Proton Polarization in Elastic Electron-Proton Scattering

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The proton polarization in elastic electron-proton scattering is calculated. In order to calculate the imaginary part of the scattering amplitude, unitarity is used taking only the electron-proton state as the intermediate state. The maximum magnitude of the polarization for energies near about 1 GeV/c is $\sim$0.2%. The energy value at which the polarization vanishes independently of scattering angles is found.

I. INTRODUCTION

It is well known that the electron-proton elastic scattering process is well described by the Rosenbluth formula with phenomenological form factors.[1] This means that contributions from higher-order (in $\alpha = e^2/\hbar c$) diagrams to the *unpolarized* cross section are negligible and cannot be detected with present experimental techniques. In fact, Drell and Ruderman, and Drell and Fubini[2] estimated the correction for the two-photon-exchange process to the elastic electron-proton scattering and obtained for the incident electron energy up to 1 GeV/c, a correction of $<1\%$.

Both theoretically and experimentally, the higher-order contributions from the electromagnetic interactions to the e-p scattering process are very interesting and several efforts have been made to study these effects. The most direct approaches are (1) to detect the recoil proton polarization, and (2) to detect the difference between the cross sections

$$\sigma(e^+p \to e^+p) \quad \text{and} \quad \sigma(e^-p \to e^-p).$$

Since the one-photon-exchange process gives no proton polarization and no $e^+ - e^-$ differences, the higher-order processes will be revealed in these two experiments. The polarization of the recoil proton is related to the imaginary part of the electron-proton scattering amplitudes. The simplest process whose amplitudes have nonzero imaginary part is the two-photon-exchange process (Fig. 1). Barut and Fronsdal[3] calculated the polarization for the e-μ and e-e scatterings and obtained values which are smaller than 0.1% for any energy and angle. Guerin and Piketty[4] used the isobar model and concluded that $|P| < 0.5\%$ for incident electron energy around 1 GeV/c.

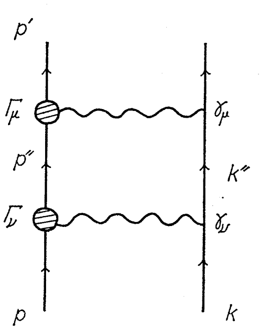

FIG. 1. Two-photon-exchange process.

For completeness, we list the available experimental data of the proton polarization:

Bizot *et al.*,[5]

$$P = (3.1 \pm 2.5)\%,$$
$$K_L = 950 \text{ MeV}/c, \quad q^2 = 16 \text{ F}^{-2} = 0.6 \text{ (GeV}/c)^2;$$

Bizot *et al.*,[6]

$$P = (4 \pm 2.7)\%,$$
$$K_L = 950 \text{ MeV}/c, \quad q^2 = 0.6 \text{ (GeV}/c)^2;$$

and Anderson,[7]

$$P = (1.3 \pm 2.0)\%,$$
$$K_L = 900 \text{ MeV}/c, \quad q^2 = 0.4 \text{ (GeV}/c)^2,$$

where P denotes the polarization, K_L is the incident electron energy in the laboratory system, and q^2 is the square of the four-momentum transfer.

II. FORMALISM AND CALCULATIONS

The polarization of the recoil proton is defined by

$$P = \text{Tr}(F^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\omega} F)/\text{Tr}(F^\dagger F), \tag{2.1}$$

[1] There are many reports on the proton form-factor measurements. For example, see a review given by S. D. Drell, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 85.

[2] S. D. Drell and M. A. Ruderman, Phys. Rev. **106**, 561 (1957); S. D. Drell and S. Fubini, *ibid.* **113**, 741 (1959).

[3] A. O. Barut and C. Fronsdal, Phys. Rev. **120**, 1871 (1960).

[4] F. Guerin and C. A. Piketty, Nuovo Cimento **32**, 971 (1964). The maximum polarization at $K_L = 105$ MeV/c is found to be 0.05% in our calculation, whereas these authors give the value 0.13%, which is about twice as large as ours. Our result is consistent with that of Ref. 3 in the limit of vanishing anomalous moment.

[5] J. C. Bizot *et al.*, Phys. Rev. Letters **11**, 480 (1963).

[6] J. C. Bizot *et al.*, Phys. Rev. **140**, B1387 (1965).

[7] R. L. Anderson *et al.*, see Ref. 1.

where F is related to the S matrix

$$S=1+iF \tag{2.2}$$

and $\boldsymbol{\omega}$ is the spin direction of the recoil proton ($\boldsymbol{\omega}^2=1$).

Expanding F in powers of α,

$$F=F_{(0)}+F_{(1)}+\cdots, \tag{2.3}$$

where $F_{(0)}$ corresponds to the one-photon-exchange process and $F_{(0)}^\dagger=F_{(0)}$. To the lowest order in α, the polarization is

$$P=2i\,\mathrm{Tr}(F_{(0)}\boldsymbol{\sigma}\cdot\boldsymbol{\omega}\,\mathrm{Im}F_{(1)})/\mathrm{Tr}(F_{(1)}F_{(0)}). \tag{2.4}$$

Using unitarity, the imaginary part of $F_{(1)}$ is given by

$$\mathrm{Im}\langle ep|F_{(1)}|ep\rangle=\sum_n \langle ep|F^+|n\rangle\langle n|F|ep\rangle, \tag{2.5}$$

where the $|n\rangle$'s are intermediate states consisting of $|ep\rangle,|eN^*\rangle$, etc. Guerin and Piketty calculated the polarization taking $|eN^*(1238)\rangle$ and $|eN^*(1520)\rangle$ as the intermediate states, and the state $|ep\rangle$ only for the incident electron energy $K_L<340$ MeV/c where no isobar could be produced.

There is no reason why the contribution to the polarization from the elastic unitarity part ($|n\rangle=|ep\rangle$) should be discarded from the inelastic unitarity part ($|n\rangle=|eN^*\rangle$), for the energy spectrum of electrons scattered by proton targets shows a very large elastic peak.

In this paper, we consider only the contribution to the polarization from the elastic unitarity part. Thus the polarization is now given by

$$P=\frac{A(k'p';kp)}{\sum_{\text{spins}}|\langle k'p'|F_{(0)}|kp\rangle|^2} \tag{2.6}$$

and

$$A(k'p';kp)=-i\int\frac{d^2k''}{(2\pi)^3}\frac{d^3p''}{(2\pi)^3}(2\pi)^4\delta^4(k+p-k''-p'')\times\sum_{\text{spins}}\langle kp|aF_{(0)}|k''p''\rangle\langle k''p''|F_{(0)}\times|k'p'\rangle\langle k'p'|F_{(0)}|kp\rangle, \tag{2.7}$$

where the summations are over spin directions. Momenta k, k', and k'' are used for the incident electron, the final electron, and the intermediate electron, and p, p', and p'' for the corresponding proton momenta. The operator a is defined so that

$$\Lambda_\pm=\tfrac{1}{2}(1\pm a),\quad a^2=1 \tag{2.8}$$

represents the projection operators of proton spin.

The Born amplitude of the elastic electron-proton scattering is given by

$$\langle k'p'|F_{(0)}|kp\rangle=\left(\frac{m}{\epsilon}\right)^{1/2}\left(\frac{m}{\epsilon'}\right)^{1/2}\left(\frac{M}{E}\right)^{1/2}\left(\frac{M}{E'}\right)^{1/2}\frac{1}{q''^2}\times\bar{u}(k')ie\gamma_\mu u(k)\bar{U}(p')eZ\Gamma_\mu(p',p)U(p), \tag{2.9}$$

where m, M; ϵ, E; and ϵ', E' are the masses, initial energies, and final energies of the electron and the proton, respectively. $u(k)$ and $U(p)$ are the Dirac spinors of the electron and the proton, respectively. With

$$q''^2=(k-k')^2=(p-p')^2, \tag{2.10}$$

the square of the four-momentum transfer, the proton current is given by[8]

$$\Gamma_\mu(p',p)=\mu_pG(q''^2)[i\gamma_\mu-(\kappa/2M\mu_p)(p+p')_\mu], \tag{2.11}$$

$$G(q''^2)=\left(\frac{1}{1+q''^2/\lambda}\right)^2, \tag{2.12}$$

$$\lambda=18.1\,F^{-2}=0.71\ (\mathrm{GeV}/c)^2, \tag{2.13}$$

where μ_p and κ are the total and anomalous magnetic moments of the proton, respectively, ($\mu_p=1+\kappa$). The function $G(q^2)$ is related to the proton form factors[1] as follows:

$$G_M(q^2)=\mu_pG(p^2),\quad G_E(q^2)=G(q^2). \tag{2.14}$$

Strictly speaking, Eq. (2.11) is not correct, and the correct form of the proton current $\Gamma_\mu(p',p)$ is obtained by the replacement

$$\frac{\kappa}{2M\mu_p}\to\frac{1}{1+q''^2/4M^2}\frac{\kappa}{2M\mu_p} \tag{2.15}$$

in Eq. (2.11). We will omit this kinematical factor for simplicity. The proton charge is given by eZ, $Z=-1$.

With Eq. (2.9) (and similar expressions for the processes $k+p\to k''+p''$, $k''+p''\to k'+p'$), we get

$$\sum_{\text{spins}}\langle kp|aF_{(0)}|k''p''\rangle\langle k''p''|F_{(0)}|k'p'\rangle\langle k'p'|F_{(0)}|kp\rangle=(e^2Z)^3\frac{1}{8\epsilon\epsilon'\epsilon''}\frac{1}{8EE'E''}\mathrm{Tr}[(-i\boldsymbol{k}+m)i\gamma_\mu\cdots i\gamma_\lambda]\times\mathrm{Tr}[(-i\boldsymbol{p}+M)i\gamma_5\gamma\cdot\omega\Gamma_\mu(p,p'')\cdots\Gamma_\lambda(p',p)]. \tag{2.16}$$

The calculation was done in two steps: (1) calculation of the traces appearing in Eq. (2.16); (2) integration over the intermediate momenta k'' and p''.

The calculation of the traces is very tedious but

[8] The metrics used in this paper are as follows: $\gamma_\mu^\dagger=\gamma_\mu$ (Hermitian), $(A\cdot B)=\mathbf{A}\cdot\mathbf{B}-A_0B_0$, where A and B are arbitrary four-vectors.

straightforward, and the result is given by

$$E_{\mu\nu\lambda}P_{\mu\nu\lambda}\equiv\mathrm{Tr}[(-ik)i\gamma_\mu(-ik'')i\gamma_\nu(-ik')i\gamma_\lambda]$$
$$\times\mathrm{Tr}[(-ip+M)i\gamma_5\gamma\cdot\omega\Gamma_\mu(p,p'')(-ip''+M)\Gamma_\nu(p'',p')(-ip'+M)\Gamma_\lambda(p',p)]$$
$$=32(pp'W\omega)\left[2(Wp)\frac{\kappa}{2M\mu_p}+M\right]\frac{1}{(k\cdot k')S}G(q^2)G(q'^2)G(q''^2)$$
$$\times[2(W\cdot k)(k\cdot k')(\kappa/2M\mu_p)^2K+4M(\kappa/2M\mu_p)L-H], \quad (2.17)$$

where the electron mass is neglected,[8] W is the total four-momentum of the system,

$$W=k+p=k'+p'=k''+p'', \quad (2.18)$$

and

$$(pp'W\omega)\equiv\tfrac{1}{4}\,\mathrm{Tr}(pp'W\gamma_5\gamma\cdot\omega). \quad (2.19)$$

In the laboratory system this becomes

$$-iM\boldsymbol{\omega}\cdot(\mathbf{k}_L'\times\mathbf{k}_L), \quad (2.20)$$

where $\boldsymbol{\omega}$ is the direction of proton spin and $\mathbf{k}_L$ and $\mathbf{k}_L'$ are incident and scattered electron three-momenta in laboratory system. K, L, and H in Eq. (2.17) are defined as follows:

$$K=-W^2P+[4(W\cdot k)^2-W^2(k\cdot k')]Q,$$
$$L=(W\cdot k)^2P-2SR, \quad (2.21)$$
$$H=2L+(W\cdot k)(k\cdot k')P,$$

where

$$P=[(k+k',k'')-(k\cdot k')](k+k',k''),$$
$$Q=(k+k',k'')-(k\cdot k'),$$
$$R=(k\cdot k'')(k'\cdot k''), \quad (2.22)$$
$$S=2(W\cdot k)^2-W^2(k\cdot k').$$

Throughout the calculations, electron mass is always neglected, so that the quantities P, Q, and R vanish whenever any two of the three electron momenta k, k', and k'' are equal.

In this case P, Q, and R must vanish in order to obtain finite values of the integrals, since they contain photon propagators which diverge at the forward direction.

From Eq. (2.17), one can conclude that the polarization vanishes for any angle, when

$$f\equiv2(W\cdot p)(\kappa/2M\mu_p)+M=0 \quad (2.23)$$

is satisfied for physical momenta. In the laboratory

$$(W\cdot p)=(k\cdot p)+p^2$$
$$=-MK_L-M^2,$$

so that from Eq. (2.23) we find the energy at which the polarization vanishes:

$$K_L=(1/\kappa)M=530\text{ MeV}/c. \quad (2.24)$$

The factorizability of the factor f can be understood by noticing that the proton helicity-flip matrix element vanishes at this energy in the Born amplitude, Eq. (2.9). The reason is as follows: If we put

$$(p+p')_\mu=2W_\mu-(k+k')_\mu$$

in the proton current, Eq. (2.11), we can neglect the second term $(k+k')_\mu$ by observing

$$(k+k')_\mu\bar{u}(k')i\gamma_\mu u(k)$$
$$=-2m\bar{u}(k')u(k)$$
$$=0\text{ (in the limit of vanishing electron mass)}.$$

Therefore, the proton current, Eq. (2.11), is equivalent to

$$\Gamma_\mu=\mu_pG(q''^2)[i\gamma_\mu-(\kappa/M\mu_p)W_\mu].$$

By making use of this form for the proton current, it is easy to demonstrate that the helicity-flip part of the proton current vanishes at this energy. Since the polarization is given by the interference of the helicity-flip and helicity-nonflip part of the amplitudes, we obtain the vanishing of the polarization for any scattering angle at this energy.

The next task is to perform the integration over intermediate momenta. Let us define the quantity

$$B(q^2,q'^2)\equiv G(q^2)G(q'^2)[-(W\cdot k)q''^2(\kappa/2M_p)^2K$$
$$+4M(\kappa/2M\mu_p)L-H]. \quad (2.25)$$

Since $B(q,q')$ is expressed in the Lorentz-invariant form, the integration can be performed in the center-of-mass system:

$$\int\frac{d^3k''}{(2\pi)^3}\frac{d^3p''}{(2\pi)^3}\frac{1}{4\epsilon''E''}(2\pi)^4\delta^4(k''+p''-W)\frac{B(q^2,q'^2)}{q^2q'^2}$$
$$=\frac{1}{(2\pi)^2}\frac{1}{4}\frac{(Wk)}{W^2}\left[\int d\Omega_{k''}\frac{B(q^2,q'^2)}{qq'^2}\right]_{\mathrm{c.m.}}. \quad (2.26)$$

Combining Eqs. (2.7), (2.16), (2.11), (2.25), and (2.26), we obtain

$$A(k'p';kp)=-i\frac{1}{(2\pi)^2}(e^2\mu_pZ)^3\frac{1}{\epsilon\epsilon'EE'}(pp'W\omega)$$
$$\times\frac{G(q''^2)}{q^2q'^2}\frac{(W\cdot k)}{W^2}\left[2(W\cdot p)\left(\frac{\kappa}{2M\mu_p}\right)+M\right]$$
$$\times\left[\int d\Omega_{k''}\frac{B(q^2,q'^2)}{q^2q'^2}\right]. \quad (2.27)$$

The angular integration of Eq. (2.26) can be done analytically, but the results are so complicated that we do not reproduce them here.

If we set $\kappa=0$, i.e., $\mu_p=1$ and replace all G functions by unity in Eq. (2.27), we get an expression for the polarization which is identical to Eq. (15) of Barut and Fronsdal[3] with $m_1=M$ and $m_2=0$ in their equation.

The results of Guerin and Piketty,[4] for the incident-electron energy smaller than 340 MeV/c, may be obtained using the Clementel-Villi form factors,

$$G(q^2)=1-h+h\lambda/(q^2+\lambda)\,,$$
$$h=1.06\,,\quad \lambda=0.36\ (\mathrm{GeV}/c)^2\,,$$

instead of ours [Eq. (2.12)]. This may be considered as a small-q^2 limit of our double-pole-type form factor.

III. RESULTS AND DISCUSSIONS

A computer was used to calculate the angular integral in Eq. (2.26). The polarizations of the recoil proton are given in Figs. 2 and 3.

(1) Since the factor f depends linearly on the energy K_L, the maximum value of the polarization increases slowly to 0.5% at 10 GeV/c. Of course, this value is not reliable, because we have neglected the effects of the kinematical factor in the proton current Eq. (2.11).

(2) Near 1 GeV/c, where the kinematical factor can be approximated by unity, the contribution of the elastic unitarity part to the polarization is comparable to that of inelastic unitarity part as calculated in Ref. 4. Furthermore, comparing our results with those of Barut and Fronsdal,[3] we may conclude that the anomalous magnetic moment plays an important role in the increase of the polarization at high energies. This can be seen from the fact that the anomalous moment appears with a factor

$$(p+p')/2M\,,$$

which increases linearly as the energy. Since the form factor decreases rapidly as energy increases, the polarization cannot reach large values.

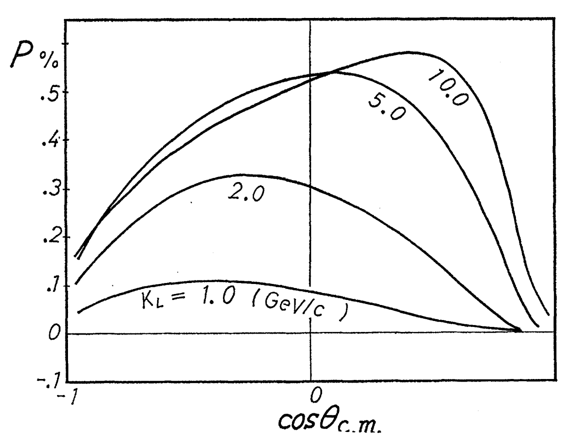

FIG. 2. Typical angular distribution of the proton polarization $P(\theta)$ due to the elastic intermediate state where θ denotes the scattering angle of electron in the center-of-mass system. The energy of the incident electron in the laboratory system is indicated.

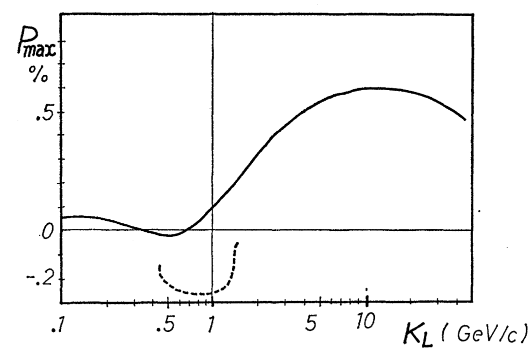

FIG. 3. Maximum magnitude of the polarization versus incident electron energies in the laboratory system. Solid line: contribution from the elastic intermediate state. Dashed line: isobar contribution taken from Ref. 4.

Combining the results of Guerin and Piketty,[4] we find that the "elastic" contribution and "isobar" contribution have opposite signs, so that in some energy region they compensate, and the magnitude of the total polarization becomes small or zero. However, the "isobar" contribution decreases rapidly as energy increases.

(3) We conclude that the detection of proton polarization will be very difficult for any available energies with present experimental accuracy.

(4) The remaining theoretical problem is to estimate the inelastic contributions at high energies.

Note added in proof. Since this paper was submitted for publication new data of the proton polarization experiments have been published: T. Powell *et al.*, Phys. Rev. Letters **24**, 753 (1970).

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