Veneziano Amplitude for πN Scattering*

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A crossing-symmetric Regge-behaved amplitude for πN scattering is constructed with a minimum number of Veneziano terms from ρ, ρ', P', and P'' trajectories in the t channel and N_α, N_γ, and Δ_δ trajectories in the s and u channels. The parameters are determined from low-energy πN resonances, isospin selection rules, and ρ universality. The model fits the high-energy charge-exchange differential cross sections in the forward direction, and also predicts polarization in reasonable agreement with experimental values. The resulting backward elastic differential cross sections are discussed.

I. INTRODUCTION

One of the most attractive features of Veneziano-type amplitudes[1] is the possibility of describing both the high-energy and the low-energy behavior of the scattering with the same set of parameters. In πN scattering, where a wealth of experimental information is available, it is interesting to investigate how well a Veneziano-type representation correlates the high-energy data with low-energy resonance parameters. This provides an interesting test for the model. Although Igi[2] extended the ideas of Veneziano to the case of πN scattering, he made no attempts to correlate the resonance parameters with the high-energy data. The recent work of Fenster and Wali[3] is the first systematic study of this problem. They proposed a representation with satellite terms[4] and made an attempt to correlate the high-energy[5] and low-energy πN scattering with the same set of parameters.

It is a well-known property of Veneziano-type amplitudes that their high-energy limit always leads to Regge residues which choose nonsense. As a consequence of this, the πN Veneziano amplitude leads to a vanishing of the $\pi^- p \to \pi^0 n$ differential cross section when the ρ trajectory goes through a wrong-signature nonsense value. The presence of satellites[5] in the Fenster and Wali[3] representation does not change the situation, since the Regge residues of satellites also choose nonsense. Furthermore, if the ρ trajectory alone dominates the asymptotic behavior of charge-exchange scattering, then the spin-nonflip and spin-flip amplitudes contribute with the same phase, which results in zero polarization. At this stage one can take the viewpoint that the so-called background terms also contribute, so that the amplitudes do not completely vanish when the trajectories go through a wrong-signature nonsense point, and one automatically has a mechanism for producing nonzero polarization. However, in this type of analysis one finds too big a polarization for $\pi^- p \to \pi^0 n$ which, in addition, is strongly energy dependent.[6]

The answer to this problem may well lie in Regge branch points. However, no one has yet shown how to introduce cuts which preserve duality or even crossing symmetry. If cuts are introduced through the absorption model as was done by Lovelace,[7] they violate the Freund-Harari[8] conjecture, which states that the Pomeranchuk trajectory is independent of the resonances. It was known before Veneziano that a model with a ρ trajectory and a second trajectory half a unit below, called the ρ' trajectory,[9–12] prevents the vanishing of the charge-exchange differential cross section when $\alpha_\rho(t)$ goes through zero. Furthermore, the $\rho+\rho'$ model provides an explanation for the nonzero polarization for $\pi^- p \to \pi^0 n$. Recently, Barger and Phillips[13] showed that a consistent solution to the πN finite-energy sum rules is obtained when they introduce a set of degenerate ρ-P' trajectories and degenerate ρ'-P'' trajectories nearly half a unit below the ρ trajectory. It should be noted that exchange degeneracy demands a P'' trajectory degenerate with the ρ' trajectory. Furthermore, Ahmadzadeh and Kauffmann[14] recently studied a model based on exchange degeneracy, $SU(3)$ symmetry, and secondary meson trajectories. They found such a model in good agreement with experiments. Logan *et al.* presented an interesting review of the independent evidence for the ρ' trajectory. The most significant points

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[2] K. Igi, Phys. Letters **28B**, 330 (1968).

[3] S. Fenster and K. C. Wali, Phys. Rev. D (to be published).

[4] We refer to those terms in Ref. 3 as satellites which (a) contain one nonleading Regge pole [example $C_x^-(\frac{3}{2},2)$ in (2.6)] or (b) dominate the s asymptotic behavior of A^- and B^- for fixed t but do not contain a ρ pole [example $C_x^-(\frac{3}{2},2)$ in (2.5)].

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[9] H. Högaasen and A. Frisk, Phys. Letters **22**, 90 (1966); H. Högaasen and W. Fischer, *ibid.* **22**, 516 (1966).

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to us seem (a) the N-N charge-exchange scattering, where the ρ' is necessary to obtain consistent fits to the differential cross sections and the total cross sections; (b) the Regge recurrence of the ρ' giving a particle $J^P=3^-$ at $\alpha_{\rho'}(t)=3$ or at a mass $\simeq\sqrt{3}$ GeV$=1732$ MeV, near the third peak of the R[15] at 1748 MeV; (c) the fits to the nucleon isovector form factor with only ρ leading to a discrepancy, whereas the addition of a ρ' with a mass of about 1000 MeV eliminates this inconsistency.[16,17]

Therefore, within the framework of duality and the Freund-Harari[8] conjecture, a model based on secondary meson trajectories seems quite attractive. In this paper we present a crossing-symmetric Regge representation which is constructed from N_α, N_γ, and Δ_δ baryon trajectories, ρ-P' meson trajectories, and ρ'-P'' secondary meson trajectories. We shall investigate how and to what extent such a model can correlate the high-energy and the low-energy behavior of πN scattering.

In Sec. II, we construct the Veneziano amplitudes from ρ, ρ', P', and P'' trajectories. We then calculate the high-s limit of these amplitudes for fixed t and fixed u, respectively. In Sec. III we evaluate the multiplicative Veneziano parameters. We impose the signature conditions on the Δ_δ, N_α, and N_γ trajectories and then show that the widths and the positions of the resonances on the parent baryon trajectories are given in terms of ten parameters. These parameters are then calculated by using isospin conditions, extrapolating the amplitudes to the nucleon and $N_\gamma(1518)$ poles, and imposing the condition that no parity partners for the $\Delta_\delta(1236)$ and $N_\gamma(1518)$ exist. The calculated width of $\Delta_\delta(1236)$, from these parameters, turns out to be 100 MeV. In Sec. IV we study the asymptotic behavior of the Veneziano amplitudes. We fit the high-energy forward differential cross section for πN charge exchange. The parameters thus evaluated are compared with their value obtained from ρ universality. The $\pi^\pm p$ differential cross sections in the backward direction are calculated using low-energy parameters. We then show how these backward differential cross sections and their dip structures change by introducing complex baryon trajectories. Finally, in Sec. V we discuss our results.

II. CONSTRUCTION OF AMPLITUDE

In this section we construct a Veneziano amplitude from the ρ, ρ', and P', P'' trajectories in the t channel and the N_α, N_γ, and Δ_δ trajectories in the s and u channels. We follow Igi's construction for the Veneziano-type amplitudes and keep only the leading Veneziano terms. The addition of satellite terms does not help us in resolving the problem mentioned at high energy, since their Regge residues also choose nonsense. Furthermore, we keep a minimum number of Veneziano terms in order to have the least number of parameters.

We thus write the Veneziano amplitudes which satisfy s-u crossing, display Regge behavior, and have resonance poles on Regge trajectories:

$$A^-/4\pi=A_\rho{}^-+A_{\rho'}{}^-, \tag{1a}$$

$$\begin{aligned}A_\rho{}^-=p_1{}^-C_{N_\alpha}{}^-(\tfrac{3}{2},1)+p_2{}^-C_\Delta{}^-(\tfrac{3}{2},1)+p_3{}^-C_{N_\gamma}{}^-(\tfrac{3}{2},1)\\+p_4{}^-C_{N_\alpha\Delta}{}^-(\tfrac{3}{2},\tfrac{3}{2})+p_5{}^-C_{N_\gamma N_\alpha}{}^-(\tfrac{3}{2},\tfrac{3}{2});\end{aligned} \tag{1b}$$

$$B^-/4\pi=B_\rho{}^-+B_{\rho'}{}^-, \tag{2a}$$

$$\begin{aligned}B_\rho{}^-=q_1{}^+B_{N_\alpha}{}^+(\tfrac{1}{2},1)+q_2{}^+B_\Delta{}^+(\tfrac{1}{2},1)+q_3{}^+B_{N_\gamma}{}^+(\tfrac{1}{2},1)\\+q_4B_{N_\alpha}(\tfrac{1}{2},\tfrac{1}{2})+q_5B_\Delta(\tfrac{1}{2},\tfrac{1}{2})+q_6B_{N_\gamma}(\tfrac{1}{2},\tfrac{1}{2});\end{aligned} \tag{2b}$$

$$A^+/4\pi=A_{P'}{}^++A_{P''}{}^+, \tag{3a}$$

$$\begin{aligned}A_{\rho'}{}^+=p_1{}^+C_{N_\alpha}{}^+(\tfrac{3}{2},1)+p_2{}^+C_\Delta{}^+(\tfrac{3}{2},1)+p_3{}^+C_{N_\gamma}{}^+(\tfrac{3}{2},1)\\+p_4C_{N_\alpha N_\alpha}(\tfrac{3}{2},\tfrac{3}{2})+p_5C_{\Delta\Delta}(\tfrac{3}{2},\tfrac{3}{2})\\+p_6C_{N_\gamma N_\gamma}(\tfrac{3}{2},\tfrac{3}{2}),\end{aligned} \tag{3b}$$

$$B^+/4\pi=B_{P'}{}^++B_{P''}{}^+, \tag{4a}$$

$$\begin{aligned}B_{P'}{}^+=q_1{}^-B_{N_\alpha}{}^-(\tfrac{1}{2},1)+q_2{}^-B_\Delta{}^-(\tfrac{1}{2},1)+q_3{}^-B_{N_\gamma}{}^-(\tfrac{1}{2},1)\\+\gamma_4B_{N_\alpha\Delta}{}^-(\tfrac{1}{2},\tfrac{1}{2})+\gamma_5B_{N_\alpha N_\gamma}{}^-(\tfrac{1}{2},\tfrac{1}{2}).\end{aligned} \tag{4b}$$

We have used the notation of Fenster and Wali[3] and let

$$\begin{aligned}B_x{}^\pm(\tfrac{1}{2}m,n)=B(\tfrac{1}{2}m-\alpha_x(s),\,n-\alpha(t))\\\pm B(\tfrac{1}{2}m-\alpha_x(u),\,n-\alpha(t)),\end{aligned} \tag{5a}$$

$$\begin{aligned}B_{xy}{}^\pm(\tfrac{1}{2}m,\tfrac{1}{2}n)=B(\tfrac{1}{2}m-\alpha_x(s),\,\tfrac{1}{2}n-\alpha_y(u))\\\pm B(\tfrac{1}{2}m-\alpha_x(u),\,\tfrac{1}{2}n-\alpha_y(s)),\end{aligned} \tag{5b}$$

$$B_x(\tfrac{1}{2}m,\tfrac{1}{2}n)=B(\tfrac{1}{2}m-\alpha_x(s),\,\tfrac{1}{2}m-\alpha_x(u)), \tag{5c}$$

$$B(\mu,\nu)=\frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)}=\frac{C(\mu,\nu)}{(\mu+\nu-1)}. \tag{5d}$$

In Eqs. (1)–(4) the ρ', P'' contributions are defined in a similar fashion as ρ-P' contributions with $\alpha_{\rho,P'}(t)$ replaced by $\alpha_{\rho',P''}(t)$ and the multiplicative constants p, q, and r replaced by P, Q, and R.

The s asymptotic behavior of these amplitudes for fixed t is given by

$$\begin{aligned}&A^-(s,t)/4\pi\\&=-(p_1{}^-+p_2{}^-+p_3{}^-)\xi_\rho(\alpha's)^{\alpha_\rho(t)}\Gamma(1-\alpha_\rho(t))\\&\quad-(P_1{}^-+P_2{}^-+P_3{}^-)\xi_{\rho'}(\alpha's)^{\alpha_{\rho'}(t)}\Gamma(1-\alpha_{\rho'}(t)),\end{aligned} \tag{6}$$

$$\begin{aligned}&A^+(s,t)/4\pi\\&=(p_1{}^++p_2{}^++p_3{}^+)\xi_{P'}(\alpha's)^{\alpha_{P'}(t)}\Gamma(1-\alpha_{P'}(t))\\&\quad+(P_1{}^++P_2{}^++P_3{}^+)\xi_{P''}(\alpha's)^{\alpha_{P''}(t)}\Gamma(1-\alpha_{P''}(t)),\end{aligned} \tag{7}$$

$$\begin{aligned}&B^-(s,t)/4\pi\\&=(q_1{}^++q_2{}^++q_3{}^+)\xi_\rho(\alpha's)^{\alpha_\rho(t)-1}\Gamma(1-\alpha_\rho(t))\\&\quad+(Q_1{}^++Q_2{}^++Q_3{}^+)\xi_{\rho'}(\alpha's)^{\alpha_{\rho'}(t)-1}\Gamma(1-\alpha_{\rho'}(t)),\end{aligned} \tag{8}$$

[15] M. N. Focacci *et al.*, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

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$$
\begin{aligned}
B^+(s,t)/4\pi &\\
= -(q_1^- + q_2^- + q_3^-)&\xi_{P'}(\alpha' s)^{\alpha_{P'}(t)-1}\Gamma(1-\alpha_{P'}(t)) \\
&- (Q_1^- + Q_2^- + Q_3^-)\xi_{P''}(\alpha' s)^{\alpha_{P''}(t)-1}\Gamma(1-\alpha_{P''}(t))\,,
\end{aligned} \tag{9}
$$

where $\xi_{\rho,\rho'} = 1 - e^{-i\pi\alpha_{\rho,\rho'}(t)}$ and $\xi_{P',P''} = 1 + e^{-i\pi\alpha_{P',P''}(t)}$. It should be noted that ρ, ρ' contribute only to "minus" amplitudes and P', P'' only to "plus" amplitudes, because of the connection between s-u crossing and t-channel signature.[18]

For large s and fixed u, the baryon trajectories contribute with degenerate signature. However, in Sec. II we shall enforce the condition that fermion trajectories have definite signature which will give us relations between the multiplicative parameters. Thus amplitudes have the following s asymptotic behavior for fixed u:

$$
\begin{aligned}
A^-/4\pi = {}& [(-p_2^- + p_4^- - P_2^- + P_4^-)\xi_\Delta^+ + (-p_2^- - p_4^- - P_2^- - P_4^-)\xi_\Delta^-]\Gamma(\tfrac{3}{2} - \alpha_\Delta(u))(\alpha' s)^{\alpha_\Delta(u)-1/2} \\
& + [(-p_1^- - p_4^- + p_5^- - P_1^- - P_4^- + P_5^-)\xi_{N_\alpha}^+ + (-p_1^- + p_4^- - p_5^- - P_1^- + P_4^- - P_5^-)\xi_{N_\alpha}^-] \\
& \times\Gamma(\tfrac{3}{2} - \alpha_{N_\alpha}(u))(\alpha' s)^{\alpha_{N\alpha}(u)-1/2} + [(-p_3^- - p_5^- - P_3^- - P_5^-)\xi_{N_\alpha}^+ + (-p_3^- + p_5^- - P_3^- + P_5^-)\xi_{N_\gamma}^-] \\
& \times\Gamma(\tfrac{3}{2} - \alpha_{N_\gamma}(u))(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\,,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
B^-/4\pi = {}& [(q_2^+ + q_5 + Q_2^+ + Q_5)\xi_\Delta^+ + (q_2^+ - q_5 + Q_2^+ - Q_5)\xi_\Delta^-]\Gamma(\tfrac{1}{2} - \alpha_\Delta(u))(\alpha' s)^{\alpha_\Delta(u)-1/2} \\
& + [(q_1^+ + q_4 + Q_1^+ + Q_4)\xi_{N_\alpha}^+ + (q_1^+ - q_4 + Q_1^+ - Q_4)\xi_{N_\alpha}^-]\Gamma(\tfrac{1}{2} - \alpha_{N_\alpha}(u))(\alpha' s)^{\alpha_{N\alpha}(u)-1/2} \\
& + [(q_3^+ + q_6 + Q_3^+ + Q_6)\xi_{N_\gamma}^+ + (q_3^+ - q_6 + Q_3^+ - Q_6)\xi_{N_\gamma}^-]\Gamma(\tfrac{1}{2} - \alpha_{N_\gamma}(u))(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\,,
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
A^+/4\pi = {}& [(p_2 + p_5 + P_2 + P_5)\xi_\Delta^+ + (p_2 - p_5 + P_2 - P_5)\xi_\Delta^-]\Gamma(\tfrac{3}{2} - \alpha_\Delta(u))(\alpha' s)^{\alpha_\Delta(u)-1/2} \\
& + [(p_1^+ + p_4 + P_1^+ + P_4)\xi_{N_\alpha}^+ + (p_1 - p_4 + P_1^+ - P_4)\xi_{N_\alpha}^-]\Gamma(\tfrac{3}{2} - \alpha_{N_\alpha}(u))(\alpha' s)^{\alpha_{N\alpha}(u)-1/2} \\
& + [(p_3^+ + p_6 + P_3^+ + P_6)\xi_{N_\gamma}^+ + (p_3^+ - p_6 + P_3^+ - P_6)\xi_{N_\gamma}^-]\Gamma(\tfrac{3}{2} - \alpha_{N_\gamma}(u))(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\,,
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
B^+/4\pi = {}& [(-q_2^- + \gamma_4 - Q_2^- + R_4)\xi_\Delta^+ + (-q_2^- - \gamma_4 - Q_2^- - R_4)\xi_\Delta^-]\Gamma(\tfrac{1}{2} - \alpha_\Delta(u))(\alpha' s)^{\alpha_\Delta(u)-1/2} \\
& + [(-q_1^- - \gamma_4 - \gamma_5 - Q_1^- - R_4 - R_5)\xi_{N_\alpha}^+ + (-q_1^- + \gamma_4 + \gamma_5 - Q_1^- + R_4 + R_5)]\Gamma(\tfrac{1}{2} - \alpha_{N_\alpha}(u))(\alpha' s)^{\alpha_{N\alpha}(u)-1/2} \\
& + [(-q_3^- + \gamma_4 - Q_3^- + R_4)\xi_{N_\gamma}^+ + (-q_3^- - \gamma_4 - Q_3^- - R_4)\xi_{N_\gamma}^-]\Gamma(\tfrac{1}{2} - \alpha_{N_\gamma}(u))(\alpha' s)^{\alpha_{N\gamma}(u)-1/2}\,,
\end{aligned} \tag{13}
$$

where

$$2\xi_x^\pm = 1 \pm e^{-i\pi[\alpha_x(u)-1/2]}\,.$$

III. DETERMINATION OF PARAMETERS

In this section we discuss the evaluation of the multiplicative parameters in our Veneziano amplitude. There are a total of 44 constants in Eqs. (1)–(4). The determination of such a large set of parameters would lead to a large linear system; the stability of the solution may then well be questioned. We remark therefore that each of these numbers individually has no physical meaning. We observe that as resonance parameters of leading trajectories or as Regge residues at high energies, they occur always in certain combinations so that the important physical predictions depend on ten parameters only. We now impose various physical conditions to determine these parameters.

We demand that in Eqs. (10)–(13), the trajectories have the correct signature:

(a) Δ trajectory:

$$
\begin{aligned}
&p_2^- + P_2^- = p_4^- + P_4^-\,, \quad q_2^+ + Q_2^+ = -(q_5 + Q_5)\,, \\
&p_2^+ + P_2^+ = -(p_5 + P_5)\,, \quad q_2^- = Q_2^- = r_4 + R_4\,;
\end{aligned} \tag{14}
$$

(b) N_α trajectory:

$$
\begin{aligned}
&p_1^- + P_1^- = p_4^- - p_5^- + P_4^- - P_5^-\,, \\
&\qquad\qquad q_1^+ + Q_1^+ = q_4 + Q_4\,, \\
&p_1^+ + P_1^+ = p_4 + P_4\,, \quad q_1^- + Q_1^- + r_4 + R_4 + r_5 + R_5\,;
\end{aligned} \tag{15}
$$

(c) N_γ trajectory:

$$
\begin{aligned}
&p_3^- + P_3^- = -(p_5^- + P_5^-)\,, \quad q_3^+ + Q_3^+ = -(q_6 + Q_6)\,, \\
&p_3^+ + P_3^+ = -(p_6 + P_6)\,, \quad q_3^- + Q_3^- = r_5 + R_5\,.
\end{aligned} \tag{16}
$$

Let us now define a very useful combination of these parameters:

$$p_4^- + P_4^- = C_4\,, \quad p_5^- + P_5^- = C_5\,; \tag{17}$$

$$q_4 + Q_4 = D_4\,, \quad q_5 + Q_5 = D_5\,, \quad q_6 + Q_6 = D_6\,; \tag{18}$$

$$p_4 + P_4 = E_4\,, \quad p_5 + P_5 = E_5\,, \quad p_6 + P_6 = E_6\,; \tag{19}$$

$$r_4 + R_4 = F_4\,, \quad r_5 + R_5 = F_5\,. \tag{20}$$

We first note that our 44 parameters, using the 12 signature conditions (14)–(16), can be expressed in terms of the above ten parameters denoted by C, D, E, and F.

We shall now show that the parameters (width and position) of the πN resonances which lie on the *parent* N_α, N_γ, and Δ_δ trajectories can be expressed in terms of these ten parameters. Furthermore, these ten constants completely describe the high-energy behavior of πN scattering for fixed u.

We now impose the isospin conditions that no $I=\frac{1}{2}$ resonances lie on the leading Δ trajectory and also no $I=\frac{3}{2}$ state on leading N_α and N_γ trajectories.

[18] See, for example, J. D. Jackson, UCRL Report No. UCRL-19351 (unpublished).

(a) N_α trajectory:

$$p_1^- + P_1^- = p_1^+ + P_1^+, \quad E_4 = C_4 - C_5,$$
$$q_1^- + Q_1^- = q_1^+ + Q_1^+, \quad D_4 = F_4 + F_5; \tag{21}$$

(b) N_γ trajectory:

$$p_3^- + P_3^- + p_3^+ + P_3^+, \quad E_6 = C_5,$$
$$q_3^- + Q_3^- = q_3^+ + Q_3^+, \quad D_6 = -F_5; \tag{22}$$

(c) Δ trajectory:

$$F_4 = 2D_5, \quad E_5 = 2C_4. \tag{23}$$

It should be noted that in view of the signature conditions, Eqs. (14)–(16), not all ten isospin conditions, Eqs. (21)–(23), are independent and we get only six independent isospin conditions which reduce the ten parameters to four.

To evaluate the four parameters, we use the following low-energy conditions:

(a) Nucleon pole:

The extrapolation to the nucleon pole yields

$$2q_1^+ + 2q_4 + q_1^- + \gamma_4 + \gamma_5 + 2Q_1^+ + 2Q_4 + Q_1^- + R_4 + R_5 = 3\alpha'(g_{\pi NN}^2/4\pi);$$

using signature and isospin conditions, we get

$$D_4 = (g_{\pi NN}^2/4\pi)\alpha'. \tag{24}$$

(b) N_γ pole:

We here use two conditions, the width of $N_\gamma[1518, J^P = \frac{3}{2}^-]$,[19] and the experimental fact that the parity partner of N_γ has not been observed. Thus we have

$$\Gamma_{N_\gamma} = [(E_\gamma - M)/M_\gamma^2] \times 2q_\gamma^3[(M_\gamma + M)F_5 - E_6], \tag{25}$$

$$E_6 = (M_\gamma - M)F_5. \tag{26}$$

In the above equations, q_γ and E_γ are the c.m. momentum and the nucleon energy at the position of the N_γ resonance, respectively.

(c) Δ pole:

We first use the condition that the parity partner of $\Delta_\delta[J^P = \frac{3}{2}^+, 1238]$ has not been observed experimentally, i.e.,

$$C_4 = (M_\Delta + M)D_5. \tag{27}$$

We have now determined all the parameters on the leading trajectories. Consequently, we calculate the width of Δ_δ resonance,

$$\Gamma_\Delta = \tfrac{4}{3}(q_\Delta^3/M_\Delta)(E_\Delta + M)D_5 \tag{28a}$$

$$\approx 100 \text{ MeV}, \tag{28b}$$

where in Eq. (25) we used N_γ parameters from Ref. 18 and $g_{\pi NN}^2/4\pi = 15$.

However, there are four resonance conditions which do not depend on these ten parameters because they lie on daughter trajectories. Such conditions are (i) no $\frac{1}{2}^+$ and $\frac{1}{2}^-$ particles on the Δ trajectory at M_Δ and (ii) $\frac{1}{2}^+$ and $\frac{1}{2}^-$ resonances [the $S_{11}(1550)$ and the Roper resonance] on N_γ trajectories at M_γ, whose widths are known.[19] These four conditions depend on eight parameters and give us the value of another combination of these parameters. Since the Regge recurrences of these resonances are not known and they do not contribute to high-energy behavior for fixed u, we do not mention the value of this combination of parameters. These new equations do not lead to any inconsistant situation. In addition to resonance conditions there are further constraints in our problem, such as the Adler condition and the scattering lengths in the $I = \frac{3}{2}, \frac{1}{2}$ states.[3] However, each of these constraints depends on one particular combination of the parameters and does not lead to any interesting predictions.

[19] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).

IV. HIGH-ENERGY BEHAVIOR

In our model, the high-energy behavior of the N charge-exchange scattering, in the forward direction, is dominated by ρ and ρ' trajectories. Thus, using Eqs. (6)–(9), we get

$$\frac{d\sigma}{dt} = \frac{2\pi}{q^2}\left[\frac{M^2}{s}\left(1 + \frac{t}{2q^2}\right) \times \left|\left(a - \frac{b}{2M\alpha'}\right)F_\rho + \left(m - \frac{W}{2M\alpha'}\right)F_{\rho'}\right|^2 - \frac{t}{16q^2}|aF_\rho + mF_{\rho'}|^2\right], \tag{29}$$

where

$$F_x = (1 - e^{-i\pi\alpha_x(t)})(\alpha' s)^{\alpha_x(t)}\Gamma(1 - \alpha_x(t)), \tag{30}$$

$$a = \sum_{n=1}^{3} p_n^-, \quad b = \sum_{n=1}^{3} q_n^-, \tag{31a}$$

$$m = \sum_{n=1}^{3} P_n^-, \quad n = \sum_{n=1}^{3} Q_n^-. \tag{31b}$$

The polarization for this reaction is given by

$$P = 2\frac{\operatorname{Im}(f_1^{-*}f_2^-)}{d\sigma/d\Omega}\sin\theta, \tag{32}$$

where

$$f_1 = [(E+M)/8\pi W][A + (W-M)B], \tag{33}$$

$$f_2 = [(E-M)/8\pi W][-A + (W+M)B]. \tag{34}$$

Using Eqs. (6) and (7) in Eq. (32), we get

$$P = -\left(\frac{E^2 - M^2}{\sqrt{s}}\right)\frac{1}{s}\frac{(mb - na)\operatorname{Im}(F_\rho F_{\rho'}^*)}{d\sigma/d\Omega}. \tag{35}$$

We now evaluate the parameters a and b from ρ universality and vector dominance[20]:

$$A^- = \left(\frac{g_\rho^2}{4M}\right)\frac{\kappa(s-u)}{m_\rho^2 - t}, \tag{36}$$

[20] B. Dutta-Roy, I. R. Lapidus, and M. J. Tausner, Phys. Rev. **181**, 2091 (1968).

$$B^{-}=g_{\rho}{}^{2}\frac{(1+\kappa)}{m_{\rho}{}^{2}-t}, \tag{37}$$

where the isovector anomalous magnetic moment of nucleon $\kappa=3.7$. Using

$$g_{\rho\pi\pi}{}^{2}/4\pi\sim 2.5, \tag{38}$$

we have

$$a=2.31\ \mathrm{GeV}^{-1},\quad b=5.85\ \mathrm{GeV}^{-2}. \tag{39}$$

On the other hand, we fit the experimental data for πN charge-exchange differential cross sections using the four free parameters a, b and A, B. Our best fits in Figs. 1 and 2 are obtained for

$$a=1.50\ \mathrm{GeV}^{-1},\quad b=2.68\ \mathrm{GeV}^{-2}, \tag{40}$$

$$A=1.00\ \mathrm{GeV}^{-1},\quad B=3.80\ \mathrm{GeV}^{-2}. \tag{41}$$

The above values of a and b correspond to a ρ width of about 80 MeV and a ρ' width of about 100 MeV. It should be noted that if we introduce satellite terms in our representation, we can obtain a much better agreement with the ρ width of 120 MeV. With the satellite terms, however, one has two more parameters which contribute to the high-energy behavior of the scattering amplitude and are not related to the ρ width. Consequently, one can adjust the satellite parameters in the high-energy region in such a way as to obtain reasonable agreement with ρ universality. Such satellite terms would alter the signature condition and change our result regarding the Δ width to some extent.

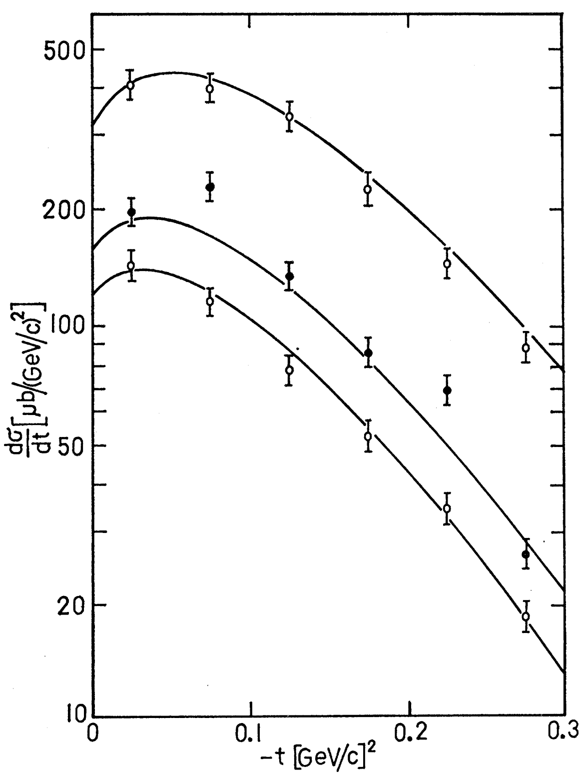

FIG. 1. Differential cross section for $\pi^{-}p\rightarrow\pi^{0}n$ at $p_{\mathrm{lab}}=5.85$, 13.3, and 18.2 GeV/c. The data are from Ref. 21.

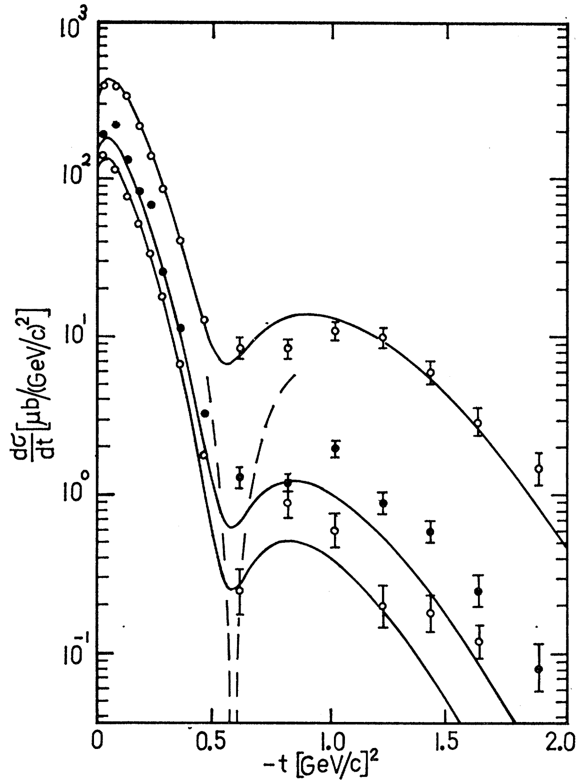

FIG. 2. Wide-angle differential cross section for $\pi^{-}p\rightarrow\pi^{0}n$ at $p_{\mathrm{lab}}=5.85$, 13.3, and 18.2 GeV/c. Data from Ref. 21.

In our analysis, we obtain a best fit for $\alpha_{\rho}(0)=0.58$, which also gives the correct s variation of the differential cross section.[22] This result is in agreement with the recent work of Höhler, Steiner, and Strauss,[23] who show that the "soft-pion intercept" $\alpha_{\rho}(0)=0.482$ is not compatible with the total and forward πN cross sections.

We now use the values a, b and A, B from Eqs. (40) and (41) in Eq. (35) and obtain the polarization as shown in Fig. 3. The magnitude of the polarization can be increased if we introduce satellite terms in our representation. This is because the parameters of the satellite terms are not related to the ρ width and can be adjusted in the high-energy region only.[3] We find a dip in the polarization at $\alpha_{\rho}(t)=0$, and this dip persists even if we add satellite terms. (See Fig. 4.)

We now study the $\pi^{\pm}p$ differential cross sections at high energies and for fixed u. Using Eqs. (10)–(13), we obtain

$$\frac{d\sigma}{d\Omega}(\pi^{\pm}p)=|\varphi_1{}^{\pm}|^2+\frac{1}{4q^2}\left[\frac{(M^2-\mu^2)^2}{s}-u\right]\times(|\varphi_1{}^{\pm}|^2-|\varphi_2{}^{\pm}|^2), \tag{42}$$

[21] A. V. Stirling *et al.*, Phys. Rev. Letters **14**, 763 (1965); P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966).

[22] G. Höhler *et al.*, Phys. Letters **20**, 79 (1966).

[23] G. Höhler, F. Steiner, and R. Strauss, University of Karlsruhe report (unpublished).

where

$$\varphi_1^+ = \sqrt{s}(-2D_5F_\Delta - 2D_4F_{N_\alpha} + 2D_6F_{N_\gamma}), \tag{43a}$$

$$\varphi_1^- = \sqrt{s}(-3D_5F_\Delta), \tag{43b}$$

$$\varphi_2^+ = \tfrac{1}{2}[-E_5G_\Delta - 4E_6G_{N_\gamma} + 4E_4G_{N_\alpha} + M(-2D_5F_\Delta - 4D_4F_{N_\alpha} + 4D_6F_{N_\gamma})], \tag{43c}$$

$$\varphi_2^- = \tfrac{1}{2}(-3E_5G_\Delta - 6MD_5F_\Delta), \tag{43d}$$

$$(\tfrac{1}{2} - \alpha_x)F_x = G_x = \Gamma(\tfrac{3}{2} - \alpha_x)\xi_x(\alpha' s)^{\alpha_x - 1/2}. \tag{43e}$$

We have already evaluated [Eqs. (17)–(27)] all the parameters appearing in the above equations. Now if we take $\alpha_{N_\alpha}(0) = \alpha_{N_\gamma}(0) = -0.5$ and $\alpha_{\Delta_\delta}(0) = 0$,[3] we obtain for the $\pi^\pm p$ cross sections a factor 1500–2000 times larger than their experimental value. In connection with the Veneziano representation, this situation was first realized by Fenster and Wali.[3]

The analysis presented here can be simply extended to the Fenster-Wali (FW) representation. Our preliminary analysis indicates that if secondary meson trajectories are added to the FW representation, it is possible to fit both the forward and backward cross sections with the same set of parameters. However, in this situation we have 59 parameters. The stability of the solutions of these equations is under investigation, and the results will be presented elsewhere.

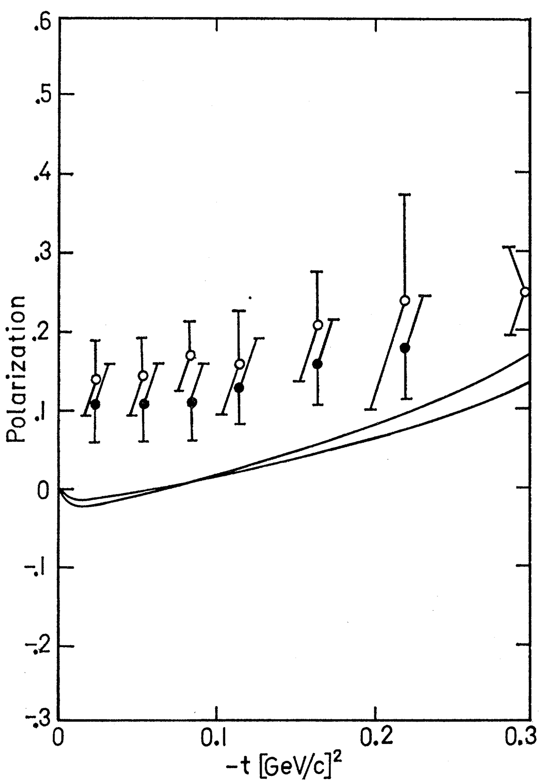

FIG. 3. Polarization for $\pi^- p \to \pi^0 n$. Data from Ref. 24 at $p_{\mathrm{lab}} = 5.9$ and 11.2 GeV/c. The curve is computed for 5.9 GeV/c. The polarization in the present model has very little energy dependence, so that a curve at 11.2 GeV/c would overlap the one drawn to a large extent.

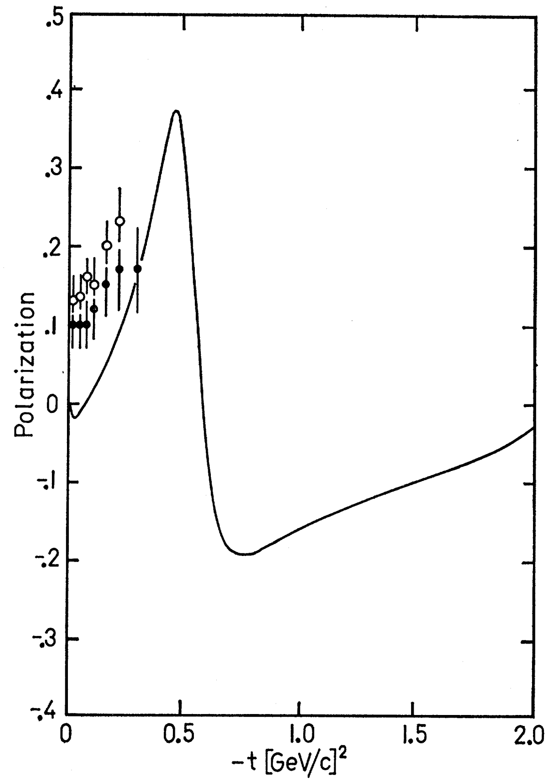

FIG. 4. Prediction of our model for the wide-angle polarization in $\pi^- p \to \pi^0 n$. Same data as in Fig. 3.

At this stage we study the effect of introducing an imaginary part into the trajectories. Since the Δ trajectory is the dominant trajectory in the backward direction, we take $\alpha_\Delta(x) = a + bx + i\lambda[x - (M+\mu)^2]^{1/2}$, with $a \approx 0$, $b \approx 1$, and study the variation of λ. When $\lambda \approx 1$, we get the correct mangitude for the $\pi^\pm p$ differential cross sections; however, the dips at wrong-signature nonsense points, associated with the signature of the Δ trajectory, also move away. Consequently, when $\lambda \approx 1$, we get a dip in the $\pi^+ p$ differential cross section at $u = -0.5$, and in $\pi^- p$ at $u = -1.5$; both these dips contradict the experiments. Thus complex trajectories give us the correct magnitude for backward $\pi^\pm p$ differential cross sections; however, the dips move away from wrong-signature nonsense points.

V. CONCLUSIONS

We have studied in this paper a Veneziano-type amplitude for πN scattering. In our construction, we have used the secondary meson trajectories to ensure nonvanishing of the amplitudes at the nonsense wrong-signature points. We have followed Igi's construction of crossing-symmetric amplitudes with Regge behavior.

[24] P. Bonamy *et al.*, in *Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967*, edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 171; D. Drobnis *et al.*, Phys. Rev. Letters **20**, 274 (1968).

We find that with $\rho+\rho'$ Regge exchange, a satisfactory fit to the charge-exchange differential cross section can be obtained. Our fits indicate a ρ intercept larger than the "soft-pion limit" $\alpha_\rho(0)=0.482$,[25] in agreement with a recent analysis of Höhler *et al.*[21] The calculated polarization from these values of the parameters has a dip when $\alpha_\rho(t)=0$. This observation is consistent with the results of Glebov *et al.*,[26] who fit the charge-exchange differential cross section with a ρ Regge pole and a Regge branch point. However, in their study of πN finite-energy sum rules, Barger and Phillips[13] find a peak in the polarization when $\alpha_\rho(t)=0$. This is because their Regge residues are t dependent, whereas we use Regge residues which are constant [apart from $\Gamma(1-\alpha_\rho(t))$]. However, as shown in the FW calculation,[3] the introduction of satellite terms will improve agreement with ρ universality. The two parameters of the satellite terms are not related to ρ width,[3] and they can be adjusted in the high-energy region in such a way as to produce the correct polarization.

In a recent paper, Ahmadzadeh and Kauffmann studied a model based on $\rho+\rho'$ trajectories, with Regge residues given by the Veneziano representation. They obtain a good fit to the πN charge-exchange polarization. It should be noted, however, that their ρ' parameters correspond to a negative-width particle.

In the low-energy region we find that the width and position of πN resonances which lie on the leading baryon Regge trajectories can be expressed in terms of ten parameters. These parameters are then evaluated using various low-energy conditions. From these parameters we calculate the N^* width, which turns out to be about 100 MeV. However, these parameters result in too large a backward differential cross section.[3,5] We show that it is possible to obtain a reasonable magnitude for the $\pi^\pm p$ differential cross section by introducing large imaginary parts in the baryon trajectories. This procedure, however, moves the dips far away from wrong-signature nonsense points, in contradiction with experiment.

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[25] C. Lovelace, Phys. Letters **28B**, 264 (1968).

[26] V. Yu. Glebov, A. B. Kaidalov, S. T. Sukhorykov, and K. A. Ter-Martirosyan, ITEP, Moscow, report (unpublished).

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Pion Photoproduction, Continuous Dispersion Sum Rules, and Regge Intercepts*

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We have formulated the continuous dispersion sum rules (CDSR) for pion photoproduction. Attempts are made to find the intercepts of the Regge trajectories at $t=0$, using the result of a partial-wave analysis by Walker, and by Berends, Donnachie, and Weaver. We have, in all, analyzed the twelve regularized t-channel helicity amplitudes. While reasonable results are obtained for the trajectories associated with the antisymmetric isovector [$(-)$] amplitudes, results for the symmetric isovector [$(+)$] amplitudes (in particular, the ω trajectory) are inconsistent with what we expect from hadron physics. This phenomenological analysis for $\gamma N\to\pi N$ is plagued with the additional degrees of freedom allowed by the electromagnetic nature of the reaction. It is also possible that the cutoff energy used in the CDSR is not high enough. Further investigation in the analysis of the data is necessary. In connection with the study of the helicity amplitudes for pion photoproduction, we also obtain, by eliminating the invariant amplitudes, the crossing relation between the s- and t-channel amplitudes, without recourse to the crossing matrix.

I. INTRODUCTION

It is well known that studies on single-pion photoproduction ($\gamma N\to\pi N$) have added understanding to low-energy pion physics; information for the latter comes mainly from elastic pion-nucleon scattering $\pi N\to\pi N$. It is natural to inquire as to whether the same situation will persist at higher energies.

In the intermediate-energy region the phenomenological analysis for $\gamma N\to\pi N$ is greatly complicated by the doubling of the independent parameters. There are four invariant amplitudes,[1] in contrast to two[2] for pion scattering, $\pi N\to\pi N$. Also, there is no optical theorem, which relates the imaginary part of the forward scattering to the total cross section, and which fixes the phase of the imaginary part to be positive definite.

* Work supported in part by the National Research Council of Canada.

[1] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957); J. S. Ball, *ibid.* **124**, 2014 (1961).

[2] G. E. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).