. We have a $K^*K\gamma$ vertex with a factor $\epsilon^\mu(eG_{K^*K\gamma}\epsilon_{\mu\nu\lambda\sigma}p^\lambda k^\sigma)$ connected to a K^* propagator with a factor $(g_{\nu\tau}-Q_\nu Q_\tau/M_{K^*}{}^2)/(M_{K^*}{}^2-Q^2)$ connected to a $K^*e\nu$ vertex with a factor $(G/\sqrt{2})\sin\theta\sqrt{2}g_{K^*}l^\tau$. A similar diagram involving ρ exchange holds for π decay with the same factors, except that the subscripts K^* and K are replaced by ρ and π, and $\sin\theta$ is replaced by $\cos\theta$.

Now $G_{K^*K\gamma}$ and $G_{\rho\pi\gamma}$ are comparable, as are the numbers g_{K^*}, g_ρ and M_{K^*}, M_ρ. These numbers do not cause any large difference between π decay and K decay. In the last factor, the lepton momentum l^τ introduces one factor of (M_K/M_π) in the ratio of rates, which, however, is nearly reduced to unity by the factor $(\sin\theta/\cos\theta)$. In the first factor, $p^\lambda k^\sigma$ causes a factor of $(M_K/M_\pi)^2$ which remains. Thus, it is essentially the pseudotensor $K^*K\gamma$ (or $\rho\pi\gamma$) vertex which forces the appearance of the (mass)^2.

Returning now to the calculation of the rate, we are interested in that portion of the Dalitz plot in which the SD decays dominate those occurring via IB or via the interference of the two. As is the case for π decay,[2] the interference term may be neglected. We arbitrarily require that the bremsstrahlung rate be less than 1% of the SD rate. We find this to be true over the region of the Dalitz plot shown in Fig. 1, in which the photon energy ω is greater than 57 MeV and $\omega+E>260$ MeV, where E is the positron energy. It occupies 87% of the plot. Thus, for $K\to\gamma e\nu$, SD decays are much more probable than IB decays. IB decays are down by a factor of α compared to the nonradiative decays, but SD decays are not.

The total SD decay rate for $\omega>57$ MeV, $\omega+E>260$ MeV comes out

$$W_{\rm SD}=R\int dx\,dy(1-x)$$
$$\times[(1+\gamma_K)^2(1-z)^2+(1-\gamma_K)^2(1-y)^2] \quad (5.2)$$

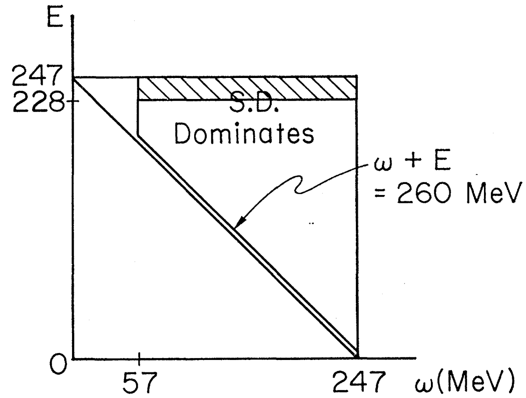

FIG. 1. Dalitz plot for $K^+\to\gamma e^+\nu$. ω is the photon energy and E is the positron energy in MeV. SD dominates IB (IB<1%) in the area indicated ($\omega>57$ MeV, $\omega+E>260$ MeV). The narrow band at top ($E>228$ MeV) is the area in which positrons cannot come from K_{l3}.

$$=(R/60)[0.98(1+\gamma_K)^2+0.75(1-\gamma_K)^2], \quad (5.3)$$

where

$$R=[2M_K/(8\pi)^3](eG\sin\theta\,M_K{}^3v_K)^2\approx 68\,000\ \text{sec}^{-1} \quad (5.4)$$

and x, y, and z are the energies of the photon, positron, and neutrino, respectively, in units of their maxima.

Since it is the positive value of γ_π that agrees with the $\pi\to\gamma e\nu$ experiment,[1] we choose the positive value for γ_K also, $\gamma_K=+0.14$, assuming that symmetry breaking is not bad enough to change the sign. Values of $W_{\rm SD}$ and the branching ratio [BR$=W_{\rm SD}(K\to\gamma e\nu)/\Gamma(K^+\to\text{all})$] are given in Table II for three values of γ_K. As claimed earlier, the rate is not very sensitive to γ_K.

Experimentally the decay will be swamped by $K^+\to\pi^0 e\nu$ (BR$=4.8\times10^{-2}$), so that the easiest way to observe the decay will be to seek positrons more energetic than allowed in K_{l3}. Such decays form only a narrow band at the top of the Dalitz plot, as shown in Fig. 1. The maximum electron energy in K_{l3} is 228 MeV, while that in $K\to\gamma e\nu$ is 247 MeV, so that there are only 19 MeV to play with. Within this band, the decay rate is

$$W=(R/60)[0.321(1+\gamma_K)^2+0.0027(1-\gamma_K)^2],$$

which is 475 sec^{-1} (BR$=5.8\times10^{-6}$) for $\gamma_K=0.14$. As the integrand in Eq. (5.2) shows, when y is near unity, the rate is largest for photons near $\frac{2}{3}$ their maximum energy, or about 165 MeV.

TABLE II. Decay rate and branching ratio (BR) of the decay $K^+\to\gamma e^+\nu$ for several values of γ_K.

γ_K	Rate ($W_{\rm SD}$)	BR
0.05	1900 sec^{-1}	2.3×10^{-5}
0.14	2050 sec^{-1}	2.5×10^{-5}
0.50	2700 sec^{-1}	3.3×10^{-5}

It is quite possible that this decay could be observed as a by-product of an accurate $K\to e\nu$ experiment. Some effort has already been made in this direction by Macek, Mann, McFarlane, and Roberts,[26] who get an upper limit of $|v_K(0)|<0.24M_K^{-1}$ (BR$<7.1\times10^{-5}$).

VI. TIME REVERSAL

In the decay $K\to\gamma\mu\nu$, IB generally dominates over SD decays. Other authors[27] have considered the effects of time-reversal noninvariance in this decay manifest in the interference between IB and SD amplitudes. In the present case we consider the electron decay mode and concentrate our attention on the SD amplitude alone, which is much larger than the IB amplitude.

If the time-reversal violation occurs in the electromagnetic interaction, the form-factor structure of the amplitude will be unaltered except that v_K and a_K may now be complex. In general the correlations $\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{p})$ and $\boldsymbol{\varepsilon}\cdot(\mathbf{k}\times\mathbf{p})$, where $\boldsymbol{\sigma}$ is the positron spin, can occur. However, it is an excellent approximation to neglect the positron mass, and to this extent, the positron has a definite helicity. The $V-A$ interaction aligns the positron along its momentum and so, in fact, no $\boldsymbol{\sigma}\cdot(\mathbf{k}\times\mathbf{p})$ asymmetry can arise, regardless of any imaginary part in the form factors. Thus we must search for photon-polarization asymmetry.

Drop the subscript K or π from v, a, and γ, and define $D=ieG\sin\theta\, M_K{}^3v(1-x)^{1/2}$. Then the amplitude for producing a left-circularly polarized photon is given by

$$A_l=D(1+\gamma)(1-z)\,,$$

and the amplitude for right-circularly polarized photons is

$$A_r=D(1-\gamma)(1-y)\,.$$

This means that the general state of polarization is elliptical with the axis of the ellipse at an angle ψ with respect to the decay plane, with ψ given by

$$\tan2\psi=2\,\mathrm{Im}\gamma/(1-|\gamma|^2)$$

and the ratio of minor axis to major axis given by

$$r=\left|\frac{\beta|1-\gamma|-|1+\gamma|}{\beta|1-\gamma|+|1+\gamma|}\right|,$$

where $\beta=(1-y)/(1-z)$. Thus if time-reversal invariance holds ($\mathrm{Im}\gamma=0$), the major axis must have $\psi=0°$ or 90°. On the other hand, if $\mathrm{Im}\gamma\neq0$, other axes are possible and, in particular, for $\gamma=i$ (maximal violation), there is a region of the Dalitz plot (where $y=z$) for which the photons are plane polarized at 45° to the decay plane. In general, $\mathrm{Im}\,\gamma\neq0$ implies an asymmetry between the number of photons plane polarized at $+45°$ and those at $-45°$.

[26] R. J. Macek, A. K. Mann, W. K. McFarlane, and J. B. Roberts, Phys. Rev. D (to be published).

[27] D. E. Neville, Ref. 7; J. L. Gervais, J. Iliopoulos, J. M. Kaplan, Phys. Letters **20**, 432 (1966); S. W. MacDowell, Phys. Rev. Letters **17**, 1116 (1966); **18**, 227(E) (1967).

VII. VECTOR FORM FACTOR IN K_{l4} DECAY

We would like to add a remark concerning F_4, the vector form factor for K_{l4}. In the soft-pion limit, it can be related to v_K. F_4 is defined by

$$\langle\pi^+(p)\pi^-(q)|V_\lambda{}^{4-i5}(0)|K^+(K)\rangle=(F_4/M_K{}^3)\epsilon_{\mu\nu\lambda\sigma}K^\mu p^\nu q^\sigma\,.$$

If we now contract the two pions, use partial conservation of axial-vector current (PCAC) to introduce the currents, let p_μ and q_μ approach zero, and use the Weinberg identity[28] for the T product of two divergences and a current, we are left with only the term

$$-\tfrac{1}{2}(\partial/\partial y^\nu-\partial/\partial x^\nu)\delta(x_0-y_0)\times\langle0|T\{[A_0{}^{1+i2}(x),A_\nu{}^{1-i2}(y)],V_\lambda{}^{4-i5}(0)\}|K^+\rangle$$

which, upon evaluating the equal-time commutator, reduces to the amplitude for the emission of an isovector photon in $K\to\gamma l\nu$. As the amplitude for emission of a real photon is two-thirds that for an isovector photon in this case,[28a] we arrive at

$$\begin{aligned}|v_K|&=\tfrac{1}{3}(F_\pi{}^2/M_K{}^2)|F_4|M_K{}^{-1}\\&=0.025|F_4|M_K{}^{-1}.\end{aligned}\qquad(7.1)$$

The situation on F_4 is unclear. A simple ρ-dominance model with $SU(3)$ symmetry predicts[29] $|F_4|=1.24$. A better meson-dominance calculation including the K^* gives[30] $0.96\lesssim|F_4|\lesssim1.37$, depending on how symmetry breaking is included. A direct hard-pion current-algebra calculation by Sarker[31] finds $|F_4|=5.07$. Our evaluation of v_K implies $|F_4|=6.9$ according to Eq. (7.1).

Ely *et al.*[32] have presented the most recent experimental results. From an analysis of 269 events they find two solutions consistent with the data: $|F_4|=18.7\pm6.8$ and 10.04 ± 7.78. All calculations of $|F_4|$ lie between 1 and 7 and are significantly smaller than the data, which are not yet very precise.

VIII. SUMMARY

We have calculated the axial-vector form factor in $K\to\gamma l\nu$ by both on-shell current algebra and dispersion relations. Choosing reasonable values of the parameters δ and F_K/F_π, our result, Eq. (4.19), is intermediate between values obtained by other theoretical methods. By observing that we needed no assumption about the large-argument behavior, we were able to conclude that $\tilde{H}_5$ cannot be unsubtracted.

Several symmetry-breaking schemes were discussed and compared in connection with the vector form factor. The κ meson has little effect on the final result. Values

[28] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[28a] We are indebted to Prof. W. Kummer for a communication on this point.

[29] F. A. Berends, A. Donnachie, and G. C. Oades, Nucl. Phys. **B3**, 569 (1967); see also Phys. Rev. **171**, 1457 (1968).

[30] L. E. Wood, Phys. Rev. **181**, 1890 (1969).

[31] A. Q. Sarker, Phys. Rev. **176**, 1971 (1968).

[32] R. P. Ely *et al.*, Phys. Rev. **180**, 1319 (1969).

for g_V, the leptonic decay constant of the vector mesons, consistent with the calculation of DMO, were obtained with the first sum rule and the ideal mixing angle, but these results disagree with those of OS.

The decay rate for $K\to\gamma e\nu$ due to SD processes comes out unexpectedly large. This is not a current-algebra result, but is rather due to the peculiar vertex structure in a simple model and the large mass of the K.

A time-reversal-violating electromagnetic interaction could be detected by studying the polarization of the photons but not that of the positrons.

A soft-pion calculation of F_4 in K_{l4} gives a result somewhat larger than other theoretical calculations, and still much smaller than the present experimental values.

APPENDIX

To demonstrate that the smoothness hypothesis[11] for the proper vertex $\Gamma_{\mu\nu\lambda}$ is a satisfactory assumption, we shall estimate the effects of higher-than-linear terms.[33] Keeping terms of third order, the most general form for this function is

$$\begin{aligned}\Gamma_{\mu\nu\lambda}(q,p)=&a_1p_\mu p_\nu p_\lambda+a_2q_\mu q_\nu q_\lambda+a_3p_\mu p_\nu q_\lambda+a_4p_\mu q_\nu p_\lambda\\&+a_5q_\mu p_\nu p_\lambda+a_6q_\mu q_\nu p_\lambda+a_7q_\mu p_\nu q_\lambda+a_8p_\mu q_\nu q_\lambda\\&+g_{\mu\nu}(p_\lambda B_1+q_\lambda B_2)+g_{\mu\lambda}(p_\nu B_3+q_\nu B_4)\\&+g_{\nu\lambda}(p_\mu B_5+q_\mu B_6)\,,\end{aligned}\quad\text{(A1)}$$

where

$$B_i=b_i+b_i^1p^2+b_i^2q^2+b_i^3k^2\,.\quad\text{(A2)}$$

Crossing symmetry implies

$$\begin{aligned}&a_1=a_2\,,\quad b_1=b_2\,,\quad b_3=b_6\,,\quad b_4=b_5\,,\\&a_3=a_6\,,\quad b_1^1=b_2^2\,,\quad b_3^1=b_6^2\,,\quad b_4^1=b_5^2\,,\\&a_4=a_7\,,\quad b_1^2=b_2^1\,,\quad b_3^2=b_6^1\,,\quad b_4^2=b_5^1\,,\\&a_5=a_8\,,\quad b_1^3=b_2^3\,,\quad b_3^3=b_6^3\,,\quad b_4^3=b_5^3\,.\end{aligned}\quad\text{(A3)}$$

Only 16 of the 32 coefficients are independent. Choose those with odd subscripts to be independent.

The Ward identity satisfied by $\Gamma_{\mu\nu\lambda}$ produces eight homogeneous equations among the a_i and b_i^j, i.e.,

$$\begin{aligned}b_1^1+b_1^2=0\,,&\quad b_1^1+b_1^3=0\,,\\b_3^1+b_5^1+\tfrac{1}{2}(a_1+a_3)=0\,,&\quad b_3^2+b_5^2-\tfrac{1}{2}(a_1+a_3)=0\,,\\b_3^3+b_5^3+\tfrac{1}{2}(a_1-a_3)=0\,,&\quad b_5^2-b_5^1+\tfrac{1}{2}(a_5+a_7)=0\,,\\a_5-a_7=0\,,&\quad b_3^1-b_3^2-\tfrac{1}{2}(a_5+a_7)=0\,,\end{aligned}\quad\text{(A4)}$$

and two inhomogeneous equations among the coefficients b_i which are the same as if no cubic terms were present. Thus one of the b_i and five of the a_i, b_i^j are undetermined. Of the b_i, let us choose b_5 as the undetermined one. Recall $\delta=-2-b_5/b_1$.

We wish corrections to the coefficient of $g_{\nu\lambda}$ in $\Gamma_{\nu\lambda}$. Since $\Gamma_{\nu\lambda}$ comes from $q^\mu\Gamma_{\mu\nu\lambda}$ in the Ward identity, the only corrections arise from the cubic terms in the coefficient of $g_{\nu\lambda}$ in $\Gamma_{\mu\nu\lambda}$ (b_5^j and b_6^j). Let us write

$$\begin{aligned}\Gamma_{\mu\nu\lambda}=g_{\nu\lambda}\bigg[&p_\mu\bigg(b_5+c_5^1\frac{p^2}{M_A^2}+c_5^2\frac{q^2}{M_A^2}+c_5^3\frac{k^2}{M_\rho^2}\bigg)\\&+q_\mu\bigg(b_6+c_6^1\frac{p^2}{M_A^2}+c_6^2\frac{q^2}{M_A^2}+c_6^3\frac{k^2}{M_\rho^2}\bigg)\bigg]+\cdots\,,\end{aligned}\quad\text{(A5)}$$

were we have exhibited masses to make the coefficients c_i^j dimensionally the same as b_i.

Nonzero values of c_i^j would be produced by higher-mass dynamics, for example, a resonance with the same quantum numbers as the ρ or A_1 but of greater mass. To estimate their effect, let us suppose they may be approximated by a resonance with the quantum numbers of the ρ at about twice the ρ mass. This resonance would supply $\Gamma_{\mu\nu\lambda}$ with an extra factor

$$(M^{*2}-k^2)^{-1}\approx\frac{1+k^2/M^{*2}}{M^{*2}}\,,\quad\text{(A6)}$$

where $M^*\approx2M_\rho$ is the resonance mass. This will make c_5^3 (or c_6^3) come out about $\frac{1}{4}$ of b_5 (or b_6). Then when b_5 and b_6 or δ are determined from the $A_1\to\rho\pi$ and $\rho\to\pi\pi$ widths, k^2 is set equal to M_ρ^2, and b_5 and b_6 will be found to change by about 25%. Instead of the limits $1\lesssim\delta+2\lesssim1.5$, as previously determined with $c_i^j=0$, the lower limit 1 will change by $\sim$25% to, let us say, 0.75 or 1.25, and the upper limit 1.5 will change by $\sim$25% to 1.13 or 1.87. Then δ will obey either

$$-1.25\lesssim\delta\lesssim-0.87\quad\text{or}\quad-0.75\lesssim\delta\lesssim-0.13\,.\quad\text{(A7)}$$

These limits are not very different from the original $-1\lesssim\delta\lesssim-0.5$.

In order to make δ change sign and change in magnitude by nearly a factor of 5, several such resonances at not too high an energy would have to conspire to produce a similar change in b_5 and b_6 in Eq. (A5). From the above estimate, we find this possibility very unlikely. Effects of higher-order terms should change δ by, let us say, not more than 30%. We conclude that the original linear approximation to $\Gamma_{\mu\nu\lambda}$ is a safe one for the determination of a_K.

[33] For other attempts at relaxing the smoothness hypothesis see P. Horwitz and P. Roy, Phys. Rev. **180**, 1430 (1969); S. G. Brown and G. B. West, *ibid.* **180**, 1613 (1969).