$r_A=2.2$, $N_A=1.85$ (curve C). Note that the fit to $G_M(t)$ is worsened by using the value $M_\rho=0.76$ GeV rather than $M_\rho=0.71$ GeV (compare Figs. 7 and 9). The results of Figs. 9 and 10 indicate $r_V=2.45\pm0.05$, $r_A=1.9\pm0.2$, again consistent with $r_V=r_A+\frac{1}{2}$.

Finally we note that we can vary $\alpha_V(0)$ in Eq. (C1) without greatly impairing the fits. For example, the parameters $\alpha_V(t)=0.4+t$, $r_V=2.4$ give a reasonable fit, as do the values $\alpha_V(t)=0.6+t/(1.32\ \text{GeV}^2)$ and $r_V=2.4$. Certainly we can easily accommodate the Lovelace intercept[11] $\alpha_V(0)=0.483$ in addition to the intercept $\alpha_V(0)=\frac{1}{2}$ that is used in the text.

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High-Energy Inelastic Neutrino-Nucleon Interactions*

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We discuss high-energy inelastic neutrino-nucleon processes in the light of recent theoretical and experimental developments for the corresponding electroproduction processes. We review the kinematics for the process in a form especially convenient for experimental analysis. We discuss sum rules and results related to current commutation relations. Consequences of the parton model and diffractive models are considered. Other results are: (1) The vector and axial-vector contributions to the total cross section are equal, provided the only symmetry-breaking term in the energy density transforms like a quark-mass term under $U(6)\otimes U(6)$. (2) Scale invariance of one of the three form factors ($\nu\beta$ or $\nu\mathbf{W}_2$) describing the process implies a neutrino total cross section which rises linearly with laboratory energy, provided the lepton current is local and there is no W boson. The effect of a W boson on this result is studied. (3) The relation of existing neutrino data and electroproduction data given by the conserved-vector-current hypothesis is studied and found compatible with experiment.

I. INTRODUCTION

RECENT experiments on inelastic electron-proton scattering[1] have stimulated considerable theoretical interest[2–8] in their interpretation. The purpose of this paper is to study the closely related neutrino-induced inelastic processes and to discuss these interpretations and implications for such experiments.

We first review the kinematics of neutrino-nucleon processes in a hopefully convenient and transparent form for experimental analysis. Sum rules and results related to current commutation relations are discussed, and then we consider the results of the parton model. Finally we discuss a few consequences of the Pomeranchuk-trajectory-exchange model, such as proposed by Harari,[7] and by Abarbanel, Goldberger, and Treiman.[6] Much in this paper has a considerable overlap with published work and we have included it in the interest of clarity and completeness. Contributions specific to this paper include the following:

(a) A kinematical analysis and choice of variables which appear to have special convenience, and which parallel the choice found to be useful in electroproduction experiments. In particular, we show that provided only *one* of the three form factors describing the neutrino process ($\nu\beta$ or $\nu\mathbf{W}_2$) is scale invariant, then the total neutrino cross section rises linearly with laboratory neutrino energy.

(b) If the only term in the energy density which breaks chiral $SU(2)\otimes SU(2)$ symmetry has the transformation properties of a quark-mass term under chiral $U(6)\otimes U(6)$, we can relate the vector and axial-vector contributions to the total neutrino cross section. This is shown to be compatible with experiment.

(c) For the quark version of the parton model, we catalogue several sum rules.

(d) We argue that in the Pomeranchuk-exchange model as defined by Harari, the axial-vector contribution to the neutrino total cross section is probably larger than the vector contribution, in order to fit the data. The contribution of the vector current can be bounded above by the electroproduction data with the use of the conserved-vector-current hypothesis.

II. KINEMATICS

We discuss in some detail the kinematics of inelastic neutrino-proton scattering in order to obtain formulas

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[1] W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 23; E. Bloom *et al.*, Phys. Rev. Letters **23**, 930 (1969); M. Breidenbach *et al.*, *ibid.* **23**, 935 (1969).

[2] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969); SLAC Report No. SLAC-PUB-571, 1969 (unpublished).

[3] R. P. Feynman (unpublished).

[4] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).

[5] S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. Letters **22**, 744 (1969); Phys. Rev. **187**, 2159 (1969).

[6] H. D. Abarbanel, M. L. Goldberger, and S. Treiman, Phys. Rev. Letters **22**, 500 (1969).

[7] H. Harari, Phys. Rev. Letters **22**, 1078 (1969).

[8] J. J. Sakurai, Phys. Rev. Letters **22**, 981 (1969).

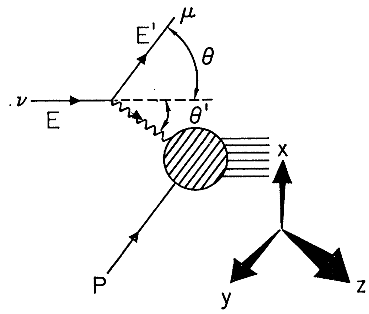

FIG. 1. Inelastic neutrino-nucleon scattering together with the coordinate system used in decomposing the leptonic current.

easily comparable with experiments. Upon neglect of the muon mass, the $V-A$ form of the leptonic current determines the polarization state of the final muon (as well as that of the incident neutrino) and thus defines a pure polarization state for the "virtual W" exchanged between the leptons and hadrons. It is therefore natural, as observed by Lee and Yang,[9] to describe the process in terms of cross sections corresponding to the three helicity states of the virtual W: right-handed (R), left-handed (L), and scalar (S). The formulas we get correspond to those widely used in inelastic electron-proton and μ-proton scattering.

The kinematics of the process is shown in Fig. 1, where

p = four-momentum of neutrino,
p' = four-momentum of muon,
$q=p-p'$ = momentum transformed from leptons to hadrons,
$\nu=E-E'$ = energy transfer, in laboratory frame,
P = four-momentum of target nucleon,
θ = angle of produced muon relative to incident neutrino,
θ' = angle of $\mathbf{q}$ relative to incident neutrino,
$Q^2=-q^2=4EE'\sin^2(\frac{1}{2}\theta)$.

Neglecting the muon mass, we can write the leptonic current as

$$j_\mu^{\text{lept}}=\bar{u}(p')\gamma_\mu(1-\gamma_5)u(p)$$
$$=2\frac{E'p_\mu+Ep_\mu'-g_{\mu 0}p\cdot p'+i\epsilon_{\mu 0\beta\gamma}p'^{\beta}p^{\gamma}}{(EE')^{1/2}\cos\frac{1}{2}\theta}. \quad (2.1)$$

From current conservation, we can eliminate one of the components and expand the current in terms of three orthonormal polarization vectors whose spatial components lie along the axes shown in Fig. 1; the z axis lies along $\mathbf{q}$. This decomposition simplifies considerably in the high-energy limit $\nu\gg 2M\approx 2$ BeV; $Q^2\ll\nu^2$, which is all we consider here. The exact formula is given at the end of this section and discussed in Appendix A. The polarization vectors are, in the high-energy approximation,

$$\epsilon_\mu^S\cong\frac{\nu}{(Q^2)^{1/2}}\left(1,0,0,1-\frac{Q^2}{2\nu^2}\right),$$
$$\epsilon_\mu^R=(\sqrt{\tfrac{1}{2}})(0,1,i,0), \quad (2.2)$$
$$\epsilon_\mu^L=(\sqrt{\tfrac{1}{2}})(0,1,-i,0),$$

while the current, evaluated in the laboratory frame, becomes (up to an over-all phase)

$$j_\mu^{\text{lept}}\approx 4\frac{(EE'Q^2)^{1/2}}{\nu}\left[\epsilon_\mu^S+\left(\frac{E'}{2E}\right)^{1/2}\epsilon_\mu^R+\left(\frac{E}{2E'}\right)^{1/2}\epsilon_\mu^L\right]. \quad (2.3)$$

The polarization vectors satisfy the conditions $\epsilon_S^2=+1$, $\epsilon_{R,L}^2=-1$; $\epsilon_{S,R,L}\cdot q=0$. The only change in (2.3) in going over to antineutrino-induced processes is the interchange $R\leftrightarrow L$.

For the hadronic current operator, we use the Cabibbo current

$$J_\mu(0)\equiv(V_\mu-A_\mu)^{\Delta S=0}\cos\theta_c+(V_\mu-A_\mu)^{|\Delta S|=1}\sin\theta_c. \quad (2.4)$$

The normalization is such that in the quark model

$$J_\mu(0)=\bar{p}'\gamma_\mu(1-\gamma_5)(n'\cos\theta_c+\lambda'\sin\theta_c), \quad (2.5)$$

where p', n', and λ' are the quark field operators. The cross section into a group of final hadronic states $|n\rangle$ is given by

$$\frac{d\sigma^{(n)}}{dQ^2d\nu}=\frac{\pi}{EE'}\frac{d\sigma^{(n)}}{d\Omega dE'}\cong\frac{G^2}{2\pi}\frac{E'}{E}\frac{Q^2}{\nu^2}\times|\langle n|j^{\text{lept}}\cdot J(0)|P\rangle|^2(2\pi)^3\delta^4(P_n-P-q), \quad (2.6)$$

where j^{lept} is the expression in (2.3) within the bracket [ ]. Using the current (2.3), we see the cross section is the sum of three helicity cross sections and three interference terms. Pais and Treiman[10] have made the following general comment: Let Γ be the set of final-state hadron momenta which are measured. (This may include a partial summation over the particle momenta in the states $|n\rangle$.) Let $\Gamma'=R\Gamma$ be the set of momenta obtained by rigid rotation of Γ about $\mathbf{q}$ by angle ϕ (the muon and neutrino momenta are *not* rotated). Then under this rotation the only change in the cross section

[9] T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).

[10] A. Pais and S. B. Treiman, in *Anniversary Volume Dedicated to Nikolai Nikolaievitch Bogoliabov* (Nauka, Moscow, 1969), pp. 257–260.

is to replace $j_\mu{}^{\text{lept}}$ in (2.3) as follows:

$$j_\mu{}^{\text{lept}} \to \frac{4(EE'Q^2)^{1/2}}{\nu}\left[\epsilon_\mu{}^S + \left(\frac{E'}{2E}\right)^{1/2}\epsilon_\mu{}^R e^{i\phi} \times\left(\frac{E}{2E'}\right)^{1/2}\epsilon_\mu{}^L e^{-i\phi}\right]. \quad (2.7)$$

Accordingly, the interference terms between S-R, S-L, and L-R are proportional to $(E/2E')^{1/2}\cos(\phi+\delta)$, $(E/2E')^{1/2}\cos(\phi+\delta')$, and $\cos(2\phi+\delta'')$, respectively. By taking appropriate moments of the data, these interference terms may be isolated. We emphasize that this "azimuthal test" for interference terms can be made for any hadron configuration, even when some particle momenta have been summed out. Likewise, if ϕ is averaged out, or if there is no ϕ dependence, the interference terms cancel. Assuming the ϕ average taken, we get, in the high-energy limit (see Appendix A),

$$\int\frac{d\phi}{2\pi}\frac{d\sigma}{dQ^2d\nu d\Gamma} = \frac{G^2}{2\pi^2}\frac{E'}{E}\frac{Q^2}{\nu}\left(1-\frac{Q^2}{2M\nu}\right) \times\left(\frac{d\sigma_S}{d\Gamma}+\frac{E'}{2E}\frac{d\sigma_R}{d\Gamma}+\frac{E}{2E'}\frac{d\sigma_L}{d\Gamma}\right). \quad (2.8)$$

The $d\sigma_i/d\Gamma$ are the appropriate helicity cross sections for virtual W-nucleon absorption into final phase space $d\Gamma$, defined analogously to the Hand cross sections[11] used in electroproduction. They depend only upon q_μ and hadron variables. Thus, in principle, they can be separately obtained by varying E and E' with q fixed and studying the dependence. This is analogous to the "Rosenbluth straight-line plot" used in electron-scattering experiments.

For cross sections with all hadron states summed over, another notation is convenient and widely used.[12,13] These use invariant form factors α, β, γ (or $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3$) instead of σ_R, σ_L, and σ_S. In electroproduction, it has been found convenient to use a "hybrid" form[14] utilizing one of these form factors, $\mathbf{W}_2$, and using the cross-section ratio $\sigma_T/(\sigma_T+\sigma_S)$ for the other. A similar form is convenient for the neutrino process. We write, at high energies only,

$$\frac{d\sigma}{dQ^2d\nu} = \frac{G^2}{2\pi}\frac{E'}{E}\beta(\nu,Q^2)\left[1+\frac{\nu}{E'}(L)-\frac{\nu}{E}(R)\right], \quad (2.9)$$

where

$$(L)=\sigma_L/(\sigma_R+\sigma_L+2\sigma_S)\leq 1\,, \quad (2.10)$$
$$(R)=\sigma_R/(\sigma_R+\sigma_L+2\sigma_S)\leq 1$$

is a convenient shorthand for the cross-section ratios. The relationship between β, $\mathbf{W}_2$ and the cross sections $\sigma_{R,L,S}$ is, in general,

$$\beta=\mathbf{W}_2=\frac{1}{2\pi}\frac{Q^2}{\nu}\frac{1}{(1+Q^2/\nu^2)}\left(1-\frac{Q^2}{2M\nu}\right) \times(2\sigma_S+\sigma_R+\sigma_L). \quad (2.11)$$

Had no approximation beyond $m_\mu\approx 0$ been made, (2.10) would be replaced by

$$\frac{d\sigma}{dQ^2d\nu} = \frac{G^2}{2\pi}\frac{E'}{E}\beta(\nu,Q^2)\left[1-\frac{Q^2}{4EE'}+\frac{\nu^2+Q^2}{2EE'}(R+L) +\frac{(E+E')(\nu^2+Q^2)^{1/2}}{2EE'}(L-R)\right], \quad (2.12)$$

and the expression (2.3) for lepton current would be replaced by

$$j_\mu{}^{\text{lept}} = \frac{4(EE'Q^2)^{1/2}}{\nu}\left\{\frac{(1-Q^2/4EE')^{1/2}}{(1+Q^2/\nu^2)^{1/2}}\epsilon_\mu{}^S + \frac{1}{2(2EE')^{1/2}}\left(\frac{E+E'}{(1+Q^2/\nu^2)^{1/2}}+\nu\right)\epsilon_\mu{}^L + \frac{1}{2(2EE')^{1/2}}\left(\frac{E+E'}{(1+Q^2/\nu^2)^{1/2}}-\nu\right)\epsilon_\mu{}^R\right\}. \quad (2.13)$$

As $Q^2\to 0$, σ_R and σ_L approach finite quantities, but σ_S diverges as $(Q^2)^{-1}$. The coefficient is proportional to $|\langle n|q_\mu J^\mu(0)|P\rangle|^2$. For $\Delta S=0$ processes, Adler's theorem[15] relates this term to $\pi^\pm$ absorption on nucleons, with the aid of the hypothesis of partially conserved axial-vector current (PCAC). The formula is ($Q^2\lesssim m_\pi{}^2$)

$$\frac{d\sigma_S}{d\Gamma}\approx\frac{F_\pi{}^2}{Q^2}\left(\frac{m_\pi{}^2}{m_\pi{}^2+Q^2}\right)^2\frac{d\sigma_\pi}{d\Gamma}, \quad (2.14)$$

with $F_\pi\approx 0.9m_\pi$, the pion decay constant, and σ_π the appropriate $\pi^\pm$-nucleon cross section.

We close this section with a comment on isotopic spin questions. For $\Delta S=0$ transitions, charge symmetry says that

$$\frac{d\sigma_i}{d\Gamma}(W^\pm p)=\frac{d\sigma_i}{d\Gamma'}(W^\mp n)\,, \quad (2.15)$$

where Γ and Γ' are related by a 180° rotation in isotopic spin space (the charge symmetry operation $e^{i\pi T_2}$). Thus $\sigma(\nu p)-\sigma(\bar\nu n)$ is a measure of $\sigma_L(\nu p)-\sigma_R(\nu p)$, because, under $\nu\leftrightarrow\bar\nu$, $R\leftrightarrow L$ in (2.3) and (2.8). Likewise $\sigma(\nu n)-\sigma(\bar\nu p)$ measures $\sigma_L(\nu n)-\sigma_R(\nu n)$. Therefore neutrino-antineutrino comparisons in D_2 or light nuclei are an excellent way to test for differences in σ_R and σ_L.

[11] L. N. Hand, Phys. Rev. **129**, 1834 (1963).
[12] S. Adler, Phys. Rev. **143**, 1144 (1966).
[13] D. J. Gross and C. H. Llewellyn Smith, Nucl. Phys. **B14**, 337 (1969).
[14] F. Gilman, Phys. Rev. **167**, 1365 (1968).
[15] S. Adler, Phys. Rev. **135**, B963 (1964).

III. SUM RULES

In this section, we catalogue in our notation the sum rules which express integrals over the data in terms of equal-time commutators of currents with each other and their time derivatives. Some of these may be written as follows:

$$\int_0^\infty d\nu\,[\beta(\nu,Q^2)-\beta(\nu,Q^2)]=J_{00}, \qquad (3.1)$$

$$\lim_{Q^2\to\infty}\int_0^\infty d\nu\,[\bar\beta(\nu,Q^2)(\bar R+\bar L) -\beta(\nu,Q^2)(R+L)]=J_{xx}, \qquad (3.2)$$

$$\lim_{Q^2\to\infty}\int_0^\infty d\nu\,[\bar\beta(\nu,Q^2)(\bar L-\bar R)+\beta(L-R)]=iJ_{xy}, \qquad (3.3)$$

where L, R, $\bar L$, and $\bar R$ are defined as in (2.10). The superscript bar refers to antineutrino-induced processes. Altogether there are 12 such sum rules for which it might eventually be practical to test; there are separate sum rules for p and n targets and for $\Delta S=0$ and $|\Delta S|=1$ transitions.

The right-hand sides of these sum rules are equal-time current commutators evaluated as $P_z\to\infty$; in particular,

$$J_{\mu\nu}=\lim_{P_z\to\infty}\int d^3x\,\langle P_z|[J_\mu(\mathbf{x},0),J_\nu^\dagger(0)]|P_\nu\rangle. \qquad (3.4)$$

Equation (3.1) is the Adler[12] (Fubini[16]–Dashen–Gell-Mann[17]) sum rule and depends on a reliable current commutator J_{00}, but not a totally reliable derivation. Equation (3.2) is the "backward" asymptotic sum rule.[18] Equation (3.3) is a sum rule of Gross and Llewellyn Smith.[13] The right-hand sides of the last two sum rules are model dependent. Furthermore, it is not clear, even given the model, that they can be calculated from the "naive" canonical commutation relations of the model. We catalogue in Table I, only as an example, the results for $J_{\mu\nu}$ in the "naive" quark model. We consider these commutators to be postulated, rather than derived, as done by Feynman, Gell-Mann, and Zweig[19] in their formulation of chiral $U(6)\otimes U(6)$. An additional hierarchy of sum rules involves commutators of space components of the current with various time derivatives of the current at infinite momentum. A prototype is that given essentially by Callan and Gross[20] and by Cornwall and Norton[21]:

$$\lim_{Q^2\to\infty}\int_0^1 dx\,[\nu\beta(\nu,Q^2)(R+\bar L)+\nu\beta(\nu,Q^2)(R+L)]=\dot J_{xx}$$
$$=\lim_{P_z\to\infty}\int\frac{d^3x}{P_0}\langle P_z|\left[i\frac{\partial J_x}{\partial t}(\mathbf{x},t),J_x^\dagger(0)\right]|P_z\rangle_{t=0}, \qquad (3.5)$$

where

$$x=Q^2/2M\nu. \qquad (3.6)$$

Notice that for $\Delta S=0$ transitions, $\bar\beta_p=\beta_n$, $\bar R_p=R_n$, etc., so that this integral can be related to the behavior of the sum of νp and νn cross sections.

The properties of commutators such as in (3.5) are a theoretical *terra incognita*. Deductions from Lagrangian models appear to be unreliable. Here we add one more such deduction in a model of commutators suggested by the "naive" quark model and to some extent the model of symmetry breaking of Gell-Mann, Oakes, and Renner.[22] We make the following assumptions. The Hamiltonian may be written as

$$H=H_R(t)+H_L(t)+H'(t),$$

with

$$\text{(a)}\quad [V_\mu(0)-A_\mu(0),H_R(0)]=0, \qquad (3.7)$$

$$\text{(b)}\quad [V_\mu(0)+A_\mu(0),H_L(0)]=0, \qquad (3.8)$$

and (c) under chiral $U(6)\otimes U(6)$, H' transforms as $(\mathbf{6},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\mathbf{6})$, i.e., in the same way as a quark-mass term: H' is the term responsible for the breaking of chiral symmetry.

As an example, the "gluon" model satisfies these conditions. From the above assumptions it is possible to prove (formally) the following theorem on "asymptotic chiral symmetry":

Theorem. Under the above assumptions,

$$\lim_{P_z\to\infty}\int\frac{d^3x}{P_0}\langle P_z|\left[\frac{\partial V_i(\mathbf{x},t)}{\partial t}-\frac{\partial A_i(\mathbf{x},t)}{\partial t},V_i^\dagger(0)+A_i^\dagger(0)\right]|P_z\rangle_{t=0}=0. \qquad (3.9)$$

This is shown in Appendix B.

Upon spin average over the nucleon state $|P_z\rangle$, it follows that the $V-A$ cross terms do not contribute to these commutators, and therefore we have the following corollary.

TABLE I. Results for $J_{\mu\nu}$ in the "naive" quark model.

	Proton target		Neutron target	
	$\Delta S=0$	$\|\Delta S\|=1$	$\Delta S=0$	$\|\Delta S\|=1$
J_{00}	$2\cos^2\theta_c$	$4\sin^2\theta_c$	$-2\cos^2\theta_c$	$+2\sin^2\theta_c$
J_{xx}	$2\cos^2\theta_c$	$4\sin^2\theta_c$	$-2\cos^2\theta_c$	$+2\sin^2\theta_c$
iJ_{xy}	$6\cos^2\theta_c$	$4\sin^2\theta_c$	$6\cos^2\theta_c$	$2\sin^2\theta_c$

[16] S. Fubini, Nuovo Cimento **43A**, 161 (1966).
[17] R. Dashen and M. Gell-Mann, Phys. Rev. Letters **17**, 340 (1966).
[18] J. D. Bjorken, Phys. Rev. **163**, 1769 (1967).
[19] R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1969).
[20] C. Callan and D. Gross, Phys. Rev. Letters **22**, 156 (1969).
[21] J. Cornwall and R. Norton, Phys. Rev. **177**, 2584 (1969).
[22] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

Corollary. The vector and axial-vector contributions to

$$\lim_{Q^2\to\infty}\int_0^1 dx\,[\nu\beta(R+L)+\nu\bar{\beta}(\bar{R}+\bar{L})] \tag{3.10}$$

and to

$$\lim_{Q^2\to\infty}\int_0^1 dx\,[\nu\beta(S)+\nu\bar{\beta}(\bar{S})] \tag{3.11}$$

are equal.

It is possible to test this corollary, using the neutrino and electroproduction data. But first we note that "scale invariance," as evidenced in electroproduction data[1] and quite possibly in the existing neutrino data,[23] implies that $\nu\beta$ and $\nu\bar{\beta}$ are nontrivial functions of x for large Q^2. The cross-section ratios R, L, $\bar{R}$, and $\bar{L}$ are also scale invariant, barring pathologies. Such a behavior is clearly compatible with the sum rules (3.1)–(3.3), (3.5), and the corollary (3.10) and (3.11). It also leads to a total neutrino cross section rising linearly with laboratory neutrino energy. We discuss next the total neutrino cross section and obtain bounds for the integral over $\nu\beta$, which then is used in testing the corollary.

Using (2.9) and scale invariance (i.e., $\nu\beta$ a function of x alone), we find

$$\frac{d\sigma}{d\nu}\cong\frac{G^2}{2\pi}\frac{E'}{E}\int_{\sim 0}^{2M\nu}\frac{dQ^2}{\nu}\,\nu\beta(\nu,Q^2)\left[1+\frac{\nu}{E'}(L)-\frac{\nu}{E}(R)\right]$$

$$\cong\frac{G^2M}{\pi}\frac{E'}{E}\left[1+\frac{\nu}{E'}\langle L\rangle-\frac{\nu}{E}\langle R\rangle\right]\times\int_0^1 dx\,\tfrac{1}{2}(\nu\beta_p+\nu\beta_n)\,, \tag{3.12}$$

where $\langle R\rangle$ and $\langle L\rangle$ imply that the appropriate averages over x have been taken. Then the total cross section is

$$\sigma_{\rm tot}=\frac{G^2ME}{\pi}\int_0^1 dx\,\tfrac{1}{2}(\nu\beta_p+\nu\beta_n)\times(\tfrac{1}{2}+\tfrac{1}{2}\langle L\rangle-\tfrac{1}{6}\langle R\rangle)\,. \tag{3.13}$$

The right-hand factor in parentheses lies between 1 and $\frac{1}{3}$. In particular,

$$\begin{aligned}\tfrac{1}{2}+\tfrac{1}{2}\langle L\rangle-\tfrac{1}{6}\langle R\rangle&=1 &&\text{if}\quad \sigma_R=\sigma_S=0\\ &=\tfrac{2}{3} &&\text{if}\quad \sigma_R=\sigma_L,\quad \sigma_S=0\\ &=\tfrac{1}{2} &&\text{if}\quad \sigma_R=\sigma_L=0\\ &=\tfrac{1}{3} &&\text{if}\quad \sigma_L=\sigma_S=0\,.\end{aligned} \tag{3.14}$$

From (3.13) we see that a linear rise in $\sigma_{\rm tot}$ depends only on the assumption that $\nu\beta$ be scale invariant. The neutrino measurements[23] give

$$\sigma_{\rm tot}=(G^2ME/\pi)(0.6\pm0.15)\,, \tag{3.15}$$

and we get

$$0.6\pm0.15\leq\int_0^1 dx\,\tfrac{1}{2}(\nu\beta_p+\nu\beta_n)\leq1.8\pm0.45\,. \tag{3.16}$$

Neglecting $\Delta S\neq0$ transitions, the vector $\Delta S=0$ part of the neutrino cross section can be related via the conserved-vector-current hypothesis to the isovector contribution of the electroproduction data. For $\Delta S=0$ transitions, we have, from an isotopic rotation,

$$\beta_p^{V,\Delta S=0}(\nu,Q^2)+\beta_n^{V,\Delta S=0}(\nu,Q^2)=2[W_{2p}(\nu,Q^2)+W_{2n}(\nu,Q^2)]^{\text{isovector only}}\,, \tag{3.17}$$

where $W_{2p,n}$ are the corresponding electroproduction structure functions. Using the results of the corollary,

$$\int_0^1 dx\,[\nu\beta_p+\nu\beta_n]^{\Delta S=0}=2\int_0^1 dx\,[\nu\beta_p+\nu\beta_n]^{V,\Delta S=0}=4\int_0^1 dx\,[\nu W_{2p}+\nu W_{2n}]^{\text{isovector only}}\leq4\int_0^1 dx\,[\nu W_{2p}+\nu W_{2n}]\,. \tag{3.18}$$

The electroproduction data,[1] with the assumption $\sigma_S\ll\sigma_T$, give

$$\int_0^1 dx\,\nu W_{2p}=0.18\pm0.02\,. \tag{3.19}$$

The inequalities (3.16) and (3.19) read

$$0.6\pm0.15\leq\int_0^1 dx\left(\frac{\nu\beta_p+\nu\beta_n}{2}\right)\leq4\int_0^1 dx\,\nu W_{2p}\left(\frac{W_{2p}+W_{2n}}{2W_{2p}}\right)=0.72\pm0.08\langle(W_{2p}+W_{2n})/2W_{2p}\rangle\,, \tag{3.20}$$

where $\langle\ \rangle$ again implies that the appropriate average over x has been taken. The agreement is satisfactory albeit inconclusive in view of the statistics of the neutrino data, the uncertainties in $\langle R\rangle$ and $\langle L\rangle$, the uncertainties in W_1/W_2 and in W_{2n}/W_{2p}, and the unknown magnitude of the isoscalar contribution in the electroproduction process.

IV. POMERANCHUK EXCHANGE

Abarbarnel, Goldberger, and Treiman,[6] and Harari[7] have argued that the ν dependence of the electroproduction data suggests that the dominant dynamical mechanism for large ν/Q^2 is exchange of the Pomeranchuk trajectory. Harari,[7] by using a duality argument, has suggested that for large Q^2 and all ν only the Pomeranchuk trajectory contributes. The most characteristic

[23] D. H. Perkins, in Proceedings of the Topical Conference on Weak Interactions, CERN, 1969 (unpublished), pp. 1–42; I. Buganov *et al.*, Phys. Letters **30B**, 364 (1969).

predictions of the Pomeranchuk-exchange class of models are the equality of ep and en cross sections, and likewise of νp, νn, $\bar{\nu} p$, and $\bar{\nu} n$ cross sections, both total and differential. In addition, $\nu W_2 \to f(Q^2)$ for large ν at fixed Q^2, and $\langle R\rangle$ and $\langle L\rangle$ likewise tend to constants. The feature of scale invariance, i.e., $f(Q^2) \to$ const, is more difficult to explain in such models. Furthermore, in these models there is no $V-A$ cross term, and consequently $\sigma_R = \sigma_L$. Ignoring $\Delta S \neq 0$ transitions, the vector (as opposed to axial-vector) contribution to the total neutrino cross section can be obtained from electroproduction data, as we did in Eq. (3.18). Taking that result and using the notation $\nu\beta_p = \nu\beta_n = \nu\beta$, we find

$$\int dx\ \nu\beta = \int dx\ (\nu\beta^V + \nu\beta^A) \approx 0.9 \pm 0.2\,, \qquad (4.1)$$

where we have taken $\langle S\rangle = 0$, as suggested by the data.[1,23] We can now estimate the vector contribution to (4.1) and thus obtain a value of the axial-vector part. From the conserved-vector-current argument,

$$\int dx\ \nu\beta^V = 2\int dx\ \nu W_2^{\text{isovector}} \leq 0.36 \pm 0.04\,. \qquad (4.2)$$

An $SU(3)$ or quark-model estimate would give

$$\int dx\ \nu W_{2p}^{\text{isoscalar}} \sim \frac{1}{3}\int dx\ \nu W_{2p}^{\text{isovector}}\,. \qquad (4.3)$$

Thus a "best" estimate for the isovector contribution might be

$$\int dx\ \nu W_{2p}^{\text{isovector}} \approx \frac{3}{4}\int dx\ \nu W_2 \approx 0.13 \pm 0.02\,, \qquad (4.4)$$

giving

$$\int dx\ \nu\beta^V \approx 0.26 \pm 0.04\,. \qquad (4.5)$$

This would imply that the axial-vector contribution is

$$\int dx\ \nu\beta^A \approx 0.64 \pm 0.2\,, \qquad (4.6)$$

indicating that it is *larger* than the vector contribution. Without assuming (4.4), we still obtain the bound

$$\int dx\ \nu\beta^A \geq 0.54 \pm 0.2\,. \qquad (4.7)$$

It is perhaps surprising that the axial-vector contribution should be larger than the vector, owing to the fact that the axial-vector current is mediated by heavier states (e.g., $A1$ versus ρ) than the vector current. However, in the present state of the data and theory, none of this can be considered as very conclusive.

V. PARTON MODEL

In the parton model,[3,4] the scattering process is described in an infinite-momentum frame. In such a frame, we visualize that the proton consists of N pointlike constituents (partons) with probability $P(N)$. The parton longitudinal momentum distribution in this frame is given by $f_N(x)$, where x is the fraction of the proton longitudinal momentum carried by the parton. The physical cross section is obtained by assuming that the lepton scatters incoherently, with the point cross section, from the partons. The point cross section is then averaged over the parton momentum distributions $f_N(x)$ and over the proton configurations N. These ideas are discussed more fully in Refs. 2 and 4. For definiteness, we shall hereafter assume the partons to have spin $\frac{1}{2}$, and in most cases we shall take them to be "point quarks."

We begin by cataloguing the high-energy cross sections for neutrinos and antineutrinos on (point) spin-$\frac{1}{2}$ partons and antipartons. The results are given in Table II. In Table II we have omitted the factors of $\cos^2\theta_c$ or $\sin^2\theta_c$ coming from the Cabibbo structure of the weak current. We have also assumed that the contributing partons have spin $\frac{1}{2}$, isospin $\frac{1}{2}$, and are coupled by $V-A$ to the leptons.

For spin-$\frac{1}{2}$ partons, only σ_L contributes to the neutrino cross section as ν, $Q^2 \to \infty$; i.e., $\sigma_R = \sigma_S = 0$. To see this, we observe that in this limit, it is always possible to find a Breit frame for which the parton is extreme relativistic before and after the collision (Fig. 2). The $V-A$ structure of the weak current guarantees that it be left handed. Therefore the "virtual W" must also be left handed. Furthermore, for the case of backward scattering in the center-of-mass frame, the cross section vanishes unless the incident lepton is left handed. This condition corresponds to $E' \to 0$ (or $\nu \to E$) in the laboratory frame. Therefore, under these circumstances $\bar{\nu}$-parton (and ν-antiparton) scattering vanishes. This same argument reveals why in the general formula (2.9) only the contribution of σ_L survives as $\nu \to E$ for neutrino-induced and σ_R for antineutrino-induced processes.

We now may compute the neutrino cross sections in the parton model. Following the procedure of Refs. 2 and 4, and assuming that each kind of parton has the same distribution $f_N(x)$ of longitudinal momentum xP_μ, we find (see also Ref. 13)

$$\beta(\nu,Q^2)\langle R\rangle = \sum_N P(N) N_{\bar{p}'} \int_0^1 dx\ f_N(x) 2\delta\left(\nu - \frac{Q^2}{2Mx}\right)$$
$$= \frac{2}{\nu}\sum_N P(N) N_{\bar{p}'} x f_N(x)\,, \qquad (5.1)$$

with

$$x = Q^2/2M\nu \qquad (5.2)$$

and $\langle R\rangle = \sigma_R/(\sigma_R + \sigma_L + 2\sigma_S)$ as defined in (2.10).

TABLE II. Results for the parton model.

	$d\sigma/dQ^2d\nu$	Helicity of neutrino	Helicity of recoiling parton	Nonvanishing helicity cross section
ν+parton (isospin down)	$\frac{G^2}{\pi}\delta\left(\nu-\frac{Q^2}{2M}\right)$	L	L	σ_L
$\bar{\nu}$+parton (isospin up)	$\frac{G^2}{\pi}\delta\left(\nu-\frac{Q^2}{2M}\right)\left(1-\frac{\nu}{E}\right)^2$	R	L	σ_L
ν+antiparton (isospin down)	$\frac{G^2}{\pi}\delta\left(\nu-\frac{Q^2}{2M}\right)\left(1-\frac{\nu}{E}\right)^2$	L	R	σ_R
$\bar{\nu}$+antiparton (isospin up)	$\frac{G^2}{\pi}\delta\left(\nu-\frac{Q^2}{2M}\right)$	R	R	σ_R

N is the number of partons (here taken to be quarks—antiquarks) in a given configuration, $N_{\bar{p}'}$ is the number of $\bar{p}'$ antiquarks (or, more generally, isospin-down antipartons) in the same configuration. According to Table II, only $\bar{p}'$ antiquarks contribute to $\beta(R)$. In the same way, we find

$$\nu\beta(R)=2\sum_N P(N)N_{\bar{p}'}xf_N(x)\,,$$
$$\nu\bar{\beta}(\bar{R})=2\sum_N P(N)[N_{\bar{n}'}\cos^2\theta_c+N_{\bar{\lambda}'}\sin^2\theta_c]xf_N(x)\,,$$
$$\nu\beta(L)=2\sum_N P(N)[N_{n'}\cos^2\theta_c+N_{\lambda'}\sin^2\theta_c]xf_N(x)\,,\qquad(5.3)$$
$$\nu\bar{\beta}(\bar{L})=2\sum_N P(N)N_{p'}xf_N(x)\,.$$

The integral over β or $\bar{\beta}$ times the cross-section ratio therefore measures the mean number of the appropriate kind of partons in the nucleon (this integral may well diverge logarithmically):

$$\int_0^\infty d\nu\,\beta(\nu,Q^2)(R)=\int_0^1\frac{dx}{x}(\nu\beta)(R)=2\sum_N P(N)N_{\bar{p}'}\times\int_0^1 dx\,f_N(x)=2\sum_N P(N)N_{\bar{p}'}=2\langle N_{\bar{p}'}\rangle\,.\qquad(5.4)$$

We get the results ($Q^2\to\infty$)

$$\int_0^\infty d\nu\,\beta(\nu,Q^2)(R)=2\langle N_{\bar{p}'}\rangle\,,$$
$$\int_0^\infty d\nu\,\bar{\beta}(\nu,Q^2)(\bar{R})=2\langle N_{\bar{n}'}\cos^2\theta_c+N_{\bar{\lambda}'}\sin^2\theta_c\rangle\,,$$
$$\int_0^\infty d\nu\,\beta(\nu,Q^2)(L)=2\langle N_{n'}\cos^2\theta_c+N_{\lambda'}\sin^2\theta_c\rangle\,,\qquad(5.5)$$
$$\int_0^\infty d\nu\,\bar{\beta}(\nu,Q^2)(\bar{L})=2\langle N_{p'}\rangle\,.$$

The sum rules (3.1)–(3.3) have a simple meaning in this (quark) parton model (remember $\sigma_S=0$). The Adler[12] sum rule (3.1) is

$$\begin{aligned}\int d\nu\,(\bar{\beta}-\beta)&=2\langle N_{p'}+N_{\bar{n}'}\cos^2\theta_c-N_{\bar{p}'}-N_{n'}\cos^2\theta_c\rangle\\&=2(\cos^2\theta_c+2\sin^2\theta_c)\text{ for a proton target}\\&=2(-\cos^2\theta_c+\sin^2\theta_c)\text{ for a neutron target},\end{aligned}\qquad(5.6)$$

in agreement with Table I. Because $\sigma_S=0$, (3.2) is a special case of (3.1). The Gross–Llewellyn Smith[13] sum rule (3.3) becomes ($Q^2\to\infty$)

$$\begin{aligned}\int_0^\infty d\nu\,[\bar{\beta}(\bar{L}-\bar{R})+\beta(L-R)]&=2\langle N_{p'}+N_{n'}\cos^2\theta_c+N_{\lambda'}\sin^2\theta_c-N_{\bar{p}'}-N_{\bar{n}'}\cos^2\theta_c-N_{\bar{\lambda}'}\sin^2\theta_c\rangle\\&=2(3\cos^2\theta_c+2\sin^2\theta_c)\text{ for a proton target}\\&=2(3\cos^2\theta_c+\sin^2\theta_c)\text{ for a neutron target}.\end{aligned}\qquad(5.7)$$

We can obtain another set of sum rules using the stronger assumption that all partons in a configuration have the same distribution of longitudinal fraction $f_N(x)$. It then follows that

$$\int_0^1 dx\,xf_N(x)=\frac{1}{N}\,,\qquad(5.8)$$

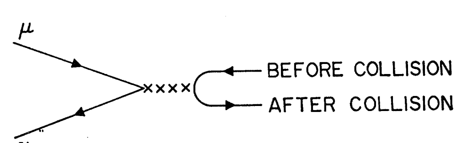

FIG. 2. Breit frame for the lepton-parton collision.

and we find ($Q^2\to\infty$)

$$\int_0^1 dx\ \nu\beta(R)=2\sum_N P(N)N_{\bar{p}'}\int_0^1 dx\ xf_N(x)$$
$$=2\sum_N P(N)(N_{\bar{p}'}/N)=2\langle N_{\bar{p}'}/N\rangle. \quad (5.9)$$

This gives us the results ($Q^2\to\infty$)

$$\int_0^1 dx\ \nu\beta(R)=2\langle N_{\bar{p}'}/N\rangle,$$
$$\int_0^1 dx\ \nu\beta(\bar{R})=2\langle (N_{\bar{n}'}/N)\cos^2\theta_c+(N_{\bar{\lambda}'}/N)\sin^2\theta_c\rangle,$$
$$\int_0^1 dx\ \nu\beta(L)=2\langle (N_{n'}/N)\cos^2\theta_c+(N_{\lambda'}/N)\sin^2\theta_c\rangle, \quad (5.10)$$
$$\int_0^1 dx\ \nu\beta(\bar{L})=2\langle N_{p'}/N\rangle.$$

Using the measurement of the total neutrino cross section (3.15) and assuming scale invariance and $\sigma_S=0$, we have from (3.13)

$$\int_0^1 dx\ \nu\beta[\langle L\rangle+\tfrac{1}{3}\langle R\rangle]=0.6\pm0.15, \quad (5.11)$$

where β is averaged over neutron and proton target nucleons. Therefore, from (5.10),

$$(\cos^2\theta_c)\langle N_{n'}/N\rangle+\tfrac{1}{3}\langle N_{\bar{p}'}/N\rangle+(\sin^2\theta_c)\langle N_{\lambda'}/N\rangle=0.3\pm0.08. \quad (5.12)$$

The average $\langle\ \rangle$ now includes an average over neutron and proton target nucleons, and it implies

$$\langle N_{n'}/N\rangle=\langle N_{p'}/N\rangle=\langle N_{\bar{p}'}/N\rangle+\tfrac{3}{2}\langle 1/N\rangle. \quad (5.13)$$

We can now rewrite (5.12) as

$$(\cos^2\theta_c+\tfrac{1}{3})\langle N_{p'}/N\rangle+(\sin^2\theta_c)\langle N_{\lambda'}/N\rangle-\tfrac{1}{2}\langle 1/N\rangle=0.3\pm0.08, \quad (5.14)$$

and find

$$\langle N_{p'}/N\rangle\approx(0.22\pm0.06)+\tfrac{3}{8}\langle 1/N\rangle, \quad (5.15)$$

a reasonable value when it is compared with the electroproduction data and their interpretation in terms of the (quark) parton model.

A difference in neutrino and antineutrino total cross sections, even when averaged over n and p targets, is characteristic of parton models.[4,5] Using only charge symmetry, some scale invariance, and the high-energy approximation, we have

$$\sigma_{\rm tot}{}^{\nu}-\sigma_{\rm tot}{}^{\bar{\nu}}=\frac{2}{3}\frac{G^2ME}{\pi}\cos^2\theta_c\int_0^1 dx\ \nu\beta(L-R)+(|\Delta S|=1\text{ contribution}). \quad (5.16)$$

In the (quark) parton model, we find from (5.10), (5.13), and (3.15)

$$\sigma_{\rm tot}{}^{\nu}-\sigma_{\rm tot}{}^{\bar{\nu}}=(0.6\pm0.15)\frac{G^2ME}{\pi}\left(1-\frac{\sigma_{\rm tot}{}^{\bar{\nu}}}{\sigma_{\rm tot}{}^{\nu}}\right)\cong 2\frac{G^2ME}{\pi}\langle 1/N\rangle. \quad (5.17)$$

Therefore,

$$\langle 1/N\rangle=(0.3\pm0.08)(1-\sigma_{\rm tot}{}^{\bar{\nu}}/\sigma_{\rm tot}{}^{\nu}). \quad (5.18)$$

Thus the model predicts at least

$$\sigma_{\rm tot}{}^{\nu}(E)\geq\sigma_{\rm tot}{}^{\bar{\nu}}(E). \quad (5.19)$$

For $\langle 1/N\rangle\gtrsim 0.1$, as perhaps suggested by the shape of the electroproduction data for νW_{2p}, we find

$$\sigma_{\rm tot}{}^{\bar{\nu}}(E)\lesssim(0.7\pm0.1)\sigma_{\rm tot}{}^{\nu}(E), \quad (5.20)$$

where, again, the cross sections are averaged over neutron and proton.

One cannot overestimate the crudity of this model. However, what can be emphasized is the richness to be found in the comparison of the various kinds of neutrino-induced processes, both with regard to the internal quantum numbers of target and projectile and the helicity states of the "virtual W" exchanged between lepton and hadron.

VI. EFFECT OF INTERMEDIATE BOSON ON SCALE INVARIANCE

Throughout this paper we have assumed that the intermediate boson does not exist, or if it does, that it is sufficiently heavy so its effects are not observable. As a last topic it is interesting to study how our considerations are modified if a W exists. The basic formulas are only changed by the replacement

$$G^2\to\frac{G^2}{(1+Q^2/m_W{}^2)^2}. \quad (6.1)$$

If scale invariance remains valid, when $s=2ME\gtrsim m_W{}^2$, then the total cross section no longer rises linearly with energy. To estimate the modification we go back to (3.12) and change variables from (Q^2,ν) to (x,y) with

$$x=Q^2/2M\nu \quad\text{and}\quad y=\nu/E. \quad (6.2)$$

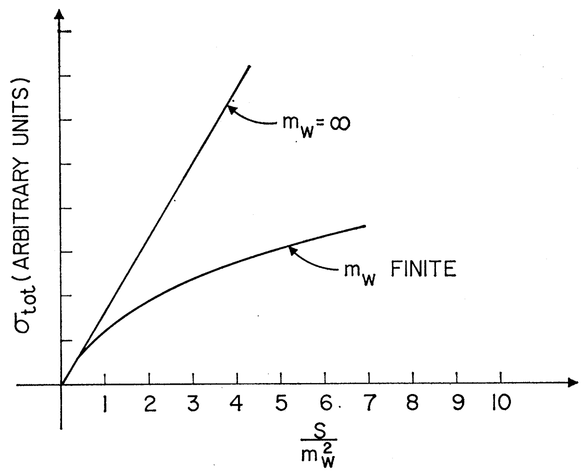

FIG. 3. Deviations of the total neutrino cross section from the linear energy dependence due to the exchange of a massive W boson.

We then obtain

$$\sigma_{tot}^{\nu} = \frac{G^2ME}{\pi}\int_0^1 dxdy\, F(x)\frac{1}{[1+(s/m_W^2)xy]^2} \times[1-y+y\langle L\rangle - y(1-y)\langle R\rangle]. \quad (6.3)$$

For $s/m_W^2 \gg 1$,

$$\sigma_{tot}^{\nu} \approx \frac{G^2 m_W^2}{2\pi}F(0)\ln\left(\frac{s}{m_W^2}\right). \quad (6.4)$$

We chose for $F(x)$ the same functional form as in electroproduction and also $\langle L\rangle = 1$, $\langle R\rangle = 0$ (to simplify the estimation). In Fig. 3 we plot σ_{tot}^{ν} as a function of s/m_W^2.

The most that can be stated is that an observed linear rise in cross section would be evidence *against* the existence of a W with a mass below a certain value. Were the cross section *not* to rise linearly with energy, a breakdown of scale invariance, due to a mechanism other than W exchange, could also be responsible.

VII. CONCLUSIONS

High-energy neutrino-nucleon interactions provide a rich and complementary study to that of "deep inelastic" electroproduction. Some of the subjects which should be practical to study experimentally are the following: (1) the linear rise of total cross section with energy, which is a strong indicator for the scale invariance of Adler's form factor $\nu\beta$; (2) a difference in neutrino-nucleon and antineutrino-nucleon cross sections, which measures $\langle(\sigma_L-\sigma_R)/(\sigma_L+\sigma_R+2\sigma_S)\rangle$, a model-sensitive quantity; and (3) the class of interactions for which $\nu/E\approx 1$ (large energy transfer, low secondary muon energy), which are highly sensitive to the presence of σ_L in neutrino-induced processes and σ_R in antineutrino-induced processes.

The magnitude and energy dependence of the measured neutrino cross section is approximately what might have been expected from the electroproduction data by using the conserved-vector-current hypothesis along with various combinations of auxiliary hypotheses. If anything, it is a little larger ($\lesssim 50\%$) than might have been anticipated. However, theory is in much too crude a condition to allow an incisive comparison.

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APPENDIX A

The steps involved in deriving the exact (to within the $m_\mu \approx 0$ approximation) results of (2.12) and (2.13) are algebraically lengthy and we give here some of the intermediate steps. In replacing the trigonometric functions of θ and θ' by the more convenient variables Q^2, ν, E, and E', we note that

$$\sin^2(\tfrac{1}{2}\theta) = Q^2/4EE', \quad (A1)$$

and the corresponding cosine follows trivially. To obtain $\sin\theta'$ and $\sin(\theta'+\theta)$, we use conservation of momentum in two different directions, *viz.*,

$$\text{perpendicular to } \mathbf{p}: E'\sin\theta = (\nu^2+Q^2)^{1/2}\sin\theta', \quad (A2)$$

$$\text{perpendicular to } \mathbf{q}: E\sin\theta' = E'\sin(\theta'+\theta), \quad (A3)$$

so that

$$\sin\theta' = \frac{Q}{\nu}\left(\frac{E'}{E}\right)^{1/2}\left(\frac{1-Q^2/4EE'}{1+Q^2/\nu^2}\right)^{1/2}. \quad (A4)$$

The components of the leptonic current can be read from (2.1) with the help of (A1)–(A4). Equation (2.1) itself may be obtained by trace techniques[24]:

$$j_0^{\text{lept}} = 4(EE')^{1/2}\frac{(1-Q^2/4EE')}{\cos\frac{1}{2}\theta} = 4(EE')^{1/2}\left(1-\frac{Q^2}{4EE'}\right)^{1/2}, \quad (A5)$$

$$j_x^{\text{lept}} = 2(EE')^{1/2}\frac{(1+E/E')}{\cos\frac{1}{2}\theta}\sin\theta' = 2\frac{Q}{\nu}\frac{E+E'}{(1+Q^2/\nu^2)^{1/2}}, \quad (A6)$$

[24] J. D. Bjorken and M. C. Chen, Phys. Rev. **154**, 1335 (1967).

$$j_y^{\text{lept}}=2i(EE')^{1/2}\frac{\sin\theta}{\cos\frac{1}{2}\theta}=2i\frac{Q}{\nu}\frac{(\nu^2+Q^2)^{1/2}}{(1+Q^2/\nu^2)^{1/2}}=2iQ. \quad \text{(A7)}$$

The z component is obtained from j_0 by using current conservation

$$j_0=j_z(1+Q^2/\nu^2)^{1/2}, \quad \text{(A8)}$$

while the right- and left-hand combinations follow from (A6) and (A7):

$$j_{R,L}^{\text{lept}}=\frac{1}{\sqrt{2}}(j_x\mp ij_y)=\sqrt{2}(Q/\nu)(1+Q^2/\nu^2)^{-1/2}\times[(E+E')\pm(\nu^2+Q^2)^{1/2}]. \quad \text{(A9)}$$

By collecting (A5), (A8), and (A9), Eq. (2.13) follows. The cross section follows by analogy to (2.6):

$$\frac{d\sigma}{dQ^2d\nu d\Gamma}=\frac{G^2}{2\pi}\frac{E'}{E}\frac{Q^2}{\nu^2}\sum{}' \,|\langle n|j_\mu^{\text{lept}}J^\mu(0)|P\rangle|^2\times(2\pi)^3\delta^4(P_n-p-q). \quad \text{(A10)}$$

The summation $\sum'$ is over all final-state variables except for the set of final-state hadron momenta Γ, which are measured. We define the helicity cross sections for the "virtual" W-nucleon absorption into final hadronic space spanning the phase space $d\Gamma$ by

$$\frac{d\sigma^{(i)}}{d\Gamma}=\frac{\pi}{\nu}\frac{1}{1-Q^2/2M\nu}\times\sum{}' \,|\langle n|\,\epsilon_\mu{}^iJ^\mu(0)|P\rangle|^2(2\pi)^2\delta^4(P_n-p-q), \quad \text{(A11)}$$

and by arguments described in Sec. II we can obtain (2.8) and (2.12).

We finally give the relations between the cross sections defined in this paper and the form factors[2,12,13] $\mathbf{W}_1$ and $\mathbf{W}_3$:

$$\mathbf{W}_1=\beta(1+\nu^2/Q^2)[(R)+(L)], \quad \text{(A12)}$$

$$\mathbf{W}_3=\beta(2M/Q)(1+\nu^2/Q^2)^{1/2}[(L)-(R)]. \quad \text{(A13)}$$

APPENDIX B

In proving the theorem, Eq. (3.9), it is sufficient to take the case for which

$$H'=\int d^3x\, H'(x)=\int \bar{q}(x)Mq(x)d^3x,$$

with M a 3×3 "mass matrix" and $q=(p',n',\lambda')$ a quark field operator satisfying canonical commutation relations. This is because all that we shall use is the Lorentz-transformation property of the double commutator in (3.9); this property depends only on the group structure and not the specific representation we use here. Then at $t=0$,

$$[(V_i{}^\alpha-A_i{}^\alpha, H'), V_i{}^\beta+A_i{}^\beta]=q^\dagger\{[\lambda^\alpha\alpha_i(1-\gamma_5), \beta M], \lambda^\beta\alpha_i(1+\gamma_5)\}q=\bar{q}(A+B\gamma_5)q,$$

where α and β are $SU(3)$ labels, and A and B 3×3 $SU(3)$ matrices. Consequently,

$$\langle P|\bar{q}(A+B\gamma_5)q|P\rangle=(M/P_0)\bar{u}(p)(a+b\gamma_5)u(p)\xrightarrow[P_z\to\infty]{} O(1/P_z),$$

and the double commutator (3.9) is $O(1/P_z{}^2)$ as $P_z\to\infty$.