Comments and Addenda

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General Fermion Reggeization without Parity Doubling

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The Reggeization scheme of Carlitz and Kislinger is extended to daughter trajectories.

RECENTLY, Carlitz and Kislinger[1] showed that parity doubling of fermion trajectories can be avoided if there are cuts in the complex angular momentum plane. Their results were given for the leading trajectory. In this note we extend their solution to general daughter trajectories.

In CK, the solution without parity doubling was presented in the context of the Van Hove model. The solution, however, is independent of any model. The main justification for investigating Regge parametrizations without parity doubling is not because of a model but because—experimentally—the baryons seem to lie on linear nondoubled trajectories.[2] A Regge expansion without parity doubling which satisfies the analyticity and unitarity constraints may be successful in explaining experimental data.

Our results are based on the most general form for Regge-pole expansions compatible with analyticity and factorization. A general helicity amplitude can be written as[3]

$$T_{\lambda_3\lambda_4,\lambda_1\lambda_2}(u,\theta)=\frac{1}{2\pi i}\oint dL \times\sum_k\left[\frac{\gamma_k^+(L,W)}{g_k^+(L,W)}+\frac{\gamma_k^-(L,W)}{g_k^-(L,W)}\right]D_{\lambda_1-\lambda_2,\lambda_3-\lambda_4}{}^L(\theta).\tag{1}$$

The function $D_{\lambda,\lambda'}{}^\alpha(\theta)$ is the appropriate analytic continuation of the rotation matrix $d_{\lambda,\lambda'}{}^J(\theta)$, and $\gamma_k^\pm(L,W)$ is the Regge residue [including the factor

$$\{1\pm\exp(i\pi(L+\tfrac{1}{2}))\}/\sin\pi(L+\tfrac{1}{2})].$$

The most general form of the residue which guarantees analyticity at $W^2=u=0$ and pseudothresholds is given in an earlier paper[3,4] and will not be needed here. The superscript ($\pm$) labels the parity of the trajectory.

The factor $g_k^\pm(L,W)$ provides the poles and cuts in the L plane which will be picked up by the contour integral. The Regge expansion *with* parity doubling[3,4] requires

$$g_k^\pm(L,W)=L-\alpha_k^\pm(L,W),\tag{2}$$

where

$$k+\alpha_k^\pm(L,W)=f_{1,k}(L,u)\pm W(L+\tfrac{1}{2})f_{2,k}(L,u)\tag{3}$$

and

$$f_{i,k}(L,u)=\sum_{k'=0}^{k}\frac{k!\Gamma(2L+k+2)}{(k-k')!\Gamma(2L+k-k'+2)}u^{k'}a_{i,k'}(L+k,u).$$

The functions $a_{i,k}(L+k,u)$ are analytic at $u=0$ and must be given by dynamics.

The model of Carlitz and Kislinger is obtained by using a different form for $g_k^\pm(L,W)$:

$$g_k^\pm(L,W)=(L+k-\alpha_c)^{1/2}-f_{3,k}(L,u)\mp W(L+\tfrac{1}{2})f_{4,k}(L,u).\tag{4}$$

In addition to the moving poles at

$$L=\alpha_k^\pm=-k+\alpha_c+[f_{3,k}(\alpha_k^\pm,u)\pm W(\alpha_k^\pm+\tfrac{1}{2})f_{4,k}(\alpha_k^\pm,u)]^2,\tag{5}$$

the contour integral also picks up a fixed cut whose end point is α_c-k. The proof that (1) and (4) do not violate the analyticity constraints is essentially the same as the proof given previously.[3]

The parametrization we have presented in (4) gives a single parent pole and single daughter poles. However, Carlitz and Kislinger have pointed out that *two* parent poles (which intersect at $u=0$) are needed to

[1] R. Carlitz and M. Kislinger, Phys. Rev. Letters **24**, 186 (1970). We refer to this paper as CK.

[2] The $D_{15}(1680)$ and $F_{15}(1688)$ nucleon resonances are often cited as examples of parity doublets. However, their branching ratios indicate that these two resonances have quite different F/D $SU(3)$ coefficients. In addition, quark models do not predict doubling of parent trajectories.

[3] S. A. Klein, Phys. Rev. D **1**, 609 (1970).

[4] L. Durand, III, P. M. Fishbane, S. A. Klein, and L. M. Simmons, Jr., Phys. Rev. Letters **23**, 201 (1969).

fit the backwards peak in π^+p scattering.[5] The second trajectory, which can have either parity, should be fairly flat in order to avoid low-lying resonances.

Multiple poles can be accommodated in our general formalism by letting the index k in (1) label the multiple parent trajectories as well as the daughters. If parent trajectories of the same parity are degenerate at $u=0$, then their residues are able to be singular at this point.

We have chosen the cut to have an end point $\alpha_c(u)$, which is fixed so that the term $[L+k-\alpha_c(u)]^{1/2}$ does not contribute unwanted singularities in u. However, a cut which arises from *dynamics can* have a moving end point. The most general expression for the end point of a moving cut must be of the same form as that given in (3) for the location of a pole.

If the cut has a dynamic origin we would expect it to be due to the exchange of the pole trajectory plus the Pomeranchuk trajectory. The location of the parent branch point at $u=0$ would then be

$$\alpha_c=\alpha_c(0)=\alpha_0+\alpha_P-1,$$

where α_P is the intercept of the Pomeranchuk trajectory. Since α_P is very close to unity, we have

$$0\leq\alpha_0-\alpha_c\ll1.$$

[5] Reference 13 of CK mentions the need for a double pole; it also shows how a single pole can arise.

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Brans-Dicke Theory under Transformation of Units and Its Relation to the Jordan Theories*

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The relation between the original form of the Brans-Dicke (BD) theory and a particular form of the Jordan theory has been pointed out by Dicke. Here we establish the relation between the various unit-transformed BD theories and other Jordan theories. Although this formal identification is possible, the two theories are not equivalent, because of the different interpretations given to the matter Lagrangian and conservation laws in the various forms of the two theories.

THE Brans-Dicke (BD) and the Jordan theories[1–3] of gravitation both involve a scalar as well as a tensor field to describe the gravitational interaction. In the BD theory, the scalar field is introduced in a manner strongly motivated by the ideas of Mach. On the other hand, a scalar field appears quite naturally in the formal development of Jordan's unified field theory.

In the BD theory the introduction of the scalar is *ad hoc*, while its interpretation is evident from the manner in which it is introduced. On the other hand, the presence of a scalar field is manifest in the five-dimensional formulation of Jordan's theory, but its identification with G^{-1} is *ad hoc*.

Furthermore, within the framework of unifying gravitation and electromagnetism, the introduction of a phenomenological matter Lagrangian into the Jordan theory represents a new formal element, while the BD theory naturally includes matter in its initial formulation.

It has already been observed[1] that the BD theory is equivalent to the Jordan theory with the parameters $\eta=-1$, $b=0$, and the identification $\zeta=\omega$, $\kappa=\phi^{-1}\sim G$. The additional fact that the Jordan theory is formally invariant under a Pauli conformal transformation[4] lends weight to the idea that the unit-transformed BD theories[5,6] may also be identified with other choices of the Jordan parameters η and b. In this paper we shall establish the connection between the various BD and Jordan formalisms.

The variational principle for the original BD theory is[1]

$$0=\delta\int d^4x\,(-g)^{1/2}[\phi_0\lambda R+(16\pi/c^4)L_m{}^{\mathrm{BD}}-\omega\phi_0\lambda^{-1}\lambda_{,i}\lambda^{,i}],\tag{1}$$

where $L_m{}^{\mathrm{BD}}$ is the matter Lagrangian and $\phi=\phi_0\lambda$. If we perform a unit transformation (UT) by scaling lengths, times, and reciprocal masses by the space-time dependent factor $\lambda^{(1-\alpha)/2}$ (α arbitrary), we have

$$\begin{aligned}g_{ij}&\rightarrow\bar{g}_{ij}=\lambda^{1-\alpha}g_{ij},\\ m&\rightarrow\bar{m}=\lambda^{-(1-\alpha)/2}m.\end{aligned}\tag{2}$$

Thus, upon transforming the appropriate quantities

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[1] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[2] C. Brans, Phys. Rev. **125**, 2194 (1962).

[3] P. Jordan, *Schwerkraft and Weltall* (Braunschweig, Berlin, 1955).

[4] Reference 3, Chap. IV, §28.

[5] R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

[6] R. E. Morganstern (unpublished).