c-numbers, and the term proportional to Λ^2 is absent by the argument presented in Refs. 6 and 7. Thus, we have shown that nonleptonic decays are finite to lowest order. Note that the vacuum term is not there if α and β are different states. We have also applied the extended field algebra to study divergences in higher-order weak matrix elements, making use of Ward identities and making repeated use of the multiple Bjorken technique.[10] In particular, we have been able to show that to order g^4, the worst divergence is of Λ^2 type or, in other words, Λ^4 and $\Lambda^2 \ln\Lambda$ divergences are absent. This result makes the $K_L{}^0 - K_S{}^0$ mass difference calculations[11] more reliable, and opens up a way for study of nonleading divergences. We do not reproduce the details of the g^4-$_\mu$rder calculation here since it is only a tedious application of the well-known techniques.

VI. CONCLUSION

In conclusion, we would like to summarize our results. We have extended the minimal current algebra of Bjorken and Brandt to include scalar and pseudoscalar densities in the algebra. The algebra is different from the quark $[SU(3)\times SU(3)]_\beta$ algebra in that in our model scalar densities commute, whereas in quark-type models

$$[U_i(x),\, U_j(0)]_{x_0=0} = if_{ijk}V_{0,k}\delta^3(\mathbf{x}). \qquad (41)$$

We have shown that in a certain limit, the commutators take a simplified form. Using these simplified commutators, we have shown that nonleptonic weak processes, when treated in the intermediate vector-boson model, are divergence free to lowest order in the weak coupling constant G $(G/\sqrt{2} = g^2/m_W{}^2)$. We have also commented about the applicability of our results to the study of nonleading divergences in higher-order nonleptonic processes.

ACKNOWLEDGMENTS

I would like to thank Professor B. W. Lee and Dr. H. T. Nieh for discussions.

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PHYSICAL REVIEW D VOLUME 1, NUMBER 10 15 MAY 1970

Subsidiary Conditions and Ghosts in Dual-Resonance Models*

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(Received 17 December 1969)

We investigate the possible subsidiary conditions compatible with the equations of motion of dual-resonance models for the unrealistic choice of $\alpha(0)=1$. In the language of four-dimensional harmonic oscillators, we find one subsidiary condition for each mode of oscillation. All time components (in the c.m. system) can therefore be eliminated. We discuss the possibility of relaxing the condition $\alpha(0)=1$.

INTRODUCTION

One of the more serious problems which appears in the dual-resonance models is the presence of imaginary coupling when the residue at a particular value of the energy is written as a sum of factorized terms.[1] This is in fact a disease characteristic of a large class of amplitudes which satisfy superconvergent relations in the narrow-resonance approximation.[2] Therefore it is particularly discouraging that in this case one encounters the same situation in spite of the infinite number of degrees of freedom. Fubini and Veneziano already noticed a possible solution to this problem. In analogy with quantum electrodynamics they found that the fourth component (in the c.m. system of the hadron) of one of the modes of oscillation of the hadron could be considered redundant. Therefore the ghost is only apparent, and by restricting the Hilbert space by a subsidiary condition or by introducing a metric operator, one could handle this mode. However, the same result was not true for the other modes of vibration. Of the infinite number of kinds of ghosts, only one kind could be solved in this way.

Two ingredients are necessary in order that this method of resolving the problem of ghosts be feasible: (1) A linear relation should exist between the coupling of the supposedly redundant state and some other states, and (2) the cancellation must be such that the sum of terms gives real couplings.

In this paper we wish to show that at least for one particular value of the intercept [i.e., $\alpha(0)=1$] there exists an infinite class of relations which make the time components of all the modes of oscillation redundant. While we cannot claim to have solved the com-

* Supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U.S. Atomic Energy Commission under Contract No. AT(11-1)-888, #C00-267.

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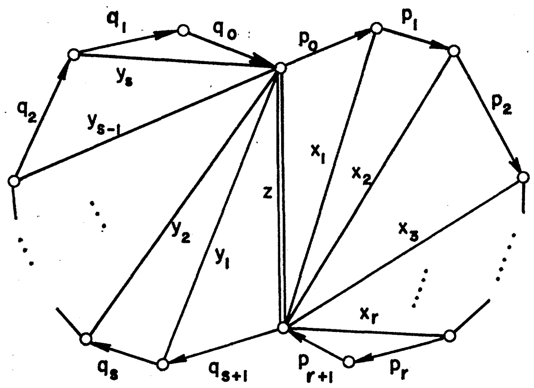

FIG. 1. Dual diagram for the $(r+s+4)$-point function showing the Koba-Nielsen choice of variables.

plete problem of ghosts [because of the unrealistic choice of $\alpha(0)$], it is our belief that these linear relations, perhaps with some additional complications, will also exist in more realistic models, and therefore the solution of the problem of ghosts can be achieved in this way.

In Sec. I we derive the general expression for the Ward identities of the first mode. These results have been obtained independently by other authors.[3] In Sec. II we show how to derive relations for the second mode and by a generalization obtain these relations for the nth mode. In the last section we discuss the problems arising when the condition $\alpha(0)=1$ is relaxed.

I. REDUNDANT COMPONENTS OF FIRST MODE

We begin by considering the Koba-Nielsen[4] choice of variables for the n-point dual amplitude[5] (see Fig. 1). Following Bardakci and Ruegg,[6] we write the amplitude will all the exponents added in such a way that only numerators appear:

$$A_{r+2,s+2}=\int_0^1 d\mathbf{y}du d\mathbf{x}\,\phi_{r+2}(\mathbf{x},p)\phi_{s+2}(\mathbf{y},q)u^{-\alpha(s)-1}\times\prod_{1\leq i\leq r}\prod_{1\leq j\leq s}[1+u(1-y_1\cdots y_{s+1-j})\times(1-x_1\cdots x_i)]^{-2p_iq_j}. \quad (1)$$

In this formula the variable $u=z(1-z)^{-1}$ is integrated from 0 to ∞, while the remaining variables are integrated from 0 to 1, and $\phi_{r+2}(\mathbf{x},p)$ and $\phi_{s+2}(\mathbf{y},q)$ are the integrands of the $(r+2)$- and $(s+2)$-point functions.

We now make a change of variables which leaves ϕ invariant:

$$y_0y_1\cdots y_l=1-y_0'\cdots y_{s+1-l}',\quad x_0=x_0'=y_0=y_0'=1,$$
$$x_0x_1\cdots x_l=1-x_0'\cdots x_{r+1-l}',\quad x_{r+1}=x_{r+1}'=y_{s+1}=y_{s+1}'=0.$$

Therefore Eq. (1) becomes

$$A_{r+2,s+2}=\int_0^1 dy'dx'\int_0^\infty du\,\phi_{r+2}(x',p)\phi_{s+2}(y',q)u^{-\alpha(s)-1}\times\prod_{n=1}^\infty\exp\left[\sum_{0\leq i\leq r+1}\sum_{0\leq j\leq s+1}2\frac{(-u)^n}{n}(y_0'\cdots y_j')^nq_j\times(x_0'\cdots x_i')^np_{s+1-i}\right]. \quad (2)$$

We define

$$Q^{(m)}=\sum_{j=0}^{s}(y_0'\cdots y_j')^mq_j\sqrt{2},$$
$$P^{(m)}=\sum_{i=0}^{r}(x_0'\cdots x_i')^mp_{s+1-i}\sqrt{2}, \quad (3)$$

and finally obtain

$$A_{r+2,s+2}=\int_0^1 dy'dx'\,\phi_{r+2}(x',p)\phi_{s+2}(y',q)\int_0^\infty du\,u^{-\alpha(s)-1}\times\exp\sum_{m=1}\frac{(-u)^m}{n}P^{(m)}Q^{(m)}. \quad (4)$$

We note that both vectors $P^{(m)}$ and $Q^{(m)}$ correspond to a clockwise rotation. Equation (4) gives immediately the twisted propagator[7] which is diagonal in the usual modes but has a different off-mass-shell behavior to the untwisted propagator. The result of Gallardo, Galli, and Susskind[8] for the twisted vertex function is obtained if one disregards the integral from 1 to ∞, arguing that it only changes the behavior off the mass shell, and then uses as the propagator

$$-(1)\Sigma/[\alpha(s)+\Sigma], \quad (5)$$

where

$$\Sigma=\sum_{n=1}^\infty na_n^\dagger a_n.$$

For our purposes we are interested in the equivalence between Eq. (4) and the usual expression,[1]

$$A_{r+2,s+2}=\int_0^1 dy'dx'\int_0^1 dz\,\phi_{r+2}(x',p)\phi_{s+2}(y',q)z^{-\alpha(s)-1}\times\exp\sum_{n=1}^\infty\frac{z^n}{n}\bar{P}^{(n)}Q^{(n)}. \quad (6)$$

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From the result,[1]

$$P^{(n)}=\sum_{k=1}^{\infty}\binom{n}{k}(-1)^k\bar{P}^{(k)}+\bar{P}^{(0)}, \tag{7}$$

where $P^0=p_0+p_1+\cdots+p_{r+1}=\sqrt{2}P_{\text{c.m.}}$, we obtain

$$\sum_{n=1}^{\infty}\frac{(-u)^n}{n}P^{(n)}Q^{(n)}$$

$$=\sum_{n,k}\frac{u^n}{n}\frac{n!}{(n-k)!k!}(-1)^{k-n}\bar{P}^{(k)}Q^{(n)}+\sum_n\frac{(-u)^n}{n}\bar{P}^{(0)}Q^{(n)}$$

$$=\sum_n\left(\frac{uQ}{1+uQ}\right)^{(n)}\frac{\bar{P}^{(n)}}{n}+\sum_n\frac{(-u)^n}{n}\bar{P}^{(0)}Q^{(n)}, \tag{8}$$

where the product $[uQ(1+uQ)^{-1}]^{(n)}$ is to be understood symbolically as an expression for the infinite series

$$\sum_{k=0}^{\infty}\binom{-n}{k}u^{n+k}Q^{(n+k)}.$$

We take the functional derivative of both Eqs. (4) and (6) with respect to the tensor

$$\prod_{i=1}^{\infty}\frac{[\bar{P}^{(i)}]^{\mu(\lambda_i)}}{\lambda_i!i^{\lambda_i}},$$

and obtain

$$\left\langle\int_0^1 z^{\Sigma n\lambda_n-\alpha(s)-1}dz\prod_{i=1}^{\infty}[Q^{(i)}]^{\mu(\lambda_i)}\right\rangle$$

$$\equiv\left\langle\int_0^{\infty}u^{\Sigma m\lambda_m-\alpha(s)-1}du\prod_{i=1}^{\infty}\{[Q(1+uQ)^{-1}]^{(i)}\}^{\mu(\lambda_i)}\right.$$

$$\left.\times\exp\sum_1^{\infty}\frac{(-u)^n}{n}\bar{P}^{(0)}Q^{(n)}.\right\rangle \tag{9}$$

The equality must be satisfied for all values of z [we put $u=z/(1-z)$ in order that both integrals have the same end points]. We choose z infinitesimal, with the result that

$$\langle\prod_{i=1}^{\infty}[Q^{(i)}]^{\mu(\lambda_i)}\rangle=\langle\prod_{i=1}^{\infty}[Q^{(i)}]^{\mu(\lambda_i)}$$

$$+z[\sum n\lambda_n-\alpha(s)+1-\bar{P}^{(0)}Q^{(1)}]\prod[Q^{(i)}]^{\mu(\lambda_i)}$$

$$-z\sum_{n=1}^{\infty}[Q^{(1)}]^{\lambda_1}[Q^{(2)}]^{\lambda_2}\cdots[Q^{(n)}]^{\lambda_n-1}\lambda_n[Q^{(n)}]^{\lambda_n+1}\cdots\rangle, \tag{10}$$

and therefore

$$\langle[\sum_{n=1}^{\infty}n\lambda_n-\alpha(s)+1-\bar{P}^{(0)}\cdot Q^{(1)}]\prod_{i=1}^{\infty}[Q^{(i)}]^{\mu(\lambda_i)}$$

$$-\sum_{n=1}^{\infty}\{\prod_{i=1,i\neq n}^{\infty}[Q^{(i)}]^{\mu(\lambda_i)}[Q^{(n)}]^{\mu(\lambda_n-1)}[Q^{(n)}]^{\mu(\lambda_n+1)}\lambda_n\}\rangle=0. \tag{11}$$

This tensor relationship can be written in the operator formalism of Ref. 9:

$$\langle 0|\exp\left(\sum_n i\frac{a_nQ^{(n)}}{n^{1/2}}\right)\{\sum_{n=1}[n^{1/2}a_n{}^\dagger-(n+1)^{1/2}a_{n+1}{}^\dagger]n^{1/2}a_n$$

$$+\tfrac{1}{2}\bar{P}^{(0)2}+i\bar{P}^{(0)}\cdot a_1{}^\dagger\}\prod_{i=1}^{\infty}\frac{[a_i{}^\dagger]^{\lambda_i}}{\lambda_i!i^{\lambda_i}}|0\rangle=0. \tag{12}$$

Thus the operator

$$O_{(1)}=\sum_{n=1}^{\infty}[-(n+1)^{1/2}a_{n+1}{}^\dagger+n^{1/2}a_n{}^\dagger]a_nn^{1/2}$$

$$+P_{\text{c.m.}}{}^2-iP_{\text{c.m.}}\cdot a_1{}^\dagger\sqrt{2} \tag{13}$$

has the property of transforming any state into a redundant state, that is, a state which is not coupled to scalars.

II. CONSTRAINTS FOR nth MODE

In order to generalize the constraint relationships to the nth mode, we summarize the results of Sec. I in a more systematic way. All the dynamics of the uncrossed tree diagrams are included in the equation of motion[9]

$$[-\partial^\mu\partial_\mu+\sum_n na_n{}^\dagger a_n+1$$

$$+g{:}\phi(\sum_{n=1}^{\infty}(2/r)^{1/2}(a_r+a_r{}^\dagger)+x){:}]\psi(a_r{}^\dagger,x)$$

$$=H_T\psi(a_r{}^\dagger\,x)=0, \tag{14}$$

where ϕ is the external scalar neutral field and ψ the master wave function defined by Nambu. The operator $O_{(1)}{}^\dagger$, Hermitian conjugate of $O_{(1)}$, has the following property:

$$[H_T-1][O_{(1)}{}^\dagger+1]\psi=O_{(1)}{}^\dagger[H_T]\psi=0. \tag{15}$$

Therefore the subsidiary condition $(O_{(1)}{}^\dagger+1)\psi=0$ is compatible with the equation of motion and can be chosen as a constraint. This condition shows explicitly that the scalar component of the first mode is redundant. We now show how to derive the corresponding relation for the nth mode. By analogy we write

$$O_{(2)}{}^\dagger+2=\sum_{n=1}^{\infty}a_n{}^\dagger n^{1/2}[-(n+2)^{1/2}a_{n+2}+n^{1/2}a_n]$$

$$-\partial_\mu\partial^\mu+1-\sqrt{2}\partial_\mu\sqrt{2}a_2{}^\mu+\tfrac{1}{2}(a_1\cdot a_1). \tag{16}$$

The additional term $\frac{1}{2}(a_1\cdot a_1)$ corresponds to the scalar component (the tract) of a tensor of rank 2. It is trivial to verify that

$$(H_T-2)(O_{(2)}{}^\dagger+2)\psi=O_{(2)}{}^\dagger H_T\psi=0. \tag{17}$$

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[9] S. Fubini, D. Gordon, and G. Veneziano, Phys. Letters **29B**, 679 (1969); Y. Nambu, in Proceedings of the International Conference on Symmetries and Quark Models, Wayne University, 1969 (unpublished). We follow Nambu's notation: $g^{\mu\nu}=(1,-1,-1,-1)$ and $[a_\mu{}^{(r)},a_\nu{}^{(s)\dagger}]=-\delta_{rs}g_{\mu\nu}$.

In general, for the mth mode we write

$$O_{(m)}{}^{\dagger}+m=\sum_{n=1}^{\infty} a_n{}^{\dagger}n^{1/2}[-(n+m)^{1/2}a_{n+m}+n^{1/2}a_n]-\partial_\mu\partial^\mu$$

$$+1-a_m{}^{\mu}(2m)^{1/2}\partial_\mu+\sum_{l=1}^{m} a_l\cdot a_{m-l}\lambda_{l,m}[l(m-l)]^{1/2} \quad (18)$$

and attempt to determine $\lambda_{l,m}$ so that

$$(H_T-m)(O_{(m)}{}^{\dagger}+m)=O_{(m)}{}^{\dagger}H_T. \quad (19)$$

By construction it is true that for any $\lambda_{l,m}$

$$(-\partial_\mu\partial^\mu-m+1+\sum_n na_n{}^{\dagger}a_n)(O_{(m)}{}^{\dagger}+m)$$
$$=O_{(m)}{}^{\dagger}(-\partial_\mu\partial^\mu+1+\sum_n na_n{}^{\dagger}a_n). \quad (20)$$

Therefore we must prove that

$$:\phi(\sum (2/r)^{1/2}(a_r+a_r{}^{\dagger})+x):(O_{(m)}{}^{\dagger}+m)=O_{(m)}{}^{\dagger}:\phi:. \quad (21)$$

With the expression

$$:\phi:=\{\exp i[\sum_r (2/r)^{1/2}a_r{}^{\dagger}\cdot\partial]\exp i[\sum_r (2/r)a_r\cdot\partial]\}\phi(x), \quad (22)$$

using the fact that[9]

$$a_l\{\exp[\sum_n (2/n)^{1/2}a_n{}^{\dagger}\cdot k]\}$$
$$=\{\exp[\sum_n (2/n)^{1/2}a_n{}^{\dagger}\cdot k]\}[a_l-k(2/l)^{1/2}], \quad (23)$$

we obtain

$$:\phi:(O_{(m)}{}^{\dagger}+m)-O_{(m)}{}^{\dagger}:\phi:=i\sqrt{2}\sum_{n=1}^{m-1} n^{1/2}a_n$$
$$\times(1+\lambda_{n,m}+\lambda_{n,m-n}):\partial\phi:+[(1-m)-\sum_{l=1}^{m-1}2\lambda_{lm}]:\phi:. \quad (24)$$

Thus we find that the appropriate choice is $\lambda_{lm}=-\frac{1}{2}$.

If we now restrict ourselves to the subspace of the solutions of the wave equation which satisfies all the constraints, we eliminate all scalar excitations. We can also understand these relations at the level of the tree diagrams. In general, any state which can be written as $O_{(m)}|\psi\rangle$ can be shown to be decoupled from the vacuum, i.e., scalar external particles.

CONCLUSIONS

To solve the complete problem of ghosts, one must consider a more realistic model in which the masses of the scalar external particles are positive. Unfortunately, while the constraint for mode (1) can be easily generalized, this is not the case for the other modes. In particular, we discuss the case in which all trajectories are equal but the intercept is not fixed. We consider the n-point amplitude written in the operation formalism

$$\langle 0|VBVB\cdots BV,$$

where

$$V=\exp[ik\cdot\sum_{r=1}^{\infty}(2/r)^{1/2}a_r{}^{\dagger}]$$
$$\times\exp[ik\cdot\sum_{r=1}^{\infty}(2/r)^{1/2}a_r]=g:\phi(k):,$$

$$B=B(s+\mu^2-\sum na_n{}^{\dagger}a_n,\,-\mu^2)$$

is the usual Beta function, and $\mu^2=k^2$ is the mass of the external scalar particle.

We consider a state $O_{(m)}|\psi\rangle$ and calculate the coupling of that state to the vacuum. Equations (20) and (21) are modified in the following manner:

$$VO_{(m)}=(O_{(m)}-m\mu^2)V,$$
$$B(-s+\mu^2-\sum na_n{}^{\dagger}a_n,\,-\mu^2)(O_{(m)}-m\mu^2)$$
$$=O_{(m)}B(-H_{\text{free}}+m,\,-\mu^2)+Y, \quad (25)$$

where Y can be written as

$$(1+\mu^2)[\sum_{r=0}^{m-1}B(-H_{\text{free}}+r-\mu^2)-m]$$
$$=(1+\mu^2)\int_0^1 x^{-H_{\text{free}}-1}(1-x)^{-\mu^2}(\sum_{r=0}^{m-1}x^r-m)\,dx. \quad (26)$$

Therefore, if we label by x_i the integration parameters corresponding to the different propagators, we find the coupling of a particular state to the vacuum is equal to

$$(1+\mu^2)\int_0^1 dx\,\phi(xP)[\sum_{r=0}^{m-1}x_1{}^r-m+x_1{}^m(\sum_{r=0}^{m-1}x_2{}^r-m)$$
$$+x_1{}^m x_2{}^m(\sum_{r=0}^{m-1}x_3{}^r-m)+\cdots][Q^{(1)}]^{i_1}[Q^{(2)}]^{i_2}\cdots. \quad (27)$$

These correspond to scalar excitations which do not exist in the model. Therefore no cancellation is possible. However, it is now plausible, as conjectured by Fubini and Veneziano, that in a more complicated model (i.e., one that includes those excitations from the beginning) generalized subsidiary conditions can be found such that all time components will become redundant.

ACKNOWLEDGMENTS

I would like to thank B. Sakita for many interesting discussions, from which some of the ideas of this paper originated. I would also like to thank H. Lipinski for a critical reading of the manuscript.