Radiative Corrections to the $e^-e^\pm$ Scattering Amplitude at Infinite Energy*

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(Received 8 December 1969)

In the high-energy limit of *ee* elastic scattering, as given by all possible multiphoton exchanges, we have considered a large class of radiative corrections. This class of radiative corrections is one in which one ignores processes involving pair creations and anihilations. The resultant amplitude is found to be proportional to the square of the c.m. energy, multiplied by the product of the e.m. form factors and the eikonal form.

IN an earlier paper in *Physical Review Letters*,[1] we studied, through the infinite-momentum technique,[2] high-energy ee, $e\gamma$, and $\gamma\gamma$ elastic scattering amplitudes by summing all possible multiphoton exchange diagrams. These amplitudes are found to be proportional to s, the square of the c.m.-system energy, multiplied by simple combinations of the Glauber forms of high-energy scattering.[3] This is consistent with some earlier finite-order calculations,[4] as well as with studies of the same processes with different techniques.[5] In the above calculations, however, no radiative corrections to the infinite-energy electron lines were included.

Recently, Yao[6] has made an important advance by showing that the second-order radiative correction to the $e^-e^\mp$ scattering does *not* change the eikonal form of the spin-nonflip part of the amplitude. The only modification to the amplitude is to give the electron a structure $F_1(\mathbf{k})$, where $F_1(\mathbf{k})$ is just the electric form factor up to order e^2 and $\mathbf{k}=(k_1, k_2)$ is the momentum transfer. The resultant spin-nonflip amplitude is simply

$$M(e^-e^\pm)=\tfrac{1}{2}isF_1(\mathbf{k})F_1'(\mathbf{k})E_\pm'(\mathbf{k}), \quad (1)$$

$$E_\pm'(\mathbf{k})=\int d^2b\exp(-i\mathbf{b}\cdot\mathbf{k})\{\exp[\pm i\chi(\mathbf{b})]-1\}, \quad (2)$$

$$\chi(\mathbf{b})=-e^2\int d^2q(2\pi)^{-2}(\mathbf{q}^2+\lambda^2)^{-1}\exp(i\mathbf{q}\cdot\mathbf{b}). \quad (3)$$

λ is a fictitious photon mass introduced to avoid possible infrared divergence. The amazing simplicity of expression (1) suggests that it might be valid in a more general class of radiative corrections.

The purpose of the present note is to show that a generalized formula holds for an entire class of radiative corrections:

$$M(e^-e^\pm)=\tfrac{1}{2}is[F_1(k)+(i/2m)F_2(k)(\boldsymbol{\sigma}\times\mathbf{k})_3]_{a'a}\times[F_1(k)+(i/2m)F_2(k)(\boldsymbol{\sigma}\times\mathbf{k})_3]_{b'b}E'(k), \quad (4)$$

where $F_{1,2}$ are electric and magnetic form factors, and a, b are helicities of e^-, $e^\pm$. This class of radiative corrections is one in which one ignores processes involving pair creations and annihilations [e.g., radiative corrections corresponding to Figs. 1(a)–1(c) are allowed, and 1(d) is ignored].[7] Since the above class of radiative corrections is quite large, our result strongly suggests that Eq. (4) may also be relevant to strong interactions.

The derivation of Eq. (4) follows closely the technique developed in Ref. 1, and the identical kinematics are used throughout. In the infinite-s limit, each diagram which contributes to the leading terms consists of two parts joined by photon lines. One part includes internal lines with infinite $p_+=p^0+p^3$, which is proportional to $\sqrt{s}$ in the c.m. system, describing particles moving in the positive 3-direction with infinite momenta (hereafter, we shall refer to it as the p_+ part of the diagram). The other part includes lines with infinite $p_-=p^0-p^3$, also proportional to $\sqrt{s}$, describing particles moving in the negative 3-direction with infinite momenta. The momenta of the photons joining these two parts are finite. For each part, one writes down a factor which is the product of coupling constants, propagators, etc. These factors transform in a well-defined way under Lorentz transformation.

To analyze the part with infinite p_+, we choose a standard reference frame moving with the particle so that the momentum variable p' in this frame becomes

* Work supported in part by the National Science Foundation under Contract No. NSF GP 8081.

[1] S. J. Chang and S. Ma, Phys. Rev. Letters **22**, 1334 (1969); Phys. Rev. **188**, 2385 (1969).

[2] S. Weinberg, Phys. Rev. **150**, 1313 (1966); S. J. Chang and S. Ma., *ibid.* **180**, 1506 (1969); L. Susskind and G. Frye, *ibid.* **165**, 1535 (1968); K. Bardakci and M. B. Halpern, *ibid.* **176**, 1686 (1968).

[3] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by Wesley E. Brittin *et al.* (Interscience, New York, 1959), Vol. I, p. 315; G. Molière, Z. Naturforsch. **2**, 133 (1947); J. Schwinger, Phys. Rev. **94**, 362 (1954); L. I. Schiff, *ibid.* **103**, 443 (1956); D. S. Saxon and L. I. Schiff, Nuovo Cimento **6**, 614 (1957); K. T. Mahanthappa, Phys. Rev. **126**, 329 (1962); R. L. Sugar and R. Blankenbecler, *ibid.* **183**, 1387 (1969).

[4] M. Lévy, Phys. Rev. **130**, 791 (1963); R. Torgersen, *ibid.* **143**, 1194 (1966); H. Cheng and T. T. Wu, Phys. Rev. Letters **22**, 666 (1969); Phys. Rev. **182**, 1852 (1969).

[5] H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Letters **23**, 53 (1969); F. Englert *et al.*, Nuovo Cimento **64A**, 561 (1969); Cheng and Wu, Phys. Rev. **186**, 1611 (1969); Y. P. Yao, ICTP Report No. Ic/69175 (unpublished); M. Levy and J. Sucher, Phys. Rev. **186**, 1656 (1969).

[6] Y. P. Yao, University of Michigan Report, 1969 (unpublished). The second-order radiative correction to an electron which interacts twice with an external potential was considered by H. Cheng and T. T. Wu, Phys. Rev. **185**, 1868 (1969).

[7] The class of allowed radiative corrections is somewhat larger than the class mentioned above. It actually includes the entire class of radiative corrections in which only one electron (or positron) is interacting with the exchange photons.

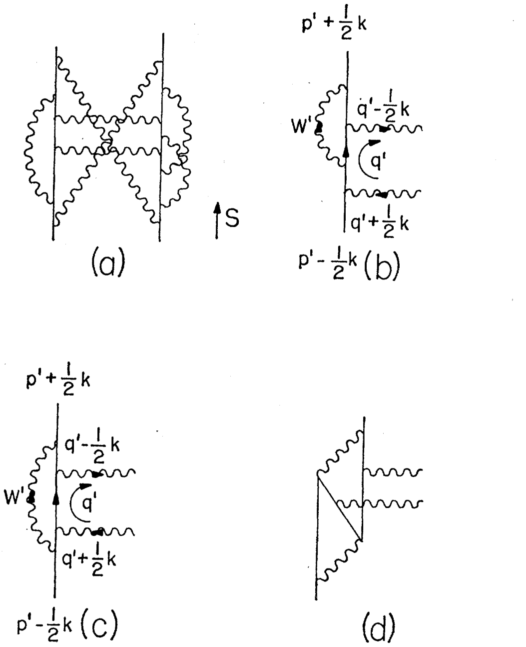

FIG. 1. (a) Typical multiphoton exchange diagram for *ee* scattering with radiative corrections; (b), (c), and (d) examples of the p_+ part of a diagram. Note that these are Weinberg's infinite-momentum diagrams. See Ref. 2.

finite. The p' introduced here is related to the corresponding p in the c.m. frame through

$$p_+' = p_+/\sqrt{s}, \qquad p'^{1,2} = p^{1,2}, \qquad p_-' = (\sqrt{s})\, p_-. \quad (5)$$

In particular, q_+' is of order $1/s$ and can be ignored. Now the s dependence becomes explicit and can be factored out. Notice that, upon boosting the standard frame to infinite momentum, the plus component (i.e., the $0+3$ component) of a vector or a tensor picks up a large factor $\sqrt{s}$ while the remaining components are small. With the s dependence explicitly exhibited, it is simple to ignore all but the leading term. Such a term contains only the plus components. The algebra is thus greatly simplified. Some of the typical p_+ parts are shown in Figs. 1(b)–1(d).

Our first conclusion after analyzing these p_+ parts is that processes with exchanged photons attaching to more than one line segment of a forward moving electron line will not contribute. For example, Fig. 1(b) does not contribute because the photons do not attach to the same electron line, while Fig. 1(c) does.

Verification of the above result is quite straightforward in a superrenormalizable theory such as the $\lambda\varphi^3$ theory. Let us use Figs. 1(b) and 1(c) as examples to illustrate the essential arguments in the proof for $\lambda\varphi^3$ theory. In Fig. 1(b), the propagators which contain q_-' as a factor are

$$(q_-' - \mathbf{q}^2 + \tfrac{1}{4}\mathbf{k}^2 + i\epsilon)^{-1}[(1-W_+')(q_-' + \tfrac{1}{4}\mathbf{k}^2 + m^2 - W_-') - (\mathbf{q}-\mathbf{W})^2 - m^2 + i\epsilon]^{-1}.$$

According to Weinberg's infinite-momentum rules,[2] $1-W_+'$ must always be positive.[8] This implies that both poles of q_-' are on the same side of the real axis, and consequently the amplitude vanishes after q_-' integration.[9] The result given here can be readily generalized to N-photon processes. In Fig. 1(c), however, the q_-' integral does not vanish because of the additional contribution from the semicircle as we enclose the contour of integration.

For a renormalizable theory, care has to be taken to ensure the convergence of the intermediate steps. There are two possible difficulties in verifying our result in quantum electrodynamics. First, the procedure of renormalization may invalidate our conclusion. Second, the possible existence of q_-' factors in the numerators may affect the convergence of the q_-' integrals. However, it is easy to see that the second difficulty never arises because only the plus components enter into our expression.[10] As an example of carrying out the renormalization, let us consider all possible two-photon processes as given in Fig. 2. In these diagrams, a mass counter term δm is included to cancel the divergent part in the proper self-energy diagram. As it is well known, the divergent part in the vertex and in the wave-function renormalization should cancel each other, and the remaining divergence can be disposed of by a corresponding wave-function renormalization on the external particles. To be more specific, we introduce a regulator with mass M to the photon, and modify the photon propagator from $i(p^2-\lambda^2+i\epsilon)^{-1}$ to $i(p^2-\lambda^2+i\epsilon)^{-1} - i(p^2-M^2+i\epsilon)^{-1}$ in all the loop integrals.[11] Now the regulator mass M serves as a cutoff in these processes. With the presence of the regulator (s), all integrals converge. The proper self-energy and other would-be divergent parts can be canceled by a proper choice of δm. The regularized amplitude is well defined and has a well-defined limit at $M=\infty$. We now look into this *regularized* amplitude and carry out the q_-' integrals. It is easy to see that Figs. 2(d), 2(e), and 2(f) do not contribute, for the same reasons as in $\lambda\varphi^3$ theory. Figures 2(a), 2(b), and 2(c), on the other hand, can be viewed as an insertion of a two-photon vertex into various parts of the electron line. For N-

[8] In general, Weinberg's rules cannot be applied to quantum electrodynamics because of the extra p dependence in the numerator. However, for the "good components" (here the plus components), Weinberg's rules usually apply. This can be readily verified after W'_- integration. See discussions in Chang and Ma's paper in Ref. 2; S. Drell, D. Levy, and T. M. Yan, Phys. Rev. Letters **22**, 744 (1969); Phys. Rev. **187**, 2159 (1969). However, for the regularized amplitudes introduced in this paper, Weinberg's rules do apply.

[9] The author wishes to thank W. Pardee for his assistance in clarifying this point.

[10] This is due to the presence of the γ_+ factors at the vertices. These γ_+ factors annihilate the possible q_-' terms through $\gamma_+^2=0$ such that one can always enclose the q_-' integration contours without picking up extra contributions from $|q_-'|=\infty$.

[11] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949). One has to regularize the photon propagator in *all* the loop integrals, whether the original integrals diverge or not, to preserve the gauge invariance of the intermediate steps.

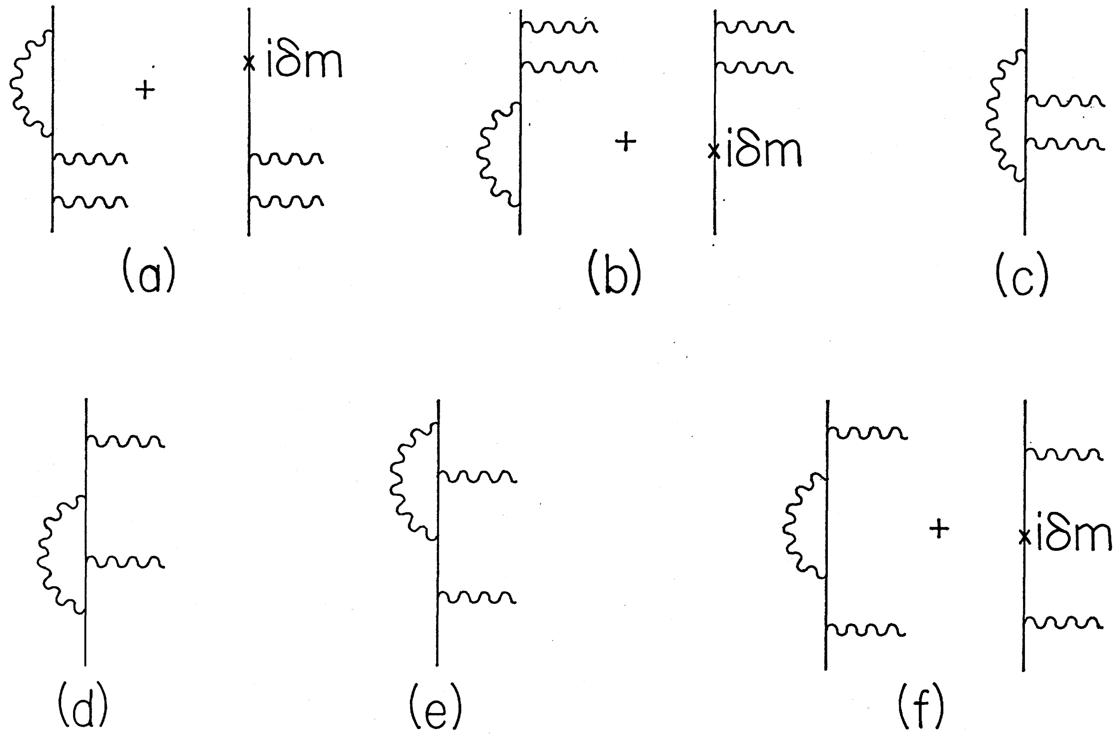

FIG. 2. Diagrams describing the lowest-order radiative correction on an electron line with two photons attached.

photon processes, the only modification is to replace a 2-photon vertex in Figs. 2(a)–2(c) by an N-photon vertex. A γ_+ factor always appears at this N-photon vertex.[1] We take the limit $M\rightarrow\infty$ *after* performing the q_-' integrals. The N-photon vertex is renormalized *exactly* as though it were a single photon vertex. Therefore, the resultant N-photon amplitude is identical, to within a simple kinematical factor, to the corresponding *renormalized* one-photon amplitude.

The advantage of the above result is enormous. We can now take the N-photon vertex as a single unit and insert it into various places on the electron line in the p_+ part of the diagram. The summing of N-photon processes with photon vertices permuted in all possible ways is now well known,[1,5] and leads precisely to the eikonal form $E'(k)$ mentioned earlier. The summing of all possible insertions in the electron lines, with proper counter terms included, is precisely the total electromagnetic vertex due to the particular class of radiative corrections described here. Combining these results and after including a kinematic factor $\frac{1}{2}ism^2$, we obtain Eq. (4).

The important question of the legitimacy of performing the q_-' loop integral before taking the limit $M\rightarrow\infty$ is not settled. However, the fact that our final result is finite and does not require additional subtractions indicates the correctness of our procedure. In addition, this result agrees with the rigorous work of Yao in the spin-nonflip case of $O(e^2)$.

Physically, our result is very reasonable. *In the case of radiative corrections considered* in this paper, it corresponds to the picture of a bare electron interacting with a cloud of photons. There is only one interacting charged constituent in each electron. Hence, there is only one eikonal form in the final amplitude. The form factors simply reflect the probability distribution of the interacting electron in the cloud. For processes involving Fig. 1(d), where virtual pairs are present, the charge structure becomes much richer. There are three charged constituents in the p_+ part of this diagram. Simple calculation indicates that we need at least three eikonal forms to describe the corresponding scattering amplitude. Similar situations have arisen in the Compton and $\gamma\gamma$ scattering amplitudes as given in Ref. 1.

The author wishes to thank Professor Y. P. Yao for sending him a copy of his work before publication. He is indebted to Dr. Paul Fishbane for many stimulating discussions and for his critical reading of the manuscript.

Note added in manuscript. After the completion of this paper, we were informed by Dr. Y. P. Yao that he has obtained similar results by a different method.