From Eq. (C6) we get the explicit expression[44]

$$d_{l+\,+}{}^{\lambda}(\theta)=\pi^{-1}\int_{\cot\theta}^{\infty} dx\, i^{l+1}Q_l(ix)\,(\sin\theta\, x-\cos\theta)^{\lambda-1}$$

$$=2^l\frac{\Gamma(\lambda)\Gamma(l+1)}{\Gamma(l+\lambda+1)}\frac{(\sin\theta)^l}{\sin\pi(\lambda-l)}C_{\lambda-l-1}{}^{l+1}(\cos\theta),\tag{D11}$$

where $C_n{}^\nu$ are the Gegenbauer functions.[45]

After the manipulations of the end of Sec. III, we can explicitly calculate the Regge-pole eigenfunctions $f_K(t,\theta)$ of the Kth daughter $l_K=\lambda_0-K-1$ corresponding to a given Lorentz pole of eigenfunction $f_0(t)$. The result is, apart from inessential factors,

$$f_K(t,\theta)\propto f_0(t)\frac{\Gamma(l_K+1)i^K}{\Gamma(\frac{1}{2}-\frac{1}{2}K)\Gamma(\frac{1}{2}-\frac{1}{2}K+\lambda_0)}\times(\sin\theta)^{l_K+1}C_K{}^{l_K+1}(\cos\theta).\tag{D12}$$

Note that the odd daughters are absent because, due to (D7) and (D10), b^l is even under $\theta\leftrightarrow\pi-\theta$ ($w\leftrightarrow -w$). Note also that (D11) gives a result similar to the Bethe-Salpeter calculation[8] when the initial particles are put on-shell. The latter circumstance explains why only amplitudes even in w are obtained in this simple case.

[44] Bateman Manuscript Project, *Higher Transcendental Functions*, edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. I, Eqs. 3.7 (31), 3.3 (13), and 3.15 (4).
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Possible Extension of Minimal Current Algebra and Applications

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An attempt has been made to extend the minimal current algebra of Bjorken and Brandt starting from a gauge-field Lagrangian and including in it nonets of scalar and pseudoscalar fields and making use of canonical communtation relations both for spin-zero and spin-one fields. To apply it to the problem of weak-interaction divergences, we identify suitably normalized fields with weak currents and scalar and pseudoscalar densities introduced by Gell-Mann. As in the case of Bjorken and Brandt, we go to the limit $m_0\to 0$, $g_0\to 0$ such that $g_0/m_0{}^2=\text{const}\neq 0$, where m_0 and g_0 are masses and coupling constants of the Yang-Mills field. In the extended minimal algebra, the nonleptonic weak processes are free of all divergences to lowest order and of a class of leading divergences to all orders in the weak-coupling constant.

I. INTRODUCTION

THE minimal algebra of Bjorken and Brandt[1] has the particularly attractive feature that it makes the electromagnetic mass differences of hadrons finite to lowest order in the fine structure constant. It has been shown in Ref. 1, that this algebra can be obtained as a particular limit of the massive Yang-Mills theory, i.e., as $m_0\to 0$ and $g_0\to 0$ such that m_0/g_0 is nonzero and finite, where m_0 is the mass and g_0 is the coupling constant in the theory. Of course, one uses the field-current identity of Kroll, Lee, and Zumino.[2] The purpose of the present paper is to extend the minimal algebra to include the scalar and pseudoscalar densities defined by Gell-Mann. A convenient way to achieve this goal is to work with a Yang-Mills Lagrangian with the scalar and pseudoscalar fields as matter fields and go to the limit prescribed above. To this end, we first construct an $SU(3)\otimes SU(3)$ symmetric Lagrangian out of vector, axial-vector, scalar, and pseudoscalar fields. We then identify the vector and axial-vector fields with currents and scalar and pseudoscalar fields with corresponding densities introduced by Gell-Mann, Oakes, and Renner.[3] We assume canonical commutation relations for fields, and by the limiting procedure introduced above, we obtain a simpler set of commutation rules for currents and densities. We then apply the resulting commutation relations to study the problem of the leading divergences in weak interaction. We show that nonleptonic processes are finite to lowest order in the weak-coupling constant and are free of leading divergences to all orders. We also show that, to order G^2, there are no Λ^4 and $\Lambda^2\ln\Lambda$ divergences in $\Delta S=1$ processes, where G is the weakcoupling constant. It is obvious from the above that radiative corrections to nonleptonic decays are also free of leading divergences to order G.

II. ALGEBRA OF SCALAR AND VECTOR FIELDS

We start with the following Lagrangian in the simple case with $SU(2)$ symmetry:

$$\mathfrak{L}=\mathfrak{L}_0+\mathfrak{L}_B,\tag{1}$$

where $\mathfrak{L}_0$ is $SU(2)$ symmetric and $\mathfrak{L}_B$ is the symmetry-breaking part. We work in terms of a triplet of vector

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fields and a triplet of scalar fields. The vector fields $\phi_{\mu i}$ ($\mu=1$–4; $i=1, 2, 3$) are assumed to be the gauge fields; the scalar fields σ_i are assumed to transform as triplet representations of the gauge group:

$$\mathcal{L}_0=-\tfrac{1}{4}F_{\mu\nu,i}F_{\mu\nu,i}-\tfrac{1}{2}m_0^2\phi_{\mu i}\phi_{\mu i}-\tfrac{1}{2}D_\mu\sigma_i D_\mu\sigma_i-\tfrac{1}{2}m^2\sigma_i^2,$$
$$\mathcal{L}_B=c_3\sigma_3, \tag{2}$$

where

$$F_{\mu\nu,i}=[\partial_\mu\phi_{\nu,i}-\partial_\nu\phi_{\mu,i}+\tfrac{1}{2}g_0\epsilon_{ijk}(\phi_{\mu,j}\phi_{\nu,k}+\phi_{\nu,k}\phi_{\mu,j})],$$
$$D_\mu\sigma_i=[\partial_\mu\sigma_i+\tfrac{1}{2}g_0\epsilon_{ijk}(\phi_{\mu,j}\sigma_k+\sigma_k\phi_{\mu,j})]. \tag{3}$$

We have the following canonical commutation relations among the fields:

$$[\phi_{m,i}(x), F_{0n,j}(0)]_{x_0=0}=i\delta_{mn}\delta_{ij}\delta^3(x), \tag{4}$$

where m, n denote the space part of μ, ν, etc. and i, j, k, l are used for internal symmetry indices:

$$[\sigma_i(x), D_0\sigma_j(0)]_{x_0=0}=i\delta_{ij}\delta^3(x). \tag{5}$$

Furthermore, since the fields are independent objects, we have

$$[\sigma_i(x), \sigma_j(0)]_{x_0=0}=0, \qquad [D_0\sigma_i(x), D_0\sigma_j(0)]_{x_0=0}=0, \tag{6}$$

$$[\phi_{m,i}(x), \phi_{n,j}(0)]_{x_0=0}=0. \tag{7}$$

The field equations for the $\phi_{\mu,i}$ and σ_i fields are the following:

$$-m_0^2\phi_{\mu,i}(x)=\partial_\nu F_{\nu\mu,i}(x)+g_0 J_{\mu,i}(x), \tag{8}$$

where

$$J_{\nu,i}=\tfrac{1}{2}\epsilon_{ijk}[(F_{\nu\mu,j}\phi_{\mu,k}+\phi_{\mu,k}F_{\nu\mu,j})+(D_\nu\sigma_j\sigma_k+\sigma_k D_\nu\sigma_j)], \tag{9}$$

$$-m_0^2\sigma_i(x)=-\partial_\mu D_\mu\sigma_i(x)+\tfrac{1}{2}g_0\epsilon_{ijk}$$
$$\times(D_\mu\sigma_j(x)\phi_{\mu,k}(x)+\phi_{\mu,k}(x)D_\mu\sigma_j(x)). \tag{10}$$

Using the field equations and canonical commutation relations, Eqs. (4)–(7), one sees that if we define

$$V_{\mu,i}=(m_0^2/g_0)\phi_{\mu,i} \tag{11}$$

and

$$U_i=c\sigma_i, \tag{12}$$

where c is any nonzero constant, then we get the usual field algebra of Lee, Weinberg, and Zumino,[2] i.e.,

$$[V_{0,i}(x), V_{0,j}(0)]_{t=0}=i\epsilon_{ijk}V_{0,k}\delta^3(\mathbf{x}), \tag{13}$$

$$[V_{0,i}(x), V_{m,j}(0)]_{t=0}=i\epsilon_{ijk}V_{m,k}(0)+i(m_0^2/g_0^2)\delta_{ij}\partial_m\delta^3(\mathbf{x}), \tag{14}$$

$$[\partial_0 V_{m,i}(x)-\partial_m V_{0,i}(x), V_{n,j}(0)]_{t=0}$$
$$=-i(m_0^4/g_0^2)\delta_{ij}\delta_{mn}\delta^3(\mathbf{x})+i\epsilon_{ijk}V_{m,k}\partial_n\delta^3(\mathbf{x})$$
$$-i(g_0^2/m_0^2)\epsilon_{ilk}\epsilon_{jlk'}V_{m,k}V_{n,k'}\delta^3(\mathbf{x}). \tag{15}$$

Apart from these, we get

$$[U_i(x), V_{0,j}(0)]_{x_0=0}=i\epsilon_{ijk}U_k(0)\delta^3(\mathbf{x}), \tag{16}$$

$$[U_i, V_{m,j}]=0, \tag{17}$$

$$[U_i(x), \partial_0 U_j(0)]_{x_0=0}=ic^2\delta_{ij}\delta^3(\mathbf{x})$$
$$-(g_0^2/m_0^2)i\epsilon_{ilk'}U_k U_{k'}\delta^3(\mathbf{x}). \tag{18}$$

Also, using the above commutation relations, it is easy to see that Eqs. (10) and (6) are consistent. Note that we have not taken any extra matter field as in the case of Lee, Weinberg, and Zumino, and taking them does not affect our commutation relations.

III. GENERALIZATION TO CASE OF $SU(3)\otimes SU(3)$

To generalize to the case of chiral $SU(3)\otimes SU(3)$, we assume an octet of vector and axial-vector fields and a nonet of scalar and pesudoscalar fields to belong to the $(3,3^*)\oplus(3^*,3)$ representation of $SU(3)\otimes SU(3)$. We can write down the Lagrangian as follows:

$$\mathcal{L}_0=-\tfrac{1}{4}F_{\mu\nu,i}F_{\mu\nu,i}-\tfrac{1}{4}F_{\mu\nu,i}'F_{\mu\nu,i}'$$
$$-m_0^2(\phi_{\mu,i}\phi_{\mu,i}+\phi_{\mu,i}'\phi_{\mu,i}')-\tfrac{1}{2}D_\mu\sigma_i D_\mu\sigma_i$$
$$-\tfrac{1}{2}D_\mu\pi_i D_\mu\pi_i-m_0^2(\sigma_i\sigma_i+\pi_i\pi_i), \tag{19}$$

$$\mathcal{L}_B=(c_3\sigma_3+c_8\sigma_8+c_0\sigma_0)c, \tag{20}$$

where $\phi_{\mu,i}$ is the vector field, $\phi_{\mu,i}'$ is the axial-vector field, and σ_i and π_i are scalar and pseudoscalar fields, respectively.

$$F_{\mu\nu,i}=\partial_\mu\phi_{\nu,i}-\partial_\nu\phi_{\mu,i}+\tfrac{1}{2}g_0 f_{ijk}$$
$$\times(\phi_{\mu,j}\phi_{\nu,k}+\phi_{\nu,k}'\phi_{\mu,j}'\cdots+\phi_{\nu,k}\phi_{\mu,j}+\phi_{\mu,j}'\phi_{\nu,k}'), \tag{21}$$

$$F_{\mu\nu,i}'=\partial_\mu\phi_{\nu,i}'-\partial_\nu\phi_{\mu,i}'+\tfrac{1}{2}g_0 f_{ijk}(\phi_{\mu,j}'\phi_{\nu,k}+\phi_{\nu,k}\phi_{\mu,j}'), \tag{22}$$

$$D_\mu\sigma_i=\partial_\mu\sigma_i+\tfrac{1}{2}g_0 f_{ijk}(\phi_{\mu,j}\sigma_k+\sigma_k\phi_{\mu,j})$$
$$+\tfrac{1}{2}g_0 d_{ijk}(\phi_{\mu,j}'\pi_k+\pi_k\phi_{\mu,j}'), \tag{23}$$

$$D_\mu\pi_i=\partial_\mu\pi_i+\tfrac{1}{2}g_0 f_{ijk}(\phi_{\mu,j}\pi_k+\pi_k\phi_{\mu,j})$$
$$-g_0\tfrac{1}{2}d_{ijk}(\phi_{\mu,j}'\sigma_k+\sigma_k\phi_{\mu,j}'). \tag{24}$$

Thus, the canonical commutation relations and field equations similar to the case of $SU(2)$ can be written down, and by using them and defining

$$V_{\mu,i}=(m_0^2/g_0)\phi_{\mu,i}, \qquad A_{\mu,i}=(m_0^2/g_0)\phi_{\mu,i}',$$
$$U_i=(c/g_0)\sigma_i, \qquad V_i=(c_2/g_0)\pi_i, \tag{25}$$

where c is any constant, we obtain commutation relations for the currents and the densities. The above definition allows us to identify the symmetry-breaking $\mathcal{L}_B$ with the one given by Gell-Mann, Oakes, and Renner.[3] To save space, we do not write down the commutation relations. They are just the generalizations of Eqs. (13)–(18) with axial-vector currents and with ϵ_{ijk} replaced by f_{ijk} and d_{ijk} at suitable places; for example, an analog of Eq. (16) would be

$$[U_i(x), A_{0,j}(0)]_{x_0=0}=id_{ijk}V_k\delta^3(x), \tag{26}$$

and an analog of Eq. (18) is

$$[U_i(x), \partial_0 U_j(0)]_{x_0=0}=ic^2\delta_{ij}{}^3(x)-i(g_0^2/m_0^2)$$
$$\times f_{ilk}f_{jlk'}U_k U_{k'}+i(g_0^2/m_0^2)d_{ilk}d_{jlk'}V_k V_{k'}, \tag{27}$$

$$[U_i(x), \partial_0 V_j(0)]_{x_0=0}=0, \qquad [U_i(x), U_j(0)]_{x_0=0}=0, \tag{28}$$

and so on.

IV. LIMIT OF MASSIVE YANG-MILLS FIELD

Bardakci, Frishman, and Halpern[4] took the limit

$$m_0 \to 0, \quad g_0 \to 0 \quad \text{such that } g_0^2/m_0^2 = \text{const} \tag{29}$$

and showed that the massive Yang-Mills theory in this limit is equivalent to Sugawara's theory. However, we take the limit in Ref. 1 so that

$$m_0^2/g_0^2 \to \infty\,. \tag{30}$$

As a result, in Eq. (15) we have a term of the form

$$\lim_{m_0^2/g_0^2\to\infty} (g_0^2/m_0^2)\, V_{m,k}(0)\, V_{n,k}(0)\,, \tag{31}$$

and in Eq. (18) we have a term of the form

$$\lim_{m_0^2/g_0^2\to\infty} (g_0^2/m_0^2)\, U_k(0)\, U_k(0)\,. \tag{32}$$

These are products of field operators at the same space-time point, and hence are highly singular operators. However, we take all local field products as limits of nonlocal products[5] and assume that the divergence is mild enough so that

$$\lim_{m_0^2/g_0^2\to\infty} (g_0^2/m_0^2)\, V_{m,k}(x)\, V_{n,k}(x) \to c\text{-numbers}, \tag{33}$$

and similarly for the scalar field term.[1] However, to be rigorous we should define all local products occurring in the theory as limits of nonlocal products and compute all commutators and in the end take all limits. It is easy to see that the Jacobi identity is not satisfied in this limit. After these limits, all the commutators except those of the type $[V_{m,i}, \dot{V}_{n,j}]$ and $[U_i, \dot{U}_j]$ remain unaffected and these commutators now become the following:

$$[\partial_0 V_{m,i}(x) - \partial_m V_{0,1}(x),\, V_{n,j}(0)]_{x_0=0} = -i(m_0^4/g_0^2) \times \delta_{ij}\delta_{mn}\delta^3(\mathbf{x}) + (c\text{-number terms}) + i\epsilon_{ijk} V_{m,k}\partial_n \delta^3(\mathbf{x})\,, \tag{34}$$

$$[U_i(x),\, \partial_0 U_j(0)]_{x_0=0} = ic^2 \delta_{ij}\delta^3(\mathbf{x}) + c\text{-numbers}. \tag{35}$$

From the Lagrangian, one can write down the stress-energy tensor[4] $\theta_{\mu\nu}$ and show that θ_{00}, which is the Hamiltonian density, can be written as follows:

$$\theta_{00} = H_0 + H_B\,, \tag{36}$$

where H_0 is $SU(3)\times SU(3)$ symmetric, i.e., it commutes with A_{0i} and V_{0i}, and H_B is the symmetry-breaking part such that

$$H_0 = -\mathcal{L}_B\,. \tag{37}$$

V. APPLICATION TO PROBLEM OF WEAK-INTERACTION DIVERGENCES

(a) We first discuss the leading divergences in nonleptonic weak processes. It has been shown in a number of papers[6,7] that a part of the leading divergences (i.e., $G^n\Lambda^{2n}$-type divergences) in nonleptonic weak processes treated in the intermediate vector-boson model can be expressed in terms of time-ordered products of σ commutators to all orders in the weak-coupling constant. The assumption that goes into this is that the chiral $SU(3)\times SU(3)$ is broken by terms given in Eq. (20) and the scalar and pseudoscalar densities belong to $(3,3^*)\oplus(3^*,3)$ representation of $SU(3)\otimes SU(3)$. Furthermore, in the present model we know some more commutators like Eqs. (34) and (35), etc. Thus, the present model only reproduces the results of Refs. 6 and 7. Moreover, apart from the σ commutators described above, there is another class of leading divergences proportional to $[\partial_0 V_{m,i}(x) - \partial_m V_{0,i}(x),\, V_{n,j}(0)]_{x_0=0}$, where the internal indices are such that, using Eq. (34), the above commutator becomes a c-number, and hence they do not make any contribution to the leading divergences. Also, as shown in Ref. 6, the time-ordered products of σ's can be absorbed by a suitable counterterm in the Lagrangian. So the very first consequence of minimal current algebra is the absence of a new class of leading divergences in nonleptonic processes to all orders. Gatto *et al.*[7,8] have arrived at the same conclusion by working with compound-field algebra, in which the bare mass of the vector mesons is taken to infinity.

(b) We further apply the above algebra to lowest-order nonleptonic processes. In the intermediate vector-boson model, with only charged weak currents, one can show that the lowest-order nonleptonic process has the following type of divergence. For that, we define

$$M(\alpha\to\beta) = -g^2 \int d^4q\, \Delta_{\mu\nu}(q)\, M_{\mu\nu}(q)\,, \tag{38}$$

where

$$M_{\mu\nu}(q) = \int \exp(iq\cdot x)\, d^4x [\langle\alpha\mid T(J_\mu(x)\, J_\nu^\dagger(0) + \text{H.c.})\mid\beta\rangle - \langle 0\mid T(J_\mu(x)\, J_\nu^\dagger(0) + \text{H.c.})\mid 0\rangle]. \tag{39}$$

Using the Ward identity and the Bjorken technique, we obtain[9]

$$M(\alpha\to\beta) = -(g^2\Lambda^2/m^2)\langle\alpha\mid\Sigma_i\mid\beta\rangle - g^2\ln\Lambda \times \int\langle\alpha\mid[\dot{\Sigma}_i, \Sigma_j]_{x_0=0}\mid\beta> d^3x - g^2\ln\Lambda \times \int\langle\alpha\mid[\partial_0 j_{m,i}(x) - \partial_m j_{0,i}(x), j_{m,j}(0)]_{x_0=0}\mid\beta\rangle d^3x + \text{finite terms} - \text{above terms for the vacuum.} \tag{40}$$

Λ is the cutoff, which we take to infinity. Using Eq. (35), we see that $\ln\Lambda$ terms are absent since they are

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c-numbers, and the term proportional to Λ^2 is absent by the argument presented in Refs. 6 and 7. Thus, we have shown that nonleptonic decays are finite to lowest order. Note that the vacuum term is not there if α and β are different states. We have also applied the extended field algebra to study divergences in higher-order weak matrix elements, making use of Ward identities and making repeated use of the multiple Bjorken technique.[10] In particular, we have been able to show that to order g^4, the worst divergence is of Λ^2 type or, in other words, Λ^4 and $\Lambda^2 \ln\Lambda$ divergences are absent. This result makes the $K_L{}^0 - K_S{}^0$ mass difference calculations[11] more reliable, and opens up a way for study of nonleading divergences. We do not reproduce the details of the g^4-$_\mu$rder calculation here since it is only a tedious application of the well-known techniques.

VI. CONCLUSION

In conclusion, we would like to summarize our results. We have extended the minimal current algebra of Bjorken and Brandt to include scalar and pseudoscalar densities in the algebra. The algebra is different from the quark $[SU(3)\times SU(3)]_\beta$ algebra in that in our model scalar densities commute, whereas in quark-type models

$$[U_i(x),\, U_j(0)]_{x_0=0} = i f_{ijk} V_{0,k}\delta^3(\mathbf{x}). \tag{41}$$

We have shown that in a certain limit, the commutators take a simplified form. Using these simplified commutators, we have shown that nonleptonic weak processes, when treated in the intermediate vector-boson model, are divergence free to lowest order in the weak coupling constant G $(G/\sqrt{2} = g^2/m_W{}^2)$. We have also commented about the applicability of our results to the study of nonleading divergences in higher-order nonleptonic processes.

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Subsidiary Conditions and Ghosts in Dual-Resonance Models*

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We investigate the possible subsidiary conditions compatible with the equations of motion of dual-resonance models for the unrealistic choice of $\alpha(0)=1$. In the language of four-dimensional harmonic oscillators, we find one subsidiary condition for each mode of oscillation. All time components (in the c.m. system) can therefore be eliminated. We discuss the possibility of relaxing the condition $\alpha(0)=1$.

INTRODUCTION

One of the more serious problems which appears in the dual-resonance models is the presence of imaginary coupling when the residue at a particular value of the energy is written as a sum of factorized terms.[1] This is in fact a disease characteristic of a large class of amplitudes which satisfy superconvergent relations in the narrow-resonance approximation.[2] Therefore it is particularly discouraging that in this case one encounters the same situation in spite of the infinite number of degrees of freedom. Fubini and Veneziano already noticed a possible solution to this problem. In analogy with quantum electrodynamics they found that the fourth component (in the c.m. system of the hadron) of one of the modes of oscillation of the hadron could be considered redundant. Therefore the ghost is only apparent, and by restricting the Hilbert space by a subsidiary condition or by introducing a metric operator, one could handle this mode. However, the same result was not true for the other modes of vibration. Of the infinite number of kinds of ghosts, only one kind could be solved in this way.

Two ingredients are necessary in order that this method of resolving the problem of ghosts be feasible: (1) A linear relation should exist between the coupling of the supposedly redundant state and some other states, and (2) the cancellation must be such that the sum of terms gives real couplings.

In this paper we wish to show that at least for one particular value of the intercept [i.e., $\alpha(0)=1$] there exists an infinite class of relations which make the time components of all the modes of oscillation redundant. While we cannot claim to have solved the com-

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