Electromagnetic Contributions to the Charge Asymmetry in the Semileptonic Decays of Neutral Kaons*

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While the measured charge asymmetry in the semileptonic decays of $K_L{}^0$ is clear-cut evidence for CP violation in these processes, the detailed theoretical interpretation of this result is somewhat complicated by the presence of electromagnetic effects. It is shown, however, that whereas these electromagnetic effects can produce sizable differences in decay distributions for decays into charge-conjugate states, their contribution to the asymmetry parameter $\delta_L{}^\pi$ is not greater than $\alpha\Gamma(K_{l4}{}^0)/\Gamma(K_{l3}{}^0)$.

I. INTRODUCTION

It is well known that the charge asymmetry in the semileptonic decays of neutral kaons is an important source of information concerning CP violation.[1] This asymmetry is defined as the difference between the rate of decay of a neutral kaon state into a state consisting of a charged lepton, a neutrino, and any number of hadrons and photons, and the rate of decay of the same state into the corresponding charge-conjugate state. In the case of $K_L{}^0$ decays, the existence of such an asymmetry automatically implies CP violation, since if CP were not violated by any interaction, then $K_L{}^0$ would be a CP eigenstate and its decay rates into charge-conjugate states would be equal. By now, four mutually consistent experiments have established the existence of such an asymmetry in $K_L{}^0$ decay[2–5]; this confirms the earlier finding of CP violation in the 2π decays of $K_L{}^0$.[6]

Now, while the fact that CP is violated is a simple deduction from the above-mentioned experimental asymmetry, the detailed theoretical interpretation of this result is not so straightforward. In the first place, there is a complication arising from the possible presence of $\Delta S=-\Delta Q$ transitions, but this can be overcome, as is known, by working with a regenerated kaon beam.[4] The second difficulty, and the one on which we wish to focus here, has to do with the effect of the electromagnetic interaction on these semileptonic decays. Since there are charged particles involved, it is of course clear that the electromagnetic interaction plays a role, but in most treatments of the charge asymmetry question it is assumed that this role is unimportant.[7] We should like to examine this assumption in order to see how far it is justified. We shall discover that while electromagnetic effects can produce sizable differences between the decay distribution for $K_L{}^0\to\pi^-l^+\nu_l$ and $K_L{}^0\to\pi^+l^-\bar{\nu}_l$, the electromagnetic contribution to the difference in the rates is extremely small, the correction to the usual formula for $\delta_L{}^\pi$ being of the order $\alpha\Gamma(K_{l4}{}^0)/\Gamma(K_{l3}{}^0)\simeq10^{-3}\alpha$. Hence the usual procedure of neglecting electromagnetic effects when computing $\delta_L{}^\pi$ is well justified.

We shall proceed as follows: In Sec. II we outline the basic facts about $K_{l3}{}^0$ and $\bar{K}_{l3}{}^0$ matrix elements. Sections III and IV we devote to a discussion of the charge asymmetry parameter in the $\pi l\nu$ decays of $K_L{}^0$, first neglecting electromagnetic effects (Sec. III) and then including them (Sec. IV). Finally, in Sec. V we show quite generally that electromagnetic contributions to this asymmetry are no greater than $\alpha\Gamma(K_{l4}{}^0)/\Gamma(K_{l3}{}^0)$.

II. $K_{l3}{}^0$ AND $\bar{K}_{l3}{}^0$ MATRIX ELEMENTS

We shall base our discussion on two fundamental assumptions:

(a) All interactions are TCP-invariant.

(b) Strangeness-changing semileptonic decays are first-order processes in a semileptonic weak-interaction Hamiltonian

$$H^{SL}=\sum_{l=e,\mu}H^l \quad\text{with}\quad H^l=\int d^3x\,H^l(x).$$

We are aware that either or both of these assumptions may be wrong, in which case our discussion will be no longer valid; generally, however, they are regarded as a sufficiently sound basis for discussions of the present kind and we shall accept them.

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[1] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[2] S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters **19**, 993 (1967).

[3] D. Dorfan, J. Enstrom, D. Raymond, M. Schwartz, and S. Wojcicki, Phys. Rev. Letters **19**, 987 (1967).

[4] S. Bennett, D. Nygren, H. Saal, J. Sunderland, and J. Steinberger, Phys. Letters **27B**, 239 (1968).

[5] J. Steinberger, in Proceedings of the Topical Conference on Weak Interactions, CERN, 1969 (unpublished).

[6] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[7] L. B. Okun and C. Rubbia have touched on this matter in their report in *Proceedings of the International Conference on Elementary Particles, Heidelberg, 1967*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968). Related work is to be found in N. Byers, S. McDowell, and C. N. Yang, *High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 953; I. B. Khriplovich and L. B. Okun, Phys. Letters **26B**, 672 (1967).

From assumption (a) it follows that the short- and long-lived neutral kaon states are given, respectively, by

$$\begin{pmatrix} |K_S{}^0\rangle \\ |K_L{}^0\rangle \end{pmatrix} = p|K^0\rangle \mp q|\bar{K}^0\rangle\,, \quad |p|^2+|q|^2=1\,, \quad (2.1)$$

where K^0 and $\bar{K}^0$ are eigenstates of the Hamiltonian of strong and electromagnetic interactions, H^S+H^γ, with $|\bar{K}^0\rangle$ defined in the appropriate one of the following three ways:

(i) $|\bar{K}^0\rangle \equiv C|K^0\rangle$ if H^S+H^γ is invariant under C;
(ii) $|\bar{K}^0\rangle \equiv -CP|K^0\rangle$ if H^S+H^γ is not invariant under C but is invariant under CP;
(iii) $|\bar{K}^0\rangle \equiv -TCP|K^0\rangle$ if H^S+H^γ is invariant neither under C nor under CP but only under TCP.

Assumption (b) has the consequence that the reduced T-matrix elements (defined below) for the transitions $K^0\to\pi l\nu$ and $\bar{K}_0\to\pi l\nu$ are given, respectively, by $\langle\pi l\nu_{\rm out}|H^l(0)|K_{\rm in}{}^0\rangle$ and $\langle\pi l\nu_{\rm out}|H^l(0)|\bar{K}_{\rm in}{}^0\rangle$, where the states in these expressions are "in" and "out" states of the Hamiltonian H^S+H^γ.

Now, there are four basic transitions to be considered:

$$(1) \quad K^0\to\pi^-l^+\nu_l\,, \quad (2.2)$$

$$(2) \quad \bar{K}^0\to\pi^-l^+\nu_l\,, \quad (2.3)$$

$$(3) \quad \bar{K}^0\to\pi^+l^-\bar{\nu}_l\,, \quad (2.4)$$

$$(4) \quad K^0\to\pi^+l^-\bar{\nu}_l\,. \quad (2.5)$$

Transitions 1 and 3 correspond to $\Delta S=\Delta Q$, while transitions 2 and 4 correspond to $\Delta S=-\Delta Q$.

Assuming only that the neutrino involved in these decays is a left-handed particle, we may write the reduced T-matrix elements for processes 1 and 2 as[8]

$$\tilde{T}_j=\langle\pi^-l^+\nu_{l\,\rm out}|H^l(0)|K^0(\bar{K}^0)_{\rm in}\rangle = \frac{G^1}{\sqrt{2}}\frac{1}{(2\pi)^6}\left(\frac{m_lm_\nu}{4k_0\pi_0l_0\nu_0}\right)^{1/2}\bar{u}(\nu)(1-\gamma_5) \times[A_j+iB_j\gamma\cdot(k+\pi)]v(l)\,, \quad j=1,2 \quad (2.6)$$

and for the processes 3 and 4 we have, in like manner,

$$\tilde{T}_j=\langle\pi^+l^-\bar{\nu}_{l\,\rm out}|H^l(0)|K^0(\bar{K}^0)_{\rm in}\rangle = \frac{G^1}{\sqrt{2}}\frac{1}{(2\pi)^6}\left(\frac{m_lm_\nu}{4k_0\pi_0l_0\nu_0}\right)^{1/2}\bar{u}(l)[A_j+iB_j\gamma\cdot(k+\pi)] \times(1+\gamma_5)v(\nu)\,, \quad j=3,4. \quad (2.7)$$

In Eqs. (2.6) and (2.7), the reduced T-matrix element $\tilde{T}$ is defined in terms of the S-matrix element by

$$S_{fi}=\delta_{fi}\delta^4(p_f-p_i)-i(2\pi)^4\delta^4(p_f-p_i)\tilde{T}_{fi}\,.$$

G^1 is the coupling constant for strangeness-changing semileptonic processes; k, π, l, and ν are four-vectors representing the four-momenta of the corresponding particles; the γ matrices are chosen Hermitian with $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$; the factors $(1\pm\gamma_5)$ appear because of the assumed two-component nature of the neutrino; and the A_j and B_j are scalar functions of the invariant variables s and t defined by

$$s=-(k-l)^2\,, \quad t=-(k-\pi)^2\,. \quad (2.8)$$

We now make a separation of each of the functions A_j and B_j into a part which arises from the weak interaction alone and a part which comes from second-order electromagnetic corrections to the weak interaction. (In principle, we could consider higher-order electromagnetic corrections, but we shall not do so since they are probably too small to play any significant role in the phenomena we are discussing.) Thus we write

$$A_j=a_j+\alpha c_j=a_j+\alpha(c_j{}^++c_j{}^-)\,, \quad (2.9a)$$

$$B_j=b_j+\alpha d_j=b_j+\alpha(d_j{}^++d_j{}^-)\,. \quad (2.9b)$$

In the V, A theory of weak interactions, a_j and b_j are given, respectively, by

$$\begin{aligned} a_j&=m_lf_-{}^j(t) \quad (j=1,2)\\ &=-m_lf_-{}^j(t) \quad (j=3,4)\,,\\ b_j&=f_+{}^j(t)\,, \end{aligned} \quad (2.10)$$

where $f_+{}^j(t)$ and $f_-{}^j(t)$ are the familiar K_{l3} form factors. Notice that in Eqs. (2.9a) and (2.9b) we have introduced a further separation of the functions c_j and d_j into their CP-even $(+)$ and CP-odd $(-)$ parts. This separation corresponds to the fact that, to the order to which we are working, the matrix elements may be written as

$$\langle\pi l\nu_{\rm out}|H^l(0)|K^0(\bar{K}^0)_{\rm in}\rangle = \langle\pi l\nu|H^l(0)+T(H^l(0)S^{(2)\gamma})|K^0(\bar{K}^0)\rangle\,, \quad (2.11)$$

where the states on the right-hand side of this equation are eigenstates of the Hamiltonian of strong interactions, and $S^{(2)\gamma}$ is the second-order S matrix of the electromagnetic interaction. Now, in general, $H^l(0)$ and $S^{(2)\gamma}$ both contain CP-even and CP-odd parts. (This is true in the case of $S^{(2)\gamma}$, for example, if we adopt the explanation of CP violation proposed by Bernstein *et al.*,[9] according to which the electromagnetic interaction of hadrons violates C and CP.) We may express this by writing in an obvious notation:

$$H^l(0)=H_+{}^l(0)+H_-{}^l(0)\,, \quad (2.12a)$$

$$S^{(2)\gamma}=S_+{}^{(2)\gamma}+S_-{}^{(2)\gamma}\,. \quad (2.12b)$$

Then the CP-even parts of c_j and d_j arise from the CP-even part of $T(H^l(0)S^{(2)\gamma})$, namely, $T(H_+{}^l(0)S_+{}^{(2)\gamma}+H_-{}^l(0)S_-{}^{(2)\gamma})$, while their CP-odd parts arise from its CP-odd part $T(H_-{}^l(0)S_+{}^{(2)\gamma}+H_+{}^l(0)S_-{}^{(2)\gamma})$. We

[8] C. Ryan, in *Lectures in High-Energy Physics II*, edited by H. H. Aly (Gordon and Breach, Science Publishers, Inc., New York, to be published).

[9] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139B**, 1850 (1965).

remark also that the functions $c_j^{\pm}$ and $d_j^{\pm}$ are essentially complex, since they describe final-state interactions.

Next we list the relations satisfied by functions appearing in Eqs. (2.9a) and (2.9b) corresponding to the various possibilities of CP invariance or noninvariance of $H^l(0)$ and $S^{(2)\gamma}$.

(*i*) *$H^l(0)$ CP-invariant; $S^{(2)\gamma}$ CP-invariant:*

a_j, b_j real with $a_{3,4}=a_{1,2}$, $b_{3,4}=-b_{1,2}$; also $c_{3,4}^{+}=c_{1,2}$, $d_{3,4}^{+}=-d_{1,2}^{+}$;

$$c_j^{-}=d_j^{-}=0. \quad (2.13a)$$

(*ii*) *$H^l(0)$ CP-invariant; $S^{(2)\gamma}$ CP-noninvariant:*

Relations the same as for (*i*) except that now c_j^- and d_j^- are nonzero and satisfy $c_{3,4}^{-}=-c_{1,2}^{-}$,

$$d_{3,4}^{-}=d_{1,2}^{-}. \quad (2.13b)$$

(*iii*) *$H^l(0)$ CP-noninvariant; $S^{(2)\gamma}$ CP-invariant:*

a_j, b_j complex with $a_{3,4}=(a_{1,2})^*$, $b_{3,4}=-(b_{1,2})^*$; also $c_{3,4}^{\pm}=\pm c_{1,2}^{\pm}$, $d_{3,4}^{\pm}=\mp d_{1,2}^{\pm}$. (2.13c)

(*iv*) *$H^l(0)$ CP-noninvariant; $S^{(2)\gamma}$ CP-noninvariant:*

Relations the same as for (*iii*). (2.13d)

Possibility (*i*) is the one which obtains in the superweak theory of CP violation,[10] possibility (*ii*) is the one realized in the theory of Bernstein *et al.*,[9] while in the ordinary weak theory of CP violation it is possibility (*iii*) that occurs. We shall make use of this analysis in Sec. IV.

III. CHARGE ASYMMETRY IN $\pi l\nu$ DECAYS OF K_L^0 WITH ELECTROMAGNETISM NEGLECTED

For purposes of orientation it seems worthwhile at this stage to go through the usual derivation of the expression for the charge asymmetry in the decays of K_L^0 into $\pi l\nu$ where the effect of the electromagnetic interaction is neglected. We begin by noting that from the discussion of Sec. II it follows that the reduced T-matrix elements for the decays $K_L^0\to\pi^-l^+\nu_l$ and $K_L^0\to\pi^+l^-\bar{\nu}_l$ are given, respectively, by

$$\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K_{L\,\mathrm{in}}{}^0\rangle=p\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle+q\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|\bar{K}_{\mathrm{in}}{}^0\rangle, \quad (3.1)$$

$$\langle\pi^+l^-\bar{\nu}_{l\,\mathrm{out}}|H^l(0)|K_{L\,\mathrm{in}}{}^0\rangle=p\langle\pi^+l^-\bar{\nu}_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle+q\langle\pi^+l^-\bar{\nu}_l|H^l(0)|\bar{K}_{\mathrm{in}}{}^0\rangle. \quad (3.2)$$

Squaring these matrix elements, doing the polarization sums and the phase-space integration, and making use of TCP invariance in the form of the relations

$$\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K^0(\bar{K}^0)_{\mathrm{in}}\rangle=\langle\pi^+l'^-\bar{\nu}_{l\,\mathrm{in}}|H^l(0)|\bar{K}^0(K^0)_{\mathrm{in}}\rangle^* \quad (3.3)$$

and

$$\langle\pi^+l^-\bar{\nu}_{l\,\mathrm{out}}|H^l(0)|K^0(\bar{K}^0)_{\mathrm{in}}\rangle=\langle\pi^-l'^+\nu_{l\,\mathrm{in}}|H^l(0)|\bar{K}^0(K^0)_{\mathrm{in}}\rangle^* \quad (3.4)$$

(l' denotes the spin-flipped state of l), we find the following for the sum and difference of the decay rates:

$$\begin{aligned}\Gamma_L(\pi^-l^+\nu_l)\pm\Gamma_L(\pi^+l^-\bar{\nu}_l)=(2\pi)^7\sum_{\mathrm{pol}}\int\delta^4(k-\pi-l-\nu)d^3\pi d^3l d^3\nu\{|p|^2|\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle|^2\\+|q|^2|\langle\pi^+l^-\bar{\nu}_{l\,\mathrm{in}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle|^2\pm|p|^2|\langle\pi^+l^-\bar{\nu}_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle|^2\pm|q|^2|\langle\pi^-l^+\nu_{l\,\mathrm{in}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle|^2\\+pq^*[\langle\bar{K}_{\mathrm{in}}{}^0|H^l(0)|\pi^-l^+\nu_{l\,\mathrm{out}}\rangle\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle\pm\langle\bar{K}_{\mathrm{in}}{}^0|H^l(0)|\pi^-l^+\nu_{l\,\mathrm{in}}\rangle\langle\pi^-l^+\nu_{l\,\mathrm{in}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle]\\+p^*q[\langle K_{\mathrm{in}}{}^0|H^l(0)|\pi^-l^+\nu_{l\,\mathrm{out}}\rangle\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|\bar{K}_{\mathrm{in}}{}^0\rangle\pm\langle K_{\mathrm{in}}{}^0|H^l(0)|\pi^-l^+\nu_{l\,\mathrm{in}}\rangle\langle\pi^-l^+\nu_{l\,\mathrm{in}}|H^l(0)|\bar{K}_{\mathrm{in}}{}^0\rangle]\}. \quad (3.5)\end{aligned}$$

Notice that in this expression we have "in" rather than "out" $\pi l\nu$ states in every second term; this is because we have made use of the relations (3.3) and (3.4) to replace the terms (involving out states) which originally stood in those positions. Now, as we have said, the usual discussion of this problem neglects the electromagnetic effects completely. In that approximation, one can simply disregard the distinction between the in and out $\pi l\nu$ states in (3.5), because in the absence of electromagnetism there is no interaction between the particles in these states. Thus with the neglect of electromagnetism and the additional postulate that

$$\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|\bar{K}_{\mathrm{in}}{}^0\rangle=X\langle\pi^-l^+\nu_{l\,\mathrm{out}}|H^l(0)|K_{\mathrm{in}}{}^0\rangle \quad (3.6)$$

(X, which is assumed constant, can be regarded as a measure of the violation of the $\Delta S=\Delta Q$ rule in the $\pi l\nu$ decays of neutral kaons), it immediately follows that the charge asymmetry in these decays is given by

$$\delta_L{}^\pi=\frac{\Gamma_L(\pi^-l^+\nu_l)-\Gamma_L(\pi^+l^-\bar{\nu}_l)}{\Gamma_L(\pi^-l^+\nu_l)+\Gamma_L(\pi^+l^-\bar{\nu}_l)}=(|p|^2-|q|^2)\frac{1-|X|^2}{|1+X|^2+O(\epsilon)}, \quad (3.7)$$

where $\epsilon=(p-q)/(p+q)$ is probably not greater than 3×10^{-3} in absolute value,[11] so that the term $O(\epsilon)$ in (3.7) may be safely neglected.

[10] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1967).

[11] We derive this limit from the phenomenological relationship $\epsilon=\frac{2}{3}\eta_{+-}+\frac{1}{3}\eta_{00}$, the value $|\eta_{+-}|=1.90\times10^{-3}$ [J. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 281], and the assumed upper limit $|\eta_{00}|<5\times10^{-3}$ (J. Steinberger, Ref. 5).

It is on the basis of the relation (3.7) that the theoretical analysis of the experimentally measured charge asymmetry in K_L^0 has been carried out. Using this relation and another similar one for the time dependence of the charge asymmetry in the decay of a regenerated neutral kaon state (also derived with the neglect of electromagnetic effects), Bennett *et al.*[12] have deduced values for $|p|^2-|q|^2=4\,\text{Re}\epsilon$ and $(1-|X|^2)/(|1+X|^2)$ separately. We must recall, however, that in deriving the formula (3.7) we have neglected terms which in principle could be of order α, and since the measured asymmetry δ_L^π is in the range $2-4\times10^{-3}$, it is at least a possibility that the terms neglected in Eq. (3.7) might be comparable in magnitude to the effect itself. It is necessary, therefore, to have some estimate of the electromagnetic corrections to the formula (3.7).

In clarification of a point made above, we should explain that when we said that in deriving Eq. (3.7) we had neglected terms which, in principle, could be order α, we did not mean that conventional electromagnetic effects by themselves could produce such terms. What is needed to produce any correction to the formula (3.7) is obviously a combination of electromagnetic effects *and CP* violation in either the electromagnetic or weak interaction; for, on the one hand, as we have seen, (3.7) is an exact formula if electromagnetic effects are neglected while, on the other hand, if *CP* is conserved in strong, electromagnetic, and weak interactions (with *CP* violation restricted to the superweak interaction[10]), we can use the *CP* operation rather than *TCP* to obtain, instead of (3.3) and (3.4), relations of the form

$$\langle\pi^-l^+,\,\nu_{l\,\text{out}}|H^l(0)|K^0(\bar{K}^0)_{\text{in}}\rangle =\langle-\pi^+,\,-l^-,\,-\bar{\nu}_{l\,\text{out}}|H^l(0)|-\bar{K}^0(-K^0)_{\text{in}}\rangle, \quad (3.8)$$

$$\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K^0(\bar{K}^0)_{\text{in}}\rangle =\langle-\pi^-,\,-l^+,\,-\nu_{l\,\text{out}}|H^l(0)|-\bar{K}^0(-K^0)_{\text{in}}\rangle \quad (3.9)$$

(the minus signs signify changes in sign of the corresponding three-momenta), and, with the use of these relations, (3.7) again becomes an exact formula. Thus, strictly speaking, we should say that corrections to (3.7) are of order $\alpha\lambda$, where λ is a measure of the strength of the *CP*-violating interaction. Now, in weak theories of *CP* violation, λ is expected to be of order 10^{-3} and so the corrections may be very small; in the electromagnetic theory of *CP* violation,[9] λ could be of order 1, in which case the corrections might be appreciable. It is with this latter case that we are especially concerned. Let us therefore proceed to a discussion of δ_L^π which does not neglect electromagnetic effects.

IV. CHARGE ASYMMETRY IN $\pi l\nu$ DECAYS OF K_L^0 WITH ELECTROMAGNETIC EFFECTS INCLUDED

For the purpose of the present calculation, we write out the reduced T-matrix elements for the decays $K_L^0\to\pi^-l^+\nu_l$ and $K_L\to\pi^+l^-\bar{\nu}_l$ explicitly as follows:

$$\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|K_L^0\rangle =\frac{G^1}{\sqrt{2}}\frac{1}{(2\pi)^6}\left(\frac{m_lm_\nu}{4k_0\pi_0l_0\nu_0}\right)^{1/2}\bar{u}(\nu)(1-\gamma_5) \times[A_L^++iB_L^+\gamma\cdot(k+\pi)]v(l), \quad (4.1)$$

$$\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_L^0\rangle =\frac{G^1}{\sqrt{2}}\frac{1}{(2\pi)^6}\left(\frac{m_lm_\nu}{4k_0\pi_0l_0\nu_0}\right)^{1/2}\bar{u}(l)[A_L^-+iB_L^-\gamma\cdot(k+\pi)] \times(1+\gamma_5)v(\nu). \quad (4.2)$$

The superscripts + and − on the amplitudes in these equations refer to the charge of the charged lepton, and, obviously, from the definitions (2.1)–(2.7), we have

$$A_L^+=pA_1+qA_2,\quad B_L^+=pB_1+qB_2, \quad (4.3)$$

$$A_L^-=pA_4+qA_3,\quad B_L^-=pB_4+qB_3. \quad (4.4)$$

A straightforward calculation shows that the differential decay rates for the two decays under consideration are given by[8]

$$\frac{d\Gamma_L^\pm}{ds\,dt}=\tfrac{1}{2}(G^1)^2\frac{1}{(2\pi)^3}\frac{1}{4m_k^3}\{\tfrac{1}{2}(t-m_l^2)|A_L^\pm|^2\pm m_l[(s-m_\pi^2)-\tfrac{1}{2}(t-m_l^2)](A_L^\pm(B_L^\pm)^*+(A_L^\pm)^*B_L^\pm) -\tfrac{1}{2}[(t-m_l^2)^2+(2s-t)(t-m_l^2)+4(s-m_\pi^2)(s-m_k^2)]|B_L^\pm|^2\}, \quad (4.5)$$

where s and t are the invariant variables defined in Eq. (2.8). On using Eqs. (4.3) and (4.4) as well as Eqs. (2.9) and (2.13), we find for the difference between the decay distributions in (4.5) the expression

$$\frac{d\Gamma_L(\pi^-l^+\nu_l)}{ds\,dt}-\frac{d\Gamma_L(\pi^+l^-\bar{\nu}_l)}{ds\,dt}=\tfrac{1}{2}(G^1)^2\frac{1}{(2\pi)^3}\frac{1}{4m_k^3}\{\tfrac{1}{2}(t-m_l^2)[(|p|^2-|q|^2)(|a_1|^2-|a_2|^2)+2\alpha\,\text{Re}(a_1+a_2)\,\text{Re}(c_1^-+c_2^-)$$

$$+2\alpha\,\text{Im}(a_1+a_2)\,\text{Im}(c_1^++c_2^+)]+m_l[(s-m_\pi^2)+\tfrac{1}{2}(t-m_l^2)][(|p|^2-|q|^2)(a_1b_1^*+a_1^*b_1-a_2b_2^*-a_2^*b_2)$$

$$+2\alpha\,\text{Re}(a_1+a_2)\,\text{Re}(d_1^-+d_2^-)+2\alpha\,\text{Im}(a_1+a_2)\,\text{Im}(d_1^++d_2^+)$$

[12] S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters **19**, 997 (1967); Phys. Letters **27B**, 244 (1968); **27B**, 288 (1968); cf. J. Steinberger, Ref. 5.

$$+2\alpha\,\mathrm{Re}(b_1+b_2)\,\mathrm{Re}(c_1^-+c_2^-)+2\alpha\,\mathrm{Im}(b_1+b_2)\,\mathrm{Im}(c_1^++c_2^+)]$$
$$-\tfrac{1}{2}[(t-m_l^2)^2+(2s-t)(t-m_l^2)+4(s-m_\pi^2)(s-m_k^2)][(|p|^2-|q|^2)(|b_1|^2-|b_2|^2)$$
$$+2\alpha\,\mathrm{Re}(b_1+b_2)\,\mathrm{Re}(d_1^-+d_2^-)+2\alpha\,\mathrm{Im}(b_1+b_2)\,\mathrm{Im}(d_1^++d_2^+)]\}\,, \quad (4.6)$$

where we have neglected all terms of second order in CP violation as well as those of order $\alpha(|p|^2-|q|^2)$. If one integrates this quantity over the region of the Dalitz plot, one obtains the numerator of the quantity $\delta_L{}^\pi$, Eq. (3.7).

Before coming to this integration, let us look at the general features of the expression (4.6). In the first place, we see that it contains a term proportional to $|p|^2-|q|^2$, and this is the term which gives rise to the result obtained in Eq. (3.7). In addition, however, we have two terms proportional to α. The first of these involves combinations of functions like $\mathrm{Re}(a_1+a_2)\times\mathrm{Re}(c_1^-+c_2^-)$, and this term will be present as long as there is a CP-violating part in either the weak or the electromagnetic interaction. In the case that CP violation occurs only in the weak interaction, $\mathrm{Re}(c_1^-+c_2^-)$, and $\mathrm{Re}(d_1^-+d_2^-)$ will be proportional to the parameter of weak CP violation, and since this is taken to be of order 10^{-3}, the term in question is likely to be very small. If, however, there is CP violation in the electromagnetic interaction, $\mathrm{Re}(c_1^-+c_2^-)$ and $\mathrm{Re}(d_1^-+d_2^-)$ could be of the same order as $\mathrm{Re}(a_1+a_2)$ and $\mathrm{Re}(b_1+b_2)$, in which case the first electromagnetic term in (4.6) could be quite sizable and could, on integration over the Dalitz plot, give a contribution of order $\alpha\Gamma_L(\pi l\nu)$ to the numerator of $\delta_L{}^\pi$. This is the correction to the quantity $\delta_L{}^\pi$ we are mainly concerned with. Unfortunately, no calculation of the functions c_j^- and d_j^- is available for this case, and so the discussion must remain at the qualitative level. However, one very important fact which has not been appealed to in this discussion is that of TCP invariance. In Sec. V we shall show from TCP invariance and the completeness of out and in states that the electromagnetic contribution to the numerator of $\delta_L{}^\pi$ is, in fact, a great deal smaller than $\alpha\Gamma_L(\pi l\nu)$.

The second term proportional to α in (4.6) is the one which involves the quantities $\mathrm{Im}(a_1+a_2)$ and $\mathrm{Im}(b_1+b_2)$. This term is present only if there is CP violation in the weak interaction. Now, as remarked above, the parameter of this violation is expected to be about 10^{-3}, and so this second electromagnetic term in (4.6) is likely to be negligible. It is possible to check this in a model calculation. We take the radiative corrections to K^0, $\bar{K}^0\to\pi l\nu$ as calculated by Ginsberg[13] in lowest-order perturbation theory using a phenomenological vector interaction with constant form factors and neglecting $\Delta S=-\Delta Q$ transitions. In this model, all amplitudes with the subscripts 2 and 4 (i.e., the $\Delta S=-\Delta Q$ amplitudes) vanish, while the remaining ones are given by

$$a_1=f_+\,,\quad \alpha c_1=\tfrac{1}{2}f_+A\,, \quad (4.7)$$

$$b_1=m_lf_-\,,\quad \alpha d_1=\tfrac{1}{2}m_l[f_-B+f_+(A-B)]\,, \quad (4.8)$$

$$a_3=f_+{}^*\,,\quad \alpha c_3=\tfrac{1}{2}f_+{}^*A'\,, \quad (4.9)$$

$$b_3=-m_lf_-{}^*\,,\qquad \alpha d_3=-\tfrac{1}{2}m_l[f_-{}^*B'+f_+{}^*(A'-B')]\,. \quad (4.10)$$

Here, A and B, which represent the electromagnetic corrections, have the forms

$$A=\frac{\alpha}{\pi}\left[\frac{3}{2}\ln\left(\frac{\Lambda}{m_l}\right)-1+2\ln\left(\frac{m_l}{\lambda}\right)+t_1-\frac{\frac{1}{2}t_2m_l^2(1-\xi)}{u}\right], \quad (4.11)$$

$$B=\frac{\alpha}{\pi}\left[-\frac{3}{2}\ln\left(\frac{\Lambda}{m_l}\right)-\frac{7}{4}+2\ln\left(\frac{m_l}{\lambda}\right)+t_1-\frac{2t_2}{1-\xi}\right], \quad (4.12)$$

where Λ is an ultraviolet cutoff and λ is the fictitious photon mass (or infrared cutoff), ξ is the quantity f_-/f_+, u is the invariant variable $-(k-\nu)^2$, and t_1 and t_2 are two functions of s and t whose explicit forms are given in Ref. 13. A' and B' are obtained from A and B, respectively, by making the substitution $\xi\to\xi^*$.

We see from Eqs. (4.11) and (4.12) that the electromagnetic corrections are logarithmically divergent. However, it turns out that all the divergences cancel out in the term in

$$\frac{d\Gamma_L(\pi^-l^+\nu_l)}{dsdt}-\frac{d\Gamma_L(\pi^+l^-\bar{\nu}_l)}{dsdt}\,,$$

which is proportional to α. In fact, one finds for this term

$$(\alpha/\pi)\,\mathrm{Im}\xi\; m_l^2\tfrac{1}{2}(G^1)^2(2\pi)^{-3}8m_k^{-3}|f_+|^2$$
$$\times u^{-1}[(s-m_k^2-m_l^2)(m_k^2-u)+(t-m_l^2)(m_k^2+u)]$$
$$\times\pi\frac{(u+m_\pi^2-m_l^2)}{[(u-m_\pi^2-m_l^2)^2-4m_l^2m_\pi^2]^{1/2}}\,, \quad (4.13)$$

with

$$u=m_k^2+m_\pi^2+m_l^2-s-t\,.$$

We see that this quantity is proportional to $\mathrm{Im}\xi$, which is a measure of CP violation in the weak interaction. When we now integrate (4.13) over the Dalitz plot to find its contribution to the difference of the rates for $K_L{}^0\to\pi^-l^+\nu_l$ and $K_L{}^0\to\pi^+l^-\bar{\nu}_l$, we find, rather surprisingly, that the result is zero. Technically, the reason we get zero for this integral is connected with the fact

[13] E. S. Ginsberg, Phys. Rev. **171**, 1675 (1968); **174**, 2169(E) (1968); **187**, 2280(E) (1969).

that the last function in (4.13), which describes the electromagnetic correction, is a function of u only and also that the t dependence of f_+ is neglected. If either of these features did not obtain, the integral would not necessarily vanish. We may conjecture, however, that this result is not simply a consequence of this particular model calculation and, in fact, it will turn out that it is, rather, an illustration of a more general result which we shall prove in Sec. V.

In summarizing the present section, we may say that the electromagnetic contribution to the difference between the decay distributions for the decays $K_L^0\to\pi^-l^+\nu_l$ and $K_L^0\to\pi^+l^-\bar{\nu}_l$ can provide a way of distinguishing between theories in which CP violation is located in the electromagnetic interaction and those where it occurs in the weak interaction; in the former case the difference in question should be of order α, while in the latter $\alpha\times10^{-3}$ is more likely.

V. EXACT RESULTS FOR THE MAGNITUDE OF ELECTROMAGNETIC CORRECTIONS TO δ_L^π

In order to obtain an exact result for the magnitude of the electromagnetic contribution to δ_L^π, we return to the expression for $\Gamma_L(\pi^-l^+\nu_l)-\Gamma_L(\pi^+l^-\bar{\nu}_l)$ in Eq. (3.5). We may write this as

$$\begin{aligned}\Gamma_L(\pi^-l^+\nu_l)-\Gamma_L(\pi^+l^-\bar{\nu}_l)=(2\pi)^7\sum_{\text{pol}}\int\delta^4(k-\pi-l-\nu)d^3\pi d^3l d^3\nu\\
\times\{\tfrac{1}{2}(|p|^2-|q|^2)[|\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2+|\langle\pi^-l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2\\
-|\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2+\tfrac{1}{2}[|\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2\\
-|\langle\pi^-l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2+|\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^+l^-\bar{\nu}_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2]\\
+pq^*[\langle\bar{K}_{\text{in}}^0|H^l(0)|\pi^-l^+\nu_{l\,\text{out}}\rangle\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle-\langle\bar{K}_{\text{in}}^0|H^l(0)|\pi^-l^+\nu_{l\,\text{in}}\rangle\langle\pi^-l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle]\\
+p^*q[\langle K_{\text{in}}^0|H^l(0)|\pi^-l^+\nu_{l\,\text{out}}\rangle\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|\bar{K}_{\text{in}}^0\rangle-\langle K_{\text{in}}^0|H^l(0)|\pi^-l^+\nu_{l\,\text{in}}\rangle\langle\pi^-l^+\nu_{l\,\text{in}}|H^l(0)|\bar{K}_{\text{in}}^0\rangle]\}.\quad(5.1)\end{aligned}$$

Let us consider this expression term by term.

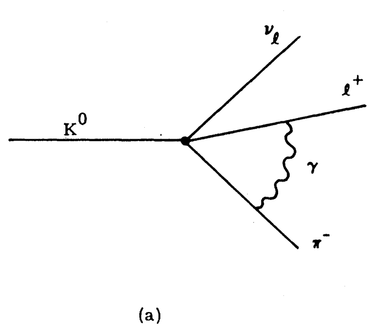
The first term is proportional to $\text{Re}\epsilon(|p|^2-|q|^2\simeq4\,\text{Re}\epsilon)$, and so neglecting the difference between in and out states in this term means neglecting terms of order $\alpha\,\text{Re}\epsilon$ which, of course, are far too small to be of any consequence in δ_L^π. This is the term which gives us the numerator of the result in Eq. (3.7) (third member).

Coming now to the second term in Eq. (5.1)—that proportional to $\frac{1}{2}$—we wish to show that this term is of order $\alpha\Gamma(K_{l4}^0)$. In order to do so we begin from the identity

$$\langle K_{\text{in}}^0|(2\pi)^3\int d^3x\,H^l(x)H^l(0)|K_{\text{in}}^0\rangle=\langle K_{\text{in}}^0|(2\pi)^3\int d^3x\,H^l(x)H^l(0)|K_{\text{in}}^0\rangle\quad(\equiv\langle K_{\text{in}}^0|\Gamma^l|K_{\text{in}}^0\rangle),\quad(5.2)$$

where Γ^l is the l-lepton part of the K-meson decay operator. Next we insert a complete set of out states of the Hamiltonian of strong and electromagnetic interactions, H^S+H^γ, between the two operators on the left-hand side of this identity, and a complete set of in states of the same Hamiltonian between the two operators on the right-hand side. It is sufficient now to consider only the three most important contributions to the sum on each side, namely, those coming from $\pi l\nu$, $\pi\pi l\nu$, and $\pi\gamma l\nu$ states. On doing this and rearranging the terms somewhat, we find

$$\begin{aligned}(2\pi)^7\sum_{\text{pol}}\int\delta^4(k-\pi-l-\nu)d^3\pi d^3l d^3\nu[|\langle\pi^-l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2\\
+|\langle\pi^+l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^-l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^+l^-\bar{\nu}_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2]\\
=-(2\pi)^7\sum_{\text{pol}}\int\delta^4(k-\pi-\pi_0-l-\nu)d^3\pi d^3\pi_0 d^3l d^3\nu[|\langle\pi^-\pi^0l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2\\
+|\langle\pi^+\pi^0l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^-\pi^0l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^+\pi^0l^-\bar{\nu}_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2]\\
-(2\pi)^7\sum_{\text{pol}}\int\delta^4(k-\pi-l-\nu-Q)d^3\pi d^3l d^3\nu d^3Q[|\langle\pi^-\gamma l^+\nu_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2\\
+|\langle\pi^+\gamma l^-\bar{\nu}_{l\,\text{out}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^-\gamma l^+\nu_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2-|\langle\pi^+\gamma l^-\bar{\nu}_{l\,\text{in}}|H^l(0)|K_{\text{in}}^0\rangle|^2].\quad(5.3)\end{aligned}$$



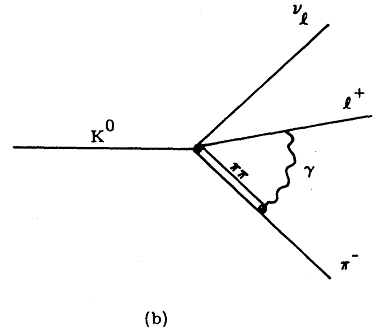


FIG. 1. Feynman diagrams for radiative corrections to $K_{l3}{}^0$ decay.

The expression on the left-hand side here is the expression which appears in the second term of Eq. (5.1).

Consider now each side of (5.3) expanded as power series in α. To order α^0, the left-hand side vanishes, and so indeed does the right-hand side. For the $\pi l\nu\gamma$ term on the right-hand side this is obvious; for the $\pi\pi l\nu$ term it follows by using the TCP relations

$$\langle \pi^-\pi^0 l^+\nu_{l\ \mathrm{in}} | H^l(0) | K_{\mathrm{in}}{}^0 \rangle = \langle \pi^+\pi^0 l'^-\bar{\nu}_{l\ \mathrm{out}} | H^l(0) | \bar{K}_{\mathrm{in}}{}^0 \rangle^* \quad (5.4\mathrm{a})$$

and

$$\langle \pi^+\pi^0 l^-\bar{\nu}_l | H^l(0) | K_{\mathrm{in}}{}^0 \rangle = \langle \pi^-\pi^0 l'^+\nu_{l\ \mathrm{out}} | H^l(0) | \bar{K}_{\mathrm{in}}{}^0 \rangle^* , \quad (5.4\mathrm{b})$$

and the fact that in the absence of electromagnetic interactions TCP invariance ensures the equality of the rates of $K^0 \to \pi^-\pi^0 l^+\nu_l$ and $\bar{K}^0 \to \pi^+\pi^0 l^-\bar{\nu}_l$, and of $K^0 \to \pi^+\pi^0 l^-\bar{\nu}_l$ and $\bar{K}^0 \to \pi^-\pi^0 l^+\nu_l$, respectively.

Turning next to the terms of order α on each side of (5.3), we see that the $\pi\gamma l\nu$ term on the right-hand side does not have any term in this order since the matrix elements themselves are of order α, while the difference between in and out states is also of order α; thus the $\pi l\nu\gamma$ term in (5.3) only contributes to order α^2. From this we deduce that the order α term of the left-hand side of (5.3) is equal to the order α term of the $\pi\pi l\nu$ part of the right-hand side; in other words, the term of order α in the second term of (5.1) is of magnitude $\alpha\Gamma(K_{l4}{}^0)$.

With regard to terms 3 and 4 of Eq. (5.1), arguments similar to that just given show that they are also of order $\alpha\Gamma(K_{l4}{}^0)$. For term 3 one starts from an identity for the quantity

$$\langle \bar{K}_{\mathrm{in}}{}^0 | (2\pi)^3 \int d^4x\, H_{-1}{}^l(x) H_1{}^l(0) | K_{\mathrm{in}}{}^0 \rangle ,$$

where $H_1{}^l(x)$ and $H_{-1}{}^l(x)$ are, respectively, the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ parts of $H^l(x)$, while for term 4 the starting point is an identity for the quantity

$$\langle K_{\mathrm{in}}{}^0 | (2\pi)^3 \int d^4x\, H_1{}^l(x) H_{-1}{}^l(0) | \bar{K}_{\mathrm{in}}{}^0 \rangle .$$

Hence, since the first term in (5.1) is of magnitude $\mathrm{Re}\,\epsilon\Gamma(K_{l3}{}^0)$, we have the result stated previously that the electromagnetic correction to $\delta_L{}^\pi$ is of order $\alpha\Gamma(K_{l4}{}^0)\Gamma/(K_{l3}{}^0)$. Actually, to be more accurate, it is of order $\alpha\lambda\Gamma(K_{l4}{}^0)/\Gamma(K_{l3}{}^0)$, where λ is the parameter of CP violation in either the weak or the electromagnetic interaction, since all terms on either side of (5.3) vanish identically if CP is conserved by both these interactions. Hence, since $\Gamma(K_{l4}{}^0)/\Gamma(K_{l3}{}^0)$ is expected to be of order 10^{-3} [$\Gamma(K_{l4}{}^0)$ has not been measured yet, but we may take it to be of the same order as $\Gamma(K_{l4}{}^+)$], it follows that the electromagnetic corrections to $\delta_L{}^\pi$ are no greater than $10^{-3}\alpha$ and hence are completely negligible.

We conclude with some comments on this result. In the first place, it is now possible to understand why we obtained the result zero for the electromagnetic corrections to $\delta_L{}^\pi$ in the calculation at the end of Sec. IV. The reason is that the correction to the difference $\Gamma_L(\pi^-l^+\nu_l) - \Gamma_L(\pi^+l^-\bar{\nu}_l)$ being calculated there was of order $\alpha\lambda\Gamma(K_{l3}{}^0)$, and, as we have seen, this correction vanishes. If the expression for the radiative corrections on which this calculation was based had included the effects of higher intermediate states, such as that shown in Fig. 1(b), as well as the Born term [Fig. 1(a)], we would not, in general, have obtained a zero result, since then the correction to $\delta_L{}^\pi$ could have had a term proportional to $\alpha\Gamma(K_{l4}{}^0)$. Technically, this would correspond to having the radiative correction term in Eq. (4.13) depend on s or t as well as u.

The second comment we wish to make is that the result about the electromagnetic corrections to $\delta_L{}^\pi$ obtained above depends crucially on summing equally over all variables in the final $\pi l\nu$ state; that is, it is a result which holds for the rates and not for the differential rates. This means that in performing an experiment to measure $\delta_L{}^\pi$ it is necessary to be sure that there is no bias in the events measured. If there is, then the radiative corrections could be significant. The existence of a bias in the events measured would mean that in the numerator of $\delta_L{}^\pi$ one was not measuring the integral of the right-hand side of (4.6) over the Dalitz plot, but rather the integral of this quantity multiplied by some weighting function. In such a case the result of this section would not hold.

Thus, in summary of what we have seen, we may say that while the electromagnetic effects may give rise to measurable differences between the decay distributions for $K_L{}^0 \to \pi^- l^+ \nu_l$ and $K_L{}^0 \to \pi^+ l^- \bar{\nu}_l$, their contribution to the difference between the rates is not greater than $\alpha\Gamma(K^0 \to \pi\pi l\nu)$. This latter result removes a possible nagging doubt about the interpretation of $K_L{}^0$ charge asymmetry experiments.

Note added in proof. After submitting this work for publication I became aware of a very similar contribution on this subject by L. B. Okun, Soviet Phys.—JETP Letters **6**, 272 (1967).

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Finite-Energy Sum Rules and Their Application to πN Scattering at Fixed u*

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The finite-energy sum rule, and a class of sum rules which can be used to probe the existence of fixed poles, are obtained for amplitudes whose left- and right-hand cuts are not related by crossing symmetry. The finite-energy sum rule is evaluated for each of four independent πN amplitudes with u fixed at (1236 MeV)^2, both sides of the resultant four sum rules being obtained from the properties of the low-energy πN resonances. Results are presented for three choices of end point: (1356 MeV)^2, (1808 MeV)^2, and (2313 MeV)^2. For the intermediate end point, all four sum rules work. For the highest one, however, they all fail. These results, while pointing to a failure of the resonance dominance approximation above 1800 MeV, give us a new confirmation of Regge high-energy behavior on the basis of low-energy data alone. In particular, they verify in some detail the relation predicted by Reggeism between the high-energy, fixed-u behavior of the amplitudes and the low-energy u-channel resonances. They also show that for $u = (1236\text{ MeV})^2$, all the πN amplitudes have Regge behavior on the average (duality) above 1800 MeV. The finite-energy sum rules are shown to be violated in a fictitious universe where the lowest particle on each of the leading πN Regge trajectories is accompanied by a degenerate partner of opposite parity.

I. INTRODUCTION

Although they follow very simply from assumptions of analyticity and Regge behavior, finite-energy sum rules (FESR) [1] provide a powerful tool for gaining new information about Regge trajectories and their residues, for obtaining theoretical insight into the nature of physical scattering amplitudes, and for constructing bootstrap models of remarkable computational simplicity.[2] As a test of the assumptions on which the FESR and their practical applications are based, we have investigated whether these sum rules are satisfied in πN scattering with the cross momentum transfer u fixed at $M^{*2} \equiv (1236\text{ MeV})^2$, the mass squared of the 3-3 resonance. Our results provide a new verification of Regge high-energy behavior from low-energy data. They also support the idea that the πN scattering amplitudes, as functions of energy, have Regge behavior on the average even in the intermediate energy region (around 2 BeV) where significant resonance structure is still present. However, the popular resonance dominance approximation appears to fail above 1800 MeV in the particular process we studied, which suggests that this approximation should be used with caution.

In Sec. II we derive the sum rules we have used. These are independent of any fixed poles that may exist at wrong-signature nonsense points in the J plane. We

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[1] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters **19**, 402 (1967); Phys. Rev. **166**, 1768 (1968); A. Logunov, L. Soloviev, and A. Tavkhelidze, Phys. Letters **24B**, 181 (1967); L. Balázs and J. Cornwall, Phys. Rev. **160**, 1313 (1967); K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967).

[2] For examples of the various applications of FESR, see Ref. 1 and also F. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters **21**, 323 (1968); S. Mandelstam, Phys. Rev. **166**, 1539 (1968); D. Gross, Phys. Rev. Letters **19**, 1303 (1967); C. Schmid, *ibid.* **20**, 628 (1968); V. Barger and R. Phillips, *ibid.* **21**, 865 (1968).