(i) The forward peak of the nucleon Compton scattering is expected to show shrinking, from an initial dependence e^{-at} shrinking to e^{-2at} eventually (with $a\simeq 4-5$ BeV^{-2}), as the energy increases. The shrinking may begin to take place at $\nu\simeq 4$ or 5 BeV.

(ii) For electroproduction, we conjecture that as Q^2 increases, the νW_2 function can begin to show diffractive features only at increasingly higher energies. The boundary of the "diffraction plateau" (for νW_2) is of the form $\nu=R(Q^2+m^2)$. We venture to guess that it is actually $\nu\simeq 3+5Q^2$.

(iii) The qualitative features of Sakurai's results are more likely to be correct in the diffractive region, i.e., to the right of the line $\nu\simeq 3+5Q^2$ in the $\nu-Q^2$ plane. This is the region where the diffraction model is more likely to be of relevance.

The physical picture we have pursued is a very simple one. But if it can provide a qualitative understanding of the data, its simplicity is its virtue.

Note added in proof. The connection between the energy uncertainty [Eq. (1)] and the longitudinal distance [Eq. (3)] has also been discussed by K. Gottfried, Cornell University report, 1969 (unpublished).

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Unitary Model of Regge Cuts*

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We propose a method for generating Regge cuts based on the multiperipheral model in the approximation of vanishing coupling of the Pomeranchuk trajectory to production processes. We utilize a formalism which explicitly satisfies the full unitarity equations, including inelastic terms, for all connected multiparticle production amplitudes as well as for the two-body amplitude. The sign of the Regge cut coincides with the sign of the cut in the absorptive model, and is opposite to that of the Amati-Fubini-Stanghellini cut. The results are applied to the recent Serpukhov pp data.

SINCE the work of Amati, Fubini, and Stanghellini (AFS)[1] appeared in 1962, it has been accepted that Regge behavior and the bilinear character of unitarity implies the existence of Regge cuts. Among the models that evaluate the cut contribution to the two-body scattering amplitude M_{22}, we should mention the absorptive corrections to Regge exchanges,[2,3] the hybrid model,[4] and the eikonal model[5] in which the Regge pole is identified with the first term in the eikonal expansion.

An early attempt to derive an expression for the cut using a detailed model of particle production and elastic unitarity was made by Amati, Cini, and Stanghellini.[6] They used, however, a nonunitary expression for the production amplitude M_{n2}, and unitary corrections to it may strongly modify the cut in M_{22}. For example, Caneschi[7] has shown that an absorptive correction to M_{n2} is sufficient to change the sign of the AFS cut.

With this in mind, we construct a formalism in which unitarity is taken into account for M_{22}, M_{n2}, and M_{nm} ($n, m>2$). To do this, we use a generalization of

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[1] D. Amati, S. Fubini, and A. Stanghellini, Nuovo Cimento **26**, 896 (1962).

[2] (a) G. Cohen-Tannoudji, A. Morel, and N. Navelet, Nuovo Cimento **48A**, 1075 (1967); (b) G. Cohen-Tannoudji, A. Morel, and P. Salin, CERN Report No. TH 1003 (unpublished).

[3] See F. Henyey, G. L. Kane, J. Pumplin, and M. Ross [Phys. Rev. **182**, 1579 (1969)] for a justification and applications of the absorptive-model corrections to Regge-pole exchanges.

[4] C. B. Chiu and J. Finkelstein, Nuovo Cimento **57A**, 649 (1968).

[5] R. Arnold, Phys. Rev. **153**, 1523 (1967); S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968); K. A. Ter-Martirosyan, Institute of Theoretical and Experimental Physics, Moscow, Report No. 681, 1969 (unpublished).

[6] D. Amati, M. Cini, and A. Stanghellini, Nuovo Cimento **30**, 193 (1963).

[7] L. Caneschi, Phys. Rev. Letters **23**, 254 (1969).

the method of Baker and Blankenbecler[8] (hereafter denoted by BB). The only limitation on our amplitudes is that they do not contain disconnected parts. They satisfy the full (elastic and inelastic) unitarity equation exactly, contain no limitation on their dependence on any variables, and have no approximation of interacting only through transitions to two-body intermediate states in spite of the formal appearance of the solution.

The formalism does not determine the S matrix uniquely, because it contains only unitarity, and additional dynamical assumptions have to be introduced. In order to construct a model for high-energy small-momentum-transfer scattering, we use the results of the multiperipheral bootstrap,[9–11] in the approximation of neglecting Pomeranchukon exchange in production processes.[9,12] We then arrive at a simple expression, completely consistent with unitarity, and explicitly exhibiting the cut corrections to Regge-pole exchanges.

We consider the simplified problem in which all external particles are spinless and choose kinematic variables for the amplitude M_{nm} as follows: (i) the total energy squared s; (ii) a set of $3n-7$ variables v_n which depend only on the final momenta; (iii) an analogous set of $3m-7$ initial variables v_m; and (iv) three Euler angles corresponding to the relative orientation of the final and initial momenta in the c.m. frame; these three parameters are denoted by g. We then project the amplitude $M_{nm}(v_n,s,g,v_m)$ on the representations of the rotation group and obtain the generalized partial-wave expansion[13]

$$M_{nm}(v_n,s,g,v_m)=\sum_{J\lambda_n\lambda_m}\frac{(2J+1)}{8\pi^2}H_{nm}{}^{J\lambda_n\lambda_m}(v_n,s,v_m)\times D_{\lambda_n\lambda_m}{}^{J*}(g)\,. \quad (1)$$

Unitarity for the partial-wave amplitudes $H_{nm}{}^{J\lambda_n\lambda_m}(v_n,s,v_m)$ reads

$$\mathrm{Im}H_{nm}{}^{J\lambda_n\lambda_m}(v_n,s,v_m)=\sum_{k=2}^{\infty}\int dv_k\,\rho_k(s,v_k)\sum_{\lambda_k=-J}^{J}H_{nk}{}^{J\lambda_n\lambda_k*}(v_n,s,v_k)\times H_{km}{}^{J\lambda_k\lambda_m}(v_k,s,v_m)\,, \quad (2)$$

where $\rho_k(s,v_k)$ is the k-body density of states. When n, m, or $k=2$, v_2 and λ_2 are absent. In the following, we restrict the use of the indices n, m, and r to be greater than 2, we drop the index J, we include the index λ_k among the variables v_k, and we use the symbol $\int_k$ to denote not only the sum over the number of particles, but also $\int dv_k\sum_{\lambda_k}$ for $k>2$. The unitarity equation (2) has the same form as that of BB, but is exact at all energies and implies no restriction on the dependence of the amplitude on its variables.

The next step in the generalized BB model is to assume knowledge of Born terms B_{22}, B_{n2}, and B_{nm}, which have no right-hand cuts, and to unitarize them with a generalized N/D method in which D is allowed to have left-hand cuts. We proceed in two steps. First, inelastic unitarity (i.e., $n>2$ intermediate states) is imposed, and then the two-body cut is introduced. The first stage is accomplished by solving the set of coupled integral equations

$$\tilde{B}_{ji}=B_{ji}+\int_{r>2}\tilde{B}_{jr}I_rB_{ri}\,, \quad (3)$$

where, at fixed v_k, $I_k(s,v_k)$ is a real analytic function of s such that its imaginary part on the physical s cut is $\rho_k(s,v_k)$. We assume that the integral equations are Fredholm, and approximate the Born term B_{nm} by a sequence of kernels of finite rank

$$B_{nm}(v_n,s,v_m)=\lim_{L\to\infty}\sum_{i=1}^{L}g_n{}^i(s,v_n)g_m{}^i(s,v_m)\,. \quad (4)$$

This is the step where connectedness of the amplitude is assumed.

The solution of Eq. (3) for $L=1$ is

$$\tilde{B}_{ji}=B_{ji}+\int_{r>2}B_{jr}I_rB_{ri}\Big/\left(1-\int_{r>2}I_rB_{rr}\right). \quad (5)$$

The form of Eq. (5) is somewhat more complicated for rank $L>1$. We assume that $B_{ij}=B_{ji}$; hence, $\tilde{B}_{ij}=\tilde{B}_{ji}$. In addition, $\tilde{B}_{ji}$ satisfies inelastic unitarity, i.e.,

$$\mathrm{Im}\tilde{B}_{ji}=\int_{r>2}\tilde{B}_{jr}{}^*\rho_r\tilde{B}_{ri}\,. \quad (6)$$

We proceed now with the second step in the unitarization by writing

$$H_{ij}=\tilde{B}_{ij}+H_{i2}I_2\tilde{B}_{2j}\,, \quad (7)$$

which can easily be inverted:

$$H_{22}=\tilde{B}_{22}/(1-I_2\tilde{B}_{22})\,, \quad (8a)$$

$$H_{n2}=\tilde{B}_{n2}/(1-I_2\tilde{B}_{22})=H_{2n}\,, \quad (8b)$$

$$H_{nm}=\tilde{B}_{nm}+\tilde{B}_{n2}I_2\tilde{B}_{2m}/(1-I_2\tilde{B}_{22})=H_{mn}\,. \quad (8c)$$

[8] M. Baker and R. Blankenbecler, Phys. Rev. 128, 415 (1962); J. W. Dash and A. Pignotti, University of Washington Report No. RLO-579, 1970 (unpublished). See also R. Arnold [Phys. Rev. 136, B1388 (1964)] and H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman [*ibid.* 177, 2458 (1969)] for alternative high-energy models in which the BB method is used.

[9] G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968).

[10] G. F. Chew, M. L. Goldberger, and F. E. Low, Phys. Rev. Letters 22, 208 (1969); I. G. Halliday, Nuovo Cimento 60A, 177 (1969).

[11] I. G. Halliday and L. M. Saunders, Nuovo Cimento 60A, 494 (1969).

[12] G. F. Chew and W. R. Frazer, Phys. Rev. 181, 1914 (1969); L. Caneschi and A. Pignotti, *ibid.* 184, 1915 (1969). The approximation in our model is that the internal Pomeranchukon coupling in production amplitudes vanishes, the nonzero Pomeranchukon couplings to external particles being responsible for the cuts.

[13] J. Werle, *Relativistic Theory of Reactions* (Interscience, New York, 1966), Chap. 5.

The form of these equations is valid for any rank L. It can be verified that H_{ji} satisfies full unitarity [Eq. (2)].

At this stage, the freedom of choice of the Born terms corresponds to the lack of crossing symmetry in the model. Instead of attempting to enforce crossing, we construct a high-energy model by appealing to the results of the multiperipheral bootstrap and guessing $\tilde{B}_{ji}$ directly, thus avoiding the choice of B_{ji}. In the multi-Regge bootstrap model, inelastic unitarity generates Regge and multi-Regge behaviors for the absorptive parts of M_{22} [9,10] and M_{n2}.[11] Very likely, a similar statement can be made for M_{nm}. In the reasonable approximation of neglecting the Pomeranchukon contribution to the production amplitudes,[9,12] the Regge cuts in the multi-Regge inelastic sum are due to lower exchanges (e.g., multimeson exchange), and occur around or below zero in the angular momentum plane, but have smaller slopes than the meson pole. We will neglect them at high energy, thus making some error at large momentum transfer. We can then incorporate the results of the multiperipheral bootstrap by identifying $\tilde{B}_{22}$, $\tilde{B}_{n2}$, and $\tilde{B}_{nm}$ with the partial-wave projections of the Regge amplitude, and the $2 \to n$ and $m \to n$ multi-Regge amplitudes. We see that the inclusion of nonzero B_{nm} into the model is necessary for the identification of $\tilde{B}_{n2}$ and $\tilde{B}_{nm}$ with multi-Regge amplitudes, because otherwise $\tilde{B}_{n2}$ would have no right-hand cut, and $\tilde{B}_{nm}$ would be zero.

Equations (8) can be interpreted at high energy as providing elastic unitarity corrections to the Regge and multi-Regge amplitudes, which already do contain inelastic unitarity. An expression of the type of Eq. (8a) has been recently proposed by Lovelace[14] as an elastic unitarization of the Veneziano model.

Our dynamical assumption is to equate the physical amplitudes to the pure Regge amplitudes in the absence of two-body intermediate states, formally achieved by setting $I_2=0$. Equations (8) now provide expressions for the fully unitarized amplitudes, but there is still an ambiguity in the choice of the real part of I_2. We can eliminate this ambiguity by requiring that the corrections to the model originating from the denominators in Eqs. (8) introduce only Regge cuts of well-defined signature. This imposes a choice of I_2 which is purely imaginary at high energy. Therefore we set

$$I_2(s)=i\rho(s)=m[(4m^2-s)/s]^{1/2}.$$

Equation (8a) with the above choices for $\tilde{B}_{22}$ and I_2 has recently been proposed by Cohen-Tannoudji *et al.*[2b] We have shown here how it arises in connection with a detailed model of particle production, and we have, in addition, simultaneously generated a prescription for unitarity corrections to multiperipheral amplitudes.

We can obtain insight into the cut structure by comparing the expansions of $\mathrm{Im}H_{22}$ to order P^2 from Eq. (8a) and from unitarity. Here P denotes the high-energy limit of $\tilde{B}_{22}$, i.e., a pure Regge-pole term. The elastic (AFS) contribution is $\rho|P|^2$, while using Eq. (6), the inelastic sum yields $\mathrm{Im}P-2\rho(\mathrm{Im}P)^2$.

The net result is

$$\mathrm{Im}H_{22}=\mathrm{Im}P+\rho[(\mathrm{Re}P)^2-(\mathrm{Im}P)^2]+O(P^3),$$

and we see that the inelastic contribution has reversed the sign of the dominant part of the AFS cut, in agree-

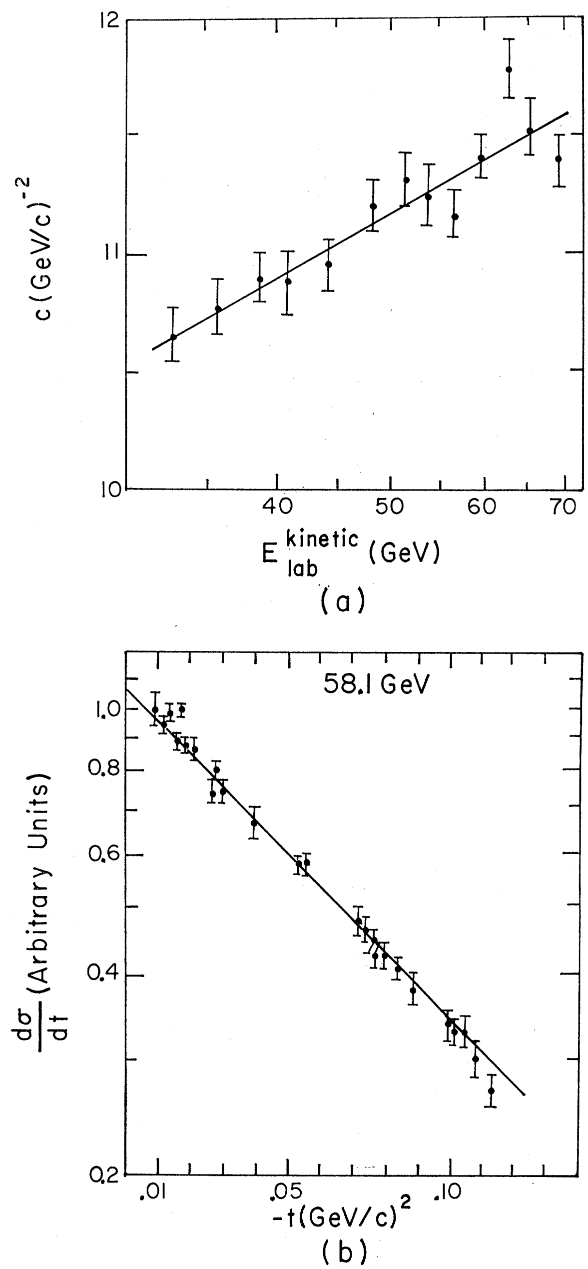

FIG. 1. (a) Fit to values of c from Ref. 15. (b) Differential cross section obtained from the above fit and comparison with the experimental values of Ref. 15 at 58.1 GeV.

[14] C. Lovelace, CERN Report No. TH 1041 (unpublished); Nucl. Phys. **B12**, 253 (1969).

ment with absorptive model results[2,3,5] and the calculations of Caneschi.[7] Therefore, in our model, absorptive corrections are interpretable as originating from the denominators in Eqs. (8).

The cut in $\mathrm{Im}H_{22}$ is, to all orders,

$$\mathrm{Im}H_{22}|_{\mathrm{cut}}=\mathrm{Im}\left(\frac{i\rho P^2}{1-i\rho P}\right),$$

and again its sign coincides with that of the absorptive model.

The formalism can easily be extended to include internal quantum numbers. For example we can treat $\pi\pi$ scattering by including several coupled two-body channels. Off-diagonal meson-pole exchanges again originate from inelastic unitarity and are corrected by two-body unitarity, which yields meson-Pomeranchukon cuts in leading order.

We illustrate the model by fitting both the pp Serpukhov data[15] and the 22–26-GeV total cross sections.[16] We approximate the partial-wave expansion by the Fourier-Bessel expansion, neglect spin dependence, and parametrize the Pomeranchuk pole by

$$M_P(s,t)=-\alpha'\gamma s_0[(s/s_0)e^{-i\pi/2}]^{1+\alpha' t},$$

which has the Bessel transform

$$P(s,b)=(i\gamma s/4\mu p^2)\exp(-b^2/4\alpha'\mu),$$

where

$$\mu=\ln(s/s_0)-\tfrac{1}{2}i\pi \quad\text{and}\quad p=\tfrac{1}{2}(s-4m^2)^{1/2}.$$

Equation (8a) then reads

$$H_{22}(s,b)=\frac{P(s,b)}{1-i\rho(s)P(s,b)},$$

and the full amplitude is

$$M_{22}(s,t)=2p^2\int_0^\infty J_0(b(-t)^{1/2})H_{22}(s,b)bdb.$$

The differential and total cross sections are given by

$$\frac{d\sigma}{dt}=\frac{4\pi m^2}{sp^2}|M_{22}(s,t)|^2$$

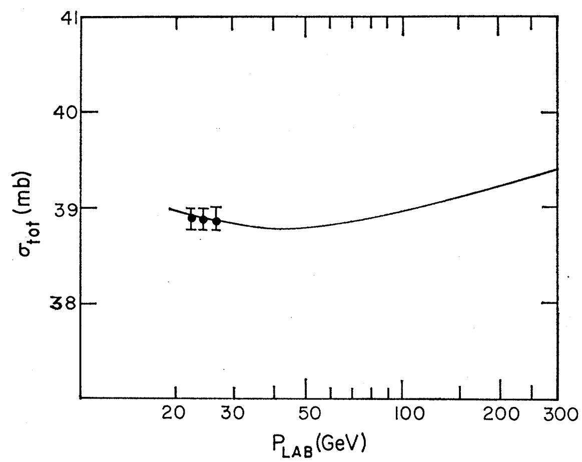

FIG. 2. Fit to the values of the total cross sections from Ref. 16 and predictions at higher energies.

and

$$\sigma^{\mathrm{tot}}=\frac{8\pi m}{ps^{1/2}}\mathrm{Im}M_{22}(s,0)=16\pi\alpha'\,\mathrm{Im}\left[i\mu\ln\left(1+\frac{\gamma m s^{1/2}}{2p\mu}\right)\right].$$

Most of the Serpukhov results[15] are presented in the form of a plot of c versus lab energy up to 70 GeV, where c is obtained by fitting $d\sigma/dt$ by $A\,e^{ct}$ over the range $0.008\leqslant|t|\leqslant0.12$ GeV^2. The fit corresponding to the parameters $\gamma=4.04$/GeV, $\alpha'=0.60$/GeV^2, and $s_0=0.025$ GeV^2 is shown in Fig. 1(a), and Fig. 1(b) shows the comparison of the differential cross section at 58.1 GeV with the experimental values.[17] The fit to the total cross section and predictions up to 300 GeV are shown in Fig. 2. The cut contributions modify the shape of the large-momentum-transfer cross sections, and are capable of producing kinks. We have obtained qualitative fits of high-t pp data at lower energies, but feel that their significance is questionable owing to additional inelastic unitarity meson-meson cut corrections, ambiguities in the Regge residues at high t, secondary trajectories, and spin dependence.

We are indebted to N. Bali, R. Blankenbecler, and L. Caneschi for many helpful discussions.

[15] G. G. Beznogikh *et al.* (unpublished).

[16] K. J. Foley *et al.*, Phys. Rev. Letters 19, 857 (1967).

[17] A fit to the Serpukhov data with a different model for cuts is reported in Ref. 15. A somewhat smaller value of α' is obtained, mainly because of the inclusion of lower-energy data.