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Study of the $\Delta S=\Delta Q$ Rule in the Leptonic Decays of the Neutral K Meson

Y. Cho,* A. Dralle,† J. Canter,‡ A. Engler, H. Fisk, R. Kraemer, and C. Meltzer
Carnegie-Mellon University, Pittsburgh, Pennsylvania§ *15213*

and

D. G. Hill, M. Sakitt, and O. Skjeggestad‖
Brookhaven National Laboratory, Upton, New York§ *11973*

and

T. Kikuchi, D. K. Robinson, and C. Tilger**
Case Western Reserve University, Cleveland, Ohio††

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The validity of the $\Delta S=\Delta Q$ rule has been tested by studying the time distributions of leptonic decays of neutral K mesons produced in the reaction $K^+d\rightarrow K^0pp$. On the basis of 215 events with known leptonic charge, we find for x, the ratio of the violating to nonviolating amplitude, $\mathrm{Re}x=0.12\pm0.09$ and $\mathrm{Im}x=-0.08\pm0.07$. Our result is consistent with no violation.

I. INTRODUCTION

Traditionally, the weak interactions have been believed to be of the four-fermion form with a weak-interaction Hamiltonian constructed of current-current terms. Generally, the Hamiltonian is written as

$$H_{\mathrm{weak}}=J_\mu J_\mu{}^\dagger+J_\mu{}^\dagger J_\mu\,.$$

The current J_μ, which is believed to have a vector and an axial-vector structure, can be separated into a nonstrange hadronic, a strange hadronic, and a leptonic part. Over a decade ago,[1] the "$\Delta S=\Delta Q$ rule" was proposed whereby the strange hadronic part of the current connects to first order only those states whose strangeness and charge are both changed by ±1.

This rule can be checked in three different particle decays. In the decay of the Σ, which is a mixture of vector and axial-vector currents, the decay $\Sigma^+\rightarrow l^+n\nu$ is a $\Delta S=-\Delta Q$ transition, while the decay $\Sigma^-\rightarrow l^-n\bar{\nu}$ is a $\Delta S=\Delta Q$ transition. The ratio of the two rates is one measure of the validity of the rule. In the decays $K^+\rightarrow\pi^+\pi^+l^-\bar{\nu}$ and $K^+\rightarrow\pi^+\pi^-l^+\nu$, which are predominantly axial vector, the ratio of decay rates is another measure of the validity of the rule. In the three-body leptonic decays of the neutral K, which is a vector interaction, the situation is more complex. Because of the mixing in the $K^0\bar{K}^0$ system, one can observe the interference between the violating and nonviolating amplitudes by studying the leptonic decays as a function of time.

The experimental verification of the $\Delta S=\Delta Q$ rule in K^0 decays has proved to be difficult. While no definite violation has been established, a ratio of the violating to the nonviolating amplitudes as large as 0.3 has not been ruled out.[2–12] In this paper we report on an analysis of 454 three-body decays of neutral K mesons. Preliminary results of this experiment have been reported previously.[11]

II. THEORY

In order to calculate the time distributions of leptonic decays for this experiment, it is sufficiently accurate to ignore the CP-violating admixture in the K_S and K_L states. We therefore define the states as

$$|K_S{}^0\rangle=(|K^0\rangle+|\bar{K}^0\rangle)/\sqrt{2}$$

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[*] Present address: Argonne National Laboratory, Argonne, Ill.

[†] Present address: Bettis Atomic Power Laboratory, West Mifflin, Pa.

[‡] Present address: State University of New York at Stony Brook, Stony Brook, N. Y.

[§] Work supported by the U. S. Atomic Energy Commission.

[‖] Permanent address: University of Oslo, Blindern, Norway.

[**] Present address: Digital Equipment Corporation, Maynard, Mass.

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and

$$|K_L{}^0\rangle=(|K^0\rangle-|\bar{K}^0\rangle)/\sqrt{2}\,,$$

with the convention

$$CP|K^0\rangle=|\bar{K}^0\rangle.$$

Let f be the amplitude for the $\Delta S=\Delta Q$ transition $K^0\rightarrow\pi^-l^+\nu$, g be the amplitude for the $\Delta S=-\Delta Q$ transition $K^0\rightarrow\pi^+l^-\bar{\nu}$, and x be the complex parameter g^*/f.

For an initially pure K^0 state, the time distribution of leptonic decays is given by

$$N^{\pm}(t,x)=\tfrac{1}{4}|f|^2[|1+x|^2e^{-\lambda_S t}+|1-x|^2e^{-\lambda_l t}$$
$$\pm 2\cos\delta t(1-|x|^2)e^{-\Lambda t}-4\,\mathrm{Im}x\sin\delta t\,e^{-\Lambda t}],\quad(1)$$

where the ± refers to the charge of the lepton and

$$\Lambda=\tfrac{1}{2}(\lambda_S+\lambda_L)\,.$$

We have used the following values[13]:

$$\delta=M(K_L)-M(K_S)=0.46\tau_S{}^{-1},$$
$$\lambda_S=1/\tau_S=1.16\times10^{10}\ \mathrm{sec}^{-1},$$
$$\lambda_L=1/\tau_L=1.86\times10^{7}\ \mathrm{sec}^{-1}.$$

The deviation of Eq. (1) assumes CPT invariance and a pure vector interaction for both f and g. Furthermore, it neglects the effect of any induced scalar contribution to either amplitude. For K_{e3} decay, which constitute most of our data, the induced scalar contribution is known to be small for the allowed amplitude f. For $K_{\mu3}$ decay, while the experimental situation is unclear,[14] most theories predict only small effects for the allowed term.[15] The inclusion of such induced scalar terms would require an additional complex parameter which cannot be justified by present experimental data.

In addition, since the experimental data cover a range of decay configurations, what is measured are averages of $|x|^2$, Rex, and Imx. Owing to the small sample of events, we will assume, as is generally done,[16,17] that

$$\langle|x|^2\rangle=\langle\mathrm{Re}x\rangle^2+\langle\mathrm{Im}x\rangle^2,$$

so that only two parameters have to be estimated. The above equality is true provided the energy dependence of the form factors can be neglected. Finally, in merging $K_{\mu3}$ and K_{e3} decays, μ-e universality is invoked.

CP invariance requires that the two amplitudes f and g be relatively real. If the $\Delta I=\frac{1}{2}$ rule were valid in leptonic K decays, this would imply the $\Delta S=\Delta Q$ rule. However, the converse is not true.

III. EXPERIMENTAL PROCEDURE

The events were obtained from the 30-in. BNL bubble chamber, filled with deuterium, exposed to a separated 600 MeV/c K^+ beam[18] at the alternating gradient synchrotron (AGS). In the first part of the exposure 250 000 pictures were obtained with an average flux of 13 K^+ per picture and an equal number of background tracks. For the second part, 500 000 additional pictures were obtained with an average flux of 25 K^+ and about two background tracks per picture.

The neutral K mesons were produced in the reaction

$$K^++d\rightarrow K^0+p+p_S\,,$$

where p_S is a spectator proton. The cross section for the reaction $K^++d\rightarrow K^0+p+p+\pi^0$ (for which the threshold is 535 MeV/c) is less than 1% of the charge-exchange cross section; these events have been removed by kinematic fitting.

The neutral K can have a visible decay into $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\gamma$, $\pi\mu\nu$, or $\pi e\nu$. The pictures were scanned for all V's and possible production vertices. No criteria concerning the association of V's to production vertices were imposed in the scanning. 75% of the pictures were scanned twice; the over-all scanning efficiency for the entire experiment was better than 95%.

Since the fraction of three-body decays expected is only about 1% of the two-body decays, extreme precautions are necessary to ensure complete removal of any background two-body decays. Once a V was found, the most likely production vertex was assumed to be its origin and the whole event was measured. The measured events were processed through standard geometry and kinematic fitting programs. The following initial geometrical criteria were then imposed:

(a) The production and decay vertices were required to be at least 5 cm from the top and bottom of the chamber and within a circle of radius 30 cm centered in the middle of the chamber.

(b) The dip angles of both the neutral track and the charged decay tracks of the V were required to be less than 70°.

(c) The projected distance to the V from the production vertex was required to be greater than 1 mm.

These initial geometrical cuts were later made more restrictive.

Each V was fitted to the two-body decay mode $K_S\rightarrow\pi^+\pi^-$ under the hypotheses that it was not associated with any production origin (one-constraint fit) and that it was associated with the production vertex (three-constraint fit). An event for which the χ^2

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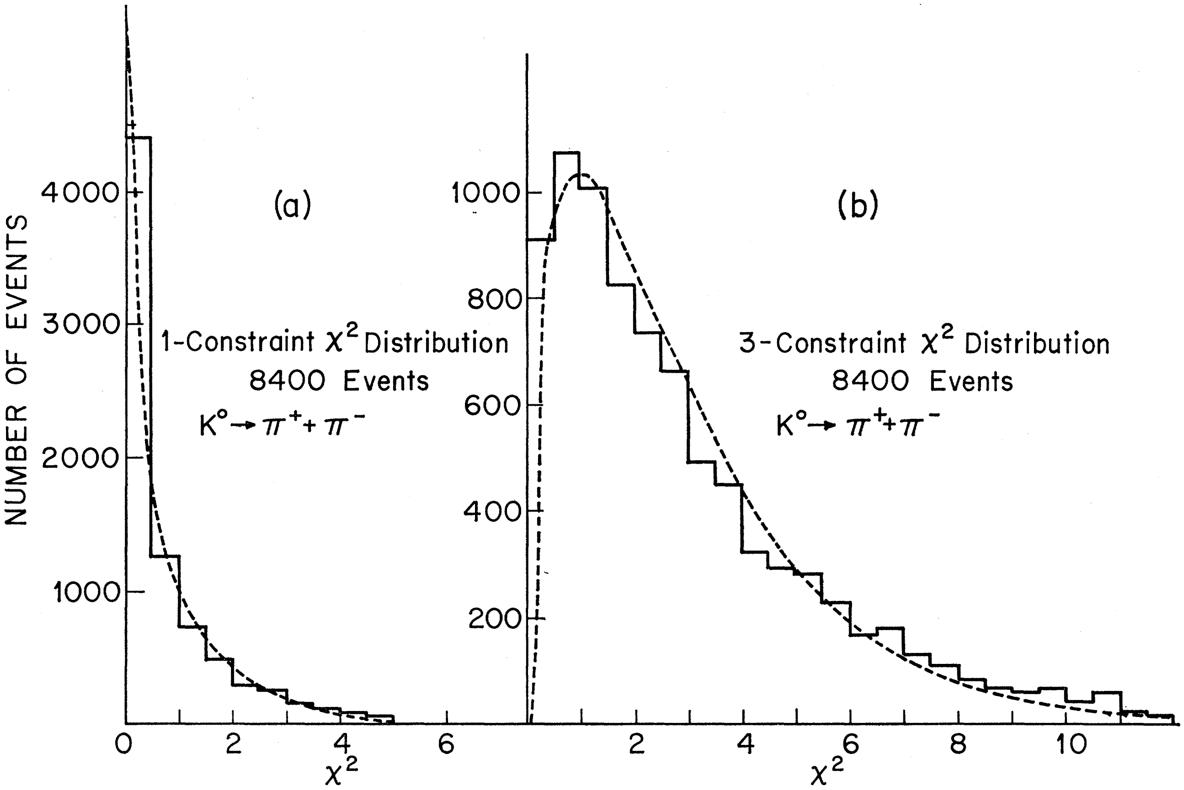

FIG. 1. χ^2 distributions for 8400 $K_S^0 \rightarrow \pi^+ + \pi^-$ decays: (a) one-constraint fit and (b) three-constraint fit. The smooth curves are the expected distributions.

probability was greater than 0.01% for the three-constraint fit was classified as a two-body decay and an over-all production and decay fit was attempted. A V which failed the three-constraint fit was fitted to other possible production origins. A small number of V's made the one-constraint decay fit at a probability of better than 2×10^{-5} but failed all production fits. These events come from three sources. Some two-body decays come from charge-exchange interactions in the chamber window. Some come from K^0's produced in the chamber which scatter before decaying; these events have been analyzed separately.[19] The remaining events are true leptonic decays which make the one-constraint 2π fit. This loss of leptonic events, which is estimated in the Appendix, is necessary in order to guarantee the removal of the dominant 2π mode. Within the final fiducial volume this loss is unbiased with respect to proper time or identification of the lepton and therefore does not affect the analysis of the time distributions, but necessitates a correction in the calculation of the total leptonic decay rate.

About 50 000 $K_S \rightarrow \pi^+\pi^-$ events were obtained and a subsample was used for a precision measurement of the K_S lifetime.[20] The χ^2 distributions for a sample of the three-constraint and the one-constraint 2π fits are shown in Fig. 1. The fraction of production origins with a visible spectator proton was 49%. The spectator momentum distribution for a sample of two-body events is shown in Fig. 2 and for our final sample of three-body events in Fig. 3. These distributions are in excellent agreement with the prediction of the impulse approximation.

An event which failed both the one- and three-constraint 2π fits was refitted to all possible production vertices under the assumption that the neutral K decayed by any one of the three-body decay modes $\pi e\nu$, $\pi\mu\nu$, $\pi^+\pi^-\pi^0$, and $\pi\pi\gamma$. The class of three-body events with visible spectators has four constraints for the over-all fit, while that with the invisible spectator has one constraint. The over-all χ^2 probability for a production of a K^0 followed by a three-body decay was required to be at least 2%. Each three-body candidate was remeasured twice, and was rejected if the χ^2 probability for the one-constraint 2π fit was greater than 2×10^{-5}, unless the fitted K^0 momentum had an unreasonable value or one of the tracks was an obvious electron. Neither of these cuts on χ^2 causes any time

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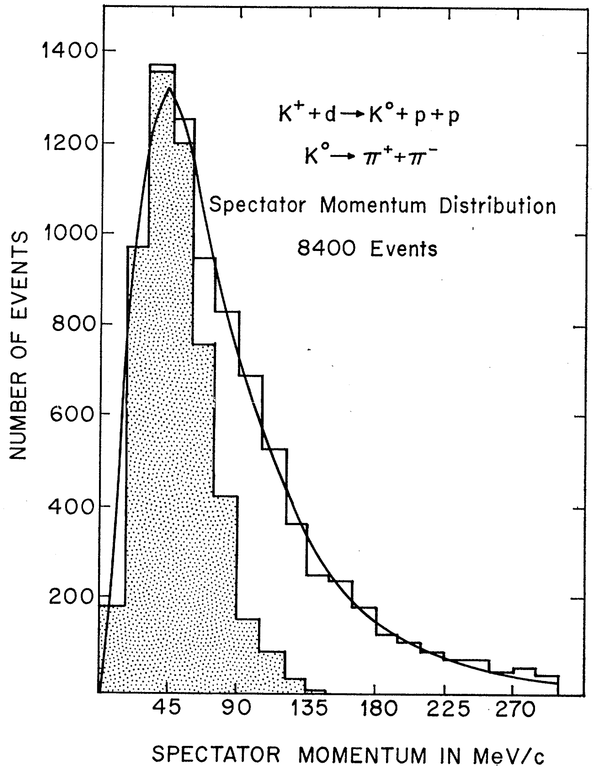

FIG. 2. Spectator momentum distribution for 8400 events $K^+ + d \rightarrow K^0 + p + p_S$ followed by $K_S{}^0 \rightarrow \pi^+ + \pi^-$. The shaded region contains events having invisible spectator protons. The smooth curve is the Hulthén distribution normalized to the total data shown.

bias. For all three-body candidates the association with a production vertex was unique. The resulting uncertainty in the fitted K^0 momentum was approximately 5%.

The kinematic overlaps among the $\pi\mu\nu$, $\pi e\nu$, and $\pi\pi\gamma$ are quite severe. In addition, a Monte Carlo study (see the Appendix) shows that the probability of an event fitting an incorrect hypothesis depends on the distance between the V and the production vertex. We, therefore, did not use kinematics to distinguish among the decay modes since it would have introduced a bias in the K^0 proper time. To identify the lepton track and separate these modes, visual ionization estimates and measurements of gap-length distributions were used. A few electrons were identified by δ rays. Muons were identified when they came to rest and decayed in the chamber; charge bias in the muon identification is negligible since the μ^- capture rate is about 0.2% of the decay rate.[21] These particle-identification techniques require a minimum track length; thus the identification may depend on the location of the decay vertex in the chamber. In order to eliminate any bias which may arise from this source, the fiducial volume was reduced by requiring all vertices to lie within a circle of 25 cm radius, centered at the middle of the chamber. With this reduced fiducial volume, which guarantees at least 8 cm of track, a Monte Carlo study indicates no time bias because of track identification.

In order to remove any possible scanning biases, we have imposed the following additional cuts, based in part upon our study of the K_S lifetime.[20] The minimum projected K^0 length was increased to 3 mm; a maximum projected length of 20 cm was imposed; the maximum dip angle of the K^0 and its charged decay products was decreased to 65°; the K^0 momentum was required to be in the interval 100–650 MeV/c; the opening angle between the charged tracks of the V was required to be less than 170°.

IV. IDENTIFIED LEPTONIC DECAYS

The experimental procedures outlined in Sec. III provide us with a sample of identified leptonic decays free of time or charge bias. This sample may also contain background events from other sources.

An event from the dominant 2π decay mode might not get rejected by the χ^2 criterion if one of the π tracks decayed in flight giving a μ in the forward direction which stopped and decayed in the chamber. For an in-flight decay of a π into a μ, $p_\pi \tan\theta$ must be less than 38 MeV/c, where p_π is the momentum of the π and θ is the space angle between the π and the μ. All possible three-body events were subjected to a two-vertex fit to the K^0 production followed by a two-pion decay in which first one and then the other pion was considered unmeasured. If p_π is the fitted momentum of the unmeasured track, and θ the angle between this fitted track and its measured direction, the above criterion will reject all π decays in flight. This cut was applied to the $K_{\mu3}$ decays and not to the K_{e3} decays.

Dalitz decays of the π^0, resulting from $K_S \rightarrow \pi^0\pi^0$, were eliminated by removing those events for which the invariant mass of the V tracks, assumed to be electrons, was less than 110 MeV/c^2.[22] The ionization of the tracks of the V was required to be consistent with that of an electron. We estimate that less than one Dalitz decay event survives this cut.

An off-beam muon or kaon which enters the chamber, comes to rest, and decays into an electron, may simulate a K_{e3} decay. Most of these events do not make a kinematic fit, and the rest were eliminated by observing track ionization, δ rays, and the change in curvature of the incoming track.

After all cuts, the sample of 215 events with known

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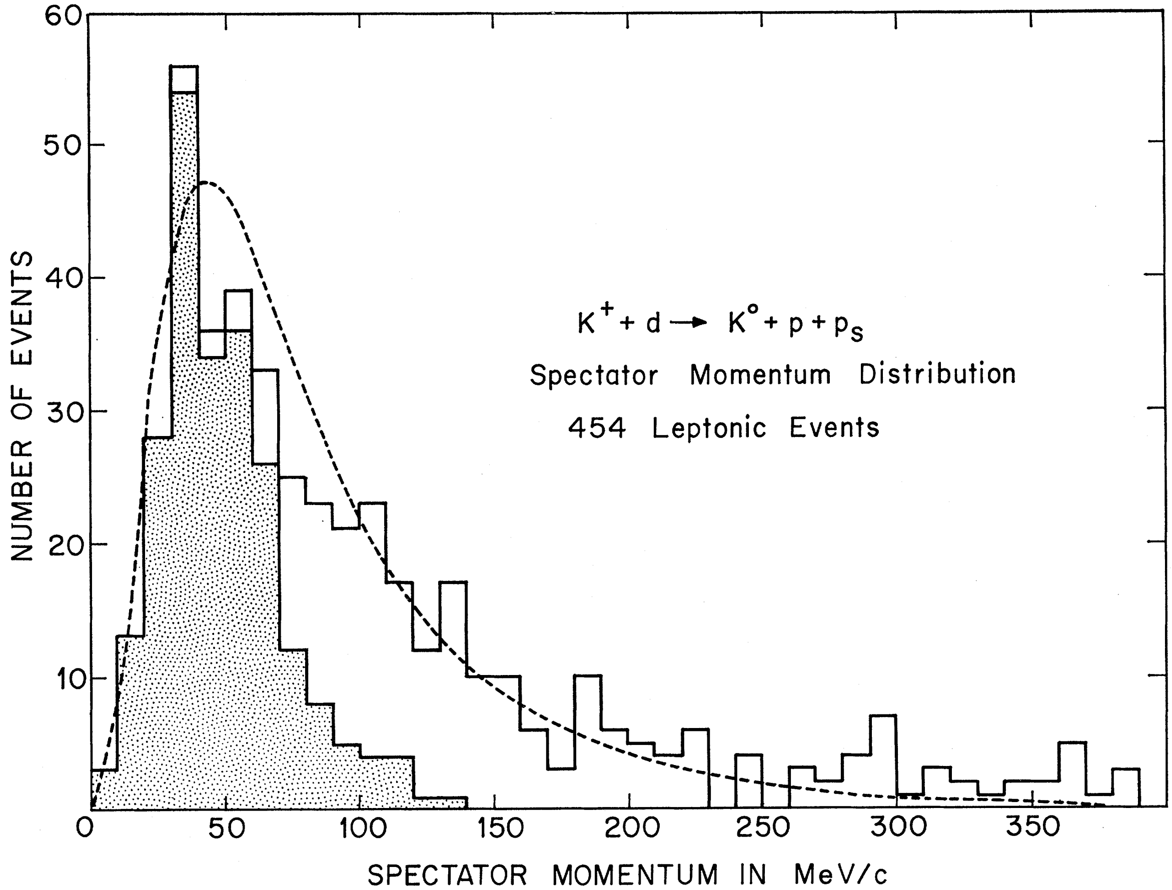

FIG. 3. Spectator momentum distribution for 454 events $K^+ + d \rightarrow K^0 + p + p_S$ followed by $K^0 \rightarrow l\pi\nu$ or $K^0 \rightarrow \pi\pi\gamma$. The shaded region contains events having invisible spectator protons. The smooth curve is the Hulthén distribution normalized to the total data shown.

lepton charge consists of

$K \rightarrow \pi^+ e^- \bar{\nu}$	69 events
$\rightarrow \pi^- e^+ \nu$	102 events
$\rightarrow \pi^+ \mu^- \bar{\nu}$	11 events
$\rightarrow \pi^- \mu^+ \nu$	15 events
$\rightarrow \pi^+ l^- \bar{\nu}$	2 events
$\rightarrow \pi^- l^+ \nu$	16 events,

where $l^\pm$ refers to a lepton for which we could identify the charge but not the mass. Events for which the charge of the lepton could not be determined were classified as ambiguous events and will be treated in Sec. V.

Figure 4 shows the distribution of pion kinetic energy in the K^0 rest frame, for the 171 identified K_{e3} decays. The curve shown is that predicted by a vector interaction with constant form factor and normalized to the observed number of events. It is in good agreement with the experimental data. Thus for this subsample of K_{e3} decays the identification is essentially independent of q^2, the four-momentum transferred to the lepton pair. On the other hand an examination of the Dalitz plot reveals that our efficiency for electron identification is optimum for $T_e \lesssim 0.4 T_e^{\max}$ and drops to zero for $T_e \simeq 0.8 T_e^{\max}$.

In order to estimate Rex and Imx, we have used the maximum-likelihood method. Equation (1) predicts for a given value of x, in addition to the time distribution of leptonic events, $N^\pm(t,x)$, the ratio of the number of positively charged to negatively charged leptons. This can be incorporated into the likelihood function[8] by writing the probability for the ith event as

$$\mathfrak{L}_i(x) = N^{q_i}(t_i,x) \bigg/ \int_{t_i^{\min}}^{t_i^{\max}} [N^+(t,x) + N^-(t,x)] dt\,,$$

where t_i is the observed proper time, $t_i^{\min}$ and $t_i^{\max}$ are the limits of the detectable time interval determined by the fiducial volume, and q_i is the sign of the lepton charge. Therefore, the joint likelihood function is given by

$$\mathfrak{L}(x) = \prod_{i=1}^{n} \mathfrak{L}_i(x)\,,$$

where n is the number of identified leptonic decays.

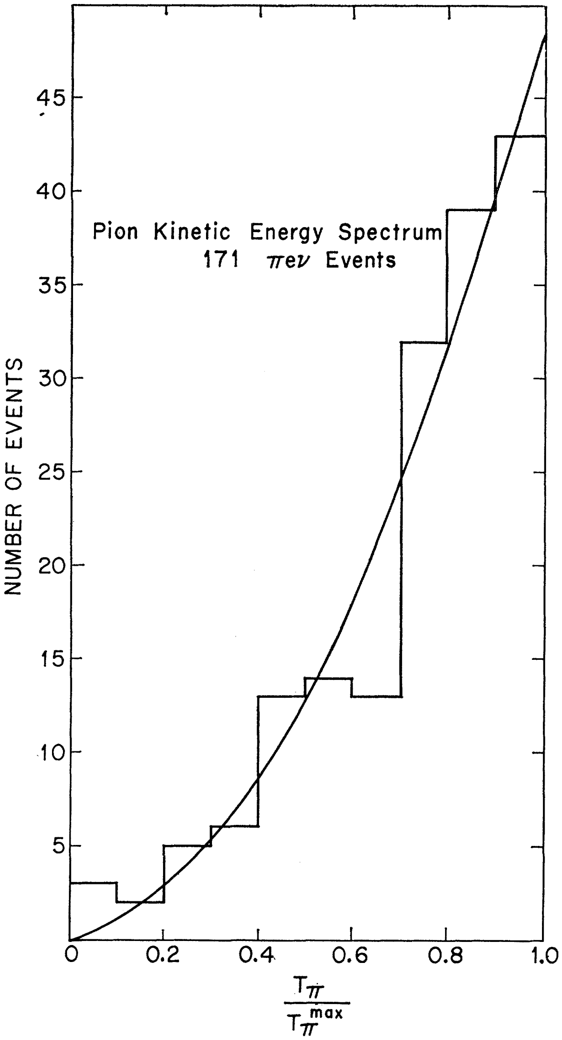

FIG. 4. Pion kinetic energy spectrum in the K^0 rest frame for 171 decays $K^0 \to \pi e\nu$. The solid curve is that predicted from a vector interaction with constant form factor.

From the 215 identified events, we obtain

$$\text{Re}x = 0.12 \pm 0.09, \quad \text{Im}x = -0.08 \pm 0.07.$$

The likelihood contours are shown in Fig. 5. Our estimate of the diagonal terms of the variance matrix comes from the $e^{-1/2}$ contour. The time distributions of positive and negative leptons are shown in Figs. 6 and 7. The dashed curves represent the expected distributions for $x=0$ folded with the geometric detection efficiency. The geometric efficiency is shown in Fig. 8. The solid curves represent the expected distributions based on our likelihood estimate of x. The χ^2 for the time distributions and charge ratio for $x=0$ is 16.1 for 12 degrees of freedom, corresponding to 18% probability. We have grouped the time intervals in such a way that in every interval at least nine counts were expected.

Using the 171 K_{e3} decays alone, we obtain

$$\text{Re}x = 0.08 \pm 0.10, \quad \text{Im}x = -0.07 \pm 0.08.$$

V. AMBIGUOUS EVENTS

In the sample of events for which the charge of the lepton is not known there are additional sources of background which must be considered.

The largest background in the ambiguous sample arises from the $\pi^+\pi^-\gamma$ decay mode. We assume the $\pi\pi\gamma$ contamination comes only from K_S decays since the present limit on the branching ratio $K_L \to \pi^+\pi^-\gamma / K_L \to$ all modes is $< 0.4 \times 10^{-3}$.[23] We also assume that the decay $K_S \to \pi^+\pi^-\gamma$ is only due to internal bremsstrahlung.[24] This has been partially confirmed by observation of the photon momentum spectrum[25] and by measurement of the total rate.[26] We reject most of the $\pi\pi\gamma$ events by removing events which fit with a momentum for the γ ray in the K^0 center-of-mass system of less than 50 MeV/c. We also exclude about 6% of the remaining $\pi\pi\gamma$ events which fit uniquely; the slight time bias introduced by this loss is negligible. We estimate that 60 ± 8 $\pi\pi\gamma$ events remain in the final sample. The effect of this contamination will be included in our analysis.

The dominant $K_S \to \pi^+\pi^-$ decay mode can give rise to three types of contamination in the ambiguous sample. The most serious type is caused by the nuclear scattering of a decay pion near the vertex of the V. This nuclear scattering is completely dominated by the $\Delta(1236)$. Half of the $\pi^- n$ scatters give rise to events with no visible spectator. We estimate that no more than two events of this type remain in our sample and our analysis will take this contamination into account.

Other background from the 2π decay mode can be caused by a Coulomb scatter or a decay in flight of a pion. The decays in flight were removed by the $p\pi \tan\theta$ cut described in Sec. IV. The small Coulomb scatters were removed in a similar manner. All events for which $p_{\text{fit}}\beta_{\text{fit}}\theta < 2500$ MeV/c degrees were rejected where p_{fit} and β_{fit} are the fitted momentum and velocity of the unmeasured track and θ is the space angle between the fitted and measured directions. These criteria reject all π decays in flight and allow at most one Coulomb scatter to survive.

Any three-body decay which fitted the $\pi^+\pi^-\pi^0$ hypothesis with a χ^2 probability greater than 2% was excluded from the sample. Estimates from our Monte Carlo calculation, described in the Appendix, show that

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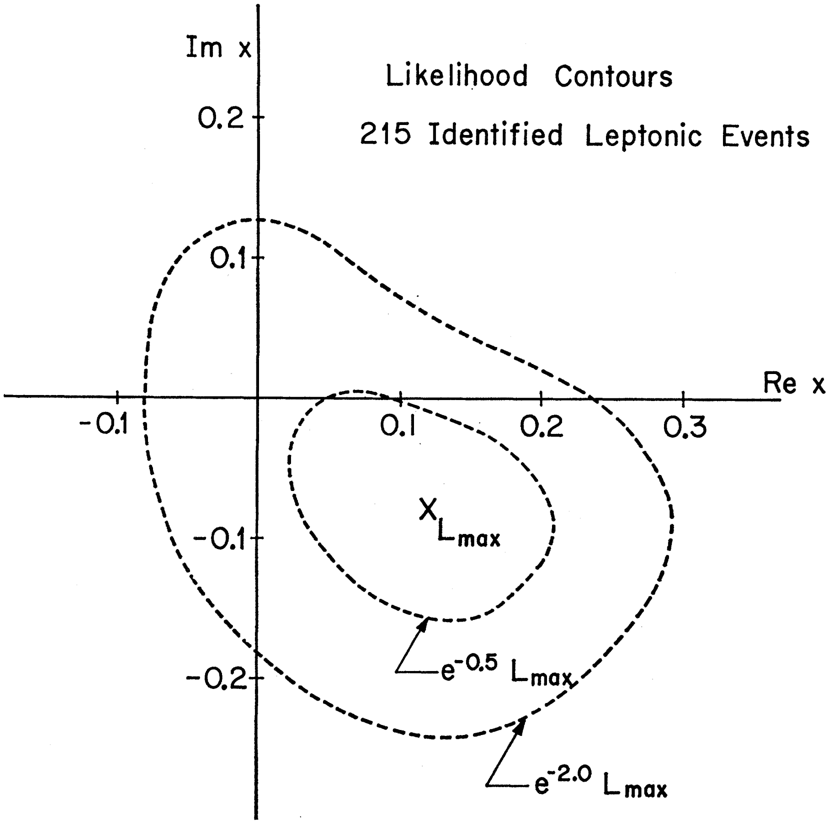

FIG. 5. Likelihood contour plot for 215 identified leptonic decays. The dashed curves are the contours for 1 and 2 standard deviations.

this criterion will eliminate about one leptonic and one $\pi\pi\gamma$ event. While this loss may be slightly time biased because of the use of kinematic identification, this bias is negligible.

We estimate that stray tracks contribute less than one event to our sample. After imposing all cuts, 239 ambiguous events remain in the sample, of which 62 ± 11 are K_S background events.

The time distribution for the ambiguous events is given by

$$N(t,x)=N^{+}(t,x)+N^{-}(t,x)+\alpha e^{-\lambda_S t},$$

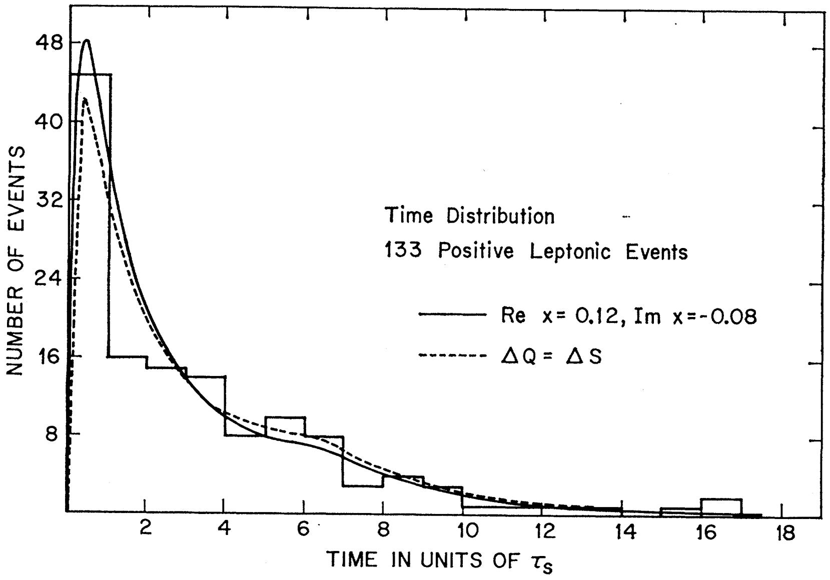

FIG. 6. Time distribution for 133 positive leptonic decays. The dashed curve is the expected distribution for $\Delta Q=\Delta S$ and the solid curve is for the best estimates of Rex and Imx.

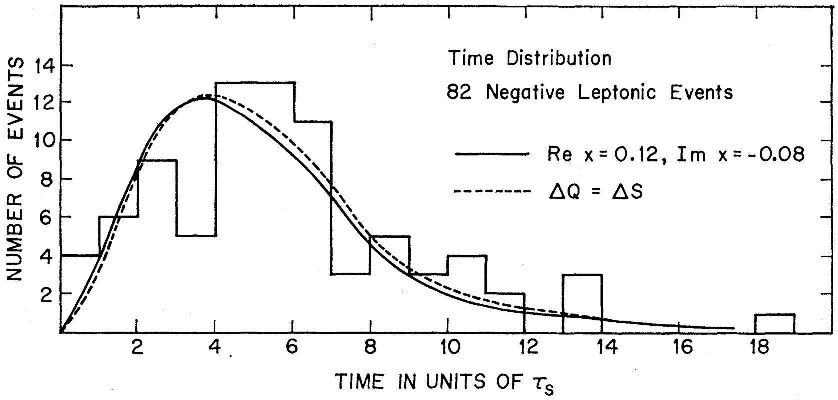

FIG. 7. Time distribution for 82 negative leptonic decays. The dashed curve is the expected distribution for $\Delta Q=\Delta S$ and the solid curve is for the best estimates of Rex and Imx.

where $N^+(t,x)$ and $N^-(t,x)$ are defined by Eq. (1) and the last term in the equation represents the sum of the contributions from the $\pi\pi\gamma$ decay mode and other K_S background. The probability for the ith event is

$$\mathcal{L}_i^{\rm amb}(x)=N(t_i,x)\Big/\int_{t_i^{\rm min}}^{t_i^{\rm max}}N(t,x)dt\,,$$

where $t_i^{\rm min}$ and $t_i^{\rm max}$ are the limits of the detectable time interval as determined by our fiducial volume. The joint likelihood function is

$$\mathcal{L}^{\rm amb}(x)=\prod_{i=1}^{n}\mathcal{L}_i^{\rm amb}(x)\,,$$

where n is the number of ambiguous events in the sample. The proper time for each ambiguous event is calculated independently of the decay mode by using the K^0 momentum from the production fit. From the ambiguous events, our likelihood estimates are

$$\text{Re}x=0.00\pm0.14\,,\quad \text{Im}x=-0.24_{-0.40}^{+0.28}\,.$$

The $e^{-1/2}$ likelihood contour is shown in Fig. 9. The estimated errors above include the effects of the uncertainty in α. The time distribution of these ambiguous events is shown in Fig. 10.

VI. CONCLUSIONS

The result from our 215 identified leptonic decays is

$$\text{Re}x=0.12\pm0.09\,,\quad \text{Im}x=-0.08\pm0.07\,. \qquad (2)$$

Since the uncertainty in the estimate of x from the ambiguous sample is large and since that sample has additional background problems, we feel it would be injudicious to merge the two samples. Our result (2) is consistent with the $\Delta S=\Delta Q$ rule. If the true value of x

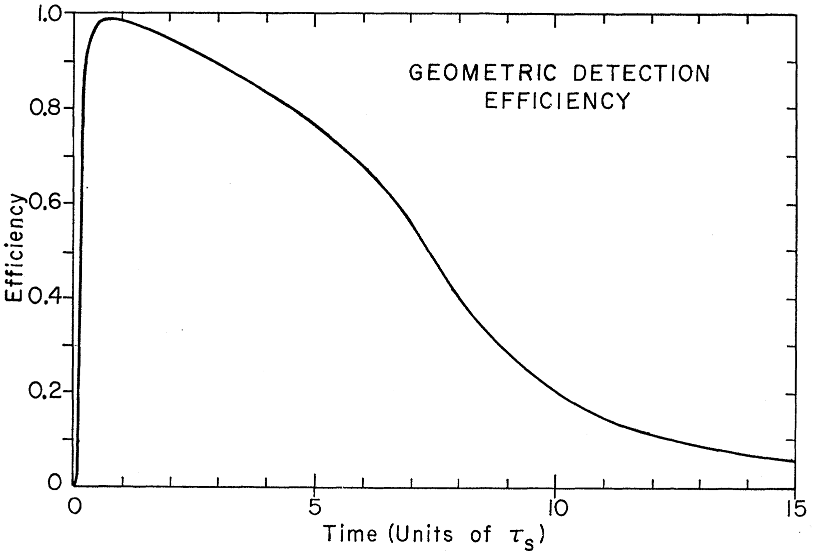

FIG. 8. Geometric detection efficiency.

is zero, the probability of getting $|x|\geq 0.15$ is 17% in a sample this size. The result of this experiment is displayed with results[4–10,12] of previous tests of this rule in Fig. 11.

We can also calculate the K_L leptonic rate from the observed number of three-body decays. Assuming the $\Delta S=\Delta Q$ rule, the number of leptonic events in the sample depends upon two parameters: $|f|^2$, the K_L leptonic rate, and α, the K_S background. The calculation of $|f|^2$ requires the efficiencies for retaining two- and three-body decays after all cuts have been imposed. These efficiencies can be calculated reliably provided that visual criteria are ignored. We have combined the identified leptonic events with the ambiguous sample using a common set of criteria for background rejection. In addition those identified leptonic decays have been removed for which the classification as three-body decays was based on visual criteria. This leaves a total sample of 393 events for the calculation of the K_L leptonic rate. We obtain

$$\Gamma_L(\text{leptonic})=(11.6\pm 0.9)\times 10^6\ \text{sec}^{-1}. \qquad (3)$$

The error in Γ_L is mainly due to the statistical uncertainty in the observed number of events, but the errors in α and $\Gamma_S(\pi^+\pi^-)$ have also been included.

Two previous experiments on leptonic K decay have yielded rates of $(9.85_{-1.05}^{+1.15})\times 10^6$ sec^{-1},[6] and $(12.2\pm 1.0)\times 10^6$ sec^{-1},[8] assuming $x=0$. The $\Delta I=\frac{1}{2}$ rule predicts from the K^+ leptonic rate[14] a rate for Γ_L(leptonic) of $(12.68\pm 0.19)\times 10^6$ sec^{-1}. Our result is consistent with these values.

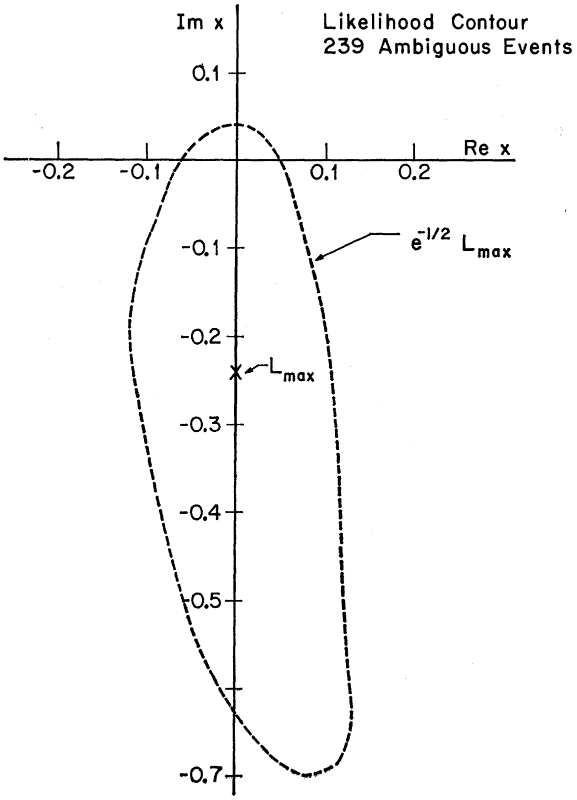

FIG. 9. Likelihood contour plot for 239 ambiguous events. The dashed curve is the contour for 1 standard deviation.

Our final results, (2) and (3), are consistent with our preliminary values.[11] Those results were based on a smaller sample of events which was subjected to less stringent cuts than the final data.

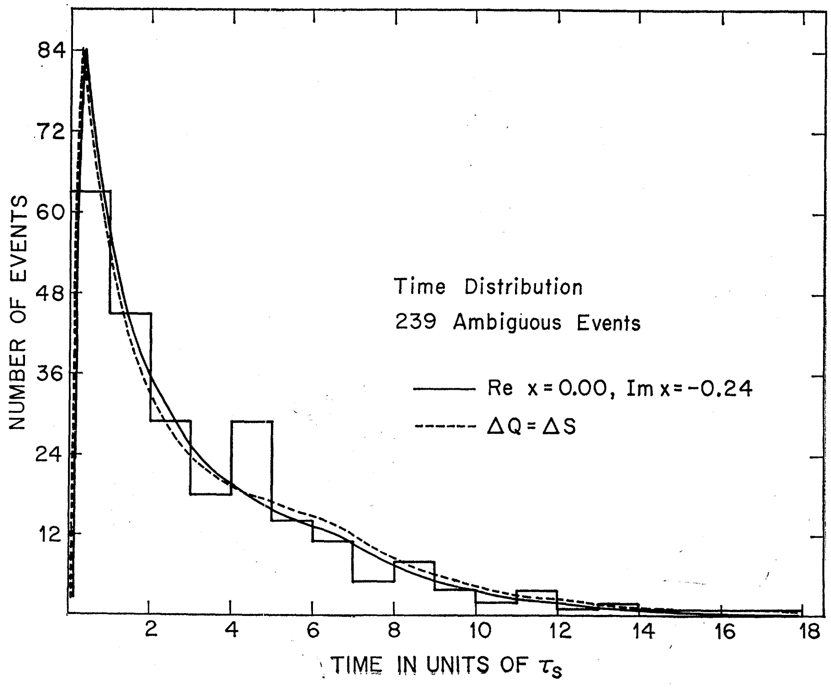

FIG. 10. Time distribution for 239 ambiguous events. The dashed curve is the expected distribution for $\Delta Q=\Delta S$ and the solid curve is for the best estimates of Rex and Imx.

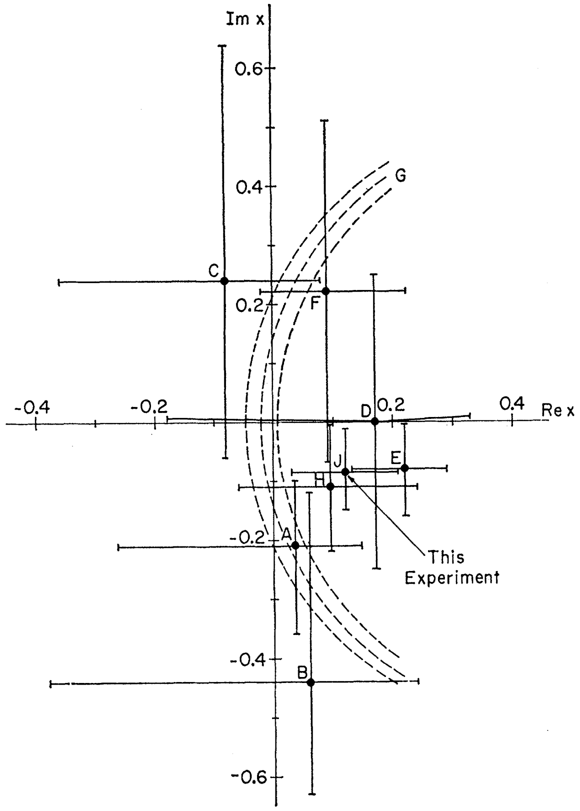

FIG. 11. Summary of the present and previous tests of the $\Delta Q=\Delta S$ rule. The experimental results shown are: *A*, Ref. 4; *B*, Ref. 5; *C*, Ref. 6; *D*, Ref. 7; *E*, Ref. 8; *F*, Ref. 9; *G*, Ref. 10; *H*, Ref. 12; *J*, this experiment.

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APPENDIX

Monte Carlo calculations have been done to study the losses of the various decay modes due to our geometric and kinematic criteria and possible biases caused by these selection criteria. For all calculations we have used a branching ratio of $K_L{}^0\to\pi\mu\nu/K_L{}^0\to\pi e\nu=0.68$ and $x=0$.

In order to incorporate the geometric detection efficiency, we have used information from a large sample of measured 2π-decay events. The K^0 momentum for each event was obtained from the kinematic fit. Decay times t_i were generated according to Eq. (1). The coordinates of the "leptonic decay" were calculated using the coordinates of the production vertex, the K^0 momentum for the 2π fit, and the generated t_i. If the generated events were within the fiducial volume they were accepted.

The decays $\pi e\nu$, $\pi\mu\nu$, $\pi\pi\gamma$, and $\pi\pi\pi$ were generated in the K^0 rest frame from appropriate matrix elements and transformed to the laboratory. For the leptonic decays, a vector interaction with $\xi=0$ was taken. For $K_S\to\pi\pi\gamma$, we have generated events according to the inner bremsstrahlung model.[24] For $K\to\pi^+\pi^-\pi^0$, we have used the following form for the matrix element squared[27]:

$$|M_{3\pi}|^2=1+2a(2T_0-T_0{}^{\max})(m_K/m_\pi{}^2)\,,$$

where T_0 is the kinetic energy of the π^0 and $a=-0.2$.

Track variables were generated in the chamber coordinate system and were changed randomly to simulate Gaussian measuring errors. These simulated events were processed through the same geometry and kinematics programs as were used to process the real data. Extensive checks were made to ensure that the errors in the track parameters of the generated events were consistent with those of measured events.

After subjecting these simulated events to the same geometric and kinematic criteria as the real data, it was found that 60% of $\pi\pi\gamma$, 57% of $\pi e\nu$, and 64% of $\pi\mu\nu$ events survived if ionization criteria are ignored. These numbers are used in our rate calculations. The kinematic overlaps among the various modes were also determined from these simulated events.

The identification criteria used in Sec. IV required a minimum length of the charged lepton track. From the Monte Carlo study, we determined that the final fiducial volume guaranteed no time bias in the identified leptonic sample.

Use of kinematics to determine the sign of the lepton charge was found to be dependent upon the length of the neutral track and therefore was time biased.

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[27] B. Aubert, in *Proceedings of the Topical Conference on Weak Interactions* (CERN, Geneva, 1969), p. 205.